Time Domain Analysis of Earth Tide Records

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Abstract

A method of time domain analysis for earth tides is described. The analysis is to correlate the observed records with the corresponding theoretical time series predicted for a laterally homogeneous solid earth, without resolving the records into a number of tidal constituents. Simple explicit expressions of theoretical tidal strains and tilts are derived for this purpose as a function of time. The least squares technique in the time domain gives the overall tidal admittance averaged over tidal frequencies. The admittance thus estimated will be useful as a diagnostic means to detect possible temporal variations in crustal rigidity due to dilatancy in seismic source regions.

1. Introduction

Earth tide records have long been customarily analyzed in the frequency domain to estimate the amplitude and phase at particular tidal frequencies. Various methods, such as classical harmonic analysis, least squares, Fourier transform and maximum entropy method, have so far been applied for this purpose, on the assumption that the tidal line spectrum can be well resolved to a sufficient degree of precision. The resolution is, however, not only severely limited by the time length of the analyzed records, but also often badly contaminated by the presence of noise. Also, these types of frequency analysis inevitably make obscure general patterns and temporal amplitude variations of the whole record.

The time domain analysis used, instead, here is to simply correlate the observed earth tide records with the corresponding theoretical time functions predicted for a solid earth, without resolving the records into a number of tidal constituents. The direct comparison between the two time series does not introduce the resolution problem, and it is not necessary to restrict the time length of analysis to longer than one lunation. The present analysis yields the overall tidal admittance (the amplitude ratio and time lag) averaged over tidal frequencies involved in the whole time series, and also immediately makes clear the departures or residuals of the observations on the real earth from the response to an idealized solid earth. The departures will in some cases be a manifestation of regional distortions of the strain and tilt field, which are expected to result from the effects of ocean-tide loading (e.g. Ozawa, 1957; Farrell, 1972; Tanaka, 1973; Beaumont and Berger, 1975), cavity effect of an observation vault (King and Bilham, 1973; Harrison, 1976), site topography (Harrison, 1976), and of local crustal structure (Beaumont and Berger, 1974), as well as from meteorological disturbances such as atmospheric loading and temperature fluctuations. We are mainly concerned here with these departures, and particularly interested in the possibility of finding temporal variations in the tidal admittance due to the change of crustal rigidity in earthquake source regions (Beaumont and Berger, 1974; Tanaka and Kato, 1974; Tanaka, 1976), and of the tectonic stress field related to seismic activity, aseismic fault movements etc..

In the above sense, solid-earth tides are used here as a reference input function to compare with the observations. In the first part of the present paper, simple explicit expressions are derived for theoretical tidal strains and tilts, as a function of time, from the tide-generating potential based on an oceanless, laterally homogeneous solid earth. Although similar expressions have been obtained by several geophysicists (e.g. Harrison, 1971; Beaumont and Berger, 1974, 1975; Lambert, 1974), on the basis of the formulations for equilibrium tides given by Munk and Cartwright (1966), our derivations are somewhat different from theirs, and may be slightly simpler for the computer use. In the second part, a method of estimating the tidal admittance is described. The observed records are bandpass-filtered to remove any powers outside the tidal frequencies as noise, and compared with the computed theoretical tidal functions. The admittance will be determined by the least squares in such a way that the residuals between the two kinds of time series be a minimum. Some examples determined by this method will be presented.

2. Theoretical Tidal Strains and Tilts on a Solid Earth

2.1. Tidal potential

The tidal gravitational potential resulting from the moon and the sun can be represented as the sum of time-variable spherical harmonics in terms of the zenith angles and distances of the two celestial bodies,

$$W=\sum_{n=2}^{\infty}\sum_{j=1}^{2} W_{jn} \tag{1}$$

where suffixes $j=1$ and 2 indicate terms for the moon (M) and the sun (S), respectively. These terms take the form (Bartels, 1957; Melchior, 1966),

$$W_{jn}=\frac{\mu M_j r^2}{R_j^3}\left(\frac{r}{R_j}\right)^{n-2} P_n(\cos\theta_j) \qquad (n=2,\ 3,\cdots) \tag{2}$$

