An Experimental Study on Transformation and Run-up of Long Period Waves on a Gentle Slope of a Beach

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Abstract

An experimental study on shoaling and run-up of long period water waves on a gentle slope in a small basin was carried out. The surge tip of the waves on the beach model was also studied experimentally in order to evaluate friction factor as a function of the wave period.

1. Introduction

On the coasts of the circum-Pacific zone, tsunamis have repeatedly caused very much suffering. To protect the coasts, several countermeasures against tsunamis have been designed and effected. It is necessary to refer to studies on shoaling and run-up of the tsunamis on a beach if the countermeasures are to be effective. There have been many studies on shoaling and run-up of long waves. The author will introduce an experiment concerning shoaling and run-up of tsunamis, which are assumed to be similar to long period waves generated in a small water basin in order to determine any experimental facts which have not yet been remarked, and to compare the experimental results to theoretical values obtained by some of the existing theories.[1]

At first, the shoaling of the wave on the slope is studied through the experimental data and the small amplitude wave theory to introduce a factor of wave steepness in the study of the waves in shallow water on a rather gentle slope, i.e., 1/50, and to find a discrepancy between the trend of the experimental result and the theory. Refering to some existing theories, the transformation of the long period waves are considered as a nonlinear and nonbreaking wave on the slope.

As for run-up of the long period waves, a comparison of the existing experimental studies to the author's experimental result is done to reveal a misleading of a simple extrapolation of shorter period waves on the beaches refering to the experiments, for example, by Saville[2], Kaplan[3] and Iwasaki et al[4]. Adding to that, the tip characteristics of the waves running up on the gentle sloping beach are studied experimentaly. An analysis on the slope and velocity of the tip are focused to study whether Cross' theory fits them or not and to evaluate a frictional factor of the surge running up on the slope refering to his theory. This is also necessary from the point of view of assuring the propriety of the treatment of tsunami surge as a surge generated by a model dam breaking[5),6)].

2. Transformation of Long Period Waves

2.1 Experimental Equipment and Instruments

A small bay facing an ocean is imagined, as shown in Fig. 1 so as to be several hundreds meter wide at the bay mouth, several hundreds meter long with a slope of one hundredth in a river. The assumed water depths are several tens of meters at the bay mouth, ten odd meters in the bay and a few meters in the river.

The water basin used in this experimental study is made of concrete mortar with dimensions of 20 m long, 0.9 m wide and 0.4 m deep. At the one end of the basin a generator of long period waves, which is able to produce waves of 0.3 to 30 min in period with the wave height of less than 3 cm. For the purpose of experimental convenience, the water depth in the basin is taken to be 10 cm. A slope of one fiftieth is prepared below the water surface and a slope of one over two hundred is arranged above the water surface in the basin. The two conditions of the slope are a rigid slope made of smoothened concrete mortar and a movable and permeable slope of fine sand.

If the Froude's similitude is applicable to the waves in the experiment, we may estimate the characteristics of the tsunamis on the gentle slope on the ground of the characteristics of the waves in the experimental basin[7),8)]. When the scale of length is one over four hundred, the period of the tsunami, 30 min, might correspond to the period of the wave, 1.5 min, in the basin.

In this study, the wave generator is arranged to form a sinusoidal wave at the toe

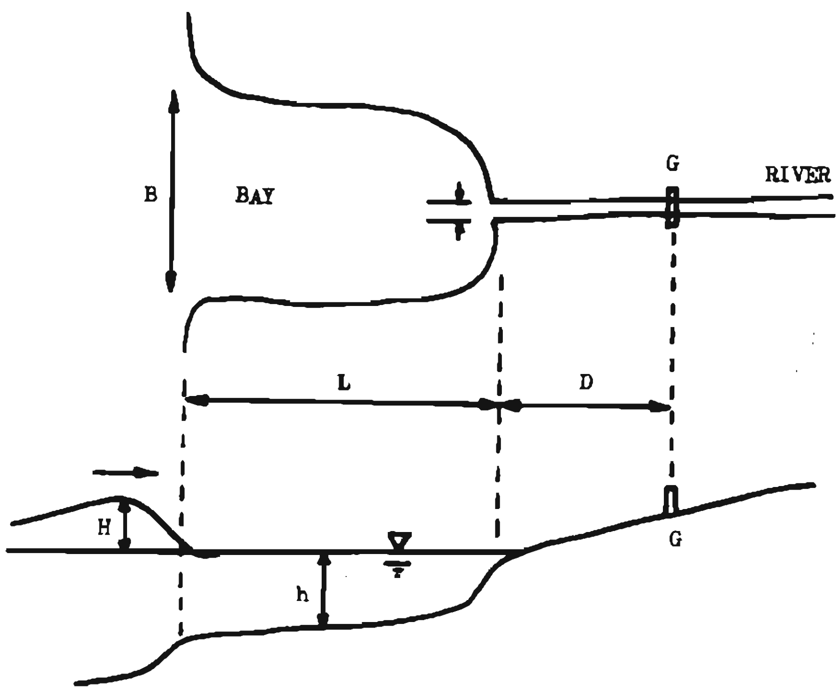

Fig. 1. Notation and schematic sketch of a model bay.

of the slope in the basin as far as possible to study how the wave transforms on the slope as the water depth decreases.

2.2 Wave Records and Observations

Wave records are obtained by use of electric resistive wave meters, which are located at 3 m off the shore line (St. 1), 1 m off the shore line (St. 3), on the shore line (St. 5) and 1 m above the shore line (St. 6).

The reference wave forms are produced and checked at St. 1 near the toe of the slope. For a long period of the wave, the breaking phenomena cannot be observed on on the slope. Only for a shorter period of the wave, the wave form is observed to transform into an undular bore or a surge. And some of the waves on the shore line form a discontinuity in advance of the arrival of the wave crests. The processes of the wave transfomrations in the basin are observed spatially and recorded on a chart of a pen oscillograph.

