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On the Large Deflections of Rectangular Glass Panes under Uniform Pressure

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Abstract

The estimation of the deflection of glass panes under wind pressure is important in designing external walls of buildings to resist wind action. A direct method to compute the deflection of rectangular panes under uniform loads is presented in this paper. With some assumptions on bending and membrane action of flat glasses, simple formulae were derived from the equations of elastic theory, and the constants in the formulae were determined in such a way that the deflections computed agree with experimental results.

1. Introduction

Large window panes have been used increasingly on tall buildings and the estimation of glass pane strength against wind forces has become a serious problem to keep windows safe in the events of storms. Although the experimental results of the strength of glass are scattered, some experimental formulae describing the strength of glass for practical use have been already known. However it is still difficult to compute the deflection of window glass under wind forces because the deflection is caused both by the bending and membrane action, and is not proportional to loading.

The estimation of the deflection of glass pane is important in designing external walls as suggested by Khan[1]. The large deflections and continuous vibrations of panes due to wind cause various troubles. The vibrational characteristics of window glass are determined by the natural frequency which can be estimated from the maximum deflection. Bowles and Sugarman[2] derived an empirical equation to express the relationship between central deflection and pressure on the square panels. But the most window panels are not square and a simple method to compute the deflection of rectangular flat glass supported on four edges is needed. A approximate method to predict the deflection is shown in the following.

2. Approximate method for pane deflection calculation

According to the results of tensile and bending tests, glass is a typical elastic material, so the deflections and stresses of flat glass can be computed by elastic theory. In practice, we can compute the deflections and stresses by the plate equation which considers membrane action together with bending action. However it is a complicated work to apply this method to every glass pane in design. A quicker and easier method must be formulated to estimate the deflections and stresses of glass panes, even if the exact deflections cannot be predicted.

Based on the experimental results, a flat glass appears to behave both as a slab and a membrane. This makes the deflection problem complex. On the other hand the boundary conditions need not be so specific because they are not effective on the central deflections when the thickness of the glass pane is very small in comparison with its size. The assumption that the bending force of the flat glass acts independently of the membrane action, has been made in working out the following equations.

Deflection δ_m and stress σ_m at the center of the rectangular membrane supported on four sides are shown by the formulae:

$$\delta_m = C_1 a \sqrt[3]{\frac{p_m}{E\lambda}} \tag{1}$$

$$\sigma_m = C_2 \sqrt[3]{\frac{E}{\lambda^2} p_m{}^2} \tag{2}$$

where

$\lambda = \frac{t}{a}$,

a: length of the shorter edge of membrane,
b: length of the longer edge of membrane,
t: thickness of membrane,
E: Young's modulus,
p_m: load applied to membrane,
C_1, C_2: constants.

Deflection δ_b and stress σ_b at the center of the rectangular slab are:

$$\delta_b = C_3 a \frac{1-\nu^2}{E\lambda^3} p_b, \tag{3}$$

$$\sigma_b = \frac{C_4}{\lambda^2} p_b, \tag{4}$$

where

a: length of the shorter edge of plate,
ν: Poisson's ratio,
p_b: load applied to plate,
C_3, C_4: constants.

To obtain an approximate solution of the problem the following assumptions are made.

$$p = p_m + p_b, \tag{5}$$

$$\delta = \delta_m = \delta_b, \tag{6}$$

Table 1. Theoretical values of C_1, C_2, C_3 and C_4. $\nu=0.25$.

b/a	C_1	C_2		C_3	C_4	
		C_{2x}	C_{2y}		C_{4x}	C_{4y}
1.0	0.80	0.258	0.258	0.0487	0.276	0.276
1.5	0.95	0.347	0.202	0.0926	0.478	0.270
2.0	1.03	0.378	0.164	0.1216	0.605	0.249
2.5	1.05	0.397	0.139	0.1379	0.674	0.224
3.0	1.07	0.409	0.126	0.1468	0.711	0.208

Suffix x denotes the direction in the shorter edge a and y the direction in the longer edge b.

where

p: load on a glass pane,

δ: deflection at the center of a glass pane.

Equation (5) means that the total load p on a pane can be considered to be divided into p_m and p_b; p_m is a part of load accepted as membrane action and p_b is the other part of load accepted as bending action. Equation (6) means that the deflections at the center as a membrane and a slab are supposed to be equal.

