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Experimental Studies on the Large Plastic Deformation of Frames Due to Horizontal Impact

— Measurements of Impact Loading and Vertical Load Effect* —

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Abstract

An experimental study is made on the nature of the large permanent deformation of the columns in a portal frame under horizontal impact loading. The column specimens were made of mild steel, aluminum alloy and copper plates. The impact load was applied by an input pendulum. The applied load and initial velocity were measured by the barium titanate ceramics, accelerometer and load cell which had been calibrated beforehand from the initial amplitude of a bifilar pendulum. The experimental results show a reasonable consensus with the analysis in which the frame restoring-force characteristics are approximated as bi-linear with a negative second slope.

Nomenclature

a	initial amplitude of bifilar pendulum
b	column width
d	column depth
E	Young's modulus
g	acceleration of gravity
H	restoring-force of frame
H_e	restoring-force corresponding to x_e
h	column height
I	impulse
$\bar{I}$	$\sqrt{gh}/(4M_0)\cdot I$
k	proportional factor of piezo-electricity
m	equivalent mass
M_j	bending moment at joint
M_0	full-plastic moment of a column
P	load
Q	resistant force of columns
Q_0	static collapse load based on perfectly-plastic theory
T	period of bifilar pendulum
t	time
U	input energy
U_c	energy absorbed in plastic deformation up to unstable collapse
U_e	maximum elastic strain energy
U_p	energy absorbed in plastic deformation
V	voltage
v	initial velocity of bifilar pendulum
W_1, W_2	weights of bifilar pendulum and beam
x	relative displacement between

* Some features of these experiments and their general results have already been described in the preliminary reports in Japanese.[1,2] This paper presents a more detailed discussion of the experimental techniques, outlines the bi-linear analysis, and makes a comparison of the predictions of that theory with the test results.

beam and pendulum

x_e elastic limit displacement in bilinear restoring-force characteristics

x_c displacement corresponding to unstable collapse state

x_0 elastic limit displacement based on elastic perfectly-plastic theory

x_1, x_2 absolute displacements of pendulum and beam

α ratio of the second slope to the first in Q-x diagram

γ material constant

δ final plastic deflection

$\bar{\delta}$ $\bar{W}_2/(4M_0)\cdot\delta$

ϵ strain

σ_0 yield limit stress

χ $x-x_e$

χ_f χ-value at termination of plastic deformation

1. Introduction

Much analytical work has been done in the last two decades to investigate the plastic deflection of structures under large-impact loading. Simplified assumptions regarding the behaviour of materials and the loading make it easy to get closed-form solutions for simple structures. The rigid-perfectly plastic type of analysis has often been adopted since the work of Lee and Symonds[3]; impact loading is treated as an abrupt change in velocity, a pure impulsive load, or as of blast-type, defining the load as a monotonously decreasing function of time.

Experiments have also been performed in order to check the validity of these analyses. The impact was recorded as a pure impulse[4~23] or a velocity change[24~26], with a few exceptions[27~30]. It has been reported that the rigid-perfectly plastic analysis over-estimates the deformation ordinarily, but is useful as a first order approximation in determining the permanent deflection[5,6,8,10,13~15,17,22,25,31], when the impact is sufficiently large. The discrepancy is mainly attributed to the fact that the yield point stress of metal is higher at high rates of strain than in a static test. Rigid-visco plastic analyses have been carried out to get a reasonable agreement with the experimental results[8,11,17,19,22,25,30,32~36]. The effect of elastic vibration is considered negligible when the work done in plastic deformation is sufficiently greater than the maximum strain energy that can be stored elastically in the structure[3,25,35,37].

The exact shape of the load-time diagram has been considered not essential in predicting the permanent deflection of a structure due to impact[38,39]. The results of a recent analysis indicate that this is the case when the load magnitude is of the order of more than ten times the static collapse load of the structure, however, if the load magnitude is smaller, the load shape plays an important role as well as the total impulse[40]. This may be valid as well when the impact is applied not as a load but as an acceleration, where the acceleration should be compared with the yield level acceleration corresponding to the static collapse.

The authors are undertaking a series of tests in order to examine the ultimate state of framed structures subjected to large impact loads. An objective of the work presented in this paper was to measure the load and acceleration magnitudes, as well as the total impulse and the velocity change, in the impact test. As a preliminary attempt, portal frame specimens were subjected to impact loads at their column tops. In the next series of tests, the portals were subjected to a large acceleration for a short duration at the column bases. The loading system in the preliminary tests was adopted as an idealization of the action of a gust or an explosion in the air, and the latter loading corresponds to the ground shock which is an idealization of the destructive action of earthquakes on buildings. Actual tall building frames are also loaded vertically due to dead loads, and their gravity causes destabilizing effects acting as an over-turning moment. This is an important factor when the deflection is large as in the ultimate state of a steel structure. In such a case the energy criterion for neglecting the elastic vibration may become questionable, because the deflection comes into play in the strength of structures. These circumstances are simulated by the beam weight in the present tests.

2. Preliminary Tests

Preliminary tests were first performed in order to know the orders of the magnitude and of the duration of the impact force by means of the piezo-electricity, and to check the suitableness of the loading system. The loading apparatus and details of the specimen and barium-titanate holder are illustrated in Fig. 1. Tests were repeated by replacing only the column specimens which were clamped by bolts to the beam and

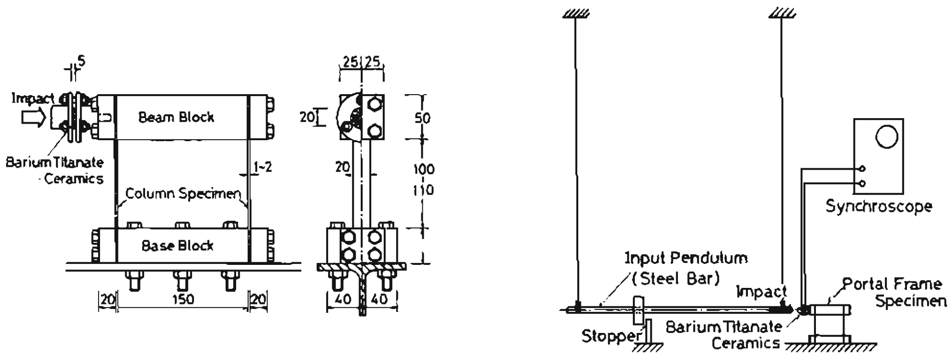


Fig. 1. Apparatus for preliminary test.

