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Harbor Oscillations Induced by Composite Waves in Rectangular Basins

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Abstract

This paper presents the results of experiments on harbor oscillations induced by composite waves in rectangular basins with a constant depth. The experiments were carried out in a two-dimensional wave tank in which a fully open harbor model was installed. In order to obtain the constant wave height the wave period was kept constant through a series of the tests but the harbor length was variable. The wave records were taken at both harbor entrance and end, and the wave amplification factor, defined as the ratio of the wave height in a certain point to that of standing waves when the entrance was closed, was derived for both regular and composite waves by the harmonic analysis.

The result of comparison between the response curves for regular and composite waves, shows that the resonant characteristics by composite waves is different from that by regular waves by mutual interaction of component waves, especially at resonant modes.

1. Introduction

Resonance in a harbor occurs when the period of waves arriving within the harbor coincides with the natural period of the harbor. Harbor oscillations, therefore, are greatly affected by not only the wave height but also the period of incident waves. This fact is generally found by obtaining the response curve of the harbor for regular waves with various periods. Since, however, ocean waves have continuous energy spectra, the influence of wave components except that of the natural period of the harbor on the harbor oscillation must be invesitgated. Because there exists the possibility of unexpected wave amplification due to mutual interaction of waves in the harbor, especially for the composite waves in the same manner as the resonant interaction of progressive waves[1)2)3)].

In this paper, the resonant characteristics of a rectangular harbor by regular waves (or monochromatic waves) and composite waves of two or three components are experimentally investigated to examine the existence and the magnitude of nonlinear effect as a first step of the study for ocean waves with continuous wave spectra.

2. Experimental Apparatus and Procedures

The wave tank used is 30 m long, 50 cm wide and 70 cm high with side walls of glass at the Department of Civil Engineering, Kyoto University. The rectangular harbor model of transparent acrylic plates was installed in the wave tank as shown in Fig. 1. The harbor length is variable from 0 cm to 80 cm. The wave generator is of piston-type and composed of eight pistons at maximum with different periods, which is in the ratio $1:\sqrt{2}$ to one another[4)5)]. The water depth was kept 10 cm through the experiments.

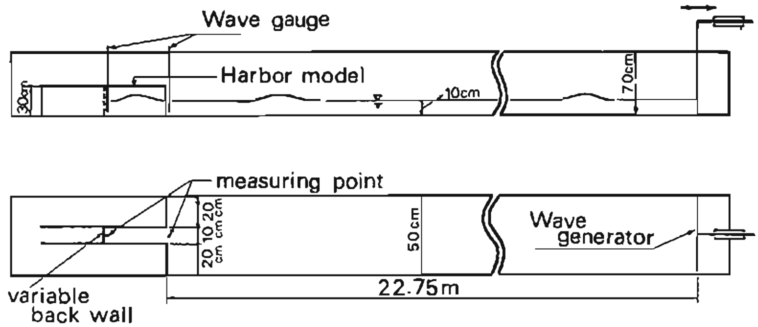

Fig. 1. Experimental Apparatus.

In making the experiment of harbor oscillation, there are two different methods: one is that the period is kept constant and the harbor length is varied, and the other is that the harbor length is kept constant and the wave period is variable. The former is inconvenient for the model study of existing harbors but have the merit to be capable of making experiments, which keeping the wave height constant. In the latter case, it is difficult to set a constant wave height. It was actually recognized in the preliminary experiment that the wave amplification factor in the harbor is changed as the incident wave height is varied. Therefore, the former method was adopted in this experiment.

The wave period and height of the long-period waves used in the experiment were chosen $T_1=2.0$sec and $H_1 \fallingdotseq 3$ mm respectively.

After a series of the experiments for each of regular waves to be composed, the experiments for composite waves were carried out for the following three cases:

Experiment I. Composition of the long-period waves (defined as T_1-wave) and the short-period waves (defined as T_2-wave) with the period $T_2=0.707$ sec and the wave height, $H_2 \fallingdotseq 6$ mm. In this case, the higher-harmonic components of T_1-wave never coincide with the components of T_2-wave. Therefore, the resonant characteristics by each component wave can easily be distinguished exactly, even when the wave height becomes large at resonance.

Experiment II. Composition of T_1-wave and the short-period wave (defined as T_3-wave) with the period $T_3=1.0$ sec and the wave height $H_3 \fallingdotseq 3.5$ mm. The first mode of oscillation by T_2-wave coincides nearly with the fundamental mode by T_1-wave in Experiment I. To avoid overlapping with the resonance point of T_1-wave, the period of T_3-wave is selected. In addition, as the second-harmonic component of T_1-wave is equal to the frequency of T_3-wave the influence of T_1-wave on T_3-wave can be discussed conveniently.

Experiment III. Composition of T_1-wave, T_2-wave and T_3-wave.