Eliminating p_m or p_b in (5) by the equations (1), (3) and (6),

$$p=\frac{1}{E^2\lambda^8}\left[\frac{C_3}{C_1}(1-\nu^2)p_b\right]^3+p_b, \tag{7}$$

$$p=\frac{C_1}{C_3}\frac{\sqrt[3]{E^2\lambda^8 p_m}}{1-\nu^2}+p_m \tag{8}$$

The relation between the load p and the deflections δ is

$$\frac{p}{E}=\frac{\lambda}{C_1^3}\left(\frac{\delta}{a}\right)^3+\frac{\lambda^3}{C_3(1-\nu^2)}\left(\frac{\delta}{a}\right) \tag{9}$$

If we obtain the values p_m and p_b from the given load p by the Equations (7) and (8), the membrane stress σ_m and the bending stress σ_b at the center of a glass pane can be computed by the formulae (2) and (4). The theoretical values of the constants in the formulae (1) and (2) of the rectangular membrane fixed on all edges and those in the formulae (3) and (4) of the rectangular plate supported on all edges are shown in Table 1. These theoretical values should be corrected for practical use.

Actually the boundary conditions above are not fulfilled for the glass pane because the pane edges are usually neither simply

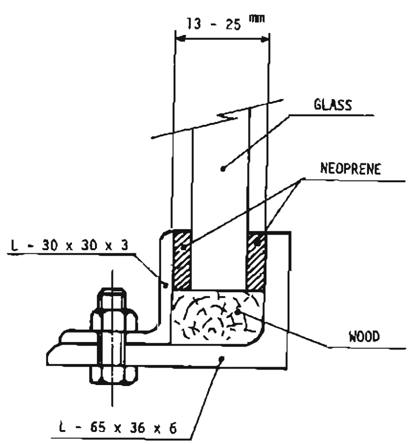

Fig. 1. Edge support of the experimented glass.

supported nor completely clamped, and sometimes the glass edges slide a little inward at the support. We performed some experiments on flat glasses to investigate the actual behaviors under uniform loads.

3. Experiments on pane deflections

Rectangular flat glasses of various sizes supported at four edges were set on a equipment of the air chamber and static uniform loads were applied by the air pressure until the panes were broken. Edge support of the tested panes is shown in Fig. 1.

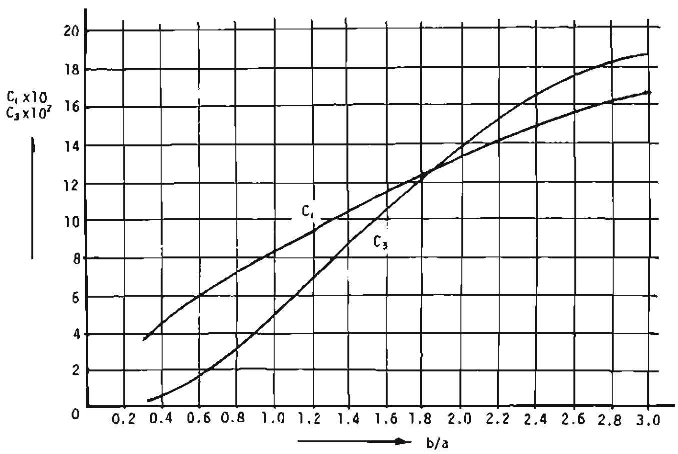

Fig. 2. Values of the constants C_1, C_3.