2.3 Transformation of Waves on a Slope and Linear Theory

When the small amplitude wave theory is taken as a reference for the analysis of the experiment, the transformation of the wave might be treated as a shoaling[9]. When the wave height is H, the wave celerity is C and the shoaling coefficient K_s is

$$K_s = \frac{H}{H_0} = \sqrt{\frac{1}{2n}\cdot\frac{C_0}{C}}, \tag{1}$$

where

$$n = \frac{1}{2}\left[1+\frac{(4\pi h/L)}{\sinh\,(4\pi h/L)}\right] \tag{2}$$

and h is the water depth and L is the wave length. The suffix 0 means the value for the deep water. And

$$\frac{C}{C_0} = \frac{L}{L_0} = \tanh\left(\frac{2\pi h}{L}\right). \tag{3}$$

So that, from (1) and (3), a formal relation of the wave steepness is reduced as follows,

$$\frac{H/L}{H_0/L_0} = \sqrt{\frac{1}{2n}}\cdot\left(\frac{C_0}{C}\right)^{3/2}. \tag{4}$$

Now, consider the waves of H_1 and H_2 at St. 1 and 2, respectively. If the small amplitude wave theory for shallow water of a constant depth is applicable as an approximation of the wave on a gentle slope, a relation

$$n_1 \approx n_2$$

is obtained for the condition of $(h/L) \ll 1$. And from (4),

$$R(K_s) = \frac{H_1/H_0}{H_2/H_0} = \sqrt{\frac{C_2}{C_1}}. \tag{5}$$

In the experiment using a water basin, it is impossible to give the ideal condition of the deep water wave. So that, the wave at St. 1 is taken as a reference in this study to ease the comparison of the experimental result and the small amplitude wave theory. From (3),

$$\frac{C_1}{C_2} = \frac{L_1}{L_2} = \frac{\tanh (2\pi h_1/L_1)}{\tanh (2\pi h_2/L_2)} \tag{6}$$

and from (5) and (6),

$$\frac{H_2/L_2}{H_1/L_1} = \left(\frac{C_1}{C_2}\right)^{3/2}. \tag{7}$$

The relations in (5) and (7) are formally independent of the wave period.

Now, let us consider the wave height H_3 at St. 3 on the slope in the experimental basin. The wave height ratio H_3/H_1 for $h=10$ cm is considered as function of a non-dimensional wave period $T\sqrt{g/h}$ to plot on Fig. 2. In Fig. 2, the experimental results are shown introducing a parameter of wave steepness (H_1/L_1) or (H/L). The height between the crest and the trough of the observed wave is defined as the wave height H_1 and the wave length L_1 is estimated by use of the relation

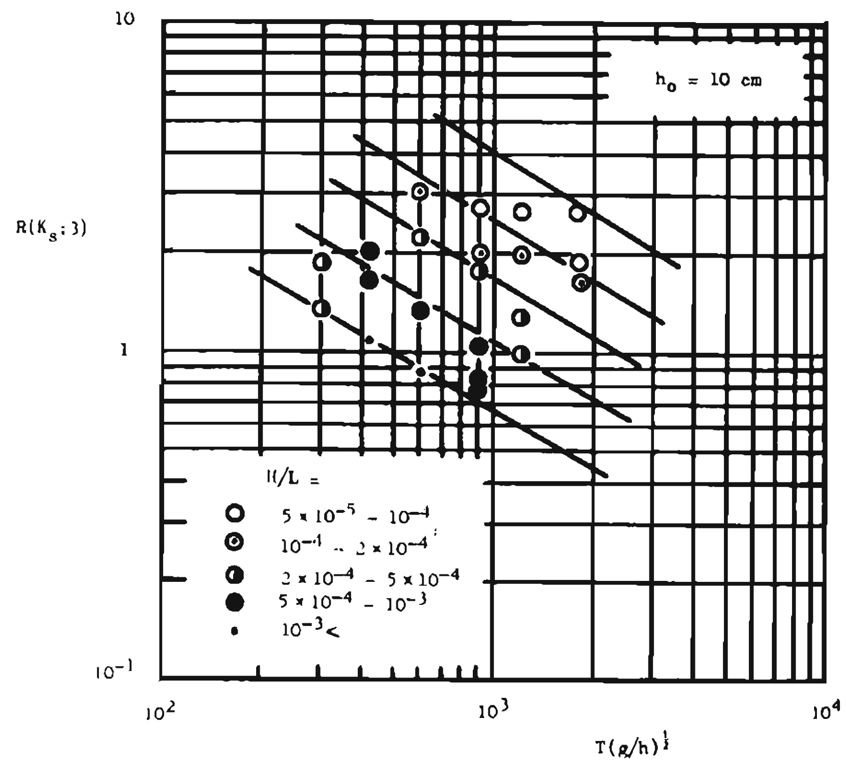

Fig. 2. Relation between $R_3(K_s)$ and $T\sqrt{g/h}$ with a parameter of (H/L).

$$L_1 = \frac{gT^2}{2\pi}\tanh\left(\frac{2\pi h_1}{L_1}\right). \tag{8}$$

Especially, in case of $(2\pi h_1/L_1) \ll 1$, an approximation is

$$L_1 = T\sqrt{gh_1} = C_1 \cdot T \tag{9}$$

When only the effect of the water depth is taken into account when using the small amplitude wave theory, an approximation is obtained as follows,

$$R_3(K_s) = \frac{H_3}{H_1} = \left(\frac{h_1}{h_3}\right)^{1/4} = \left(\frac{C_1}{C_3}\right)^{1/2} = 1.11 \tag{10}$$

for $h_1=6$ cm and $h_3=4$ cm. Though the experimental results show that the factor of $R_3(K_s)$ depends not only on the water depth but also on the wave period, as is understood in Fig. 2.