The wave height was measured by two wave gauges of electric resistance type at the harbor entrance and the harbor end with a magnetic data recorder. The records were digitalized by using a A–D converter, and after Fourier analysis for each component frequency, the wave height of the component waves at two measuring points was obtained. A 16 mm cine camera was used to determine the wave height distribution along the vertical breakwater at the harbor entrance by taking pictures of transverse oscillation in front of the breakwater.

3. Existing Theoretical Solutions for Regular Waves

Theoretical analyses of harbor oscillations for regular waves have been investigated by many researchors. Three typical solutions are briefly described herein.

Le Méhauté[6] has derived the following expression using the theory of complex variables:

$$R=(1/2)\{r\alpha_1(1+p)\}[1+(\beta_2 r^2 p)^2-2\beta_2 r^2 p\cos(\hat{\beta}_2+2\hat{r})]^{-1/2} \quad \cdots\cdots(1)$$

For the fully open harbor,

$$\alpha_1=(b/d)^{1/4},\ \beta_2=\{1-(d/b)^{1/2}\}^{1/2},$$

$$\cos\hat{\beta}_2=[2-(b/d)^{1/2}\{1+(d/b)\}]/(2\beta_2), \qquad \hat{r}=-2\pi l/L$$

in which, R: wave amplification factor, α_1: transmission coefficient, r: damping coefficient, p: reflection coefficient at the harbor end, $2b$: wave tank width, $2d$: harbor width, l: harbor length, L: wave length.

Putting $r=1$ and $p=1$ in Eq. (1), the resonant harbor length and the wave amplification factor for T_1-wave are obtained $l=32.2$ cm and $R=5.83$ respectively.

Miles and Munk[7] have developed a theory of the response of the rectangular harbor connected directly with the open sea, based on a model for the single degree of freedom oscillator, as follows:

$$\cot(k_R l)=2k_R d\{0.478-(1/\pi)\}\ln[(2k_R d) \times\{c/(2d)\}\sin\{(\pi c)/(4d)\}] \quad \cdots\cdots(2)$$

$$Q=\{1/(2k_R d)\}\{2k_R l+\sin(2k_R l)\}/\{1-\cos(2k_R l)\}-1/\pi \quad \cdots\cdots(3)$$

$$R^2=[\{1-(f/f_0)^2\}^2+(1/Q^2)(f/f_0)^2]^{-1} \quad \cdots\cdots(4)$$

in which, $k_R=2\pi/L_R$, L_R: wave length at resonance, c: entrance width of habor, f_0: resonant frequency.

Ippen and Goda[8] have derived the solution of radiated waves produced at the harbor entrance using Fourier transformation. In their theory, the wave amplification factor for the fully openning is shown as follows:

$$R=\{(\cos kl-\psi_2\sin kl)^2+\psi_1^2\sin^2 kl\}^{-1/2} \quad \cdots\cdots(5)$$

$$\psi_1=\frac{2}{\pi}\int_0^{kc/2}\frac{\sin^2\alpha\cdot d\alpha}{\alpha^2\sqrt{(kc/2)^2-\alpha^2}} \quad \cdots\cdots(6)$$

$$\psi_2=\frac{2}{\pi}\int_{kc/2}^{\infty}\frac{\sin^2\alpha\cdot d\alpha}{\alpha^2\sqrt{\alpha^2-(kc/2)^2}} \quad \cdots\cdots(7)$$

The resonant harbor length l and the wave amplification factor R for T_1-waves are obtained $l=40.6$ cm and $R=4.87$ from Miles-Munk's theory and $l=40.0$ cm and $R=6.48$ from Ippen and Goda's theory respectively.

Raichlen and Ippen[9] have also developed a theory for the rectangular harbor coupled directly to a large wave basin neglecting the friction effect and assuming boundaries where waves perfectly reflect. Detailed expressions of the theory are omitted here but the calculation for T_1-wave shows that the resonant harbor length is equal to 48.1 cm and the wave amplification factor becomes infinity.

4. Experimental Results and Discussions

Experiment I. The wave amplification factor is defined as the ratio of the wave height at any point on the centerline in the harbor to the standing wave height (called the standard wave height for convenience) when the entrance is closed. Fig. 2