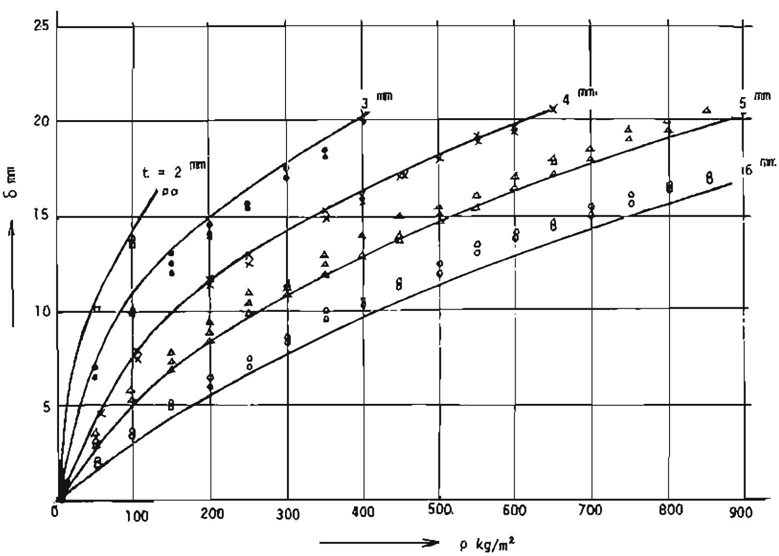

Fig. 3. Comparison of computed deflections with test results; square sheet glass (t=2, 3, 4 mm) and plate glass (t=5, 6 mm), size 1000×1000 mm.

The loading speed was about 0.5 kg/m^2/sec. The deflections at the center of glasses, the slip at the mid-point of two neighboring edges and the strains at some points of the both faces of glasses were measured.

The first experiment was conducted on square panes of two sizes. One was the plate glass of 2000 mm on a side having thicknesses of 4, 5, 8, 10 and 15 mm. The other was the plate glass of 1000 mm on a side having thicknesses of 5 and 6 mm and the sheet glass of 2, 3, and 4 mm. The second experiment was made on rectangular panes of the sizes 1000×1100, 1000×1200, 1000×1300, 1000×1500, 1000×2000 and 1000×2500 mm and they had the thickness of 5 or 6 mm. This experiment was performed in order to learn the effects of aspect ratio on deflections and strains of glasses.

4. Estimation of the constants in the formulae

According to the series of the experiments mentioned above, the values of the

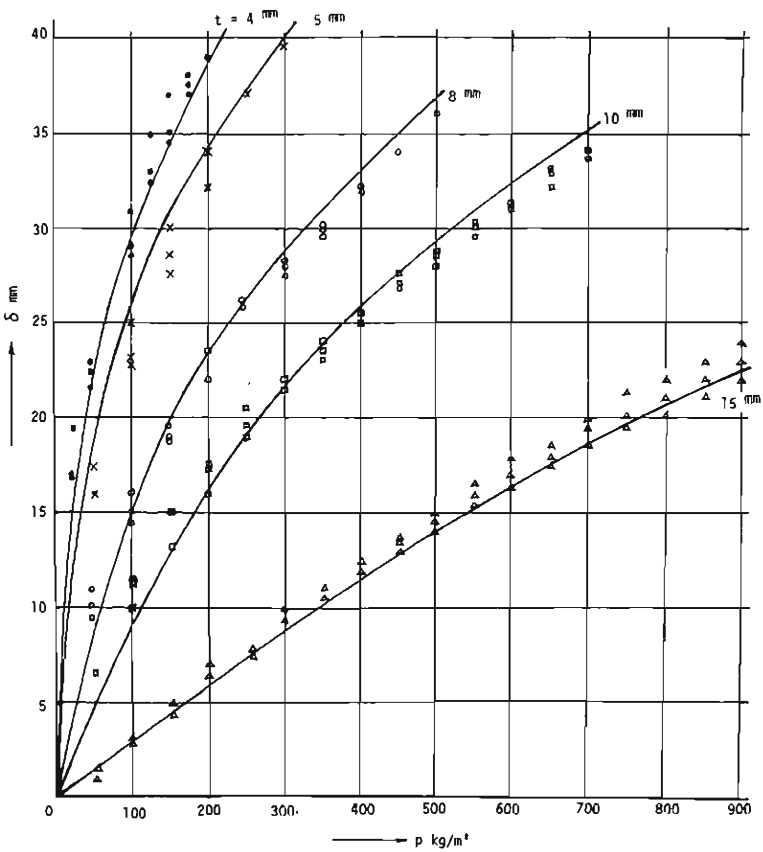

Fig. 4. Comparison of computed deflections with test results; square sheet glass ($t=4$ mm) and plate glass ($t=5$, 8, 10, 15 mm), size 2000×2000 mm.