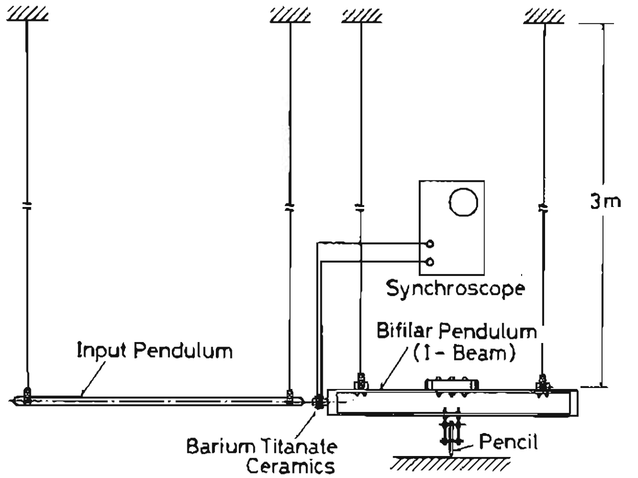


Fig. 2. Apparatus for calibration test.

base blocks. The beam weight was about 3.6 kg and the column height was 10 or 11 cm. Thin plates of mild steel and aluminum alloy were adopted as column specimens.

The impact force was applied to the portal frame specimen, of which the column bases were fixed on the ground, at the beam level by means of an input bifilar pendulum. The force was measured by recording the voltage caused by the piezo-electric effect of barium-titanate in a synchro-scope. There is a proportionality $P=kV$ between the voltage V of piezo-electricity and the input load P. A calibration test was carried out in order to estimate the proportional factor k for the barium-titanate ceramics used as shown in Fig. 2. A bifilar pendulum with barium-titanate ceramics at the head was struck by the input pendulum, and the voltage-time relation of the piezo-electricity was recorded in the synchro-scope. Based on the assumption of pure impulsive loading, the total impulse I applied is calculated by the relation, $I=2\pi W_1 a/(gT)$, from the mass W_1/g, the period $T(=3.44$ sec) and the initial amplitude a of the bifilar pendulum. The value of k is found as the ratio of the impulse I to the integration of the voltage $V(t)$ with respect to time t. The calibration test results of the two barium-titanate ceramics used in this study are shown in Fig. 3. The ordinate is the

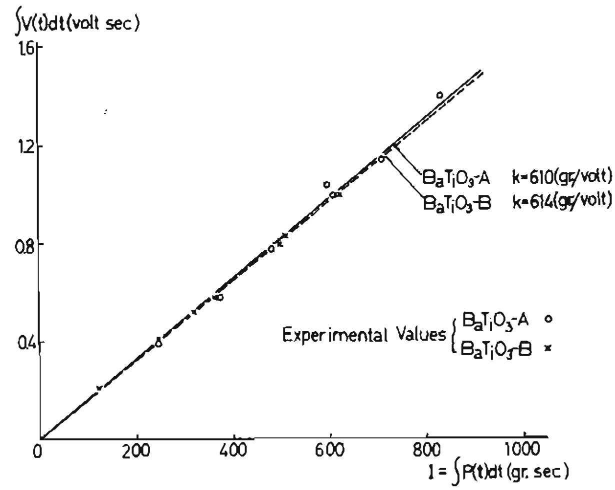


Fig. 3. Calibration test results of barium titanate ceramics.

integration of the voltage-time record and the abscissa is the total impulse obtained from the initial amplitude of the bifilar pendulum. The average values of k were 610 and 614 gram/volt, and the deviation was less than 6 per cent.

A typical voltage-time record during the impact loading test of the portal frame is shown in Photo 1. The duration of impact was nearly constant throughout the preliminary test, and was about 0.5 milli-second (henceforth abbreviated as ms), which was 1/500 to 1/100 times the duration of the plastic deformation, and the maximum value of the applied load was 200 to 2000 kg, which was 150 to 400 times the static collapse load. This indicates that the impact loading in these tests can be regarded as purely impulsive.

The final plastic deflection of the portal frame specimen was determined by a dial

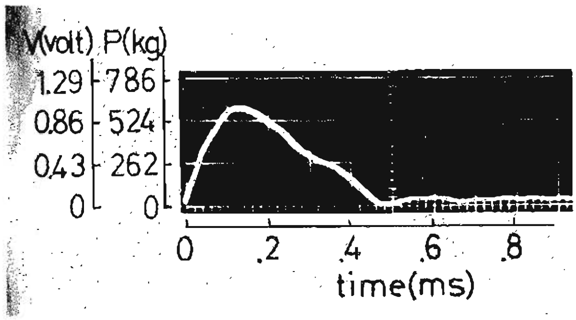
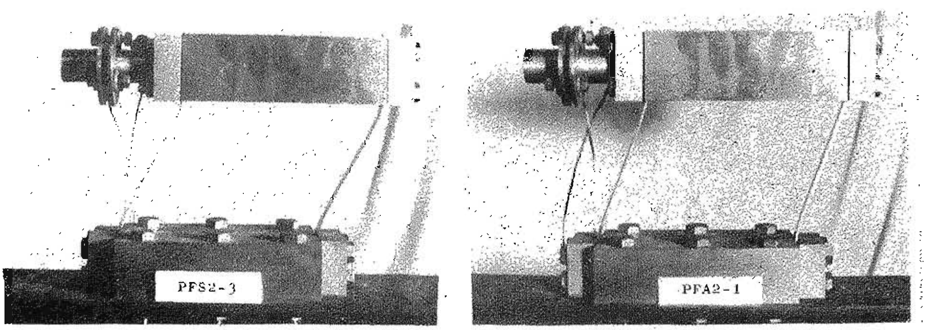


Photo 1. Voltage- and load-time record.



Mild Steel Specimen Aluminum Alloy Specimen

Photo 2. Deformed specimen in preliminary tests.

gauge as the relative displacement between the beam and the base. Typical figures of the portal frame specimens after the test are shown in Photo 2. The final deformed shapes of the two columns were almost the same, and the deformation was so concentrated at the top and the bottom of columns that the middle portions of the columns remained essentially straight. The ratio of the energy absorbed in the plastic deformation to the elastic strain energy which could be stored in the columns was 5 to 15 with a few exceptions.

