Observations of Vibrational Strains in Earthquakes, and Relations of the Maximum Amplitudes to the Seismic Magnitudes and the Epicentral Distances

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Abstract

Observations of vibrational strains in earthquakes by means of strain seismographs have been going on at Otsu Observatory and Kishu Mine since 1966.

The relations of the maximum amplitudes e_{max} of these vibrational strains to their seismic magnitudes M and their focal distances Δ have been formulated.

Assuming the azimuths of the epicenters and of the nodal lines are distributed uniformly, a compensating factor for the constant term has been obtained as -0.20.

According to the relations obtained above of e_{max} to M and Δ, the coefficients with respect to attenuations of e_{max} for Δ are almost equal to *1* except for a few events of the near earthquakes.

1. Introduction

Observations of the vibrational strains were started in *1966*. Those in the directions of *38*°W, S 52°E and the vertical have been operated by means of strain seismographs at Otsu Observatory. While that in the meridional direction at Kishu Mine has been done also.

The relations of the maximum amplitudes of these observed vibrational strains at Otsu and Kishu to their seismic magnitudes and their epicentral distances are formulated in this paper.

The relations of the maximum amplitudes of their seismic displacements A to their seismic magnitude M and their epicentral distances Δ have been studied by K. Wadati, C. F. Richter, C. Tsuboi, H. Kawasumi, and many other researchers. For example, C. Tsuboi[1] has explained studies of the relation of M to A and Δ. K. Wadati[2] has concluded that the amplitude attenuation for the epicentral distance is inversely proportional to the magnitude. H. Kawasumi[3] has obtained another type of formulae for these measures as follows,

$$M = \log A + 0.5 \log \Delta + 0.00133\Delta \qquad \Delta \leq \Delta \quad 750 \text{ km},$$

$$\text{and } M = \log A + 0.5 \log \Delta + 0.000795\Delta \qquad \Delta \geq \Delta \quad 750 \text{ km}.$$

where A and Δ are expressed in microns and kilometers, respectively. C. J. Wideman and N. W. Major[4] and S. Takemoto and M. Takada[5] have studied the attenuations of the amplitudes of the strain steps in earthquakes with regard to their epicentral distances.

In general, the seismic magnitude M is given as the following formula

$$M=\log A+\alpha \log \Delta+\beta,$$

where α and β are constants, and A and Δ are expressed in microns and in km, respectively.

Most of the values α have been in the range from 1.5 to 2.0.

If the maximum strain and the maximum displacement are in same phase, and their wave length is long, and their attenuation factor is not so small, the coefficient of the amplitude attenuation α' of the maximum strain should be $\alpha+1$ near the hypocentre. For example, if it be $\alpha'+1.7$, α' will be 2.7. But, α' is equal to α at great distance from the hypocentre.

According to our study in this paper, the coefficients α' are given as about 1.

This result is very important in the studies of seismic strain energy in the crust, as well as earthquake mechanisms, and the detection of neuclear explosions.

2. Observations

These observations of the seismic strains have been performed in the galleries of the Otsu Observatory and that of the Kishu Mine. The gallreie of the Otsu Observatory is located in 134° 59′ of the east longitude, and 34° 59′ of the north latitude. The observing room is about 80 m under the ground. That of Kishu Mine is located in 135° 53′, east longitude and 33° 52′, north latitude. The observing room is about 100 m under the ground.

Strain seismographs have been improved H-59 type and V-59 type extensometers[6),7)] which had been devised by this author (I. Ozawa) in order to make observations of the vibrational strains in earthquakes. The H-59 type is used for the observation of the horizontal component of the ground strains, and the V-59 type is employed for that of the vertical component. These instruments consist of the standerd scale made of the superinvar rod or pipe, amplifier of the relative displacement between the free end of the scale and the definite point on the ground and photographic recorder. The speed of the curve-recordings are 2 mm per minute and 16 mm per hour.

The vibrational parts of the instruments are equipped with dampers of oil or magnets in order to make their damping constants ($Q^{-1}=\epsilon/n$) be from 0.3 to 0.7. Their main vibrational parts are the standard scale in the vertical component and the pendulum of the amplifier. The standard scale in the vertical component (V-59 type) is put vertically in the well filled with water and machine oil, and the diameter of the well is about 20 cm. And so, the period of the free oscillation of the oil and water in the well is very short, and the motion of the scale is in the heavy damping vibration. The standard scale in the horizontal component is supported with many rollers which are placed at short intervals, and so its main free period is far shorter than the period of the main oscillation in the earthquakes.

The second problem of the strain seismograph is the relation between the natural periods of the seismograph and those of the main phases of the ground strains in the earthquakes. The natural periods of the strain seismographs are from *6* seconds to ten and some seconds, and most of those of the main phases of the observed seismic strains are longer than ten and several seconds. There is a problem with the estimation of the maximum amplitude of the small earthquake whose hypocentral distance is short.

The third problem is that the amplifier of the strain seismograph records not only the ground strains but also its vibrational displacement. However, the magnification of the instrument for the vibrational displacement is very much less than that for the strain. The magnification of these instrument for the strain are from 10^{-9} to 10^{-10}, that is from 10^5 to 10^6 for the relative displacement. But, the magnification for the vibrational displacement is less than *80* or *160*.

Table 1 shows the real values on the record of the maximum amplitude observed with the strain seismograph in the earthquakes, and the reported maximum amplitudes of the displacement in the earthquake of Yunnan Province are measured as $54\,\mu$ in the direction of the prime vertical (*EW*), and $65\,\mu$ in the meridional direction (*NS*); the maximum horizontal amplitude is $85\,\mu$. And so, the effect of the maximum displacement in the record of the strain seismograph is smaller than 85×80 (magnification). As the maximum amplituds in the strain seismograph in this earthquake are *51.4 mm* in the direction of *S 38°W*, *115.0 mm* in that of *S 52°E*, and *32.0 mm* in the vertical, the effects of the displacement can be neglected. Table 2 shows the constants of the instruments which are used for this study.

Table 1. The Maximum Amplitudes Observed with Strain Seismographs, and the Maximum Displacements Measured with Displacement Type Seismographs.