The notations used here are,

W; tidal gravitational potential
M_M; mass of the moon, M_S; mass of the sun
R_M; distance between the centers of the earth and the moon
R_S; distance between the centers of the earth and the sun
r; distance of an observation point to the center of the earth
θ_M; zenith angle of the moon, θ_S; zenith angle of the sun
λ; terrestrial longitude of an observation point
φ; terrestrial latitude of an observation point
μ; universal gravitational constant

It is sufficient for practical purposes (to the accuracy of 0.1%) to sum up the above terms to $n=3$ for the moon and only $n=2$ for the sun, since $r/R_M<2\times 10^{-2}$ and $r/R_S<5\times 10^{-5}$. The second and third order of the Legendre functions are,

$$P_2(\cos\theta_j)=(3\cos^2\theta_j-1)/2 \quad \text{and} \quad P_3(\cos\theta_j)=(5\cos^3\theta_j-3\cos\theta_j)/2\,.$$

The zenith angles of the moon and the sun can be related to the terrestrial latitude and longitude of an observation point on the earth's surface, in a form somewhat modified from Schureman (1941) and Longman (1959),

$$\begin{aligned}\cos\theta_M &= \sin\varphi\cdot\sin I\cdot\sin l+\cos\varphi(\cos l\cdot\cos\chi+\cos I\cdot\sin l\cdot\sin\chi)\\ \cos\theta_S &= \sin\varphi\cdot\sin\omega\cdot\sin l_1+\cos\varphi(\cos l_1\cdot\cos\chi_1+\cos\omega\cdot\sin l_1\cdot\sin\chi_1)\end{aligned} \tag{3}$$

where $$\chi=\chi_1-\nu, \quad \text{and} \quad \chi_1=15(t_0-12)-\lambda+h \tag{4}$$

All the quantities involved in the above expressions, I, l, ω, l_1, ν and h refer to Longman's paper (1959), but in numerical computations some higher- order terms are supplemented to l, l_1, R_M and R_S according to Munk and Cartwright (1966). Since θ_M and θ_S are related to T (the number of Julian centuries) (Schureman, 1941; Bartels, 1957; Longman, 1959), W is given as a function of time.

2.2. Tidal accelerations, displacements, tilts and strains

The tidal accelerations, displacements, tilts and strains on the earth's surface ($r=a$) can be derived from the gravitational potential in the following form (e.g. Melchior, 1966; Takeuchi and Alsop, 1965), where the latitude φ and longitude λ are taken as positive northward and westward, respectively, and H_n, K_n and L_n are Love numbers of the n-th order.

The vertical and two horizontal components of the tidal accelerations are,

$$f_r=-\frac{\partial W_{jn}}{\partial r}\equiv g_{jn},\ f_\varphi=\frac{1}{a}\frac{\partial W_{jn}}{\partial\varphi},\ f_\lambda=\frac{1}{a\cos\varphi}\frac{\partial W_{jn}}{\partial\lambda} \tag{5}$$

An observation of tidal gravity yields,

$$g_{obs}=G_n g_{jn} \tag{6}$$

where G_n is the tidal factor of gravity given by $G_n=1+H_n-(3/2)K_n$.

The tidal displacement components are given by,

$$u_r=\frac{H_n}{g}W_{jn},\ u_\varphi=-\frac{L_n}{g}\frac{\partial W_{jn}}{\partial\varphi},\ u_\lambda=\frac{L_n}{g\cos\varphi}\frac{\partial W_{jn}}{\partial\lambda} \tag{7}$$