Another diagram is obtained by the data at St. 3 and St. 4, where $h_1=6$ cm and $h_4=2$ cm and

$$R_4(K_s) = \frac{H_4}{H_1} = \left(\frac{h_1}{h_4}\right)^{1/4} = \left(\frac{C_1}{C_4}\right)^{1/2} = 1.32 \tag{11}$$

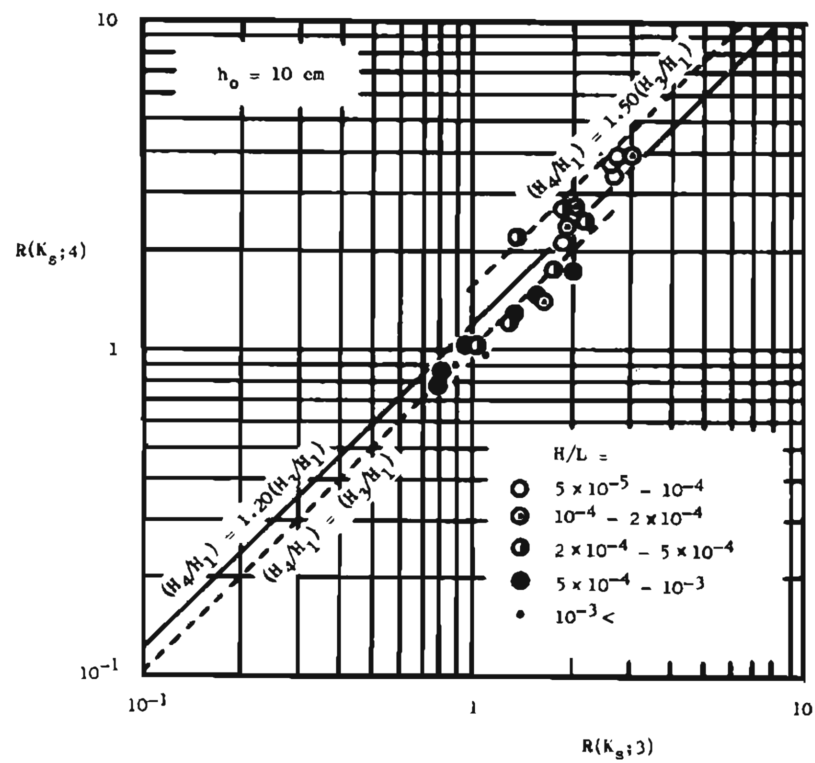

Fig. 3. A linear theory and correlation of $R_3(K_s)$ and $R_4(K_s)$ for $h_0=10$ cm.

The two factors of $R_3(K_s)$ and $R_4(K_s)$ are correlated to compare the experimental results and the small amplitude wave theory. In Fig. 3, the author can remark that the values of $R_3(K_s)$ and $R_4(K_s)$ take larger values respectively when the wave steepness is larger. Detailed analysis shows that the water depth h_1 affects the relation between $R_3(K_s)$ and $R_4(K_s)$. The reflection of the wave depends on the period of the incident wave, so that the results of the experiment as in Fig. 2 and Fig. 3 might suggest that the reflection coefficient of the wave might be an important factor for the waves in the experimental basin.

2.4 Nonlinear Transformation of Waves on a Slope

As for long waves of finite amplitude, Greenspan[10], Carrier and Greenspan[11] and others made studies on transformation of long waves in variable water depth. Similar studies have been carried out in Japan, for example, by Kishi[10] and Shuto[13]. The author refers now to the studies on non-breaking long waves by Kishi and Shuto to compare the experimental results of long period waves in a small wave basin.

From Kishi's theory, a relation between the water depth and the wave height is shown by solid curves on a diagram as in Fig. 4 with a parameter of $M_0(=H_1/h_1)$. The experimental results are also plotted on the same diagram to find a qualitative trend corresponding to Kishi's theory. It is found in Fig. 4 that the wave heights obtained by the experiments vary in a fairly wide range but Kishi's theory calls for only a narrow range. As mentioned before, the plotted data is partly from waves transformed into undular bores. It is necessary now to give criterion of wave breaking. If an undular bore is accepted as a category of breakers, the theories of non-breaking long waves cannot be suitable to discuss the experiment of long period waves in a small

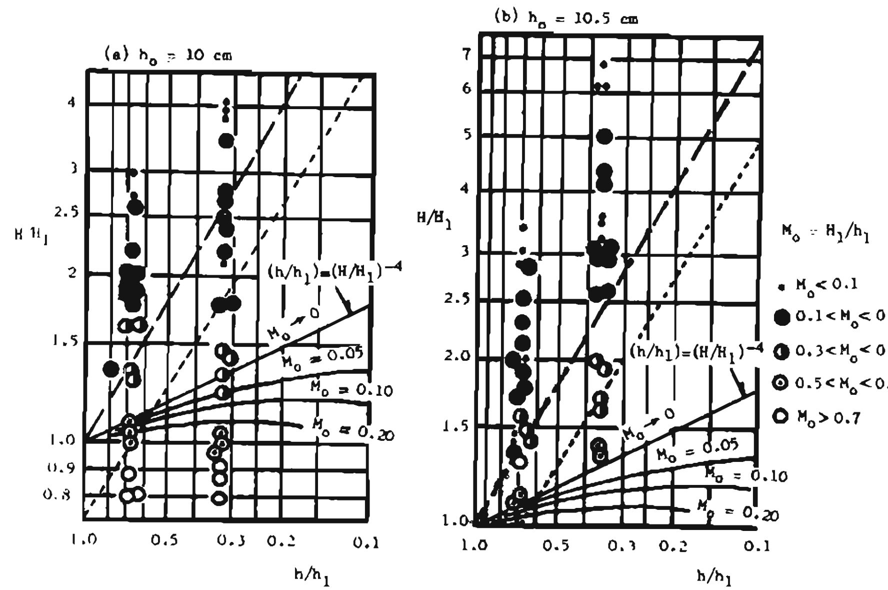

Fig. 4. Nonlinear transformation of long period wave.

basin.

Refering to studies in open channel hydraulics, an undular bore is included in a category of moving hydraulic jump which has a discontinuity of water level. And in the studies of wave breaking in shallow water (for example, Galvin)[14], an undular bore seems not necessarily considered as a breaker rather than as a phenomena of separation of a wave into solitions. The author cannot, at present, identify wave breaking as a separation into solitions refering to these studies. The reason why the wave height in the experiment scatters in the diagram is not yet solved. Confirmation should be made to assure whether the boundary conditions in the experimental basin is same as the boundary conditions in the theory.

Recently, Shuto developed a theory on shoaling of nonlinear long waves. He solved to reduce a relation

$$\left(\frac{h}{h_1}\right)=\left(\frac{H}{H_1}\right)^{-1} \tag{12}$$

for wave height of nonlinear long wave on a gentle slope. This relation (12) is shown by a chain line in Fig. 4 to compare it to the experimental trend and Kishi's theory. A dot line is also drawn in Fig. 4 which is refered to Shuto's theory for $s=0.023$.