Date of Earthquake	Location of Earthquake	Epicentral Distance	Maximum Amplitude on Strain Seismograph L_3	C_1	V_5	Displacement U-D	E-W	N-S
		km	mm	mm	mm	μ, (sec)	μ, (sec)	μ, (sec)
1970: 1: 5	China Yunnan Province	3 419	51.4	115.0	32.0	68 (11.0)	54 (13.5)	65 (13.0)
1970: 1:21	Spart of Hokkaido	1 046	52.0	91.6	27.0	97 (11.9)	83 (11.0)	65 (13.0)
1970: 5:27	Bonin Island H 440 km	971	88.0	79.0		156 (6.0)	109 (6.0)	111 (2.6)
1971: 7:14	New Ireland	4 823	56.8			Osaka 33 (3.8)	79 (4.2)	120 (4.2)
1971: 9: 6	off SW Coast of Sakhalin	1 378	33.8			15 (5.8)	19 (12.7)	16 (10.0)
1972: 1: 4	Taiwan Region	1 909	22.6		8.6	19 (15.0)	17 (14.8)	25 (4.0)

Table 2. Constants of the Instruments.

Sighn of Instrument	Type of Strain Seismograph	Direction of Observation	Length of Base Line	Record of Speed	Sensitivity in 10^{-9}/mm	Damper and Constant	Place
L_3	H-59-B	S38°W	22 m	2 mm/min.	0.46–2.85	magnetic 0.4–0.5	Otsu
C_1	H-59-B	S52°E	10	16 mm/hr.	2.72–3.12	oil 0.3–0.6	Otsu
V_5	V-59-D	Vertical	6	2 mm/min.	5.3–10.1	oil 0.4–0.6	Otsu
B	H-59-C	N	4.9	1.67 mm/hr.	7.7–9.4	oil 0.3–0.4	Kishu

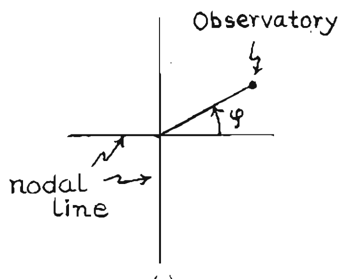
The recording speeds of L_3 and V_5 are *2 mm* per minute, and so we can measure the arrival times, the amplitudes and the periods of P, S and the surface wave in most earthquakes, except near earthquakes on the records.

That of C_1 is *16 mm* per hour, and we can discriminate the phases of p, s and the surface waves in distant earthquakes. The maximum amplitudes of these vibrational strains in all earthquakes are able to be measured in all these records.

Table 3 shows the recordings of the maximum amplitudes of the strains observed with these strain seismographs, and the dates, locations and magnitudes of the earthquakes. These dates, locations and magnitudes are quoted from the Seismological Bulletin of the Japan Meteorological Agency. Some of the readings of the maximum amplitudes in Table 3 have been added after the following analyses, and they have not been used for the calculations.

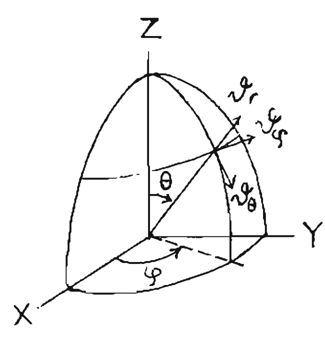
3. Theory and Analyses.

The azimuthal distributions of the amplitude of most of earthquakes whose mechanism are the quadrant type, have been given[8] as being in proportion to



(a)

Fig. 1. (a) The coordinate of the displacement.



(b)

Fig. 1. (b) The coordinate of the displacements.

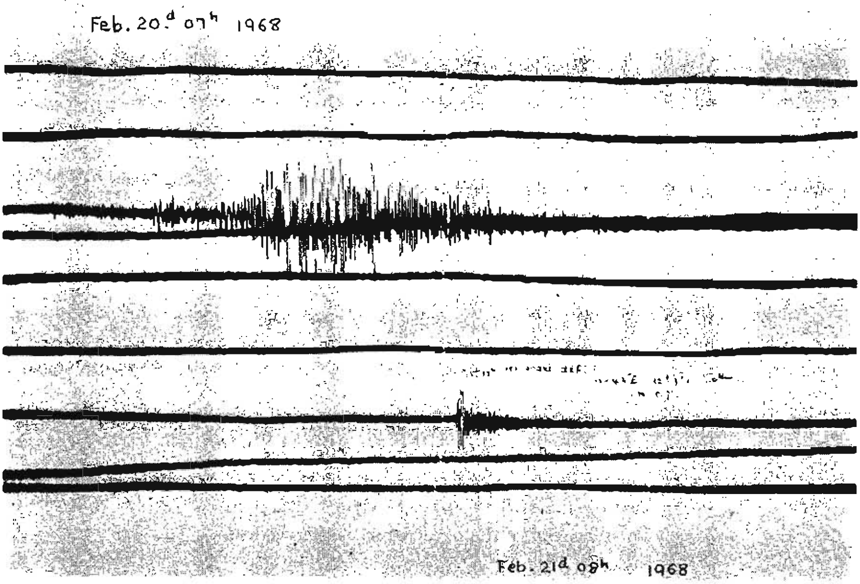
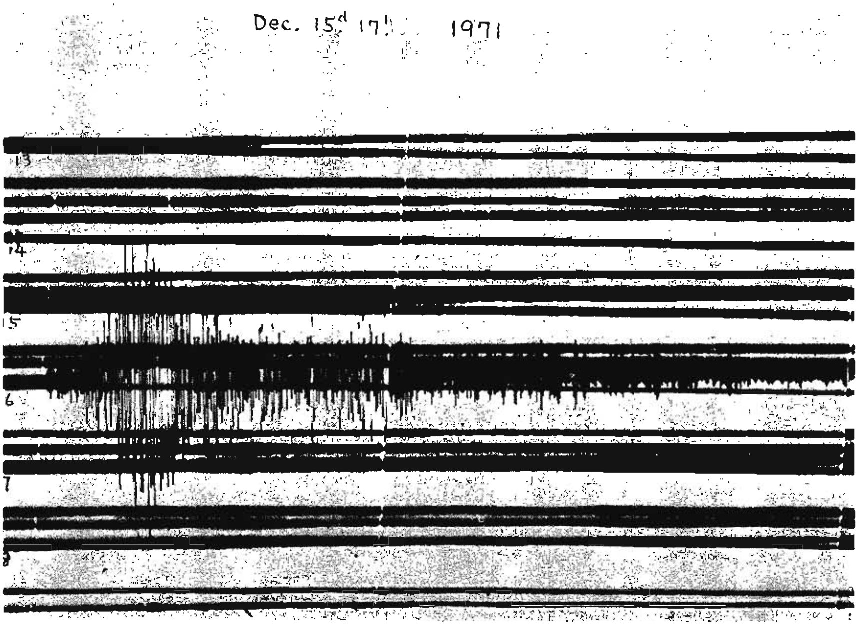


Photo. 1, (a) The records of the extensional strain at Otsu Observatory Upper is the record of the Earthquake at the Aegean Sea, February 20th 07hr., 1968 $\left(M=7\frac{1}{4}-7\frac{1}{2}\right)$. Lower is that of the Miyazaki Prefecture. Feburuary *21*th *8*hr., *1968* ($M=5.7$). The direction of the observation is *S38°W*.