The relation between the final plastic deflection of the portal frame specimen and the applied impulse is shown in Fig. 4. As the ordinate is taken the dimensionless parameter $\bar{\delta}$ proportional to the final plastic deflection δ of the columns and the abscissa the parameter $\bar{I}$ proportional to the applied total impulse I. The solid line refers to the simplified solution given by rigid-perfectly plastic theory with consideration of the over-turning moment due to the beam weight W_2:

$$\bar{\delta}=1-\sqrt{1-\bar{I}^2} \tag{1}$$

$$\bar{\delta}\equiv\frac{W_2}{4M_0}\cdot\delta; \qquad \bar{I}\equiv\frac{\sqrt{gh}}{4M_0}\cdot I \tag{2}$$

where M_0 and h designate the full-plastic moment and the height of a column, respec-

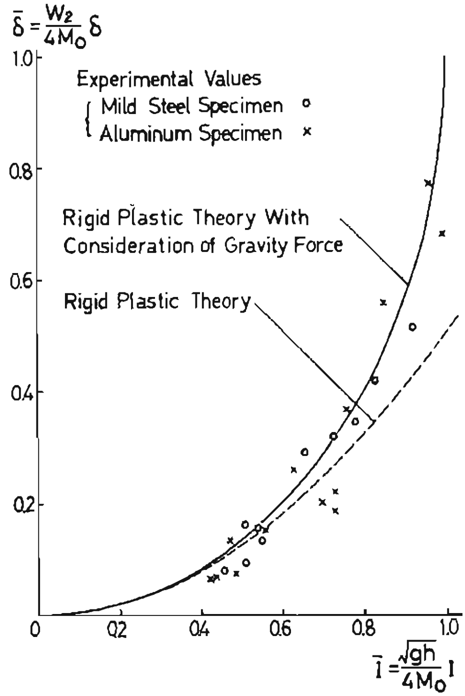


Fig. 4. Preliminary test results.

tively. Eq. (1) will be derived later in section 4. The experimental values are generally in good agreement with the simplified solution especially in the range of large deflection, and exceed the other solution illustrated by the dashed line in the figure which neglects the over-turning moment, indicating the importance of the over-turning moment in the range of the present tests.

3. Description of Tests

From the examination of the load-time relationship in the preliminary tests described in the preceding section, the following became clear: (i) The ratio of the applied load to the static collapse load is of the order of more than 100. (ii) The ratio of the duration of the load to that spent during the plastic deformation is of the order of less than 1/100. And (iii) an impact load which can be approximated as purely impulsive is experimentally reproducible by means of the collision of the two steel pendulums. As the next step of this series of experimental studies, portal frame specimens were subjected to impulsive motion at their base.

The test specimens were close to those in the preliminary tests. Since the preliminary test results showed that the effect of the over-turning moment considerably increased the final plastic deflection, the beam weight was controlled from 3.55 kg to 7.13 kg by adding an attached mass to the beam block, as shown in Fig. 5. The column heights were determined to be 15 and 20 cm, and thin plates of aluminum alloy, copper, and mild steel were adopted as the column specimens. Their typical stress-strain relations obtained from tensile tests are shown in Fig. 6. The first two materials are observed to have small strain hardening and the last to have appreciable strain hardening.

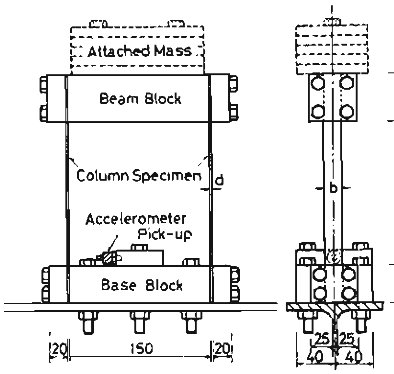


Fig. 5. Portal frame specimen.

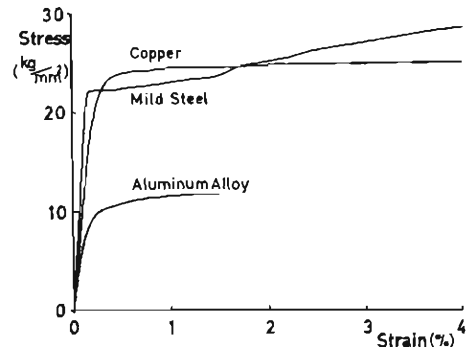


Fig. 6. Stress-strain relations.

The dimensions of the specimens and their material properties are shown in the Table. The first figure in each specimen number refers to the column material, the next to the existence of the attached mass of the beam, the next two to the column width in millimetres, and the last two to the column height in centimetres.

The experimental arrangement is illustrated in Fig. 7. The portal frame specimen was fixed to the bifilar pendulum at rest, and was subjected to the impact motion at the column bases, when an input pendulum, which was heavier than that of the preliminary tests, struck the bifilar pendulum. An accelerometer pick-up was set at the base block of the frame specimen, and the initial velocity of the bifilar pendulum was determined by integrating the acceleration-time relation recorded by a synchro-scope.

The reliability of this acceleration-time record or the velocity change was checked in the following way. A load-cell was first prepared by affixing two semi-conductor strain gauges to a head-rounded steel bar of about 10 cm long and 2.18 cm in radius, in order to measure the load-time relation of the impact applied to the bifilar pendulum. Barium-titanate ceramics were not used in this load-cell because they might have been crushed by the large shock of impact required to get a sufficiently large initial velocity

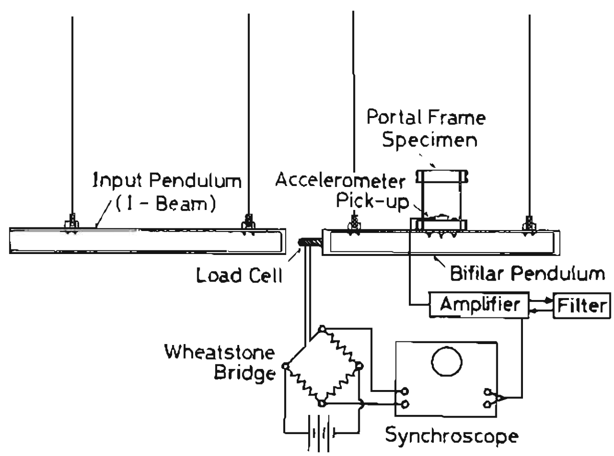


Fig. 7. Apparatus for impact loading.

Table. Properties of specimens.

Column Material	Mild Steel						Aluminum Alloy			Copper			
Specimen Number	SA 1015	SB 1015	SA 1415	SB 1415	SA 1820	SB 1820	AA 1015	AA 2015	AB 2015	CA 1015	CB 1015	CA 2020	CB 2020
Yield Stress: $\sigma_0(t/\text{cm}^2)$	2.23						1.18			2.50			
Young's Modulus: $E(t/\text{cm}^2)$	2100						800			1040			
Column Width: b(cm)	1.00	1.00	1.40	1.40	1.80	1.80	1.00	2.00	2.00	1.00	1.00	2.00	2.00
Column Depth: d(cm)	.157	.157	.157	.157	.157	.157	.150	.150	.150	.150	.150	.150	.150
Column Height: h(cm)	15.0	15.0	15.0	15.0	20.0	20.0	15.0	15.0	15.0	15.0	15.0	20.0	20.0
Beam Weight: W_2(kg)	3.55	7.13	3.55	7.13	3.55	7.13	3.55	3.55	7.13	3.55	7.13	3.55	7.13
Nominal Collapse Load: Q_0(kg)	3.66	3.66	5.12	5.12	4.94	4.94	1.77	3.54	3.54	3.75	3.75	5.63	5.63
Nominal Yield Limit Displacement: x_0(cm)	.761	.761	.761	.761	1.353	1.353	1.106	1.106	1.106	1.802	1.802	3.20	3.20
Material Constants γ	1.00						.843			.867			
Material Constants α	.035						.024			.030			
Maximum Restoring-Force: H_e(kg)	3.48	3.30	4.94	4.76	4.70	4.46	1.27	2.77	2.54	2.88	2.51	4.39	3.89
Collapse State Displacement: x_c(cm)	51.8	11.51	4152.	20.6	95.3	30.2	7.34	18.22	7.32	18.01	7.64	37.9	15.56
$x_e/x_c=\gamma x_0/x_c$	.015	.066	.000	.037	.014	.067	.127	.051	.127	.087	.073	.204	.178