Judging from the diagram in Fig. 4, the waves of $M_0<0.3$ fit fairly in to Shuto's theory rather than Kishi's theory. Shuto's theory gives a relation shown by a dot line in Fig. 4 for bed slope 0.023.

3. Run-up of Long Waves

3.1 Elevation of Run-up

As for run-up of waves on beaches, there have been many studies as found in the manuscript by Meyer and Taylor[15]. The author tried to find experimentally characteristics of run-up of long period waves in a small basin, and to compare the author's experiment to several existing experiments. One of the most interesting problems is to give an answer to the question whether a simple extrapolation of the experimental run-up for the shorter period wave is acceptable to estimate run-up of long period waves.

The experimental result is shown on a diagram of relative run-up R/H_1 and wave steepness H_1/L_1 (cf. Figs. 5 and 6). In Fig. 5, experimental run-up by Kaplan[3] is drawn by solid curves for plane slope of 1/30 and 1/60. Saville's run-up is also shown in the same diagram by dot lines for slope 1/10 with conditions of $(h/H_0)=0$ and $(h/H_0)>3$ and for slope 1/30 with $(h/H_0)>3$. Adding to them, an experimental run-up on slope 1/50 of Iwasaki's is plotted in Fig. 5.

Kaplan and Saville did their experiments for the waves of 1 or 2 sec in period and Iwasaki et al. studied for the experimental waves of several minutes as were the the waves in the author's experiment. Judging from the diagram in Fig. 5, a simple extrapolation of Kapaln's experimental formula gives an overestimation for the

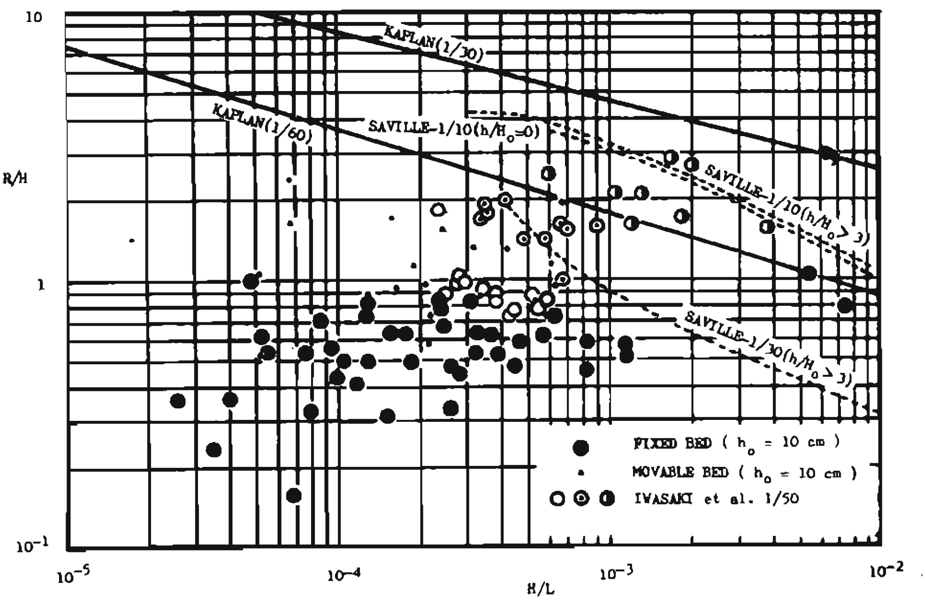

Fig. 5. Run-up (R/H) as a function of wave steepness (H/L).

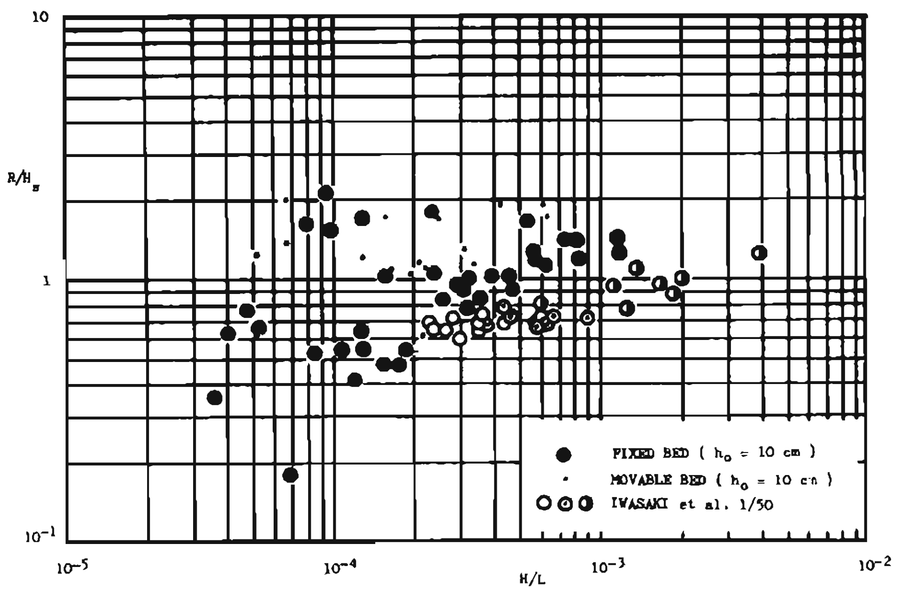

Fig. 6. Run-up refered to H_s as a function of wave steepness (H/L).

rup-up of the long period waves in the basin. As similar remark is found in the contribution by Iwasaki et al. who also studied run-up of the long period waves in an experimental basin. From Saville's and Kaplan's experimental formulae, the author can suggest that the overestimation of the run-up is in the range of $(H/L) < 10^{-3}$.