Photo, 1, (b) The record of the extensional strain observed at Otsu in the earthquake at the Near east coast of Kamchatka, December 15th *17*hr., *1971* ($M=7.3$). The direction of the observation is *S38°W*.

Photo. *1*, (c) The record of the extensional strain at Otsu Observatory in the Earthquake of the Celebes Sea June *12*th *01*hr., *1972* ($M=5.8$). The direction of the observation is the vertical.

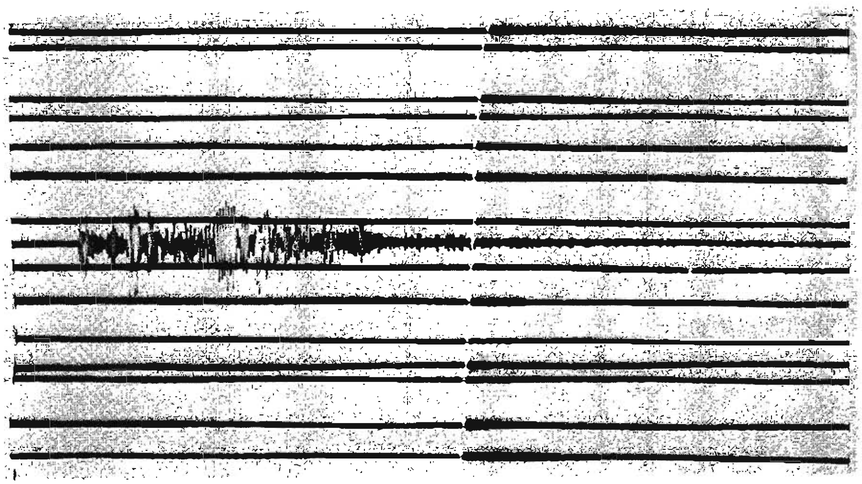


Photo. *1*, (d) The record of the extensional strain at Otsu Observatory in the Earthquake of the Philippine Is. January *08*th *14*hr., *1972* ($M=6.3$). The direction of the observation is *S38°W*.

Table 3. The example of the maximum amplitudes of the vibrational extensions in earthquakes observed with the strain seismographs, and the constants of the earthquakes. ($M \geq 6.0$)

Date of earthquake	Location: Longitude	Location: Latitude	Location: Depth	Magnitude	Maximum Amplitude: Otsu S38°W	Otsu S52°E	Otsu Vertical	Kishu N
h m	°	°	km		10^{-9}	10^{-9}	10^{-9}	10^{-9}
1966 Mar. 22 17 19	115.1 E	37.5 N	33	6.85	101			
Apr. 23 09 09	122.4 E	−0.9 S		6.85	9.6			
Jun. 07 23 03	139.6	11.3	50	6.85	13.8			
14 03 17	167.1	−12.2	259	6.2	1.74			
15 10 08	160.8	−10.4	31	7.5	31.2			
23 05 35	124.6	−7.2	507	6.1	3.44			
Jul. 01 14 51	122° 20′	24° 40′	60	6.1	4.83			
Aug. 19 21 22	41.7	39.2	26	7.0	9.84			
Sep. 09 06 15	128.4	2.4	96	6.85	4.59			
Oct. 27 23 24	145.9	22.2	29	6.0	2.57			
1967 Jan. 01 07 15	164 48	−11 18	33	7.3		66.0		
05 09 14	102 48	48 06	33	6.4	85.0	371.5		
17 20 59	142 05	38 15	30	6.3	14.7			
18 14 34	120 48	56 48	11	6.1	7.6			
19 21 40	178 48	14 48	37	6.8	1.84			
20 10 57	102 54	48 00	33	6.1	8.62			
Feb. 14 10 04	96 30	13 42	27	6.8		81.8		
10 10 00	74 54	02 54	58	7.35	1.1			
Mar. 05 02 58	24.6	39.2	33	6.75		14.3		
19 13 02	151 25	44 45	60	6.3		41.6		
27 19 01	168.1	−16.5	10	6.0		1.74		
Apr. 12 13 58	96 30	5 18	55	6.1		24.6		
May 22 03 52	101 30	01 00	173	6.3	1.38	12.5		
Jul. 23 02 07	30 48	40 42	4	7.3		103.3	214	
02 08 10	158 00	54 24	33	6.3		7.7		
Aug. 22 22 02	−22 02	−60 48	33	6	0.95			
26 09 40	140 42	12 12	33	6.1	20.6	41.6		
30 13 22	100 18	31 42	3	6.1	8.17	67.5	32.5	
Oct. 05 02 21	153 54	−5 42	52	7.3		37.0		

Date of earthquake			Location					Magnitude	Maximum Amplitude			
			Longitude		Latitude		Depth		Otsu			Kishu
									S38°W	S52°E	Vertical	N
		h m	°	′	°	′	km		10^{-9}	10^{-9}	10^{-9}	10^{-8}
	Oct.	25 10 01	122	12	24	30	38	6.0	43.3	145.0	120.0	
	Nov.	04 23 30	144	16	43	29	20	6.5	61.1	85.0	56.6	
		19 05 42	141	13	36	26	50	6.0	11.61	42.1		
		30 16 23	20	30	41	30		6.7	4.17			
	Dec.	01 22 59	154	59	47	14	120	6.4	14.3			
		11 07 51	73	54	17	42	33	6.5	5.24	13.5		
		21 11 25	−70	00	−21	48	33	7.25	6.1	16.2		
		25 10 23	153	42	−5	18	64	7.0		155.0		
	Sep.	28 13 56	153	24	−6	36	44	6.3	1.19			
1968	Jan.	19 15 13	158	24	−9	24	33	6.0	9.3	28.3	14.3	
		29 19 19	147	00	43	11	30	6.9	181	136		313
	Feb.	12 14 52	153	12	−5	30	74	7.5	32			
		21 10 44	130	43	32	01	00	6.1		40.8	61.6	
	Apr.	01 09 49	132	32	32	17	30	7.5			55.0	870
		01 16 13	132	23	32	18	00	6.3		30.4	109.0	1130
	May	16 19 39	142	51	41	25	40	7.5	24.2			1090
		17 08 05	143	29	39	46	30	6.7	65.8	51.3		
		21 06 10	149	36	44	04	10	6.2	22.3	57.9	51.7	
		23 04 29	142	34	40	15	30	6.3		49.0	23.1	183
		24 23 06	143	27	40	47	40	6.2		68.8	37.4	
		28 22 33	139	18	−02	54	65	6.1	22.8	23.4		191
	Jun.	07 21 03	120	06	−01	48	20	7.0	6.6	234	12.6	20
		12 22 42	143	08	39	25	00	7.2	196	435		870
		17 20 53	143	22	40	56	10	6.4	25.0	86.5	63.8	325
		18 03 57	144	13	38	38	40	6.0	25.2	52.5	58.3	116
	Jul.	01 19 45	139	26	35	59	50	6.1	39.8	40.8	80.3	385
		05 20 28	142	13	38	26	50	6.4			240.0	474
		12 09 44	143	29	39	34	40	6.4	30.0	95.2		48.4
	Aug.	02 05 21	122	16	16	30	36	7.3	55.2			543
		06 01 17	132	23	33	18	40	6.6				947
		10 11 12	126	12	01	24	33	7.6	57.0			230