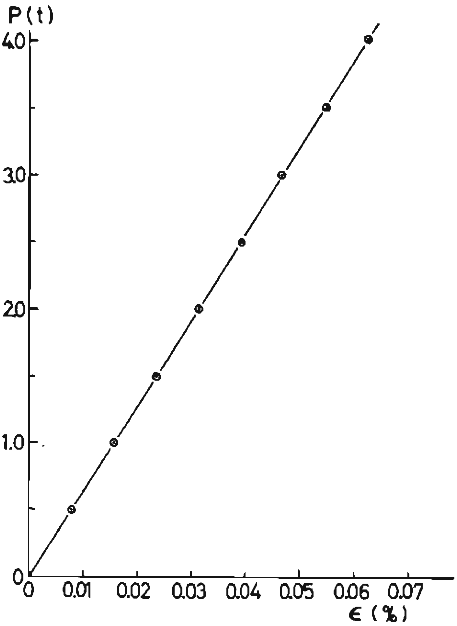


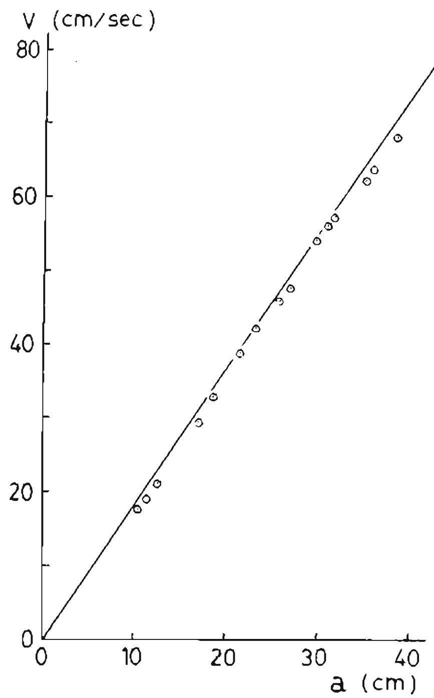
Fig. 8. Load-strain relation of load cell.

of the bifilar pendulum to cause plastic deformation of the test specimen. The static load-strain relation of the load-cell was nearly linear, as shown in Fig. 8, and the proportional factor was equal to 6.30×10^3 tons. Although the instantaneous value of the load applied to the head of the bifilar pendulum did not correspond to the base acceleration of the test frame, since the latter value contained the longitudinal wave propagation of the bifilar pendulum, the given velocity of the bifilar pendulum, computed from the momentum-impulse relation after integrating the load-time record, coincided with the one obtained from the acceleration-time record.

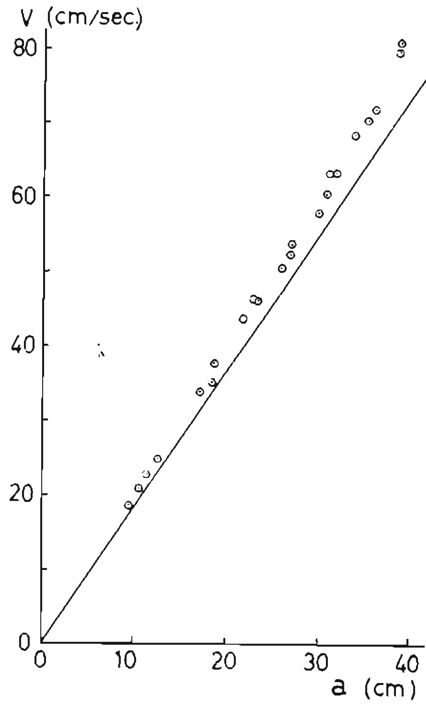
The sufficient accuracy of the given velocity obtained from the acceleration-time record was again confirmed in the same way as in the calibration of the barium-titanate ceramics described in the preceding section (Fig. 2), before the impact loading of the test frames. At the same time the load-time record was also checked. Shown in Figs. 9(a) and (b) are the relations between the given velocity v calculated from the load cell or accelerometer and the initial amplitude of the bifilar pendulum a. The solid lines refer to the theoretical values calculated by the equation $v = 2\pi a/T$.

The acceleration-time record obtained from the impact loading test of the frame specimens indicated that the duration of the impact of this loading system was approximately constant and less than 1 ms, as shown in Photo 3. The fundamental period of the elastic vibration of the test frames was found to vary from 0.15 to 0.4 sec. The maximum acceleration applied at the base of the frame specimen was ranged from 1×10^5 to 4×10^5 cm/sec^2; the ratio of the applied maximum acceleration to the yield level varied from 150 to 500. Therefore, this loading system was regarded as giving a purely impulsive load.

The final plastic deformations of the test frames were measured as in the preliminary tests. Static loading tests were also performed on portal frame specimens similar



(a) From acceleration-time records



(b) From load-time records

Fig. 9. Initial velocity-amplitude relations.

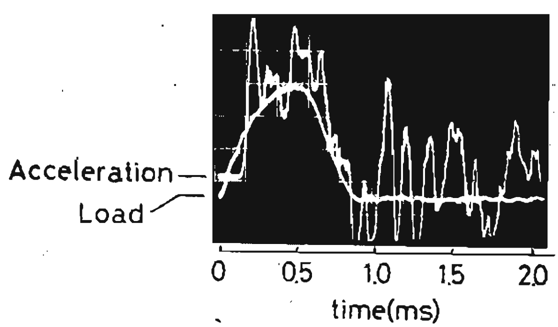


Photo 3. Acceleration- and load-time records.

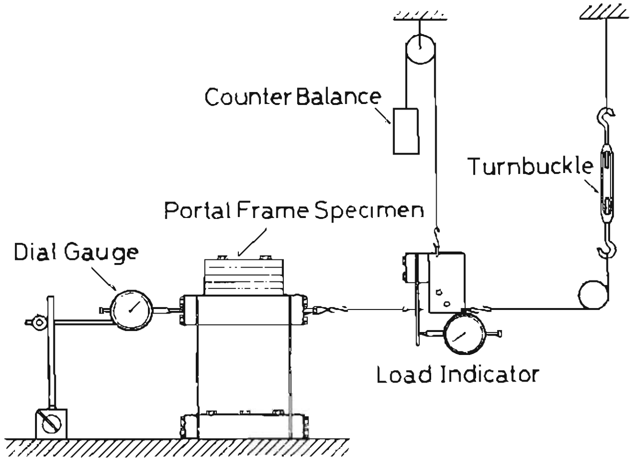


Fig. 10. Apparatus for static test.

to those of the impact loading tests, as shown in Fig. 10. The horizontal force was applied to the centre of the beam cross section by means of a turnbuckle. The applied load was measured by a cantilever-type load indicator and the beam displacement by a dial gauge.