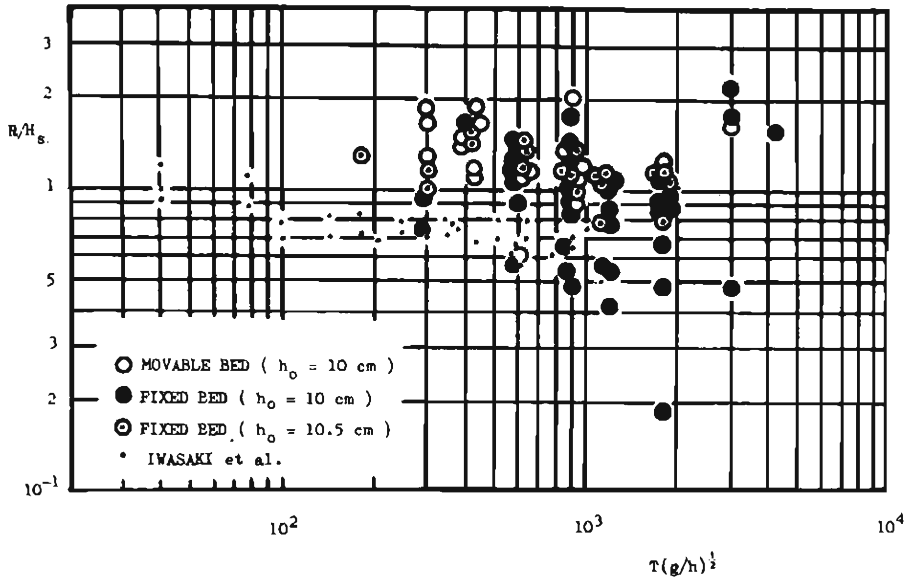

Fig. 7. Run-up refered to H_s as a function of nondimensional wave period $T\sqrt{g/h}$.

To detect why the overestimation is, Iwasaki et al. classified the wave pattern into three categories, i.e., non-breaking, breaking and surge. In the author's experiment, there was scarcely found breaker, in the form of plunging. Some of them might be in the category of spilling breakers. Iwasaki et al. considered the category of non-breaking in the range of $(H/L) \leqslant 4 \times 10^{-4}$.

When the maximum water depth at the shore line H_s is taken as the reference instead of that at St. 1, a factor R/H_s can be introduced to obtain a diagram as in Fig. 6. In Fig. 6, the experimental run-up obtained by Iwasaki et al. is also plotted for the slope of one fiftieth. Judging from Fig. 6, the increasing trend of (R/H_s) with the increase of the wave steepness (H/L) cannot be resulted as mentioned even by Kapaln and Saville.

A diagram of (R/H_s) and $T\sqrt{g/h}$ is shown in Fig. 7. Though Iwasaki et al. gave a region of $T\sqrt{g/h} > 4 \times 10^2$ as a zone of non-braaking, the author's experimental results in the above region are not necessarily non-breaking, but are in the form of undular bore. So that, Iwasaki et al. might have considered the undular bore as a separation of wave into solitons instead of as an analogy of hydraulic bore with a discontinuity of water level.

Adding to the above, an experimental effect of water depth h_0 can be found in a diagram of (R/H_2) and (H/L) as shown in Fig. 8. The experimental result shows tha the run-up is smaller when $h=10$ cm than when $h=10.5$ cm. And the effect of water depth is not so significant for the larger region of (H/L) in the diagram. The slope is taken as mentioned in section 2.1 so that the slope for $h_0=10.5$ forms a composit slope. In Fig. 8, the difference of (R/H_2) for the two values of h_0 should include

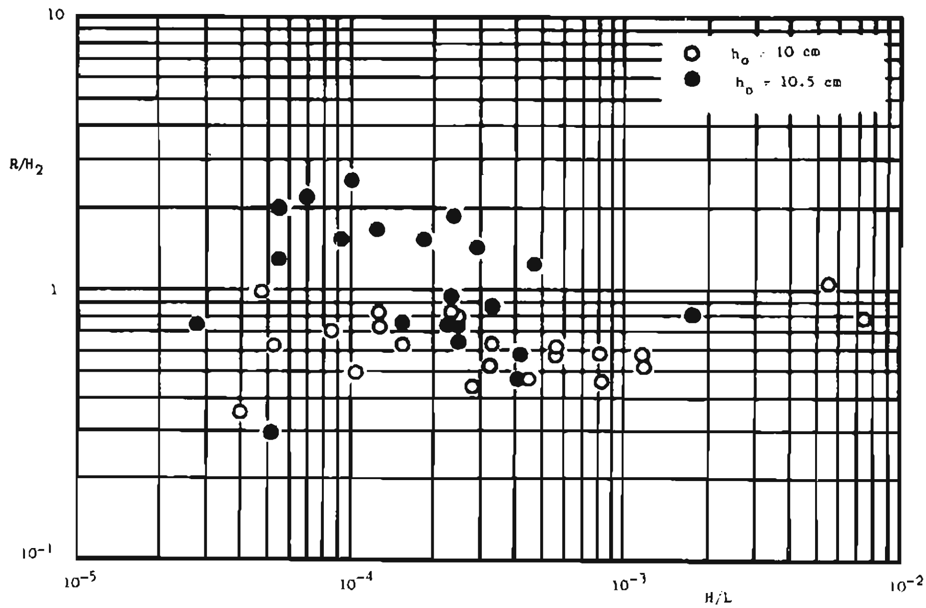

Fig. 8. Effect of water depth in the relation between (R/H_2) and (H/L).

the effect of the composite slope. If the effect of the composite slope is accepted, the experimental results for a movable bed in Figs. 5 and 6 are consistent because from the toe of the slope of the movable bed consisted by fine sand, which moves time to time under the induction of waves, and the plane slope is eroded to form a concave slope under water.

Recently, Madsen and White[16] developed a study on energy dissipation on a rough slope to show a diagram for theoretical prediction of run-up on rough slopes. In this diagram a length of the slope l_s is considered to compare wave length of the small amplitude wave. In the author's experiment, the value of l_s is 5 m from the toe to the shore line, and the wave length is in the range of 18 to 180 m. So that, the ratio l_s/L_s is in the range of 0.27 to 0.028. The prediction of run-up by Madsen and White is 1.0 to 1.2 for l_s/L_s in the range of 0.0 to 0.1 and the effect of the friction angle is not so significant. The author believes the trend of the experiment in Figs. 5 to 8 seems to be qualitatively consistent with the theoretical prediction of run-up on rough slopes.

3.2 Velocity of Surge on a Gentle Beach

In this section, characteristics of the tip of surge, which is formed by a long period wave running up a gentle slope of the beach is considered. Any spatial profile of the long period waves or the surge tip cannot be obtained through experiment, so that the estimation of the spatial profile has been carried out by utilizing the record of the wave or the surge on a chart of oscillograph and the simultaneous observation of the tip velocity under some arrangement and assumptions.