Date of earthquake	Location			Magnitude	Maximum Amplitude			
					Otsu			Kishu
	Longitude	Latitude	Depth		S38°W	S52°E	Vertical	N
h m	° ′	° ′	km		10^{-9}	10^{-9}	10^{-9}	10^{-9}
Aug. 15 07 19	119 48	00 12	23	7.4	41.4	128.0		151
19 03 46	159 54	−10 06	538	6.2	39.8	64.7		199
31 19 57	59 00	34 00	13	7.3				138
Sep. 27 13 58	129 06	−06 48	127	6.1		16.9		
Oct. 08 05 49	142 43	41 49	60	6.2	24.2	92.5	27.4	266
24 06 11	143 18	−03 18	12	6.8	85.5	88.0	73.1	124
30 08 25	150 5	66 0		6.2	6.3			
Nov. 11 23 41	143 25	40 07	30	6.3	31.4	73.5	31.9	138
14 03 41	142 47	40 07	30	6.0	14.8	27.2		143
25 06 20	142 34	40 16	50	6.0		28.6		
Dec. 07 14 04	145 54	−3 24	15	6.5	15.1	21.8		
1969 Jan. 05 22 34	158 54	−08 00	47	7.1	17.9	67.4		60
07 00 47	154 30	−10 30	32	6.8	13.7	59.8		
20 04 00	167 12	−14 54	112	6.2	14.5	19.7		
30 19 39	127 24	4 54	70	5.9	117			
Feb. 04 06 46	127 24	4 54	33	6.3	180	41.3		131.6
11 08 07	178 36	−22 42	673	6.0	55.4	39.0	39.0	
12 07 21	127 24	4 54	33	6.4	32.3	51.5	29.2	122.2
23 09 42	118 54	−3 06	13	6.9	27.1			84.6
28 11 53	−10 36	36 00	22	8.0	65.6			94
Jul. 18 14 26	119 24	33 18	33	7.3	316	524		1128
Aug. 05 02 25	125 18	−5 42	521	6.2		10.6		
05 11 18	126 12	1 18	34	7.0	22.8	44.6		94
12 06 28	147 37	42 42	30	7.8	713		181	1110
Sep. 04 01 20	140 30	30 43	60	6.2		132.0	18.7	
09 14 15	137 04	35 47	00	6.6	616	499	580	1110
Nov. 21 11 12	94 36	2 06	20	7.7	137	192	44	150
23 08 37	163 30	57 48	33	7.3	259	384		395
1970 Jan. 01 04 02	129 13	28 24	50	6.1	45.3		89.1	189
05 02 00	102 30	24 06	31	7.5	107.0	327	176	280

Date of earthquake	Location			Magnitude	Maximum Amplitude			
	Longitude	Latitude	Depth		Otsu			Kishu
					S38°W	S52°E	Vertical	N
h m	° ′	° ′	km		10^{-9}	10^{-9}	10^{-9}	10^{-9}
10 21 07	126 42	6 48	73	6.1	42.2	222	106	
20 16 31	−177 18	−25 48	80	6.5	18.3	91	40.1	111
21 02 33	143 08	42 23	50	6.7	108.0	260	148	575
Feb. 06 07 08	122 06	12 36	11	6.6		37.0	39.6	103
28 19 58	−175 06	52 42	162	6.1	28.7	22.4	37.4	163
Apr. 07 14 36	121 42	15 48	37	7.3		213		643
12 13 03	122 06	15 06	24	7.0	71.3	85	117	381
15 22 16	122 42	15 06	12	6.0	16.2	30.2	36.8	94.5
May 16 02 19	91 18	50 12	33	6.7	17.5	68.7	30.8	97.8
28 04 05	143 15	40 09	30	6.2	52.0	136		163
28 07 36	143 13	40 12	30	6.0		68.3		137
Jun. 01 05 42	−78 48	−9 12	43	7.8	16.6	56.3		38.6
05 14 00	78 48	42 30	22	6.6		45.5	25.8	240
24 22 18	−131 00	51 48	12	7.0		37.8	30.8	
Jul. 26 07 41	132 02	32 04	10	6.7	63.5	79.6	176.0	
26 16 10	132 06	32 07	10	6.1	87.7	199	143.0	
28 10 37	124 12	−8 42	41	6.2		13.6		
29 19 22	95 24	26 00	59	6.5	13.7	20.7	26.4	214
Aug. 01 02 25	−72 36	−1 30	651	7.1	18.3		29.7	77
11 19 31	166 42	−14 06	33	7.0		26.4	12.1	
26 09 40	140 42	12 12	33	6.1			26.4	
Sep. 01 14 15	146 36	17 42	42	6.4	22.4	39.2	55.0	167
14 18 45	142 20	38 41	40	6.2	52.4	56.9		214
16 10 53	144 24	13 00	47	6.0				98.7
Oct. 16 14 26	140 45	39 12	00	6.2		15.6	29.1	65.2
Nov. 01 03 00	145 30	−4 54	42	7.0	19.8	80.5	60.5	197
09 07 41	135 36	−3 24	33	6.8	14.2	62.5		103
14 16 58	121 18	22 42	28	6.1	16.6	23.3	23.1	41.2
Dec. 07 05 21	143 46	41 40	50	6.1		93.7	53.8	258
10 13 52	−80 42	−4 00	25	7.6		29.0	22.0	
29 05 11	153 36	−5 12	61	6.0		20.4		

