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Analysis of Hydraulic Resistance for Mobile Bed Channels

By

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Abstract

Extensive studies, experiments and field observations on sediment transportation in mobile bed channels have been carried out for estimating the rate of sediment transport and the bed roughness in river channels. Nevertheless, it is difficult to predict accurately these quantities. This paper treats an approach to predict friction factors in alluvial channels based on a number of experimental and observational data collected from various fields. It was clear that multiple parameters should be used to estimate accurately friction factors in alluvial channels.

1. Introduction

The prediction of hydraulic resistance in alluvial channels, which is very important for the design of river and channel improvements, is one of the most difficult problems at the present time. This difficulty arises from the occurrence of various kinds of bed configurations on the loose boundary with the change of flow stage, because these bed configurations severely affect the hydraulic resistance to flow. In other words, in rigid boundary channels the roughness height of flow does not change and remains fixed, but in alluvial channels the roughness height, which is formed by the irregularity of sand waves, changes with the intensity of flow. Therefore, since the roughness height in alluvial channels is a function of flow intensity which has not been determined yet, this fact makes a main objection to the prediction of hydraulic resistance.

Extensive studies have been carried out for establishing the approach to the prediction of friction factors in alluvial channels. Since the content of these studies is reviewed in detail in a paper by a Task Committee of ASCE[1], only outlines of these approaches are presented herein. The approaches for predicting friction factors can be classified into three types as (i) the regime theory[2),3)], (ii) the approach which divides total hydraulic resistance into two parts of resistance due to the skin friction and the form drag[4),5)] and (iii) the mean velocity formula of exponential types[6),7)].

(i) In the regime theory, mainly based on the analysis of canal data, experimental flume and natural river data have hardly been treated. Hence, to extend it to experimental flumes and natural rivers, it should first be checked by using

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various kinds of data in alluvial channels.

(ii) The approach which divides total hydraulic resistance into two parts of resistance, first proposed by Einstein and Barbarossa[4], is the most theoretical of these approaches. The adaptability to various kinds of data has been examined and it has been modified. As shown by Galay and Cheung[8] for some river data, however, it is not always consistent although it appears to be the most reliable.

(iii) The Manning formula is the most famous of all the mean velocity formulae in this kind of approach. The Manning's coefficient is not constant but changes with the stage of flow. Hence, in order to apply it to the estimate of flow velocity in alluvial channels. the coefficient should be determined based on some knowledge of hydraulics in alluvial channels. For this purpose Liu, Hwang[6] and Garde, Raju[7] attempted to investigate relations between the mean velocity formula of exponential types and bed configurations, but their results are not always satisfactory.

As shown by Cooper, Peterson and Blench[9], since many approaches described above have been investigated using the individual collection of data with narrow range in scope, they often derive incorrect results when they are applied to conditions falling outside this scope.

Peterson and Howells[10] collected various data describing the behavior of flow in alluvial channels from different places, which include measurements for rivers, canals and flumes. In this paper, the hydraulic resistance to various kinds of flow stages are discussed using their data with observations of bed configurations.

2. Dimensional analysis

The phenomena occurring in alluvial channels are controlled by many factors, and accordingly they seem to be too complex to describe them by simple mathematical equations. For that reason, when analyzing various phenomena in alluvial channels the approach of dimensional analysis has been used extensively as the most useful tool.

The variables which determine the phenomena occurring in alluvial channel flows consist of flow properties, physical properties of bed sediment and water, channel properties and gravitational acceleration. More fully, the flow properties are water depth h, mean velocity V, flow discharge Q, and concentration of transported bed material C. Physical properties of sediment and water are median diameter D_{50}, sediment density ρ_s, geometric standard deviation on size σ_b, fluid density ρ and kinematic viscosity ν. Channel properties are channel width B and bed slope S and gravitational acceleration g.