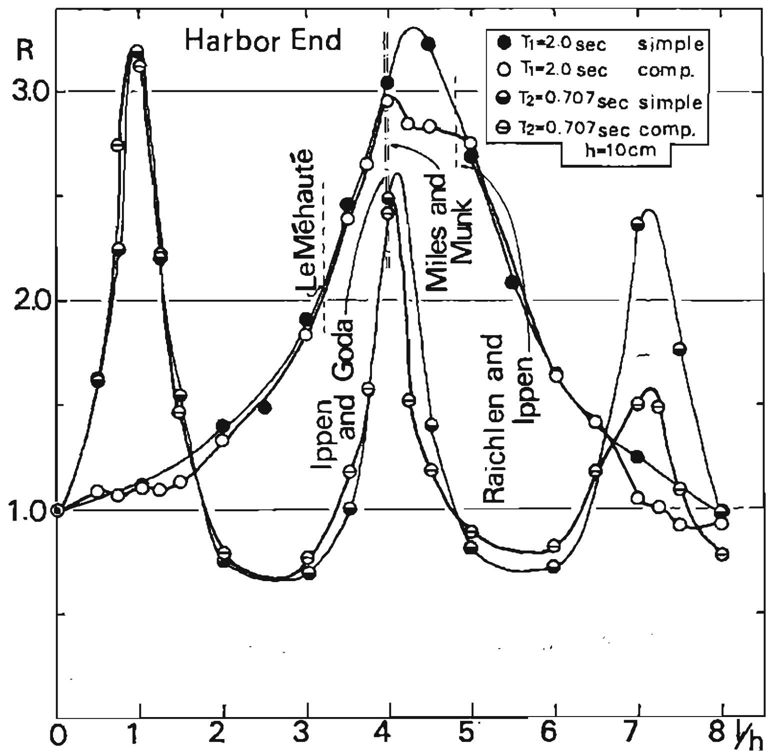

Fig. 2. Response Characteristics at Harbor End in Experiment I.

shows the variation of the wave height at the harbor end with the harbor length. The ratio of the harbor length to the water depth l/h is taken as an abscissa and the wave amplification factor R as an ordinate. It is found from this experimental result that the relative resonant harbor length is $l/h=4.3$ ($l=43$ cm), and $R\fallingdotseq 3.4$ (6.8 times of the incident wave height). The comparison between the experimental result and the calculated ones described in Section 3. shows that the experimental values of the relative resonant harbor length and the wave amplification factor are both different from the calculated ones. This may be caused by the fact that the open sea condition, the energy loss at the harbor entrance and the estimation for the friction terms do not always agree with those in the experiments. The relative resonant harbor lengths obtained based on the theories are also shown in Fig. 2.

On the other hand it is found from the experiment that the relative resonant harbor length is $l/h=1.0$, 4.1 and 7.2, and the wave amplification ratio $R=3.2$, 2.7 and 2.6 for T_2-wave.

The resonant characteristics for T_1 component wave of the composite waves differs from that for regular waves near the resonance point. In addition, near the fundamental and second modes for T_2-wave, the resonant curve for T_1 component wave also differs slightly from that for regular waves. T_2 component wave decays remarkably in comparison with regular waves at the higher order modes of oscillation. However, it is noticed that the linear relation is held except at the resonance points, even when waves are composed. The difference of the resonant curves between component and regular waves near the resonance points may be explained by introducing the concept of finite amplitude waves. The second harmonic component of waves appears as the wave height becomes very large near the resonance points. In the case of composite waves, the bound wave with addition or subtraction of each

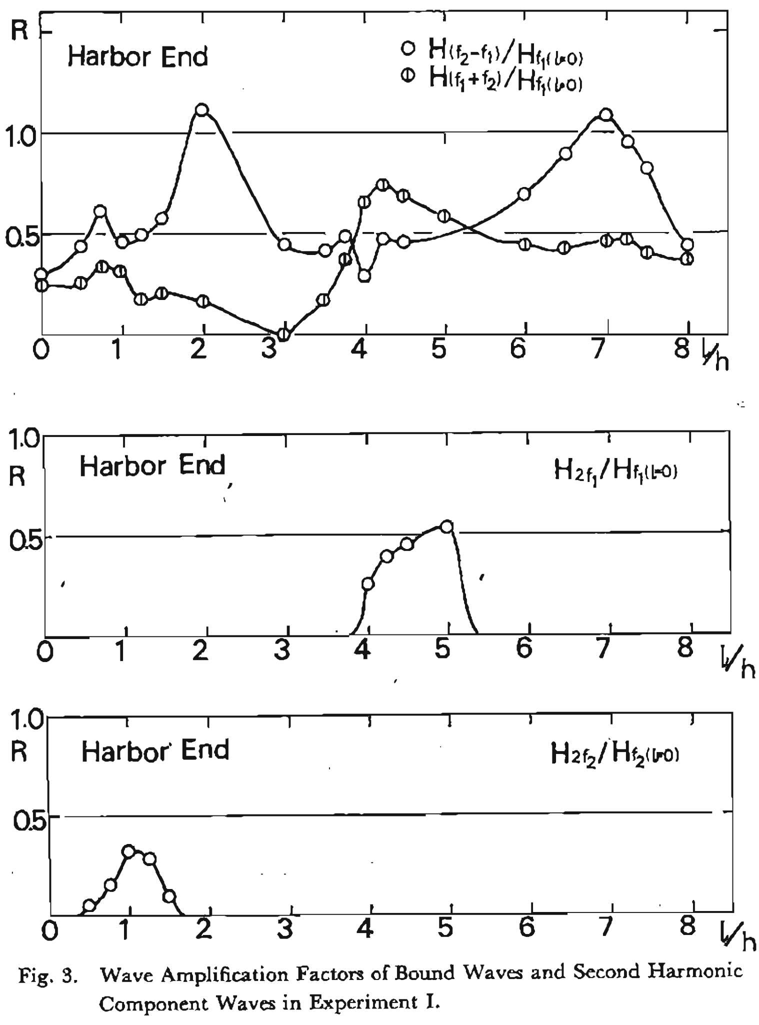

Fig. 3. Wave Amplification Factors of Bound Waves and Second Harmonic Component Waves in Experiment I.