The static load-displacement relation of the test frame is easily calculated theoretically by the following procedure, under the assumption of non-strain reversal and linearity of the longitudinal distribution of column moment. When the column top (or bottom) moment M_j is given, the distributions of bending moment and curvature in the column are uniquely determined from the moment-curvature relation of the column, and the beam displacement x is obtained by integrating the curvature along the column length. Then the resistant force of the columns Q is obtained as a function of x,

$$Q(x) = 4M_j(x)/h, \tag{3}$$

and the restoring-force of the frame H is obtained by an equation which takes into account the over-turning moment due to the beam weight (See Fig. 11),

$$H(x) = Q(x) - W_2 x/h. \tag{4}$$

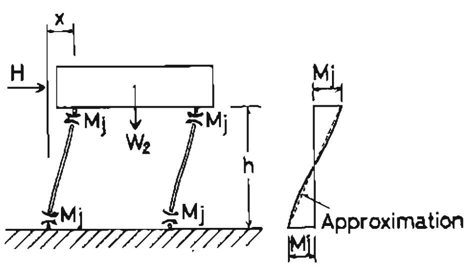


Fig. 11. Portal frame model.

The experimental Q–x relation is easily calculated from the H–x relation obtained in the static test using Eq. (4). The theoretical Q–x relation is a function of the column dimensions and the material properties, and it is independent of the beam weight.

Load-displacement relations obtained from the static test are shown in Figs. 12 (a)~(f). Test results show that the restoring-force characteristics of these test frames can be approximated to be bi-linear with a negative second slope, and that the ratios of the negative second slope to the first in the approximated bi-linear restoring-force characteristics are about 0 to 0.2*.

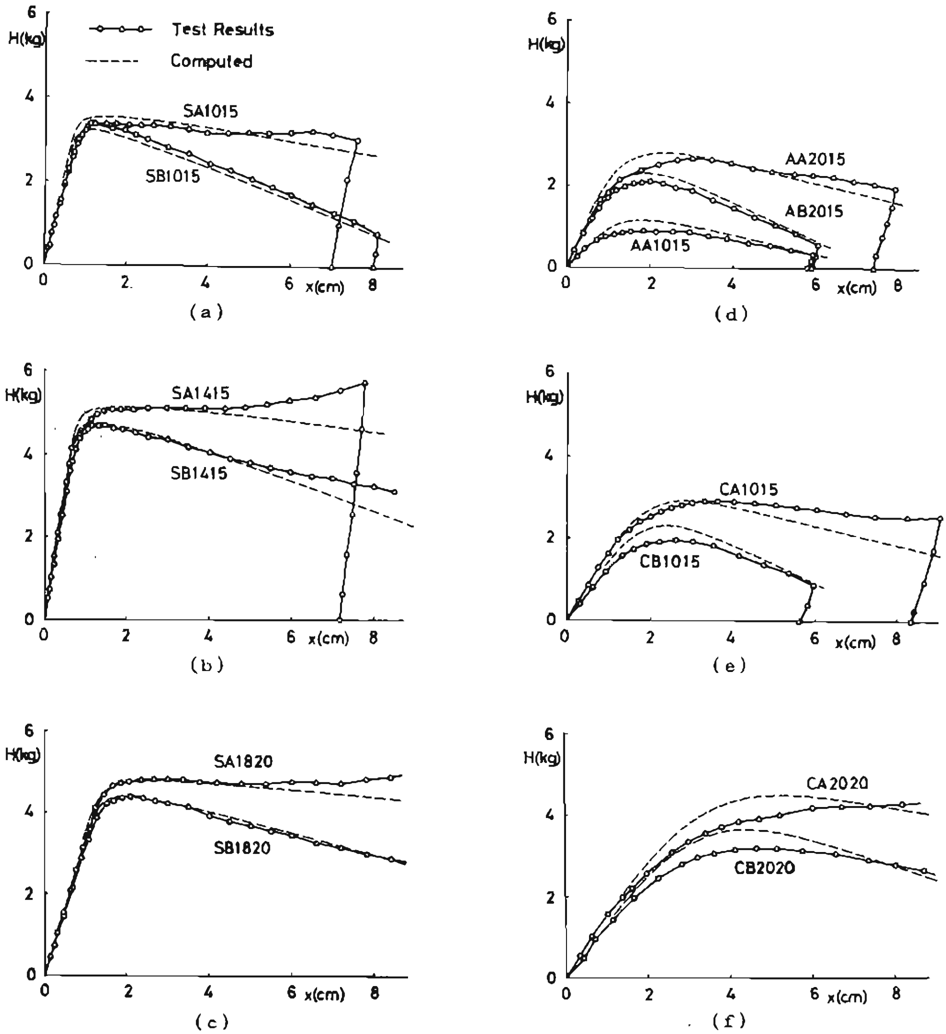


Fig. 12. Static test results.

* The theoretical restoring-force characteristics illustrated by the dashed lines in the figures of 12 are based on the observed moment-curvature relationship, and show good agreement with the test results.

4. Theoretical Analysis

In order to determine the final plastic deflection of a portal frame with a rigid beam subjected to impulsive motion at its column base, the following assumptions are employed:

1° The mass of the columns is negligibly small as compared with the beam mass, and the beam is rigid as compared with the columns.
2° Any viscous effects of the materials are ignored and the restoring-force characteristics are the same in both dynamic and static conditions.
3° Although the over-turning moment due to the beam weight is considered, the deflections are small enough so that the vertical motion of the beam is negligible.
4° The motion of the portal frame relative to the bifilar pendulum damps out sufficiently before the pendulum displacement attains a value of the order of the length of the suspending wire.

Based on the above assumptions, the dynamical system composed of the frame specimen and pendulum is considered to make a two-degrees-of-freedom motion. Denoting the displacements of the bifilar pendulum and the beam from their neutral positions by x_1 and x_2, respectively, the equations of motion for the free vibration of this dynamical system read

$$\begin{aligned} W_1\ddot{x}_1/g+H&=0 \\ W_2\ddot{x}_2/g-H&=0 \end{aligned} \tag{5}$$

These equations are rearranged into a one-degree-of-freedom motion with a equivalent mass $m=W_1W_2/\{(W_1+W_2)g\}$, if x_1 and x_2 are eliminated upon introduction of the relative displacement $x=x_1-x_2$, to be

$$m\ddot{x}+H=0 \tag{6}$$

where W_1 and W_2 are the weights of the pendulum and beam, respectively (Fig. 13). In this analysis, the restoring-force characteristics of the virgin state portal frame are approximated to be bi-linear with elastic-plastic property, and they have a negative second slope, as shown in Fig. 14.