Date of earthquake		Location			Magnitude	Maximum Amplitude			
		Longitude	Latitude	Depth		Otsu			Kishu
						S38°W	S52°E	Vertical	N
	h m	° ′	° ′	km		10^{-9}	10^{-9}	10^{-9}	10^{-9}
Dec.	29 11 34	161 24	−10 30	72	6.1		14.8		
1971 Jan.	05 06 09	137 10	34 26	40	6.1			61.3	384
	10 16 23	139 42	−3 06	38	8.0	78.2			
Feb.	05 00 40	98 48	00 36	33	7.1	7.8		25.0	
	07 11 30	−176 42	51 24	50	6.0	6.2		22.9	
Apr.	05 03 39	142 12	38 21	50	6.0	13.5			
	07 14 04	129 06	02 24	47	6.6	17.2			73
	29 00 36	101 00	22 54	15	6.3				123
May	02 15 13	−177 12	51 24	43	7.1	6.6		33.4	
	19 07 48	146 06	63 54	33	6.6	9.4		56.3	53.7
	23 01 55	40 30	38 48	3	6.7	3.0		36.9	
Jun.	11 23 04	176 06	51 30	32	6.5	7.1			
	18 16 20	−69 12	−25 30	93	6.3	4.9			
Jul.	09 04 13	129 36	−6 54	33	6.3	10.8			
	14 15 19	153 54	−5 30	47	7.9	106.1			56.4
	26 00 45	173 06	52 12	28	6.3	9.0			
	26 10 30	153 12	−4 54	48	7.9	51.9			
Aug.	20 07 17	155 24	49 18	33	6.1	7.9			
Sep.	06 03 35	141 23	46 40	00	6.9	63.2			384
	15 23 55	143 52	39 05	50	6.3	72.9			
	24 10 10	143 37	39 19	40	6.1	25.4			
	25 13 43	146 36	−6 30	115	6.3	7.9			184
Oct.	24 07 38	142 30	11 48	28	6.3	7.1			
	24 10 42	142 18	11 48	33	6.4	8.6			
	28 03 08	167 12	−15 30	40	7.1	9.9			
	29 00 21	153 12	−4 54	48	7.9	52.0			
Nov.	21 14 57	166 30	−11 48	115	6.4	12.0			
	25 04 38	159 12	52 54	106	6.3	34.2			361
1972 Jan.	04 12 17	122 06	22 36	33	6.9	31.4		45.9	92.2
	08 14 29	120 12	20 54	33	6.5	16.9		28.3	84.5
	19 07 02	145 00	−4 48	33	6.6	12.0			

Date of earthquake	Location			Magnitude	Maximum Amplitude			
	Longitude	Latitude	Depth		Otsu			Kishu
					S38°W	S52°E	Vertical	N
h m	° ′	° ′	km		10^{-9}	10^{-9}	10^{-9}	10^{-9}
Jan. 20 00 07	145 00	−4 42	33	6.4	6.2			
24 06 27	166 24	−13 12	33	7.1	7.3		9.7	
25 11 06	122 18	22 30	33	7.5	129.9		212	400
25 12 41	122 12	23 00	33	7.0	58.0		68.3	163
Feb. 15 08 38	166 18	−11 24	102	6.2	14.4			69
29 18 23	141 16	33 11	70	7.0	280.2			407
Mar. 22 19 27	153 36	49 06	134	6.3				76.8
26 07 59	146 13	43 03	50	6.1	11.8			
30 14 45	179 24	−25 42	532	6.2	34.8			58.4
Apr. 24 18 57	121 36	23 36	33	6.9		484		146
26 04 33	120 18	13 24	50	7.2		318		182
29 18 39	154 12	−5 06	409	6.0				69.2
May 04 16 57	167 30	−15 54	45	6.8	5.2	26.5		
Jul. 31 06 54	−135 41	56 49	25	7.6	54.2	184		

$$\begin{aligned} &\sin 2\phi \quad \ldots\ldots\ldots\ldots\ldots \textit{P-phase}, \\ &\cos 2\phi \quad \ldots\ldots\ldots\ldots\ldots \textit{S-phase}, \\ &\sin 2\phi \quad \ldots\ldots\ldots\ldots\ldots \textit{surface-wave}, \end{aligned} \qquad (1)$$

where ϕ is the azimuth from the nodal line to the observing point, Fig. 1 (a).

T. Matuzawa[9] has given the components of the ground displacement (ϑ_r, ϑ_θ, ϑ_φ), Fig. 1 (b), at an observatory in the earthquake caused by the double couple forces or double dipole forces at the origin $(0, 0, 0)$ as follows,

$$\begin{aligned} {}_p\vartheta_r &= -iA\frac{h}{\lambda+2\mu}\frac{e^{i(pt-hr)}}{r}\sin^2\theta\sin 2\varphi, \qquad {}_p\vartheta_\theta={}_p\vartheta_\varphi=0, \\ {}_s\vartheta_\theta &= -iA\frac{k}{\mu}\frac{e^{i(pt-kr)}}{r}\sin\theta\cos\theta\sin 2\varphi, \\ {}_s\vartheta_\varphi &= -iA\frac{k}{\mu}\frac{e^{i(pt-kr)}}{r}\sin\theta\cos 2\varphi, \qquad {}_s\vartheta_r=0, \\ h^2 &= \frac{\rho p^2}{\lambda+2\mu}, \qquad k^2=\frac{\rho p^2}{\mu}, \end{aligned} \qquad (2)$$

where the suffixes p and s of Z show the displacement components of P and S waves, respectively, and where ρ, p, λ, μ and A are the density of the ground, the frequency of the wave, the Lamé's two elastic constants as well as the constant related to the intensity of the force, respectively. From formulae (2), we have the strain components of the S-wave as follows

$$
\begin{aligned}
{}_se_{\theta\theta} &= iA\frac{k}{\mu}\frac{e^{i(pt-kr)}}{r^2}\sin 2\theta \sin 2\varphi, \\
{}_se_{\phi\phi} &= iA\frac{k}{\mu}\frac{e^{i(pt-kr)}}{r^2}(1+\sin^2\theta)\sin 2\varphi, \\
{}_se_{\theta\varphi} &= -2iA\frac{k}{\mu}\frac{e^{i(pt-kr)}}{r^2}\cos\theta\cos 2\varphi.
\end{aligned}
\tag{3}
$$