First of all, the tip velocity is considered in order to analyze the characteristics of the surges on the slope. For the ease of the analysis, no water layer is considered on the beach slope before the arrival of the surge tip. The velocity of the surge profile should be same as the velocity of the water particle at the surge tip. In the analysis, the recorded waves and surges at the shore line (St. 5) and on the beach 1 m above the shore line (St. 6) are considered to define the mean velocity of the surge tip C_s. The mean velocity C_s is defined as a quotient obtained by the distance of the two stations as the numerator while the time elapse of a surge tip between the two stations as the denominator is assumed to be the mean velocity.

The mean velocity of the surge tip defined and obtained as above is characterized, for example, by a diagram as shown in Fig. 9. In the diagram, a parameter is introduced as a wave steepness. The trend shows that the tip velocity seems intensively dependent on the wave steepness as well as on the wave period. From

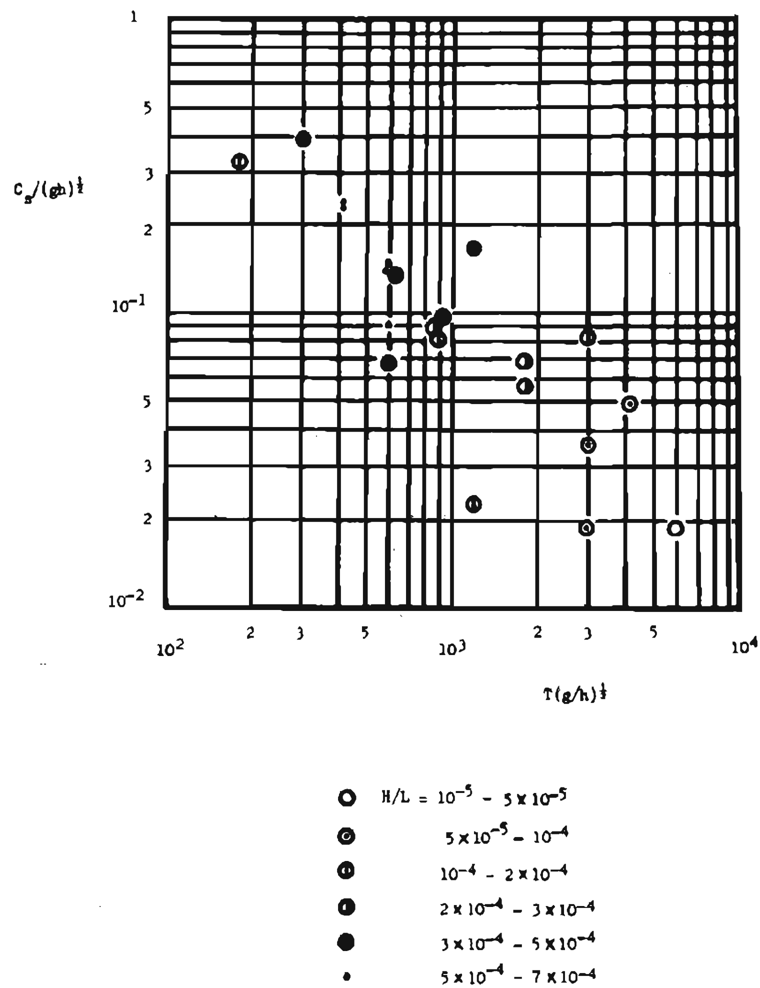

Fig. 9. Surge tip velocity $C_s/\sqrt{gh}$ as a function of $T\sqrt{g/h}$.

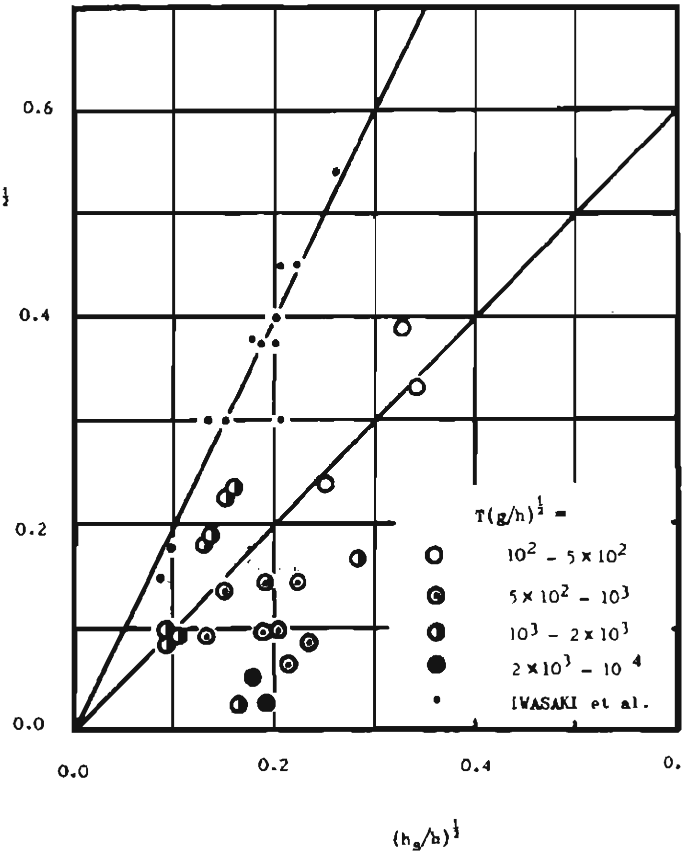

Fig. 10. Relation of $C_s/\sqrt{gh}$ and $\sqrt{h_s/h}$.

the diagram, it is left to detect the effect of the slope angle of the beach. And that the simple extrapolation for waves of several seconds cannot be suggested by the results for waves of more than twenty seconds in the diagram.

For convenience, a water depth of the surge tip h_s is introduced to define the water depth 0.5 m behind the surge tip and to relate C_s and h_s. In Fig. 10, the relation of C_s and h_s is shown. Iwasaki et al.[4] reduced a relation of $C_s = 2\sqrt{gh_s}$ after their experiments, though the author's experiment resulted to appreciate the relation of Iwasaki's as a trend of the maximum velocity of the surge tip.