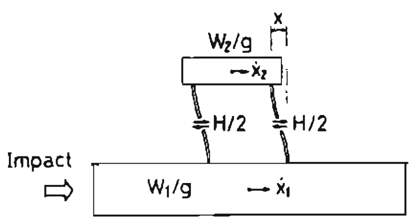


Fig. 13. Two-degrees-of-freedom model.

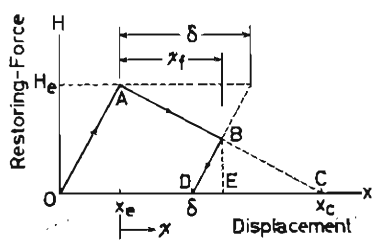


Fig. 14. Approximated restoring-force characteristics.

When the bifilar pendulum begins to move with initial velocity v, the frame responds elastically, and the relation $H=H_e x/x_e$ is substituted into Eq. (6), where x_e and H_e are the elastic limit displacement and the corresponding restoring-force, respectively.

If the input is large and x reaches x_e, the frame starts to behave plastically, and the relative velocity at this instant is obtained by integrating Eq. (6) after multiplying both members by $\dot{x}$, to be

$$\dot{x}=\sqrt{v^2-H_e x_e/m} \qquad \text{at } x=x_e. \tag{7}$$

Hereafter, the equation of motion (6), after substitution of $H=H_e\dfrac{x_c-x}{x_o-x_e}$, multiplication by $\dot{x}$ and integration, becomes

$$\frac{1}{2}m\dot{\chi}^2+H_e\chi\left\{1-\frac{\chi}{2(x_c-x_e)}\right\}=\frac{1}{2}mv^2-\frac{1}{2}H_e x_e \tag{8}$$

where $\chi\equiv x-x_e$, and x_c is the limit displacement corresponding to the unstable collapse state as shown in Fig. 14. If $\dot{\chi}=0$ is substituted into Eq. (8), χ takes the value χ_f defined in Fig. 14, and we get

$$\frac{1}{2}mv^2=H_e\chi_f-\frac{H_e\chi_f^2}{2(x_c-x_e)}+\frac{1}{2}H_e x_e. \tag{9}$$

Considering that $\chi_f\leq x_c-x_e$, we choose from the two solutions of Eq. (9) the one*

$$\chi_f=(x_c-x_e)-\sqrt{(x_c-x_e)^2-(x_c-x_e)(mv^2/H_e-x_e)}. \tag{10}$$

Eq. (9) shows that the kinetic energy $U=mv^2/2$ of the one-degree-of-freedom system with the equivalent mass m subjected to the initial velocity v is equal to the sum of the energy U_p absorbed in plastic deformation which corresponds to the area of the quadrangle OABD in Fig. 14 and the potential energy necessary to continue the elastic vibration corresponding to the area of the triangle BED. From the similarity of triangles it is found that between U and U_p there is the relation

$$\frac{x_e}{x_c}=\frac{U-U_p}{U_c-U_p}, \tag{11}$$

where $U_c=H_e x_c/2$ is the energy absorbed in plastic deformation up to the unstable collapse.

When the plastic deformation finishes, the system again behaves elastically, and since Fig. 14 gives the relation $\delta=\chi_f/(1-x_e/x_c)$, the final plastic deflection δ is obtained from

$$\frac{\delta}{x_c}=1-\sqrt{\frac{1-U/U_c}{1-x_e/x_c}}. \tag{12}$$

Eq. (12) is applicable to the region $x_e/x_c\leq U/U_c\leq 1$ in its physical sense. If U/U_c is less than x_e/x_c, the input is so small that no plastic action takes place, and if $U/U_c>1$,

* In the case of a positive second slope, $x_c<x_e$ algebraically, and Eq. (9) is also valid, but the other solution

$$\chi_f=\sqrt{(x_e-x_c)^2+(x_e-x_c)(mv^2/H_e-x_c)}-(x_e-x_c)$$

should be chosen.

the input is too large, and the frame undergoes unstable collapse. Substituting Eq. (11) into (12), we get the relation

$$\frac{\delta}{x_c}=1-\sqrt{1-U_p/U_c}\,. \tag{13}$$

The particular case of $x_e=0$, corresponds to a rigid-plastic structure. If the effect of elasticity is ignored, from the relations $x_c=4M_0/W_2$, and $U_p=U=I^2g/(2W_2)$, Eq. (13) becomes

$$\bar{\delta}\equiv\frac{W_2}{4M_0}\cdot\delta=1-\sqrt{1-\frac{ghI^2}{(4M_0)^2}}\equiv1-\sqrt{1-\bar{I}^2} \tag{14}$$

which is Eq. (1) introduced in section 2.

Eq. (12) is rewritten by defining the maximum elastic strain energy $U_e\equiv H_ex_e/2$, as

$$\frac{\delta}{x_e}=1-\sqrt{\frac{1-(x_e/x_c)(U/U_e)}{1-x_e/x_c}}\Big/\left(\frac{x_e}{x_c}\right). \tag{15}$$

5. Results and Discussion

In order to apply the bi-linear analysis described in the preceding section, the observed restoring-force characteristics in the static tests should be approximated to be bi-linear. Although the $H-x$ relation is directly affected by the over-turning moment, the $Q/Q_0-x/x_0$ relation is considered to depend only on the stress-strain relation of the material, where $Q_0=\sigma_0bd^2$ and $x_0=\sigma_0h^2/(2Ed)$ are the static collapse load and the elastic limit displacement, respectively, in the elastic-perfectly plastic bending theory, σ_0 is the yield limit stress, E is the Young's modulus, and b and d are the width and depth of columns, respectively.

The $Q/Q_0-x/x_0$ relations of the test frames rearranged from the static test results are shown in Figs. 15 (a)∼(c). Small scattering is observed in each of these figures, which reasonably confirms the fact that the $Q/Q_0-x/x_0$ relation depends only on the behaviour of the materials. These $Q/Q_0-x/x_0$ relations are taken to have approximated bi-linear characteristics which, for a loading process, consist of the straight line with the unit gradient through the origin and the straight line with the gradient α through the point (γ, γ), as follows:

$$Q/Q_0=\begin{cases} x/x_0 & \text{for} \quad x/x_0\leq\gamma \\ \alpha(x/x_0)+(1-\gamma)\alpha & \text{for} \quad x/x_0>\gamma \end{cases} \tag{16}$$

The material constants α and γ are evaluated in such a way that the work done up to the estimated maximum deflection is the same in both the actual and the approximated bi-linear relations. The maximum final plastic deflection in the impact loading tests was $\delta\approx10x_0$ for mild steel specimens, $\delta\approx7x_0$ for aluminum alloy and $\delta\approx5x_0$ for copper. The approximated bi-linear resistant force characteristics are illustrated by dashed lines in Figs. 15.