frequency appears as Phillips[10] and Hamada[11] described when treating the mutual interaction for surface waves in the perturbation solution of Stokes waves. Fig. 3 shows the variation of the height of the bound waves and the second harmonic waves with the harbor length. It is clear that the nonlinear effect appears strongly near the resonance points for T_1-wave and T_2-wave.

Experiment II. The characteristics on T_1-wave is the same as in Experiment I. The relative resonant harbor lengths and the wave amplification factors for T_3-wave are obtained experimentally $l/h=1.75$ and 6.5, also $R=3.8$ and 3.2 as shown in Fig. 4 respectively. The wave amplification factor of T_3 component wave is a little smaller than that of regular waves, except the range $l/h=3.5\sim5$. The variations of the heights of the bound wave for the additive component f_1+f_3 and the second harmonic component $2f_3$ for T_3-wave are shown in Fig. 5. At the range $l/h=6\sim8$ for the additive component in the figure, the wave with the height of 1.2 times the standard wave height for T_1-wave is produced. This is a reason why the response curve of T_1 component wave is different from that of regular waves at the same range as in Fig. 4. The height of the second harmonic component for T_3-wave is approximately equal to

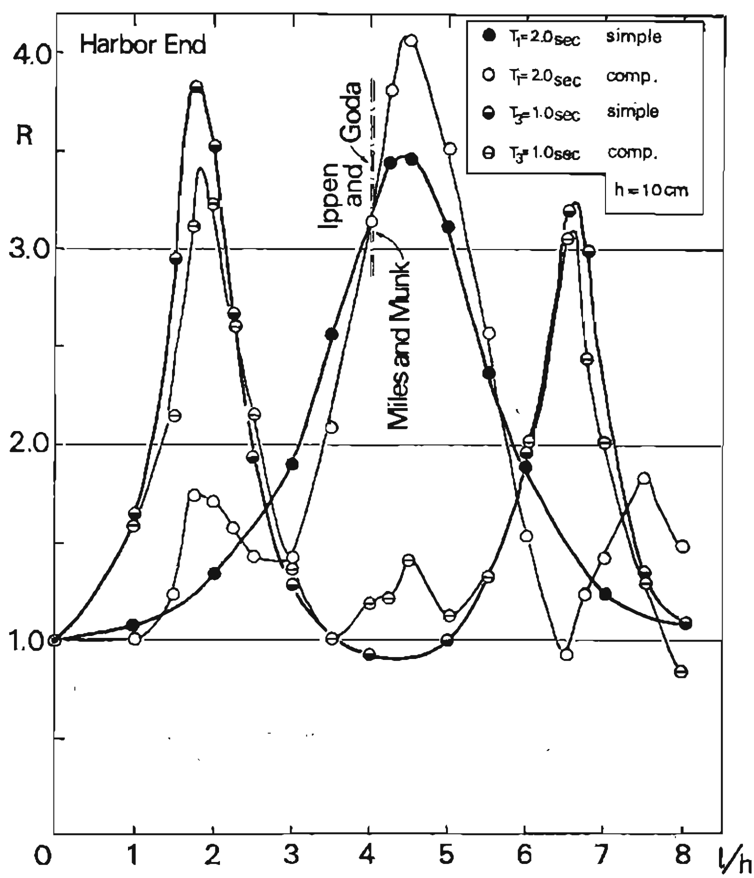

Fig. 4. Wave Amplification Factors of Bound Waves and Second Harmonic Component Waves in Experiment II.