3.3 Friction Factor at Surge Tip

One of the the existing theories of surge tip profile is Cross'[17]. Following the theory by Cross, let us consider a surge tip velocity u on a dry bed of a slope s as shown in Fig. 11. Gravitational force, body force by an accerelation, friction and hydrostatic pressure are considered.

When a propagation velocity of long wave in a shallow water of h_0 deep

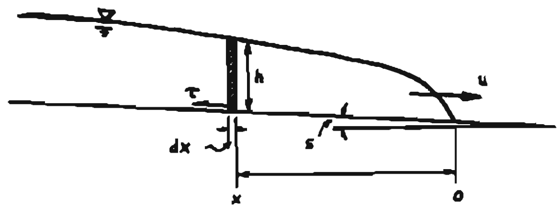

Fig. 11. Schematic sketch of surge tip.

$$u_0 = \sqrt{gh_0} \tag{13}$$

is taken to be a reference, Cross' theory might be reduced as

$$\left(\frac{h}{h_0}\right)^2 = 2\left(\frac{X}{h_0}\right)\left(\frac{g}{C_z^2}\right)\left(\frac{u}{u_0}\right)^2 \tag{14}$$

for a surge moving at a constant velocity, u on a flat bed, where h is the water depth at X distant from the surge tip and C_z is Chezy's constant. Denoting K as

$$K = 2\left(\frac{g}{C_z^2}\right)\left(\frac{u}{u_0}\right)^2 \tag{15}$$

and when $(u_0/u)=\sqrt{2}$, $C_z=40.4$ and $g=9.8$ m/sec^2,

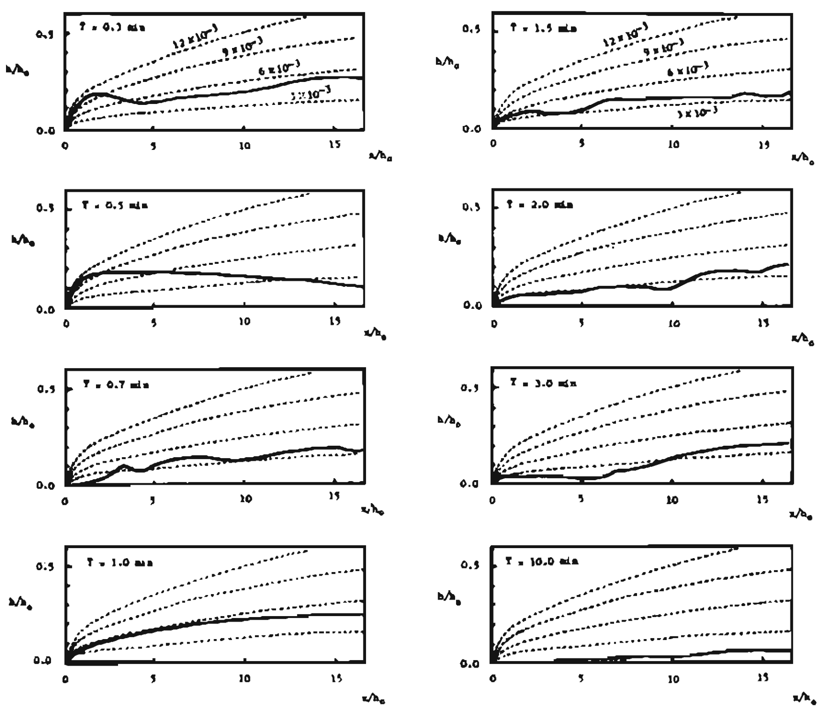

Fig. 12. Profiles of surge tip of long period wave reduced from expetiments and obtained by computations with a parameter of friction factor K.

$$K = 6\times10^{-3} \tag{16}$$

In Fig. 12, the computed surge tip profiles are shown for the K values of 3×10^{-3}, 6×10^{-3}, 9×10^{-3} and 1.2×10^{-2} by dot lines. And in Fig. 12, the estimated surge tip profiles are shown for water waves of various periods by full lines. These profiles are estimated under an assumption that the time lapse on the chart can be equivalent to the quotient of the horizontal distance by the mean velocity of the surge tip on the beach of the gentle slope.

When the surge tip profiles in the range of $0<(x/h_0)<0.5$ are considered to estimate the equivalent friction factor K, for example, from the experimental results as shown in Fig. 12, a diagram of K and $T\sqrt{g/h}$ can be obtained as in Fig. 13 to show that the estimated value of K depends on the period of the waves in the experiment. The shorter the wave period, the larger the value of the friction factor. As for the comparison between the surge tip from the experiment in the wave basin and the surge tip produced in an experiment of hydraulic bore[18], an example is shown in Fig. 14. In Fig. 14, the computed surge tips refering to Cross' theory are shown by dot lines and the estimated surge tips in the experiment of hydraulic bore are shown by solid lines with a parameter $(u^2/(C_z^2h_0))$, which corresponds to K when the value of

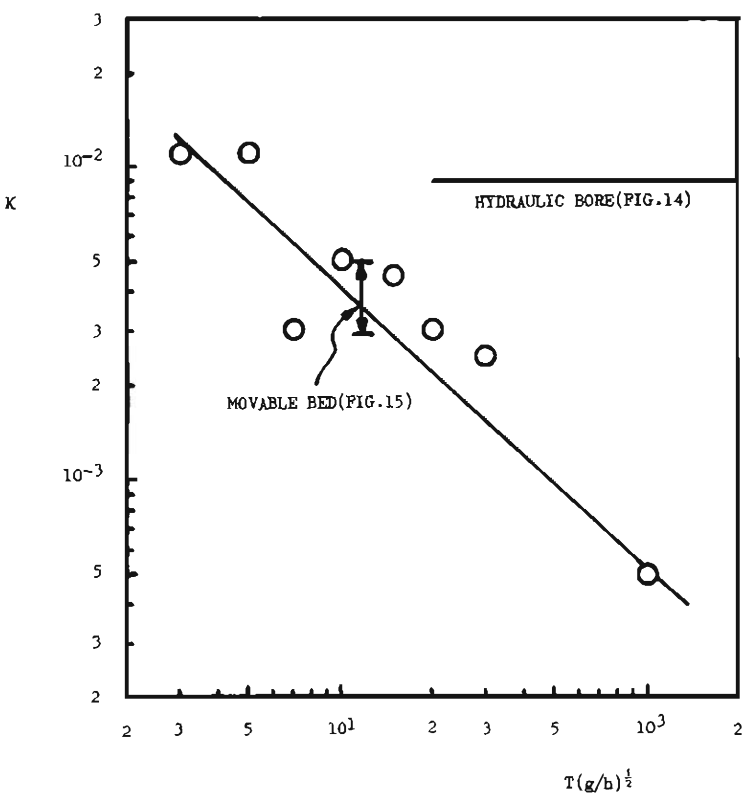

Fig. 13. Friction factor K as a function of $T\sqrt{g/h}$.