The tidal tilts along the meridian and along the latitude are,

$$i_\varphi=-\frac{\gamma_n}{ag}\frac{\partial W_{jn}}{\partial\varphi},\ i_\lambda=\frac{\gamma_n}{ag\cos\varphi}\frac{\partial W_{jn}}{\partial\lambda} \tag{8}$$

where i_φ and i_λ are taken as positive down towards the north and east, respectively, and γ_n is the diminishing factor given by $\gamma_n=1+K_n-H_n$. The tidal strain components in a spherical coordinate are given by,

$$\begin{aligned}e_{\varphi\varphi}&=\frac{1}{r}\frac{\partial u_\varphi}{\partial\varphi}+\frac{u_r}{r}=\frac{L_n}{ag}\frac{\partial^2 W_{jn}}{\partial\varphi^2}+\frac{H_nW_{jn}}{ag}\\ e_{\lambda\lambda}&=\frac{1}{r\cos\varphi}\frac{\partial u_\lambda}{\partial\lambda}+\frac{u_\varphi}{r}\tan\varphi+\frac{u_r}{r}\\ &=\frac{L_n}{ag\cos^2\varphi}\frac{\partial^2 W_{jn}}{\partial\lambda^2}-\frac{L_n}{ag}\tan\varphi\cdot\frac{\partial W_{jn}}{\partial\varphi}+\frac{H_nW_{jn}}{ag}\end{aligned} \tag{9}$$

$$e_{\lambda\varphi}=\frac{1}{r}\left(-\frac{\partial u_\lambda}{\partial\varphi}-u_\lambda\tan\varphi\right)+\frac{1}{r\cos\varphi}\frac{\partial u_\varphi}{\partial\lambda}$$

$$=-\frac{2L_n}{ag\cos\varphi}\left(\frac{\partial^2 W_{jn}}{\partial\lambda\partial\varphi}+\tan\varphi\frac{\partial W_{jn}}{\partial\lambda}\right)$$

The above expressions including suffixes j and n imply the summation over three spherical harmonics ($n=2$ and 3 for $j=1$ and $n=2$ for $j=2$) or their derivatives multiplied by the corresponding Love numbers of the second and third order. In the present article, we use the following Love numbers which have been derived by Longman (1963) and Takeuchi and Alsop (1965) for the Gutenberg earth model.

$$\begin{aligned}&H_2=0.612,\ K_2=0.302,\ L_2=0.083\\&H_3=0.290,\ K_3=0.093,\ L_3=0.014\end{aligned}\tag{10}$$

These numbers are slightly different from those given by Farrell (1972) for the Gutenberg-Bullen A model.

2.3. Derivatives of tidal potential

The next step is to calculate a number of the derivatives of the tidal potential with respect to the location of an observation point, which are involved in Eqs. (7), (8) and (9). The method we used here is to calculate the derivatives through Eqs. (3) and (4). That is,

$$\frac{\partial W_{jn}}{\partial\varphi}=\frac{\partial W_{jn}}{\partial\theta_j}\frac{\partial\theta_j}{\partial\varphi},\quad \frac{\partial W_{jn}}{\partial\lambda}=\frac{\partial W_{jn}}{\partial\theta_j}\frac{\partial\theta_j}{\partial\lambda}\tag{11}$$

If we define here $h_{jn}\equiv\frac{1}{r}\frac{\partial W_{jn}}{\partial\theta_j}$, the first and second derivatives of the tidal potential can be expressed by,

$$\begin{aligned}
&\frac{1}{r}\frac{\partial W_{jn}}{\partial\varphi}=h_{jn}\frac{\partial\theta_j}{\partial\varphi},\quad \frac{1}{r}\frac{\partial W_{jn}}{\partial\lambda}=h_{jn}\frac{\partial\theta_j}{\partial\lambda}\\
&\frac{1}{r}\frac{\partial^2 W_{jn}}{\partial\varphi^2}=\frac{\partial}{\partial\varphi}\left(h_{jn}\frac{\partial\theta_j}{\partial\varphi}\right)=\frac{\partial h_{jn}}{\partial\theta_j}\left(\frac{\partial\theta_j}{\partial\varphi}\right)^2+h_{jn}\frac{\partial^2\theta_j}{\partial\varphi^2}\\
&\frac{1}{r}\frac{\partial^2 W_{jn}}{\partial\lambda^2}=\frac{\partial}{\partial\lambda}\left(h_{jn}\frac{\partial\theta_j}{\partial\lambda}\right)=\frac{\partial h_{jn}}{\partial\theta_j}\left(\frac{\partial\theta_j}{\partial\lambda}\right)^2+h_{jn}\frac{\partial^2\theta_j}{\partial\lambda^2}\qquad (j=1,\ 2;\ n=2,\ 3)\\
&\frac{1}{r}\frac{\partial^2 W_{jn}}{\partial\varphi\partial\lambda}=\frac{\partial}{\partial\lambda}\left(h_{jn}\frac{\partial\theta_j}{\partial\varphi}\right)=\frac{\partial h_{jn}}{\partial\theta_j}\frac{\partial\theta_j}{\partial\lambda}\frac{\partial\theta_j}{\partial\varphi}+h_{jn}\frac{\partial^2\theta_j}{\partial\varphi\partial\lambda}
\end{aligned}\tag{12}$$