As shown by Kennedy and Brooks[11], various sets of dependent variables can be chosen from the variables in flow properties and bed slope. In this paper, let us choose mean velocity and sediment concentration as dependent variables:

$$C,\ V = f_{ns}\,(h, S, D_{50}, \rho, \rho_s, \nu, g, \sigma_b, B) \tag{1}$$

in which f_n means only "some function of". In Equation 1, the shape effect of particles and plan-form geometry in channels are not considered by assuming their effects are secondary. Using the method of dimensional analysis and rearranging, Equation 1 can be written in terms of non-dimensional variables as:

$$C,\ f=f_{ns}\left(\frac{h}{D_{50}},\ \tau_*,\ \frac{\sqrt[3]{\nu g}\,D_{50}}{\nu},\ \frac{B}{h},\ \sigma_b,\ \frac{\rho_s}{\rho}\right) \tag{2}$$

in which τ_*=non-dimensional tractive force$=u_*^2/(\rho_s/\rho-1)gD_{50}$, u_*=shear velocity $=\sqrt{ghI}$, and f=Darcy-Weisbach friction factor$=8/(V/u_*)^2$. Many investigators independently introduced equations of non-dimensional forms similar to Equation 2. Recently, Cooper, Peterson and Blench analyzed the flume data in alluvial channels and got function relationships as:

$$f_n\left(F_r,\ \frac{h}{D_{50}},\ C\right)=0 \tag{3a}$$

$$f_n\left(S,\ \frac{h}{D_{50}},\ C\right)=0 \tag{3b}$$

in which F_r=Froude number$=V/\sqrt{gh}$, and S is used instead of τ_*. Their scatter plots given by Equations 3a and 3b showed good correlation on analyzing flume data although $\sqrt[3]{\nu g}\,D_{50}/\nu, B/h, \sigma_b, \rho_s/\rho$ were neglected.

Most investigators analyzed the various phenomena in alluvial channels flows by using two or three variables but Vanoni[2] attempted to classify the bed configurations with a functional relationship as:

$$\text{bed configurations}=f_n\left(F_r,\ \frac{h}{D_{50}},\ \frac{\sqrt[3]{\nu g}\,D_{50}}{\nu}\right) \tag{4}$$

If we consider bed configurations as one variable, the scatter plot suggested by Equation 4 forms four-dimensional plots. It was shown from these plots that the four-dimensional plot is useful for classifications of bed configurations.

For simplifying Equation 2, B/h, σ_b and ρ_s/ρ can be omitted from this equation. These omissions are justified by reason that the effects of B/h appear to be negligible when B/h is in excess of about 4. Based on the study of Cooper[3], σ_b is ignored in Equation 2. The effects of ρ_s/ρ are omitted since the value of ρ_s/ρ for the data used in the analysis is practically constant. Based on these facts, Equation 2 can be rewritten as:

$$f_n\left(f,\ \frac{h}{D_{50}},\ \tau_*,\ \frac{\sqrt[3]{\nu g}\,D_{50}}{\nu}\right)=0 \tag{5a}$$

$$f_n\left(C,\ \frac{h}{D_{50}},\ \tau_*,\ \frac{\sqrt[3]{\nu g}\,D_{50}}{\nu}\right)=0 \tag{5b}$$

Furthermore, when $\sqrt[3]{\nu g}\,D_{50}/\nu$, which represents the effects of viscosity and sediment size on flow, is eliminated from Equations 5a and 5b, one can obtain the following equation:

$$f_n\left(f, C, \frac{h}{D_{50}}, \tau_*\right)=0 \tag{6}$$

Equations 5a, 5b and 6 are used in the analysis.

3. Analysis of data

In order to obtain scatter plots suggested by Equation 5a, only the alluvial channel data with observations of bed configurations are used in the analysis. The dramatic change of friction factor is closely related to the change of bed configurations. Hence it seems to be reasonable to investigate the fricton factor by means of the knowledge of hydraulics of bed configurations and the parameters in Equation 5a.