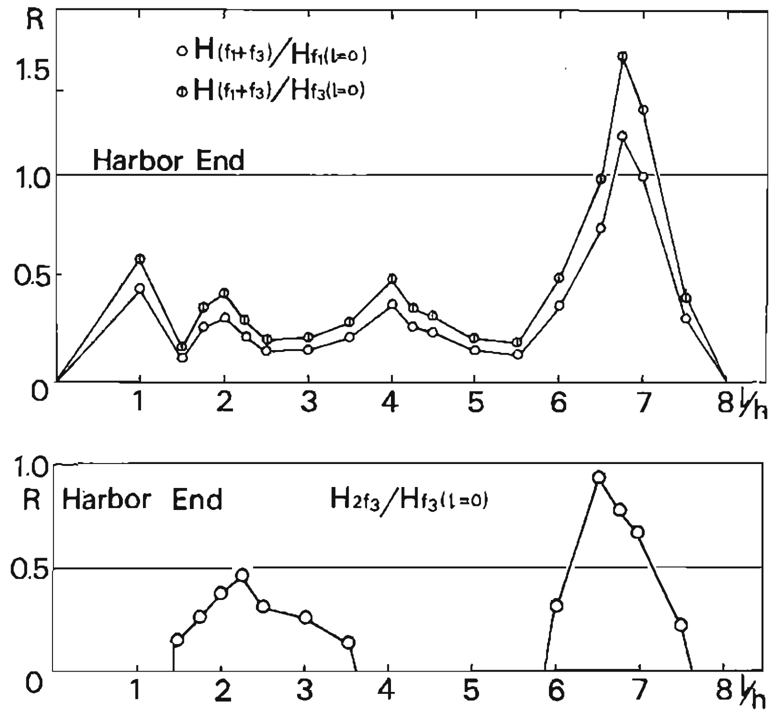

Fig. 5. Wave Amplification Factors of Bound Waves and Second Harmonic Component Waves in Experiment II.

the incident wave height near the fundamental mode of oscillation, and to the standard wave height near the first mode.

The frequency of the subtractive component coincides with the frequency of T_1-wave and also the frequency of the second harmonic component of T_1-wave is equal to that of T_3-wave. It is, therefore, impossible to pick them up independently. As shown in Fig. 4, the wave amplification factors for T_1 and T_3 component waves are larger than those for the regular waves at some values of l/h. The reason may be considered to be that the increase in the wave amplification factor for T_1 component wave in the ranges of $l/h=1.5\sim2$ and $4\sim5$ and for T_3 component wave in the range of $l/h=4\sim5$ are caused by additions of the subtractive component T_1 and T_3-waves and the second harmonic component of T_1-wave respectively.

Experiment III. Fig. 6 shows the response characteristics for composite waves of three components at the harbor end. Near the fundamental modes of oscillation for T_2- and T_3-waves, the characteristics of each component wave are not so different from that of the regular waves. However, the response curve of T_1 component wave is considerably different from that of the regular waves, because of the effects of the bound waves and second harmonic component as described in Experiments I and II. The response characteristics of the subtractive component of T_2- and T_3-waves are shown in Fig. 7. This subtractive component may play an important role to some extent for the large difference between the response characteristics of T_1 component wave and regular waves in the range of $l/h=4\sim6$ in Fig. 6.

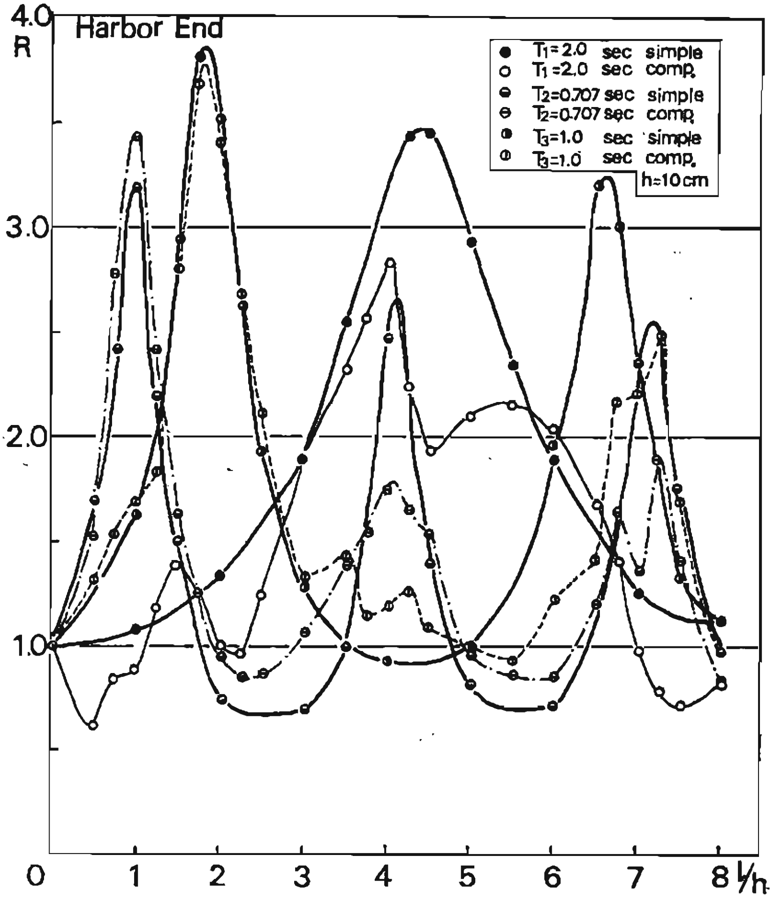

Fig. 6. Wave Amplification Factors of Bound Waves and Second Harmonic Component Waves in Experiment III.