From Eq. (2), we have,

$$\begin{aligned}&h_{j2}=-\frac{3\mu M_j r}{R_j^3}\sin\theta_j\cdot\cos\theta_j\\&h_{j3}=-\frac{3}{2}\frac{\mu M_j r^2}{R_j^4}(5\cos^2\theta_j-1)\sin\theta_j\end{aligned}\tag{13}$$

The above definition yields $\frac{1}{r}\frac{\partial^2 W_{jn}}{\partial\theta_j^2}=\frac{\partial h_{jn}}{\partial\theta_j}$, where

$$\frac{\partial h_{j2}}{\partial\theta_j}=-\frac{3\mu M_j r}{R_j^3}\cos 2\theta_j$$

$$\frac{\partial h_{j3}}{\partial\theta_j}=-\frac{3}{2}\frac{\mu M_j r^2}{R_j^4}(15\cos^2\theta_j-11)\cos\theta_j$$

On the other hand, we obtain from Eqs. (3) and (4),

$$\begin{aligned}
&\partial\theta_M/\partial\varphi=-[\cos\varphi\cdot\sin I\cdot\sin l-\sin\varphi(\cos l\cdot\cos\chi+\cos I\cdot\sin l\cdot\sin\chi)]/\sin\theta_M\\
&\partial\theta_S/\partial\varphi=-[\cos\varphi\cdot\sin\omega\cdot\sin l_1-\sin\varphi(\cos l_1\cdot\cos\chi_1+\cos\omega\cdot\sin l_1\cdot\sin\chi_1)]/\sin\theta_S\\
&\partial\theta_M/\partial\lambda=-\cos\varphi(\cos l\cdot\sin\chi-\cos I\cdot\sin l\cdot\cos\chi)/\sin\theta_M \qquad (14)\\
&\partial\theta_S/\partial\lambda=-\cos\varphi(\cos l_1\cdot\sin\chi_1-\cos\omega_1\cdot\sin l_1\cdot\cos\chi_1)/\sin\theta_S
\end{aligned}$$

since $\partial\chi/\partial\lambda=-1$ and $\partial\chi_1/\partial\lambda=-1$. The second derivatives of these terms are obtained after some transformations,

$$\begin{aligned}
&\frac{\partial^2\theta_j}{\partial\varphi^2}=\cot\theta_j\left[1-\left(\frac{\partial\theta_j}{\partial\varphi}\right)^2\right]\\
&\frac{\partial^2\theta_j}{\partial\lambda^2}=\cot\theta_j\left[1-\left(\frac{\partial\theta_j}{\partial\lambda}\right)^2\right]-S_j\frac{\sin\varphi}{\sin\theta_j} \qquad (15)\\
&\frac{\partial^2\theta_j}{\partial\varphi\partial\lambda}=-\frac{\partial\theta_j}{\partial\lambda}\left(\tan\varphi+\cot\theta_j\frac{\partial\theta_j}{\partial\lambda}\right)
\end{aligned}$$

where $\qquad S_M=\sin I\cdot\sin l, \qquad S_S=\sin\omega\cdot\sin l_1$

Theoretical tilts and strains can be directly obtained by putting all terms in Eqs. (12), (13), (14) and (15) into Eqs. (8) and (9), and taking summations over $n=2$ and 3 for $j=1$ and $n=2$ for $j=2$.