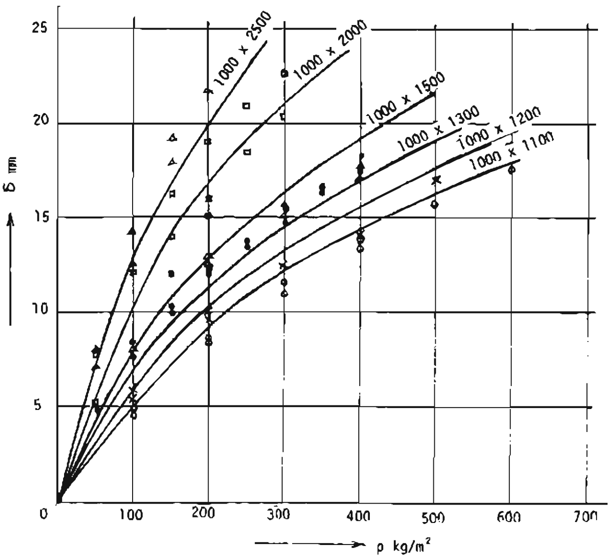

Fig. 5. Comparison of computed deflections with test results; rectangular plate glass (t=5 mm), size 1000×1100, 1000×1200, 1000×1300, 1000×1500, 1000×2000, 1000×2500 mm.

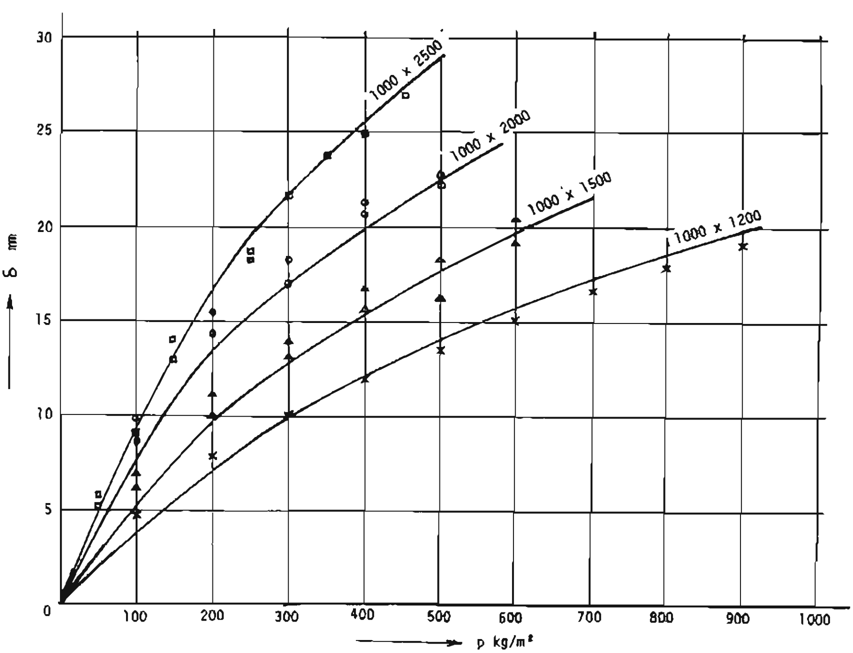

Fig. 6. Comparison of computed deflections with test results; rectangular float glass (t=6 mm), size 1000×1200, 1000×1500, 1000×2000, 1000×2500 mm.

constants C_1 and C_3 in the formulae (1) and (3) were determined in such a way that the deflections computed by equation (9) agree with the measured values. Young's modulus was assumed to be $E=7.5\times10^5$ kg/cm^2, Poisson's ratio $\nu=0.25$. Thus the obtained values of C_1, C_3 are shown in Fig. 2. They are a little larger than the theoretical values.

Some examples of the comparison between the computed deflections and the experimental results were shown in Figs. 3, 4, 5 and 6. The theoretical deflections are shown by the curves in the figures. They are in satisfactory agreement with the experimental results. The values of C_1, C_3 in Fig. 2 can be further modified to obtain a better agreement if we perform the experiments on various other glasses. The stresses or strains should be computed by the formulae (2) and (4), but to obtain good results for these seems to be more difficult than for the deflection.

5. Conclusion

The author proposed a direct method to compute the large deflections of glass panes under uniform loads such as wind pressure. This will be useful for practical design of glass panes, and is a step to know the complicated behaviour of flat glass under wind action.

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