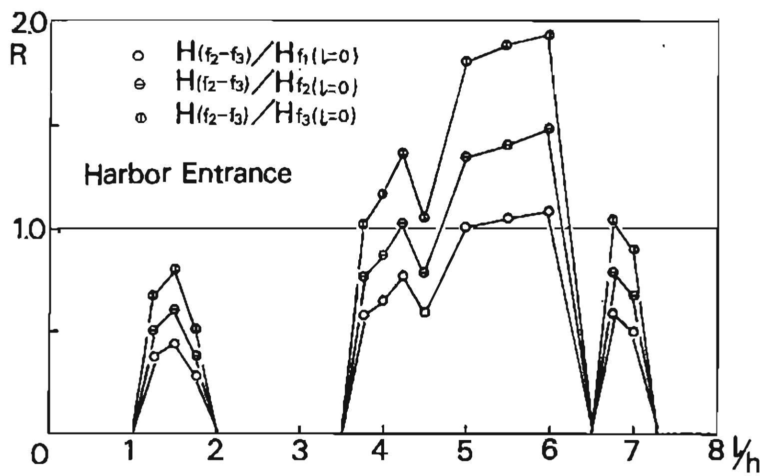

Fig. 7. Wave Amplification Factors of Bound Waves in Experiment III.

5. Discussion of Standard Wave Height

Even if incident waves can be assumed as small amplitude waves, this assumption becomes invalid with an increase in the wave height near resonance. The concept of finite amplitude waves, therefore, must be introduced, and the effect of resonant interaction of composite waves is never neglected.

In this section, some discussion will be made on the effect of the standard wave height on the response characteristics.

Fig. 8 shows the change of the ratio of the component wave height H_c of composite waves to the height H_s of regular waves with the wave period for the standard waves of Experiments I～III. It is found from Fig. 8 that the component wave height is

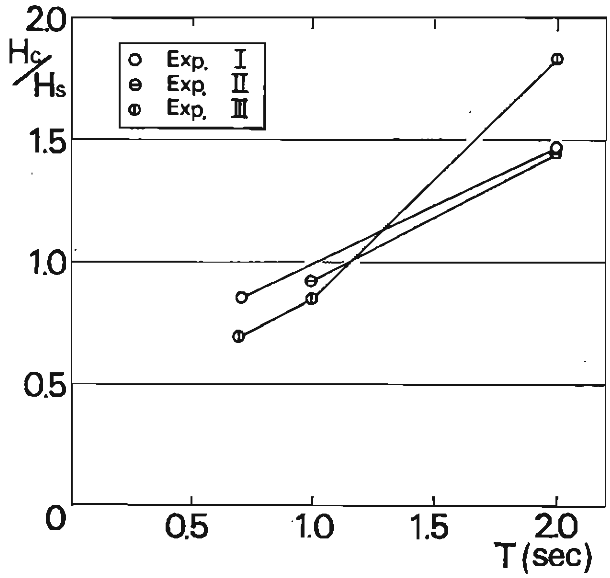

Fig. 8. Variation in Standard Wave Height with Wave Period.

smaller than the regular wave height when the wave period is short while on the other hand, the component wave height becomes larger when the period is long. No particular difference is recognized between the wave amplification factors of the component and the regular waves for T_1-wave in Experiments I and II as shown in Fig. 2 and Fig. 4, while both are much different in Experiment III as shown in Fig. 6. One of the reasons may be that the standard wave height of T_1 component wave is 1.8 times that of the regular waves as shown in Fig. 8. If such differences of the standard wave height will influence the resonant characteristics, then an important problem should be proposed, which the change of the wave amplification factor with the height of incident waves must be treated in further studies and a theory introducing its effect must be established in the future. In fact, it has been recognized experimentally that the wave amplification factor decreases with increase in the incident wave height even for the same period[12]. This means that the nonlinear effect in this problem should be made clear theoretically with the evidence of experiments.

Such explanations can be made considering the effect of the standard wave height on the resonant characteristics, that is to say that the large difference of the wave amplification factor in part between composite waves and regular waves in Experiments I~III may be caused by mutual interaction of finite amplitude waves, and on the other hand, a little difference between both response curves may be caused by the nonlinear effect of response due to the change of the standard wave height.

6. Response Characteristics at the Harbor Entrance

Fig. 9~11 show the response characteristics at the harbor entrance in Experiments I~III respectively. These response curves have almost same trend in three experiments, so only the case of Experiment I will be discussed.