When we compare the theoretical tilts and strains with the corresponding observations made by tiltmeters and strainmeters installed along an arbitrary direction α (measured clockwise from the north), these are transformed to,

$$\begin{aligned}
&i_x=i_\varphi\cdot\cos\alpha-i_\lambda\cdot\sin\alpha\\
&i_y=i_\varphi\cdot\sin\alpha+i_\lambda\cdot\cos\alpha \qquad (16)
\end{aligned}$$

$$\begin{aligned}
&e_{xx}=e_{\varphi\varphi}\cdot\cos^2\alpha+e_{\lambda\lambda}\cdot\sin^2\alpha-e_{\lambda\varphi}\cdot\sin\alpha\cdot\cos\alpha\\
&e_{yy}=e_{\varphi\varphi}\cdot\sin^2\alpha+e_{\lambda\lambda}\cdot\cos^2\alpha+e_{\lambda\varphi}\cdot\sin\alpha\cdot\cos\alpha \qquad (17)\\
&e_{xy}=(e_{\varphi\varphi}-e_{\lambda\lambda})\sin 2\alpha+e_{\lambda\varphi}\cdot\cos 2\alpha
\end{aligned}$$

The results obtained by our method have been cross-checked with those from different approaches. It has been found that the tidal tilts computed for Poorman Mine, Boulder, Colorado give a close agreement with those obtained by Harrison using his formulations (1971). Similar comparison yielded satisfactory results for the tidal strains at the Kamitakara station (lower trace in Fig. 2). Some of our theoretical results have been Fourier-analyzed and the resolved amplitudes of the M_2 and O_1 constituents have been compared with those derived exclusively for the appropriate constituents (e.g. Ozawa, 1957, with some corrections; Melchior, 1966). This comparison also gave a satisfactory agreement.

3. Comparison between Observed and Theoretical Tides in the Time Domain

3.1. Filtering of Observed tide records

Observed tide records are often contaminated by short-period noise and long-term drift resulting from meteorological disturbances, secular ground deformations and sometimes from instrumental instabilities. These noise and drift outside tidal frequencies have to be removed before correlating the records with theoretical tides. The filter used here has a trapezoid shape with dominant responses over semidiurnal and diurnal tidal frequencies as shown in Fig. 1. Bandpass-filtering is performed by convolving the uncorrected records with the impulse response of the filter as has been described in Mikumo and Nakagawa (1968). A finite time length of the impulse response introduces slight distortions from the designed filter, as indicated by broken curves in Fig. 1. We decided to use here the longer length of 169 hrs. to give satisfactory accuracies. It is better to apply the same bandpass filter to theoretical tide functions in estimating the tidal admittance, since the functions include long-period tide components.

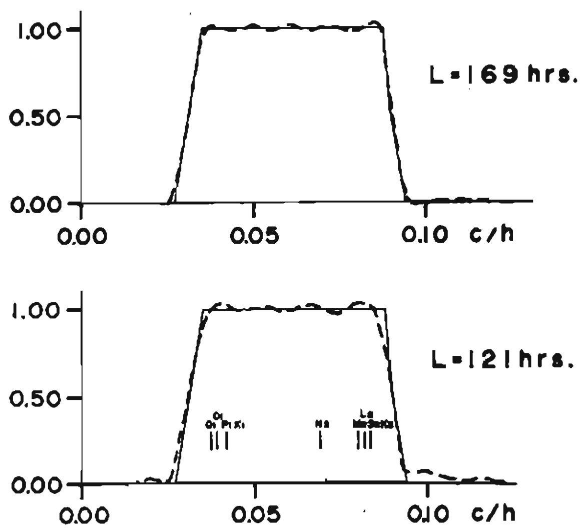

Fig. 1. Frequency response of the bandpass filter used in the present analysis.