Then, the extensional strain $\varepsilon\ (l, m)$ in the direction whose direction cosines are l for θ, and m for ϕ, is calculated as

$$\varepsilon(l, m) = iA\frac{k}{\mu}\frac{e^{i(pt-kr)}}{r^2}[\{-l^2\sin 2\theta + m^2(1+\sin^2\theta)\}\sin 2\varphi - 2lm\cos\theta\cos 2\varphi]. \tag{4}$$

As it might cause $\theta = \frac{\pi}{2}$ in the case of a shallow earthquake, we have (5)

where, α is the azimuth from the nodal line, and $m=\cos\alpha$. If the values of α and ϕ are distributed uniformly in our data, the mean value of this distribution of $\varepsilon\ (l, m)$ is calculated as,

$$
\begin{aligned}
& iA\frac{k}{\mu}\frac{e^{i(pt-kr)}}{r^2}\frac{2}{\pi}\int_0^{\pi/2}\sin 2\varphi\left\{\frac{1}{\pi}\int_0^{\pi}\cos^2\alpha\, d\alpha\right\}d\varphi \\
& \qquad = \left\{iA\frac{k}{\mu}\frac{e^{i(pt-kr)}}{r^2}\right\}\frac{2}{\pi}.
\end{aligned}
\tag{6}
$$

namely, the statistical mean is equal to $2/\pi$ times extensional strain along the nodal line, and the same distance from the origin.

Using many of our observed values of the maximum amplitude of the vibrational strain, the relation of the maximum amplitude e_{max} to the seismic magnitude M and the epicentral distance Δ is formulated by means of the method of the least square as

$$\log e_{max} = M - \alpha' \log \Delta - \beta'. \tag{7}$$

According to the formula (6), the value of β' is calculated as $2/\pi$ times the maximum strain along the nodal lines in the statistical case of the ideal distribution of the seismic focus and their nodal lines. And, the values of β' should be compensated by -0.20, which is $-\log \pi/2$.

The relations shown as (7) are calculated for every observation of the components L_3, V_5 and C_1 at Otsu, and that of B at Kishu, and are shown in Table 4.

At first analysis, all of these observations of each component are directly used for the calculations of the coefficients, α' and β' in (7). The largest value of α' is 1.37 ± 0.07, and the other values are nearly equal to *1* in these calculations. At second analysis, the observations, whose epicentral distances are shorter than *500 km* are picked up, and their α' and β' are calculated. This value of α' for L_3 is *2,19.* but the other values are slightly less than *1*. Fig. 2 (a), (b), (c) and (d) show the relations of the maximum

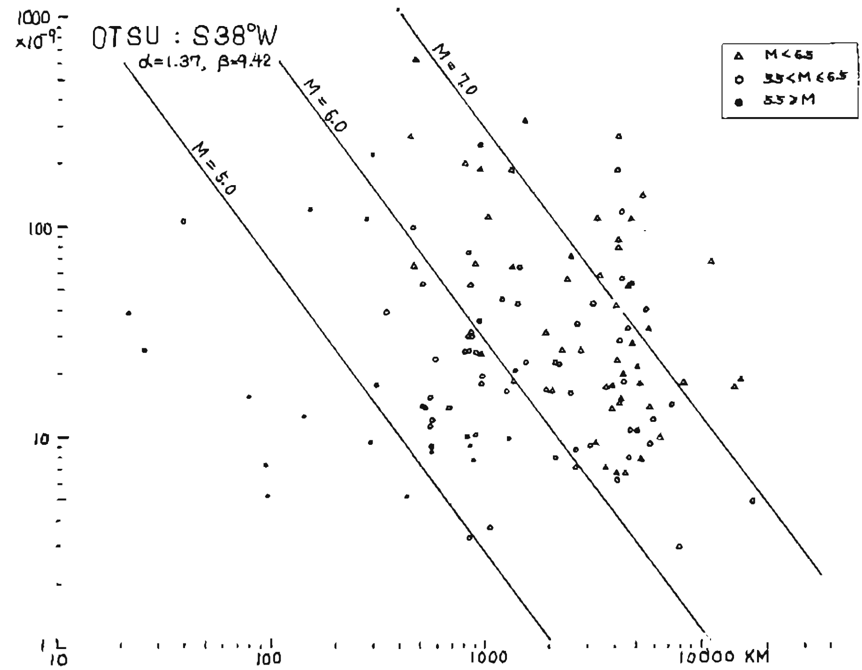


Fig. 2. (a) The relation between the maximum amplitude of the vibrational extensions in the azimuth of *S 38°W*- at Otsu Observatory versus the seismic magnitude and the epicentral distance.

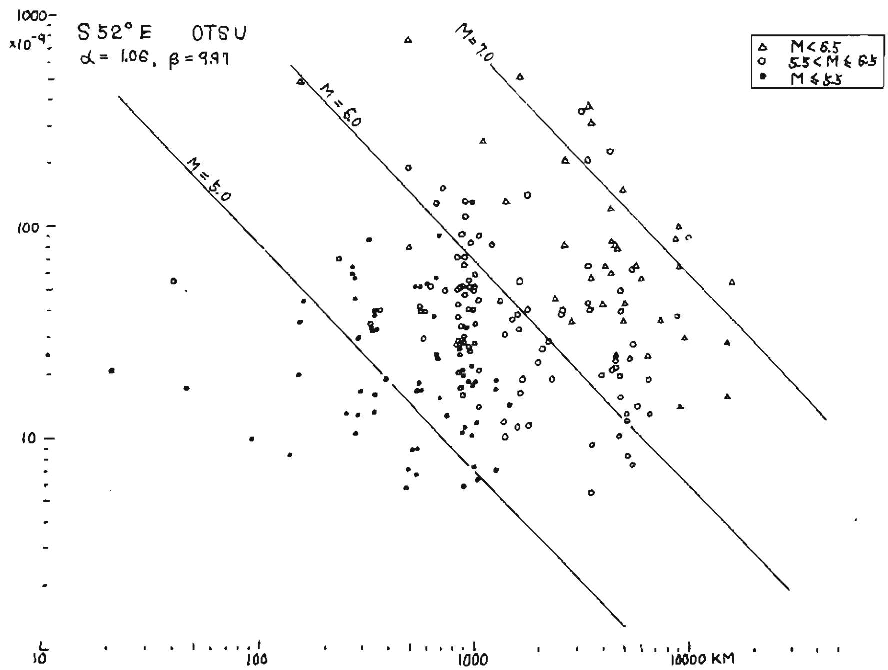


Fig. 2. (b) The relation between the maximum amplitude of the vibrational extensions in the azimuth of the *S52°E*- at Otsu Observatory versus the seismic magnitude and the epicentral distance.