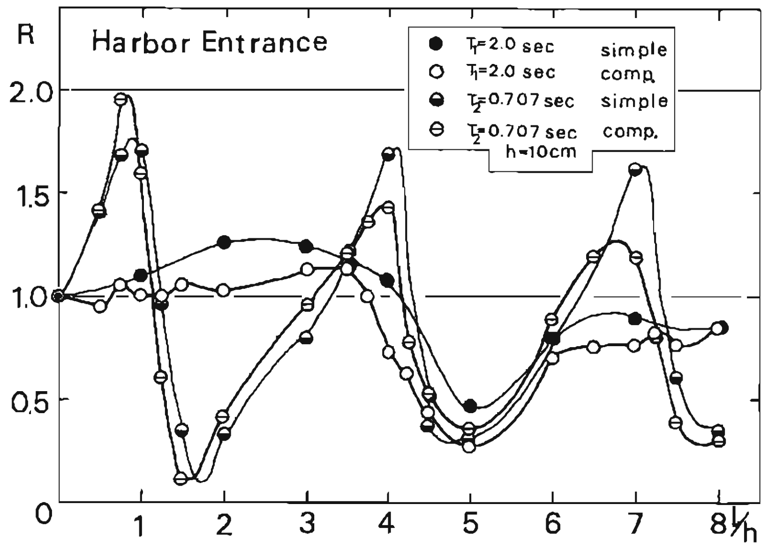

Fig. 9. Response Characteristics at Harbor Entrance in Experiment I.

As described previously, resonance of T_1-wave occurs in the case that $l/h=4.3$. At this time however, the node of the oscillation does not appear at the harbor entrance.

This can be understood from the fact that the wave amplification factor takes a minimum value in the case $l/h \fallingdotseq 5$ as seen in Fig. 9. Miles and Munk[7)] pointed out that at resonance the fully open harbor of length l would act as a quarter-wavelength resonator having an effective length $(l+\Delta l)$ and proposed the following expression for correction of the harbor length:

$$\Delta l=(2d/\pi)[1.051+\ln\{l/(2d)\}]$$
$$+(1/\pi)\ln[(2d/c)\operatorname{cosec}\{(\pi c)/(4d)\}] \quad \cdots\cdots\cdots\cdots(8)$$

Substituting the experimental conditions into Eq. (8), $\Delta l=7.84$ cm therefore $\Delta l/h=0.784$ since $h=10$ cm. The difference described above can be explained from this description. Fig. 12 shows the wave height distribution along the front of the

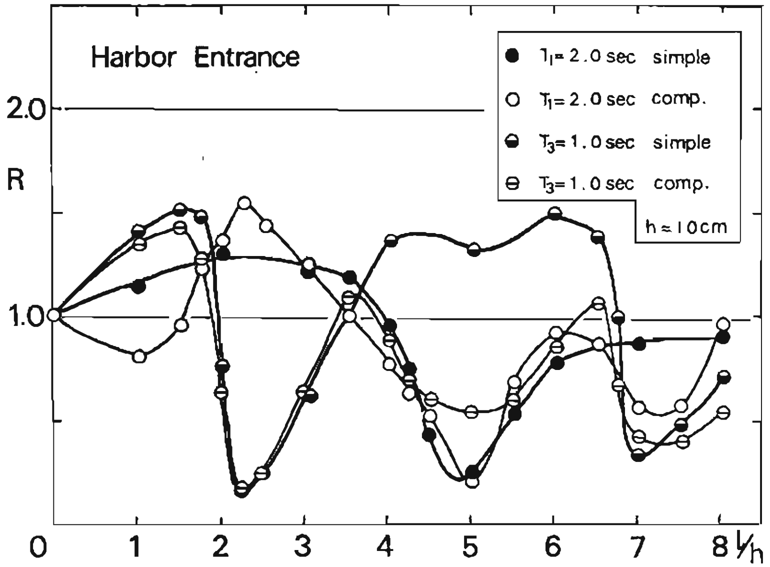

Fig. 10. Response Characteristics at Harbor Entrance in Experiment II.

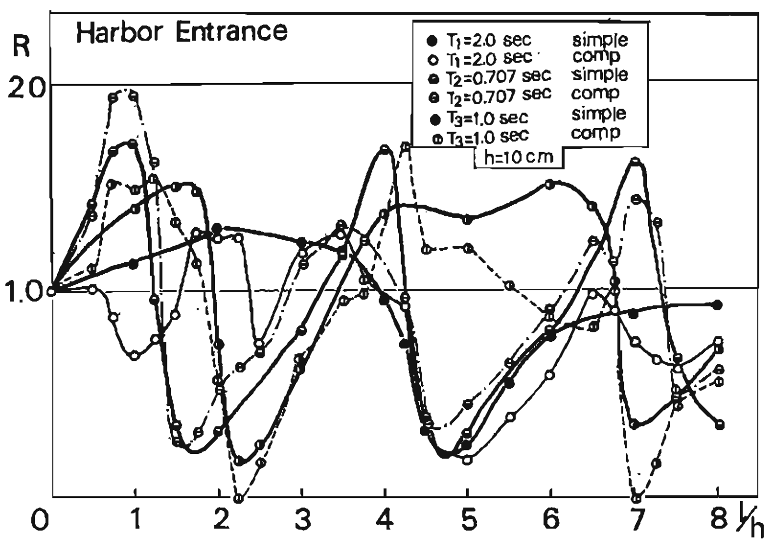

Fig. 11. Response Characteristics at Harbor Entrance in Experiment III.

breakwater at the harbor entrance for T_1- and T_2-waves, in which the abscissa x/b represents the ratio of the distance along the front of the breakwater from the bay axis to a half of the wave tank width. Remarkable transverse oscillations along the breakwater are not recognized in the case of T_1-wave, but appear in the case of T_2-wave. It can be understood from Fig. 12 that the wave height of about 2 times the standard wave height, in spite of the harbor entrance, has occurred at resonance resulting from transverse oscillation.