3.2. Comparison between the observed and theoretical tides

In Figs. 2, 3 and 4, some examples of bandpass-filtered tidal records are compared with the corresponding theoretical tides. The upper trace in Fig. 2 shows a strain

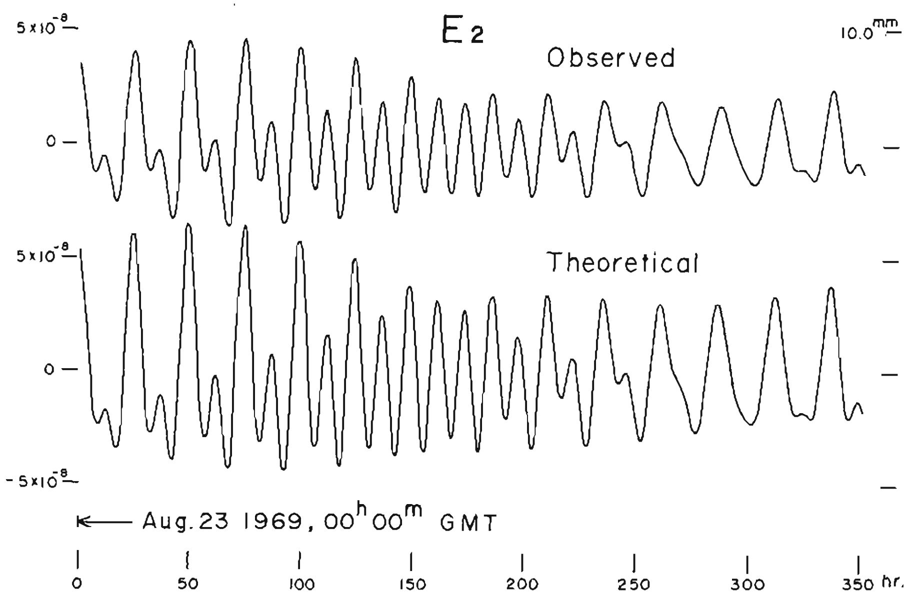

Fig. 2. Observed tidal strain record (bandpass-filtered) at Kamitakara and the corresponding theoretical tidal strains in August–September, 1969.

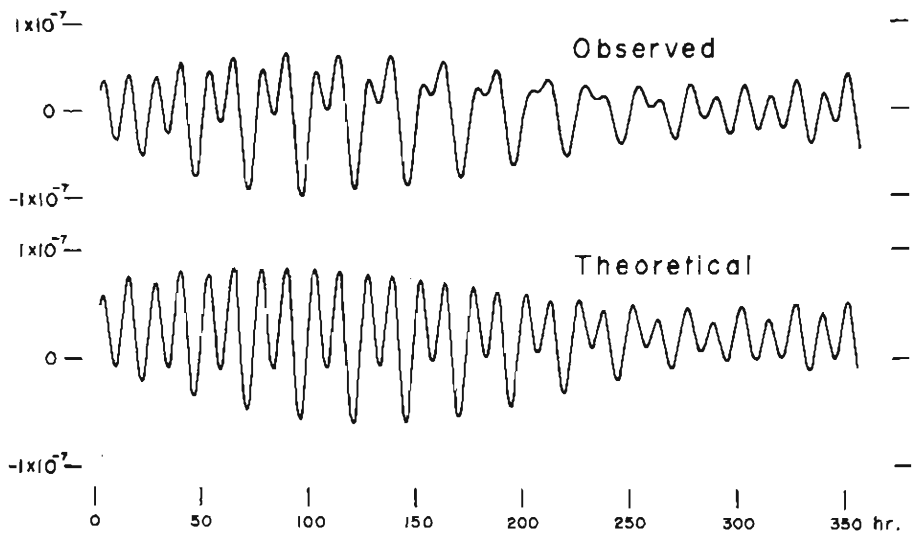

Fig. 3. Observed tidal tilt record (bandpass-filtered) at Kamitakara and the corresponding theoretical tidal tilts in June, 1976.