In Figs. 1-7, the median size D_{50} is used instead of the parameter $\sqrt[3]{\nu g}D_{50}/\nu$ in Equation 5a because $\sqrt[3]{\nu g}D_{50}/\nu$ is practically equivalent to D_{50}. Bed configurations are classified into four types: plane bed, in which bed sediment hardly moves, ripples, dunes and upper flow regime in which transition, flat bed, antidunes, standing waves and chute and pool are included. The data in which B/h is greater than 4 are plotted on graphs of f against h/D_{50} for several ranges of τ_*, and bed configurations and the ranges of D_{50} are shown by different symbols.

CASE 1 : $\tau_* \leq 0.4$

As shown in Figs. 1-4 the scatter plots of f against h/D_{50} can distinguish upper flow regime from ripples and dunes when using the parameter of h/D_{50} to determine the boundary between the former and the latter, and the boundary lines are shown by dashed lines. Furthermore, in this region f can be uniquely determined by using the parameters in Equation 5a. This fact means that the parameters suggested by Equation 5a, which are determined easily by S, h, D_{50} and other constant physical quantities, are useful to predict the friction factor in this region.

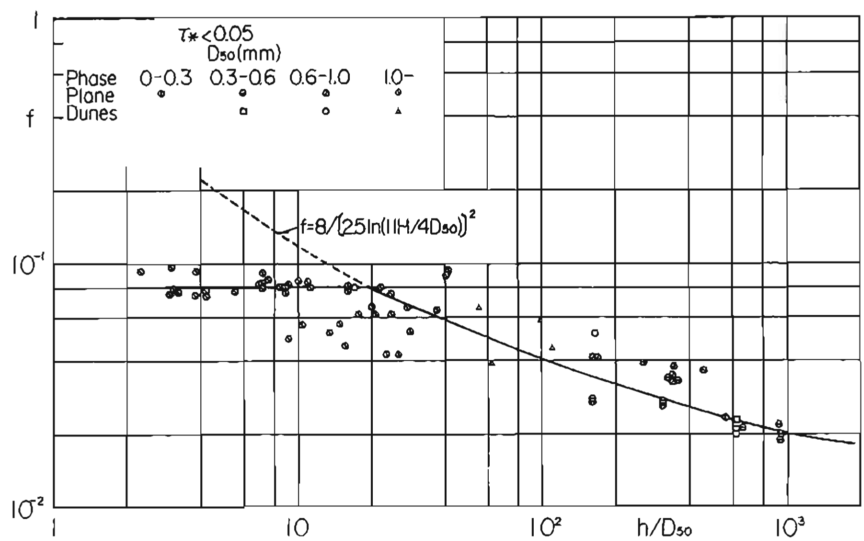

Fig. 1 Relationship between f and h/D_{50} in range $\tau_* < 0.05$

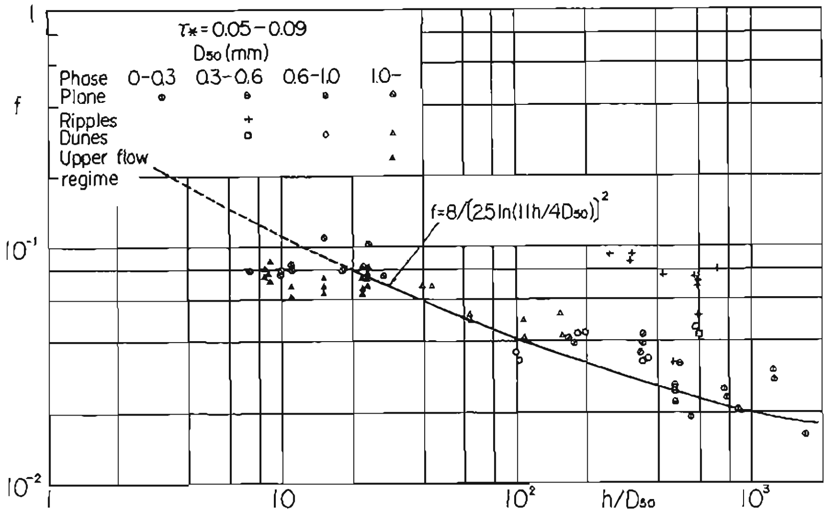

Fig. 2 Relationship between f and h/D_{50} in range $0.05 \leq \tau_* < 0.09$

In Figs. 1-2, the scatter plots of f against h/D_{50} are shown for a low value of τ_* in which sediment movement is not active. Therefore, in this region the equivalent roughness height is to be equal to the scale of sediment size when the logarithmic velocity distribution is used as a hydraulic resistance law:

$$f = \frac{8}{[2.5\, l_n(11/hk_s)]^2} \tag{7}$$

in which k_s = equivalent roughness height and l_n = natural logarithm. In the region of $\tau_* < 0.09$, k_s in Equation 7 is approximately equal to 4 D_{50} when h/D_{50} is greater than 20. When h/D_{50} is less than 20, k_s is not proportional to D_{50} but proportional to h, and f is held constant. Ripples data in Fig. 2 are different with the trend

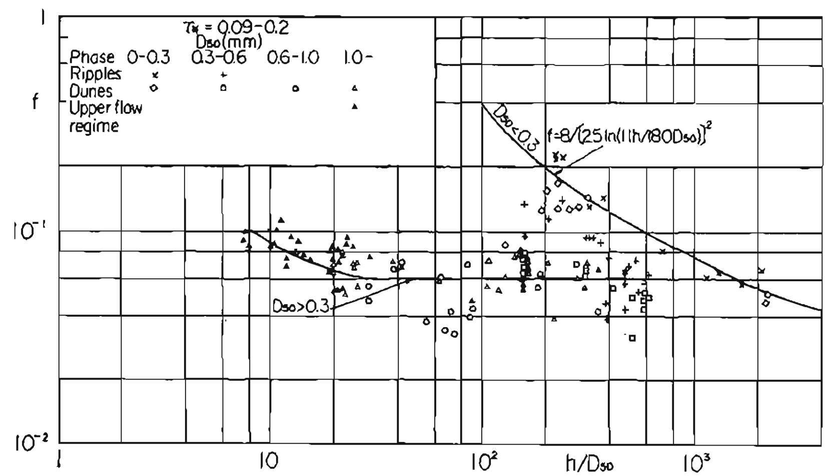

Fig. 3 Relationship between f and h/D_{50} in range $0.09 \leq \tau_* < 0.2$

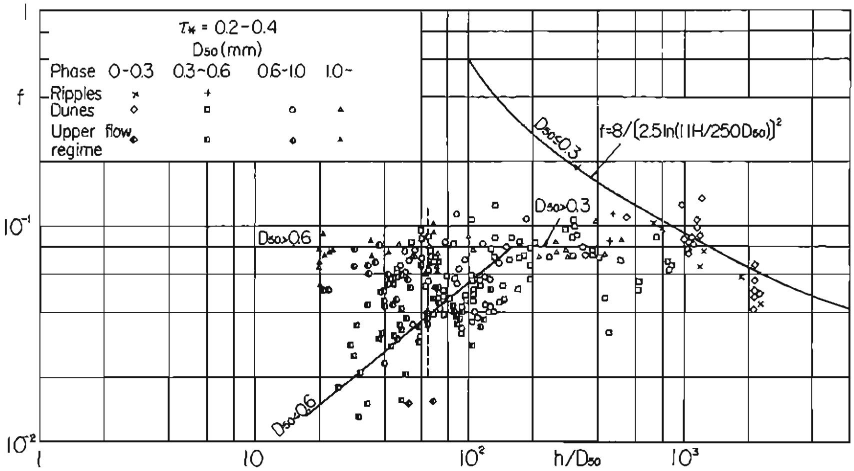

Fig. 4 Relationship between f and h/D_{50} in range $0.2 \leq \tau_* < 0.4$

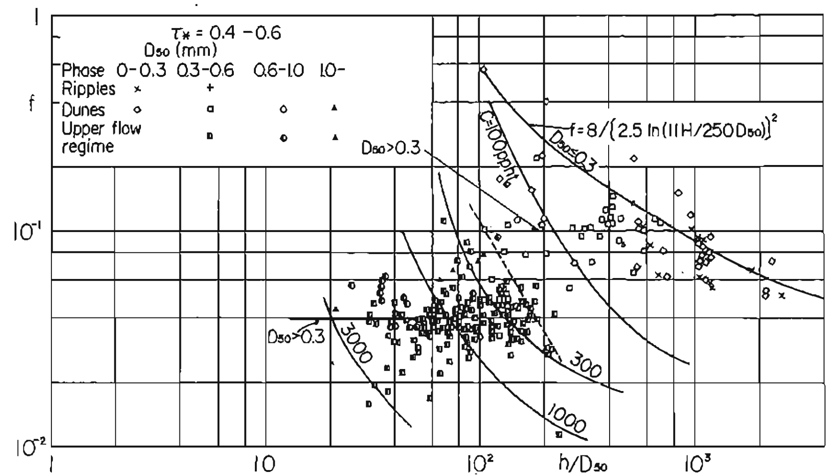

Fig. 5 Relationship between f and h/D_{50} in range $0.4 \leq \tau_* < 0.6$

described above, but this discrepancy seems to be due to the wide band width of τ_* and some instability of transition from plane bed to ripples. Furthermore, although it is difficult to distinguish upper flow regime from one part of plane bed, it does not make an objection for predicting f because f can be determined uniquely in these two bed configurations.

In Figs. 3-4, in the case of which bed sediment is transported actively, f can be represented by a single-valued function of h/D_{50} when using τ_* and D_{50} as the third and fourth parameters. In the case of $D_{50} \leq 0.3$ mm, even if both ripples and dunes are included in this region, f can be given by Equation 7 and k_s is equal to 180 D_{50} at $\tau_* = 0.09 - 0.2$ and to 250 D_{50} at $\tau_* = 0.2 - 0.4$. Even if the bed configurations in this region belong to dunes, f in dunes has the same characteristics as that of ripples.

It was shown by Yalin[14] that the scale of ripples is not proportional to water depth but proportional to sediment size. This fact herein can be proved indirectly through the consideration of f because k_s, which represents a certain kind of scale of sand waves, is proportional to D_{50}. On the other hand, in the case of $D_{50}>0.3$ mm and lower flow regime, f is independent of h/D_{50} and kept approximately constant. This fact means that k_s in Equation 7 is proportional not to D_{50} but to h. This is consistent with the results obtained by Yalin through consideration of sand waves geometry that the scale of dunes is proportional to h. Since we have little knowledge of upper flow regime, it would be difficult to have a detailed discussion of f in this region.