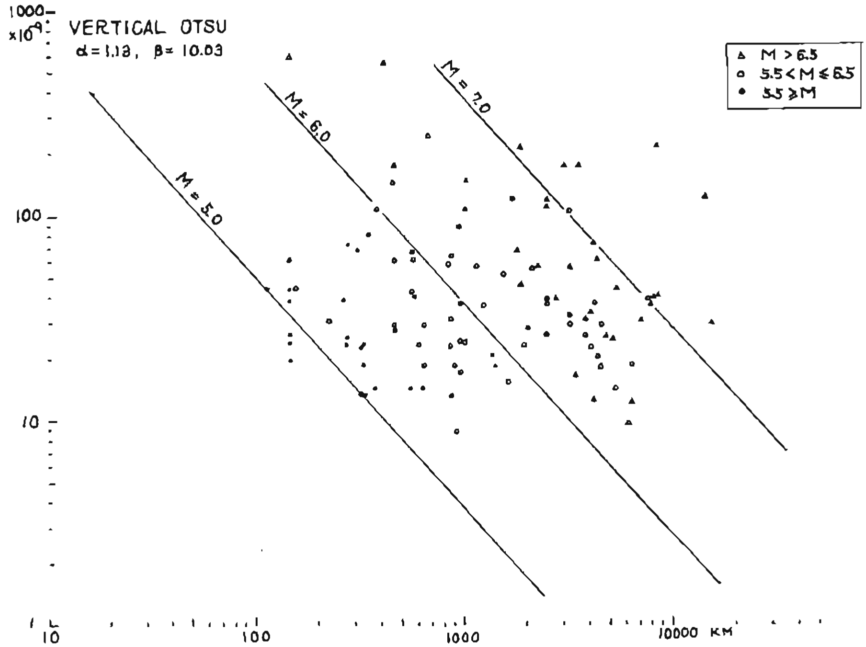


Fig. 2. (c) The relations between the maximum amplitude of the vibrational extentions in the vertical at Otsu Observatory versus the seismic magnitude and the epicentral distance.

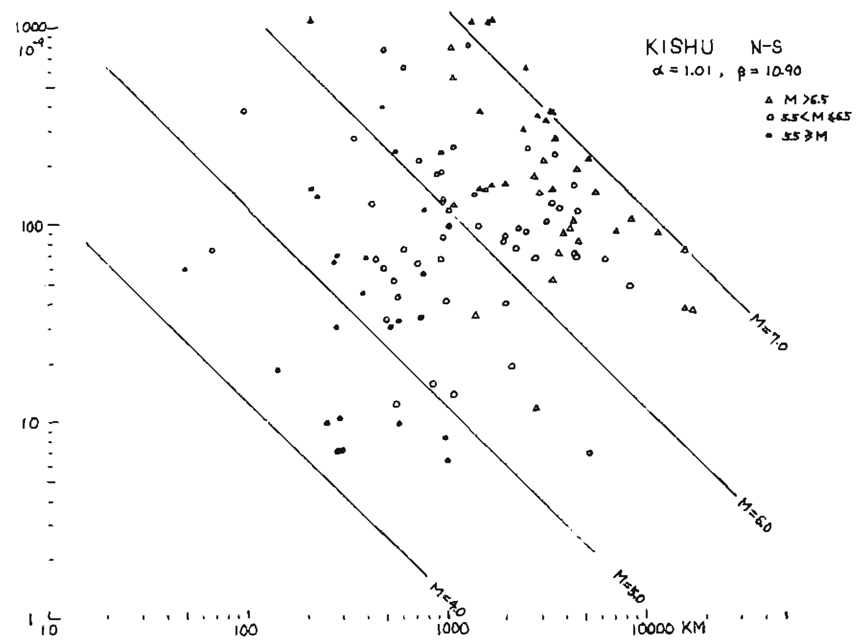


Fig. 2. (d) The relation between the maximum amplitude of the vibrational extensons in the azimuth of *N*- at Kishu Mine versus the seismic magnitude and the epicentral distance.