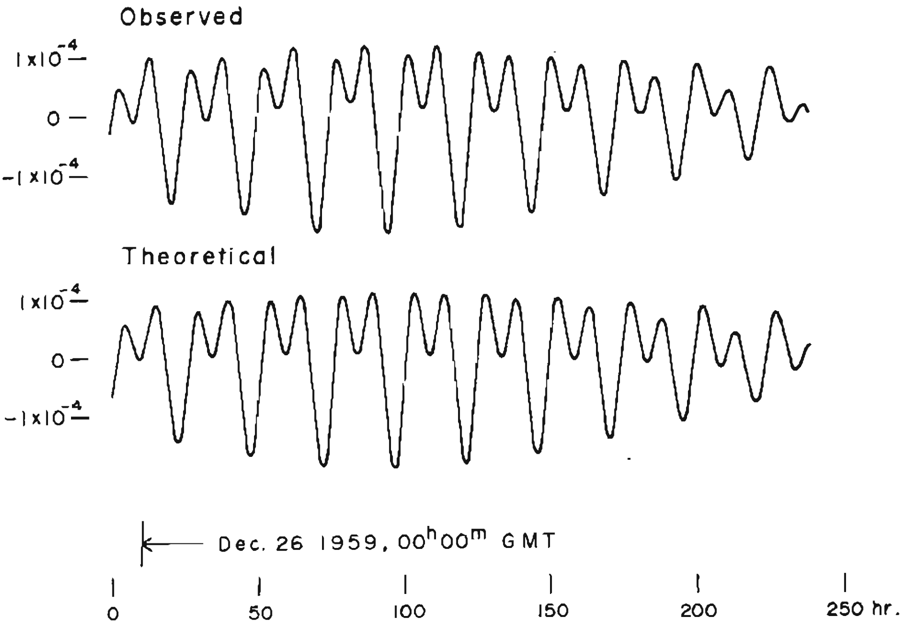

Fig. 4. Observed tidal gravity record (bandpass-filtered) at Kyoto and the corresponding theoretical gravity in December, 1959–January, 1960.

record, which has been obtained at the Kamitakara Crustal Movement Observatory ($\lambda=137°19'38''$E, $\varphi=36°16'25''$N) in the northwestern Chubu region during a period from August–September in 1969. The strainmeter used here (E_2) is of the N45°W-component, one of the instruments installed there with 28 m long quartz-tube type, and has a sensitivity of about 3.5×10^{-9} /mm on photographic recording paper (Doi *et al.*, 1976). The theoretical strains for the corresponding period are given in the lower trace. It is immediately noticed that there are excellent agreements in the waveforms and times between the observed and computed traces, except for some difference in the absolute amplitude level. These agreements assure a high quality of the results obtained for tidal admittance from the time domain analysis described below.

The record given in the upper part of Fig. 3 has been obtained at the same station in May, 1976, with a newly designed horizontal pendulum tiltmeter with a transducer of differential transformer type and an electronic recorder (Kato, 1976). The tiltmeter is installed in the S45°W direction and has an overall sensitivity of approximately 5×10^{-10} rad./digit. The corresponding theoretical tilts shown in the lower part include long-period components and thus appear to deviate from the zero level. The agreement between the two traces is not so satisfactory as in the case of strains. Although diurnal tides are predominant in the record and appear to agree with those predicted from theory, the recorded semidiurnal components are appreciably smaller than the theoretical amplitudes. The reason for this is not clear at this moment, but the predominant diurnal components might be much affected by the cavity effects as described by Harrison (1976) due to daily temperature variations, since the horizontal pendulum tiltmeter has been set up making an angle of 45° to the tunnel wall. The effects might be different in case of a tiltmeter installed paralell to the vault but more detailed discussion will be made in comparison with a long-range water-tube tiltmeter (Kato, 1976).

Fig. 4 shows a comparison between the observed and calculated tidal gravity. The observed record is a part of a long term record that has been obtained at Kyoto by an Askania gravimeter during one year from August 1, 1959 to July 31, 1960 (Nakagawa, 1962). A best agreement can be noticed in this case between the observed and predicted earth tides.

The above three comparisons indicate that the tidal admittance may be estimated to a high precison from the time domain analysis in the case of gravity and strain observations, but to somewhat less accuracy in the case of tilts.