The values of x_e, x_c and H_e introduced in the preceding section are determined from Eq. (16), to be

$$x_e = \gamma x_0 \tag{17}$$

$$x_c = (1-\alpha)\gamma Q_0 / (W_2/h - \alpha Q_0/x_0) \tag{18}$$

$$H_e = \gamma(Q_0 - W_2/hx_e) \tag{19}$$

and are listed in the Table.

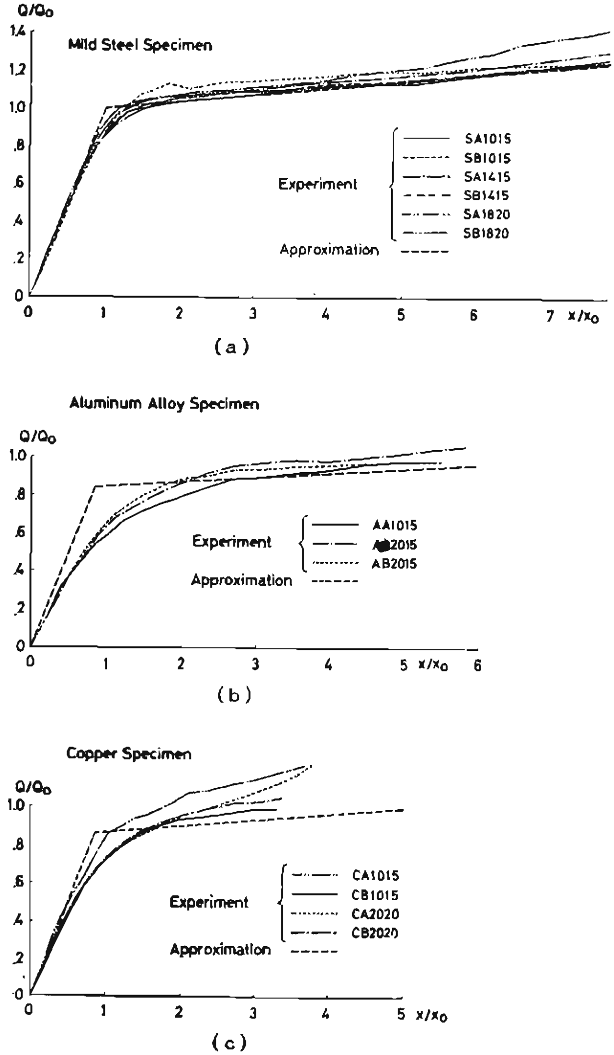


Fig. 15. $Q/Q_0 - x/x_0$ Relations from static tests.

The impact loading test results are compared with the theoretical predictions in Figs. 16 (a)~(c) and 17 using the dimensionless parameters introduced in the preceding section. In Figs. 16, for the ordinate is taken the ratio δ/x_e of the final plastic deflection to the elastic limit displacement and for the abscissa the ratio U/U_e of the input

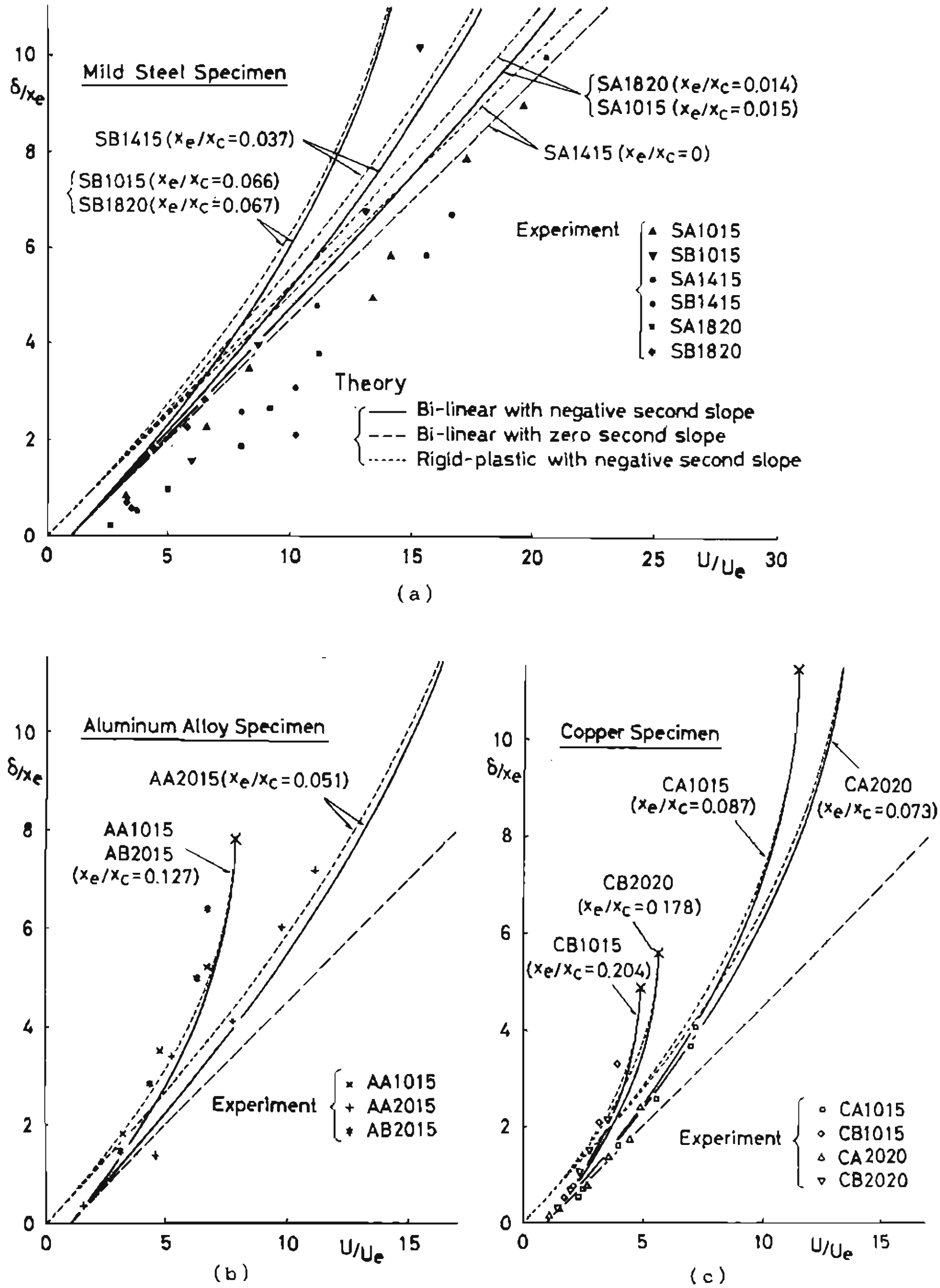


Fig. 16. $\delta/x_e - U/U_e$ relations from impact loading tests.