CASE 2 : $\tau_*\geq 0.6$

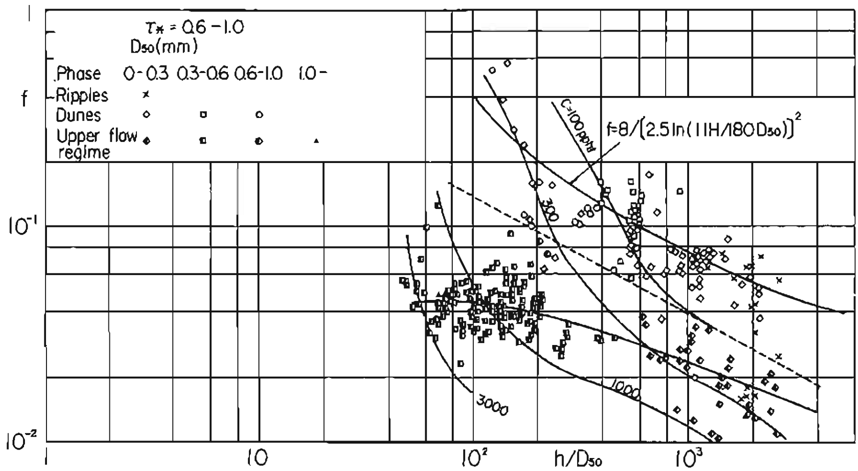

Fig. 6 Relationship between f and h/D_{50} in range $0.6\leq\tau_*<1.0$

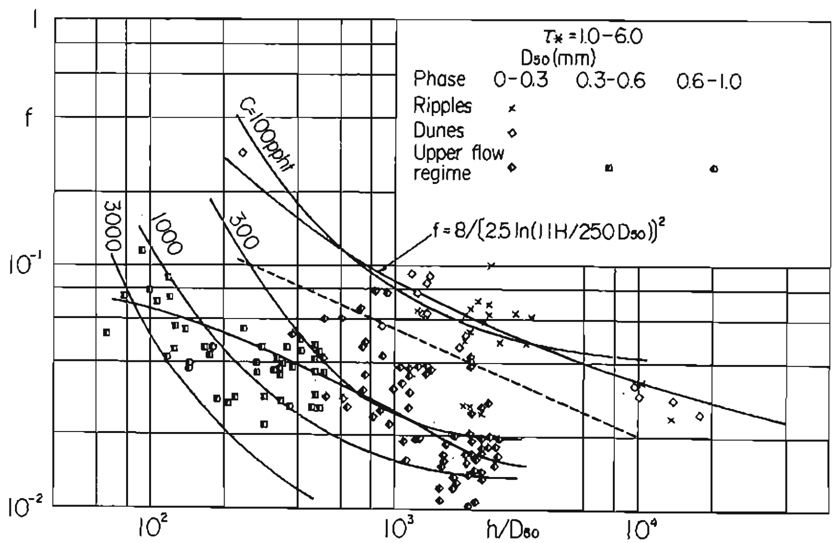

Fig. 7 Relationship between f and h/D_{50} in range $1.0\leq\tau_*<6.0$

The data in this region are shown in Figs. 6 and 7. Kennedy and Brooks suggested that in some regions f is not a single-valued function of h/D_{50} but a multiple-valued function even if τ_* and D_{50} are used as parameters. This means that in some regions two different bed configurations of lower flow regime and upper flow regime exist on scatter plots even if h/D_{50}, τ_* and D_{50} are kept constant and there exist different friction factors corresponding to each bed configuration. Therefore, in order to obtain f-diagrams, the parameters in Equation 5a are not useful and it is impossible to determine f uniquely. Figs. 6 and 7 are rewritten with the parameters of Equation 6 in which C is included instead of D_{50} since D_{50} is not so effective for classifications of bed configurations in this region. When using C as the fourth parameter, the multiplicity of f can be cancelled. Furthermore, f in lower flow regime can be represented by Equation 7 as shown in Figs. 6 and 7.

CASE 3 : $0.4<\tau_*<0.6$

This region belongs to the transition between Case 1 and Case 2 as shown in Fig. 5, and the influence of C and D_{50} on f is almost the same. In the case of $D_{50}\leq 0.3$ mm, f can be represented by Equation 7 and k_s is proportional to D_{50}. On the other hand, in the case of $D_{50}>0.3$ mm, f is kept constant on lower flow regime and upper flow regime, respectively.

The data of lower flow regime and $D_{50}\leq 0.3$ mm are plotted in Fig. 8. The average value of f in Fig. 8 can be represented by Equation 7 when substituting $k_s=200\ D_{50}$ into Equation 7. It is interesting that the value of k_s is equal to that of the maximum wave height of ripples given by Yalin. Furthermore, a certain region of the regime theory are shown by hatching parts in Fig. 8. It is concluded from this figure that Equation 7, in which $k_s=200\ D_{50}$, can be extended to the regime theory for the region of $D_{50}\leq 0.3$ mm.