3.3 Tidal admittance from least squares fitting in the time domain

The observed records are correlated here by the least squares fitting in the time domain with theoretical solid-earth tides. This procedure directly gives us the tidal admittance averaged over tidal frequencies.

Let $g(t)$ and $f(t)$ be the observed and theoretical tide functions, and α and Δt be the amplitude ratio and the phase delay time, respectively. The residuals R_j between the two time series at any time t_j are given as below, if we take up to the second term of Taylor's expansion of $f(t)$,

$$\begin{aligned} R_j &= g(t_j) - \alpha f(t_j - \Delta t) \\ &\cong g(t_j) - \alpha f(t_j) + \alpha \cdot f'(t_j)\Delta t \end{aligned} \tag{18}$$

where $f'(t_j)$ is the first derivative of $f(t)$ at t_j. The least squares conditions $\partial E/\partial\alpha=0$ and $\partial E/\partial\Delta t=0$, $(E=\sum_{j=1}^{N} R_j)$ yield the normal equations,

$$\begin{aligned} &\alpha[ff]-[gf]+\alpha\Delta t^2[f'f']+\Delta t[gf']-2\alpha\Delta t[ff']=0 \\ &\alpha[ff']-\alpha\Delta t[f'f']-[gf']=0 \end{aligned} \tag{19}$$

The first equation can be reduced to, using the second one,

$$\alpha[ff]-\alpha\Delta t[ff']-[gf]=0$$

The solutions are given by,

$$\begin{aligned} \alpha &= ([gf][f'f']-[gf'][ff'])/D \\ \Delta t &= ([gf][ff']-[gf'][ff])/\alpha D \end{aligned} \tag{20}$$

Probable errors for the above two quantities are,

$$\delta\alpha = r\sqrt{\frac{[f'f'][\varepsilon\varepsilon]}{D(N-2)}}, \quad \delta(\alpha\Delta t) = r\sqrt{\frac{[ff][\varepsilon\varepsilon]}{D(N-2)}}, \quad (r=0.67446) \tag{21}$$

wher

$$D=[ff][f'f']-[ff']^2, \; [\varepsilon\varepsilon]=[gg]-\alpha[gf]+\alpha\Delta t[gf']$$

and

$$\delta \Delta t = \delta(\alpha \Delta t)/\alpha - \Delta t \delta\alpha/\alpha$$

The admittance obtained by the present analysis for the observed record in Figs. 2 and 4 is,

$\alpha = 1.1033 \pm 0.0064$ and $\Delta t = 0.246 \pm 0.009$ hr for strain, and

$\alpha = 1.0142 \pm 0.0043$ and $\Delta t = 0.082 \pm 0.010$ hr for gravity.

The determined α values differ slightly from unity, suggesting some departures of the observations from the response to an idealized solid earth, as mentioned in the Introduction. It is to be also noted here that the time lag obtained includes not only the combined phase lags of a number of constituents involved in the observed tides but also possible time lags from insufficient time accuracy in the observation and small phase distortions in the course of filtering process. The rather large lag for the strain observation may be due to the second reason, in view of the recording paper speed of 3 mm/hr. In the case of gravity observation, the lag is reasonably small as compared with the time accuracy. A possible lag from the last reason can be avoided if the theoretical tides are also bandpass-filtered by the same filter as applied to the observed records.

The tidal factor of gravity has been calculated by the present method for every month's data over the whole observation period to check its stability. The results show some fluctuations that may be compared with those calculated by the Fourier transform method (Mikumo and Nakagawa, 1968). These fluctuations may be due to some observational reasons at that time, and have been much reduced in recent gravimetric observations. This good stability will lead us to proceed to discuss time variations of the tidal admittance from high quality observations.

The present technique is somewhat similar to the response method described by Munk and Cartwright (1966) and Lambert (1974), in that theoretical tides are used as input functions to estimate the tidal admittance. In our method, however, the admittance is not determined in a complex form as a function of frequency by allowing several numbers of lags as in their method, but only one amplitude ratio and time lag averaged over the whole time series are obtained. This would be slightly simpler, and sufficient for some particular purposes such as finding general behaviors and time variations of the observed earth tides.

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