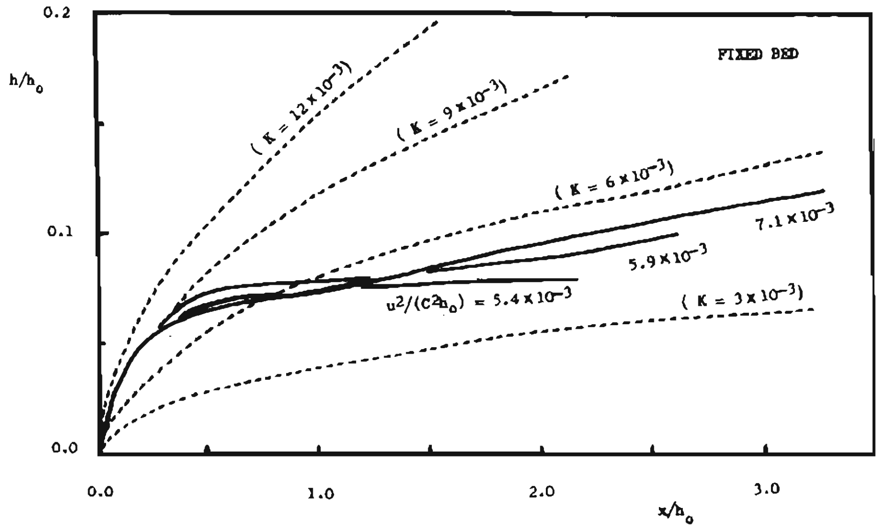

Fig. 14. Profiles of surge tip in an experiment on hydraulic bore.

$(2/u_0^2)$ equals to unity or $u_0=1.4$ m/sec.

And when the value of K in Fig. 14 is estimated as above to plot in Fig. 13, it can be found that the surge tip in the experiment of hydraulic bore is rather equivalent to the waves of shorter period than to the waves of longer period in the author's experiment.

Judging from the diagram in Fig. 13, a simple surge occuring on the slope dissipates a fairly large fraction of wave energy when the period of wave is shorter. The wave of longer period losses a small fraction of the wave energy to separate into solitones to distribute the initial wave energy on the slope. As for the longest period of wave in Fig. 13, the water surface near the shore line seems to change only the elevation as the surface is kept nearly horizontal. These results might be an important factor to detect the mechanism of the transformation of the long period waves as have studied before in this paper.

In Figs. 6 and 8, the dependency of the wave period for the run-up is considered. The trends in Figs. 6, 8 and 13 are obtained from the same source as of the experiment, though all of these trends are difficult to unify at present.

3.4 Surge Tip on a Slope of Sandy Beach

In the preceding sections, the waves on a rigid slope made of mortar have been focused and analyzed. Now, the author would like to add an analysis of the waves on a beach of a porous media, i.e., a sandy beach. For this purpose, the slope and the beach are formed of fine sand. An example of the experiment for $T=1$ min. is shown by a solid lines in Fig. 15. In these cases, the value of K of a sandy beach is apparently of a rigid beach. And that, permeability of the beach should be considered as an important factor to dissipate the wave energy because the beach permeates the water of the waves above the shore line. Although, the solid lines in

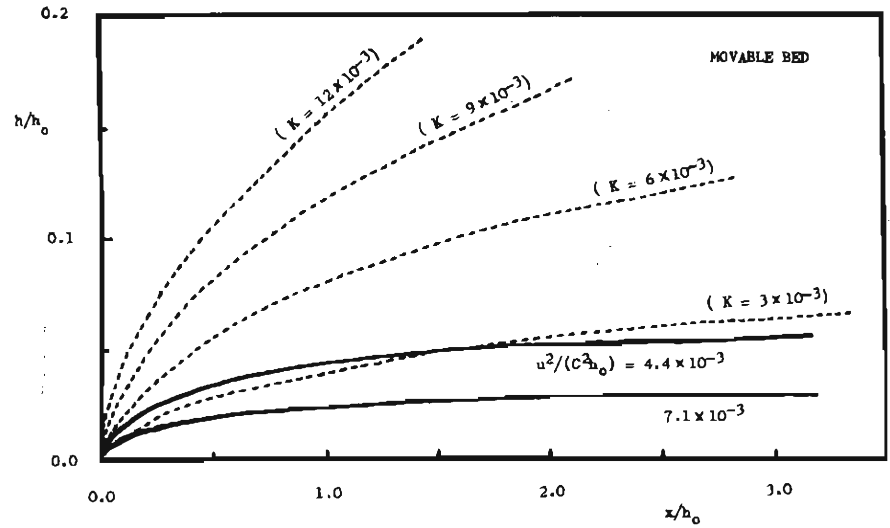

Fig. 15. Profiles of surge tip of long period wave on a sandy beach.

Fig. 15 suggest that the waves form thin surge tips on the porous beach. These might be the permeation of the water into the sandy beach. And the results of the observations show that the value of the friction factor on the sandy beach is smaller than that on the rigid beach. This may cause a misunderstanding as if the energy dissipation of the wave on the sandy beach is smaller than that on the rigid beach.

4. Concluding Remarks

The author has studied experimentally sholaing and run-up of long period waves on a gentle slope in a small basin. At first, an analysis was carried out to find the fitness of the waves in the author's experiment to a linear theory, to the existing experiments of Iwasaki's, the existing nonlinear theories developed by Kishi and by Shuto. The author noted the importance of wave steepness and introducd it as a parameter through the analysis. After that, the run-up of the long period wave was also analyzed from the point of view of frequency characteristics. The surge tip of the long period wave on the beach was also studied experimentally to evaluate friction factor of the surge tip as a function of wave period. A concentrated discussion is given on an analogy of a hydraulic bore to the surge tip of the long period wave.

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