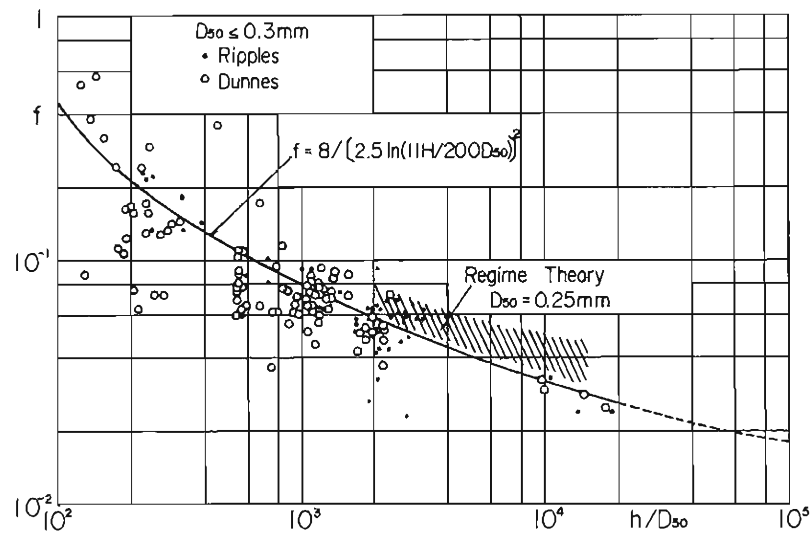

Fig. 8 Relationship between f and h/D_{50} in range $D_{50}\leq 0.3$ mm

Summary and Conclusion

The data in alluvial channel flows have been investigated by means of dimensional analysis and based on the knowledge of hydraulics on the scale of sand waves, and the following results have been obtained.

When $\tau_* \leq 0.4$, the four-dimensional scatter plots given by Equation 5a are available for predicting the friction factor. When using the parameters, the friction factor has a single-valued function of h/D_{50} and can be determined uniquely. In the case of $\tau_* < 0.09$, for the region of $h/D_{50} > 20$, f can be represented by Equation 7 when substituting $k_s = 4D_{50}$ into Equation 7. On the other hand, for the range of $h/D_{50} < 20$, f is kept constant. In the case of $0.09 \leq \tau_* \leq 0.4$, for the range of $D_{50} \leq 0.3$ mm and the lower flow regime, f has the characteristics of dunes that k_s is proportional to water depth.

When $\tau_* \geq 0.6$ and the parameters given by Equation 5a are used for the analysis of f, f has a multiple-valued function of h/D_{50}. Hence, Equation 5a is not available for predicting f but Equation 6, in which the charge of sediment C is included instead of D_{50}, can produce a single-valued function for f in this region and is useful. In the case of lower flow regime, f has the characteristics of ripples that k_s is proportional to sediment size.

When $0.4 < \tau_* < 0.6$, this region belongs to the transition between the two above cases. In the range of $D_{50} \leq 0.3$ mm, f has the characteristics of ripples and can be represented by Equation 7. On the other hand, in the range of $D_{50} > 0.3$ mm, f can be kept constant in lower flow regime and upper flow regime.

For the data of the lower flow regime and $D_{50} \leq 0.3$ mm, f can be approximately represented by Equation 7 when substituting $k_s = 200D_{50}$. It was also clear that this relationship for f can be extended to the regime theory in the range of $D_{50} \leq 0.3$ mm.

List of symbols

B	channel width
C	charge weight of transported bed material (parts per 100,000 by weight of water discharge)
D_{50}	median diameter of bed material
f	friction factor of Darcy-Weisbach $= 8/(V/u_*)$
g	gravitational acceleration
h	water depth
S	slope of energy grade line
V	mean velocity
u_*	shear velocity $= \sqrt{ghS}$
ρ	fluid density
ρ_s	sediment density
ν	kinematic viscosity of fluid
σ_b	geometric standard deviation of sediment
τ_*	non-dimensional tractive force $= u_*^2/(\rho_s/\rho - 1)g\, D_{50}$
F_r	Froude number $= V/\sqrt{gh}$
k_s	equivalent roughness height
l_n	natural logarithm

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