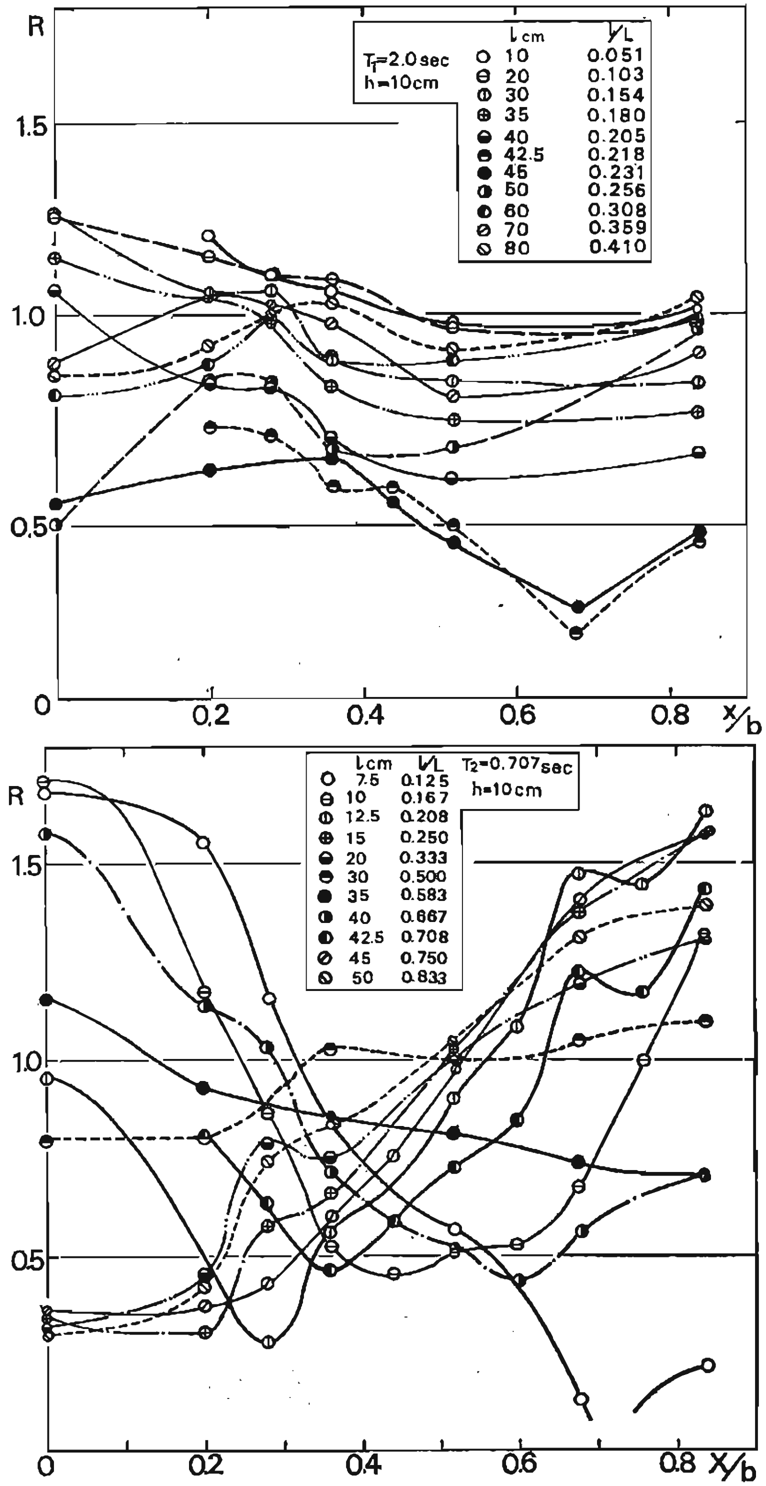

Fig. 12. Distributions of Wave Amplification Factor along Front of Breakwater for Waves with Periods T_1=2.0sec (above) and T_2=0.707sec (below).

7. Conclusion

It is concluded that the resonant characteristics due to composite waves in a harbor are different from those due to regular waves by mutual interaction between component waves, especially the nonlinear effect near resonance is very strong. It is necessary to find not only the theoretical solution involving the effect of the incident wave height on the wave amplification factor but also the method of estimating precisely energy losses at the harbor entrance and friction losses of boundaries to predict the wave amplification factor quantitatively.

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