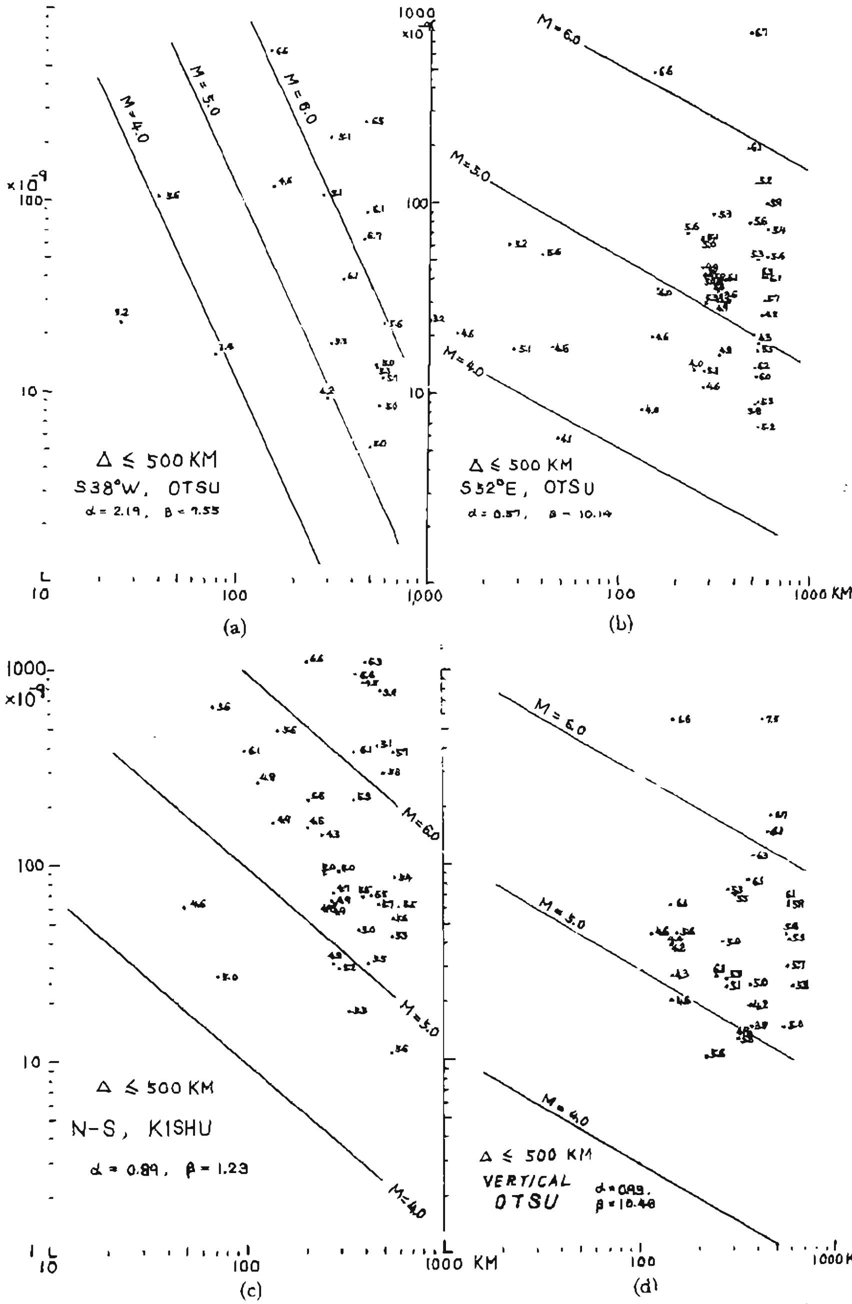


Fig. 3. (a) The maximum amplitude of the extensions in the azimuth of the *S38°W*- at Otsu Observatory versus the epicentral distances shorter than *500* km.

Fig. 3. (b) The maximum amplitude of the extensions in the azimuth of the *S52°E*- at Otsu Observatory versus the epicentral distances shotrer than *400* km.

Fig. 3. (d) The maximum amplitude of the extensions in the vertical at Otsu Observatory versus the epicentral distances shorter than *500* km.

Fig. 3. (c) The maximum zmplitude of the extensions in the azimuth of *N*- at Kishu Mine versus the epicentral distances shorter than *500* km.

Table 4. Analysed Values of α and β

Obsevatory	Direction of Observation	Division of Earthquake	Number of Earthquakes	α	β
Otsu	*S38°W*	all Magnitude	*129*	*1.37±0.07*	*9.42±0.22*
Otsu	*S52°E*	all Magnitude	*164*	*1.06±0.16*	*9.97±0.49*
Otsu	Vertical	all Magnitude	*107*	*1.13±0.11*	*10.93±0.33*
Kishu	*N*	all Magnitude	*183*	*1.01±0.25*	*9.90±0.19*
Otsu	*S38°W*	$\Delta \leq 500$ km	*18*	*2.19±0.36*	*7.55±0.99*
Otsu	*S52°E*	$\Delta \leq 500$ km	*36*	*0.57±0.13*	*10.14±0.32*
Otsu	Vertical	$\Delta \leq 500$ km	*27*	*0.93±0.18*	*10.45±0.42*
Kishu	*N*	$\Delta \leq 500$ km	*34*	*0.89±0.41*	*10.23±0.99*

amplitude e_{max} versus the focal distance Δ and the seismic magnitude M for all earthquakes. Fig. 3 (a), (b), (c) and (d) show these relations on the earthquakes whose epicentral distances are shorter than *500 km*.

4. Considerations

The values of α' are chiefly dependent on the scattering type of the wave, and it seems that the maximum phases of the strain waves propagate cylindrically, according to our analyses.

In the observations of the displacement type seismograph, the value of the coefficient α of the epicentral distance Δ is from *1.3* to *1.7*, and the maximum phases are found in the shear wave phases in most earthquakes, except teleseisms. But the maximum amplitude in the strain waves are found in the latter phases of the surface waves except in those the near earthquakes.

As the specific strain energy of the seismic waves per unit volume almost consists of the shear strain alone, the effective value of the specific energy E is approximated as[10]

$$E = 1.5\,\mu\, e^2_{max} \times \frac{1}{\sqrt{2}} \tag{8}$$

where μ is the mean rigidity of the crust.

Consequently, it may estimate that the almost energy of the maximum strain wave is confined in the surface layer of the earth, the total of the energy E is expressed as the total energy of the penetrating wave through the cylinder whose radius is equal to the special focal distance Δ_0 *cm* (the radius of the seismic origin), and whose height is equal to the thickness of the crust h cm.

Consequently, the energy E is given as follow,

$$E = \frac{3\pi}{\sqrt{2}} \Delta_0 v t_0 h \mu e^2_{max} \tag{9}$$

where v and t_0 are the velocity and the period of the maximum strain wave, respectively.

From the formula (7), we have as follow

$$e^2_{max}=10^{2(M-\beta')}\Delta^{-2\alpha'} \quad (10)$$

The unit of Δ respect for the value of α' in Table 4 is given in kilometer, and Δ in the formula (10) is rewritten as follow

$$\Delta=10^{-5}\Delta_0. \quad (11)$$

It is reasonable to be that the Δ_0 is the radius of the seismic origin or that of the ruptured crust.

I. Ozawa[11] has obtained the relation between the radius of the seismic origin Δ_0 and the seismic magnitude as follow

$$\log \Delta_0=0.45\,M+3.00. \quad (12)$$

These similar relations have been given as followings; for the radius Δ_0 *cm* of the crustal deformation

$$\log \Delta_0=0.51\,M+2.73,$$ T. Dambara[12]

for the area A cm^2 of the aftershock's region

$$\log A=1.02\,M+11.01$$ T. Utsu et al.[13]

The period t_0 of the maximum phase in the seismic waves is expressed in the form as follow

$$\log t_0=aM+b \quad (13)$$

and these coefficients a and b are given by T. Tereshima[14] and the others as follows,

	a	b	
Matsumoto et al.	0.33	−1.36	for S-phase,
Gutenberg and Richter	0.22	−1.5	for maximum phase,
Terashima	0.30	−1.40	for maximum phase.

Using the mean of our results in Otsu above mentioned, $\alpha'=1.20$, $\beta'=9.6$, that of Terashima for t_0, the values of $h=3\times10^6$ cm, $\mu=0.7\times10^{12}$ *c.g.s.* $v=4\times10^5$ *cm/sec*, and the relations (9), (10), (11), (12) and (13), we have the total energy E of the single phase of the maximum strain as follow

$$E_T=10^{1.67M+11.5}.$$

This relation has good agreement with that of Richter which is given the energy of the series of the main phases of the seismic vibrational displacement as follow

$$E=10^{1.5M+11.8}.$$

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