energy to the maximum elastic strain energy. The solid lines are drawn on the basis of Eq. (15). The crosses at the top of the theoretical curves refer to the unstable collapse state. In general, the experimental values agree reasonably well with the theoretical predictions for the final plastic deflection. A satisfactory agreement is seen especially for the aluminum alloy and the copper specimens. For the mild-steel specimens, however, the experimental values lie consistently below the theoretical curves. The difference for the mild steel specimens between the experimental and theoretical results seems to be independent of the input covered in the present tests and approximately constant ($\approx 20\%$). The discrepancy may be reasonably attributed to the strain-rate effects, because for the aluminum alloy and copper specimens, which have been reported to be of small rate-dependence[41], the rate-independent analysis and the experiment show a general agreement.

A dashed straight line is drawn for comparison, by taking the limit $x_c \to \infty$ in Eq. (15). This is the case in which the effects of strain-hardening and of the over-turning moment cancel each other. Deviation from this straight line indicates the importance of the over-turning moment, and it grows as U/U_e or δ/x_e, as it should be. The relative importance of the destabilizing effect hinges on the ratio x_e/x_c, which coincides with the ratio of the second slope to the first in the bi-linear restoring-force characteristics. The copper and aluminum alloy specimens had larger x_e/x_c-values than the mild steel specimens, and they are shown to reach unstable collapse for smaller values of U/U_e or δ/x_e than the latter.

This fact implies that when structures with a large x_e/x_c-value are subjected to impulsive motion, the structural safety should not be evaluated only by the ratio δ/x_e or U/U_e, but also by δ/x_c, U/U_c or a relevant quantity referring to the unstable collapse state. The importance of the x_c-value has been pointed out in the field of earthquake engineering in relation to the reliability of structures[42,43].

In the case of beam-like structures, the criterion for elasticity to be negligible is that the ratio U_p/U_e be sufficiently large[3,37]. As to structures in which the change in geometry has a deteriorating effect and the restoring-force characteristics have a negative slope, as in the present problem, the value of U_c is limited and U_p cannot be greater than U_c. Consequently, the effects of elastic vibration are somehow related to the ratio $U_e/U_c = x_e/x_c$ also. It became clear in the static tests that the ratio x_e/x_c ranged from zero to 0.2, and the elasticity was not negligible in the tests presented in this paper*. In Figs. 16 (a)$\sim$(c), rigid-plastic solutions are also illustrated by dotted lines. On the basis of these considerations the experimental results are again plotted in Fig. 17; the ratio U_p/U_c is taken for the abscissa and δ/x_c for the ordinate. Theoretical values corresponding to each specimen are represented by a single parabolic curve, Eq. (13).

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* If the effect of the axial force existing in a column element is neglected, for a portal frame composed of a rigid beam and two identical columns of ideally I-section this ratio is determined from

$$x_e/x_c = \frac{\sigma_0}{3E}\left(\frac{h}{d}\right)^2\frac{N}{N_0}$$

where N is the column axial force, and N_0 is the limit force in tension. Taking representative values in practical steel structural design that $\sigma_0/E \approx 1/700$, $h/d \approx 10$ and $N/N_0 \approx 0$ to 0.8[44], this ratio takes a value between zero and 0.04 in actual buildings.

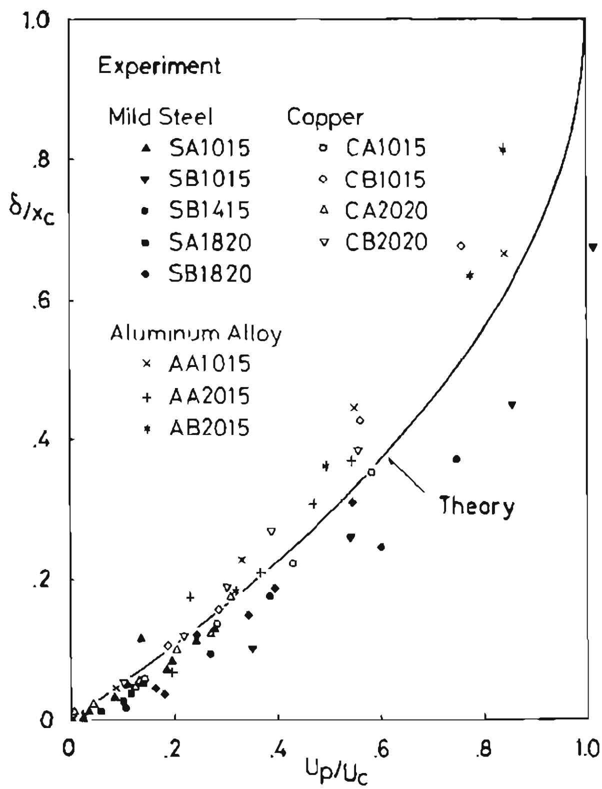

Fig. 17. $\delta/x_c - U_p/U_c$ relations from impact loading tests.

6. Concluding Remarks

Attempts have been made to develop a method of measuring input in the impact loading tests of frames. Preliminary tests were first carried out and portal frame specimens were subjected to impact loads at their column tops, and the load-time relation was observed by using the piezo-electric effects of barium-titanate ceramics. It has been found that the loading system adopted in this study can be considered to give a purely impulsive load, and that the over-turning moment considerably affects the final plastic deflection of the frame tested.

Another series of experimental studies was then performed, in which portal frame specimens fixed to a bifilar pendulum were subjected to impact motion at the column base, and the acceleration-time record of the input was taken by an accelerometer. This impact could also be regarded as purely impulsive. The theoretical predictions based on an analysis which approximates the frame restoring-force characteristics to be bi-linear with a negative second slope agreed well with the experimental values of the final plastic deflection for the aluminum alloy and the copper specimens. For the mild steel specimens the theory overestimated the deflection by about 20 per cent. The reason for this may be that strain-rate effects are ignored in the theory.

In the large deformation range or the ultimate state of normaly framed structures, the restoring-force characteristics may have a negative slope; it was caused by the over-turning moment due to the gravity force in the tests reported in this paper. It may also be caused in actual buildings by some instability effects or various local damages. Then it is dangerous to consider that the so-called ductility factor, which is normally defined to be the ratio of the maximum deflection to the yield deflection, determines the safety of a structure when it is subjected to impulsive loading. Structural safety should then be related to what is termed unstable collapse state in the present paper, and it depends greatly on the ratio U/U_c of the input energy to the maximum energy which can be absorbed in the structure up to the complete collapse.

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