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On the Use of Analog Filters in Evaluating Amplitude Spectra of Seismic Waves

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Abstract

To investigate the frequency characteristic of a particular part of a seismic-wave train, such as P, S and other prominent waves, it may be best to observe the filtered waves using filters of various frequency-bands. This paper presents a method to obtain a smoothed amplitude spectrum by means of analog-filters. The procedure is quite simple and it is easy to treat a number of data by this method.

We take a series of band-pass filters, their transfer functions $B_n(\omega)$ $n=1, 2, \cdots$ and the peak frequencies $\omega_{0,n}$ $n=1, 2, \cdots$ being defined as follow:

$$B_n(\omega)=B_1(\omega/n), \quad \omega_{0,n}=n\cdot\omega_{0,1}$$

Then the spectral density at $\omega_{0,n}$ is approximately given by

$$|F(\omega_{0,n})| \approx a_n/n\cdot\alpha,$$

where a_n and α are the maximum trace-amplitudes of the wave filtered by the n-th filter and of the impulse response of the 1-st filter, respectively. Comparison of the measured spectrum with the theoretical one for a rectangular wave shows good agreement.

This method is applied to several examples of seismic waves from microearthquakes and found to be effective for distinguishing between various parts of the wave train in the frequency characteristic.

1. Introduction

To find out origins of waves appearing on a seismogram is one of the most important subjects in seismology. We sometimes identify some prominent waves as a direct P, a direct S, a refracted P and so on judging from the wave patterns. But, strictly speaking, it is necessary to investigate each wavelet from various standpoints such as the time of onset, orbit of particle motion and the time distribution of frequency components. The last point is concerned with filtering of waves. This technique is useful for distingushing between waves of different origins.

Filtering can be also applied to evaulating the amplitude spectrum of a certain part of a wave train. This paper presents a method for this purpose utilizing analog filters.

There was a time when so called Blackman and Tukey's method[1], where autocovariance functions are connected to power spectra, was supposed to be the most practical. Nowadays, however, the Fast Fourier Transform (F.F.T.)[2] has taken the place of the above method. If digital data or time series are available, Fourier transform can be easily carried out by F.F.T..

In the above methods we must use a data window to eliminate the unnecessary parts of the waves. There seems to be no determinate way to choose the optimum window. Therefore the spectrum obtained is largely dependent on the shape and the time interval of the window used.

For oscillatory or stationary waves with a long duration time, the Maximum Entropy Spectra Estimation Method is excellent as pointed out by Ouchi & Nagumo[3]. This method involves no data window and is useful for the detection of peak frequencies, but cannot be applied to transient waves.

When we wish to obtain the spectrum of a restricted part of a wave train, it is necessary to investigate carefully the patterns of not only original waves but also filtered waves. This enables us to find the appropriate window to extract the necessary part of waves.

We shall treat seismic signals which are recorded on magnetic tapes and can be reproduced in the form of electrical signals, and use active-filters which are constructed mainly with operational amplifiers.

2. Band-pass filters

Let us use a band-pass filter (B.P.F.) to extract the specified frequency-components. This filter can be set up by connecting a high-pass and a low-pass filter in series. It is well known that a high-pass (low-pass) active filter is characterized by a cutoff frequency $\omega_H(\omega_L)$ and diminishing slope below (beyond) the cutoff frequency. Our filters are of the Butterworth type and have a slope of 24 dB/octave in the logarithmic graph.

Hereafter we shall denote frequency and angular frequency to be f and $\omega(=2\pi f)$, respectively and define the Fourier transform and its inverse transform as

$$\left.\begin{aligned} F(\omega)&=\int_{-\infty}^{\infty} f(t)\cdot e^{-i\omega t}dt \\ f(t)&=\frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\cdot e^{i\omega t}d\omega \end{aligned}\right\} \tag{1}$$

It is convenient to treat a series of band-pass filters together as shown in Fig. 1. The cutoff frequencies $\omega_{H,n}$, $\omega_{L,n}$ and the transfer function $B_n(\omega)$ for the n-th filter are defined as follow:

$$\left.\begin{aligned} &\omega_{H(L),n}=n\cdot\omega_{H(L),1} \\ &B_n(\omega)=B_1(\omega/n) \end{aligned}\right\} \tag{2}$$

From (1) the impulse response of the filter is given by

$$b(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty} B(\omega)e^{i\omega t}d\omega \tag{3}$$

Combining Eqs. (2) and (3), the following relation is established:

$$\begin{aligned} b_n(t)&\equiv\frac{1}{2\pi}\int_{-\infty}^{\infty} B_n(\omega)e^{i\omega t}d\omega=\frac{1}{2\pi}\int_{-\infty}^{\infty} B_1(\omega/n)e^{i\omega t}d\omega \\ &=\frac{n}{2\pi}\int_{-\infty}^{\infty} B_1(\omega')e^{i\omega' nt}d\omega'=nb_1(nt) \end{aligned} \tag{4}$$

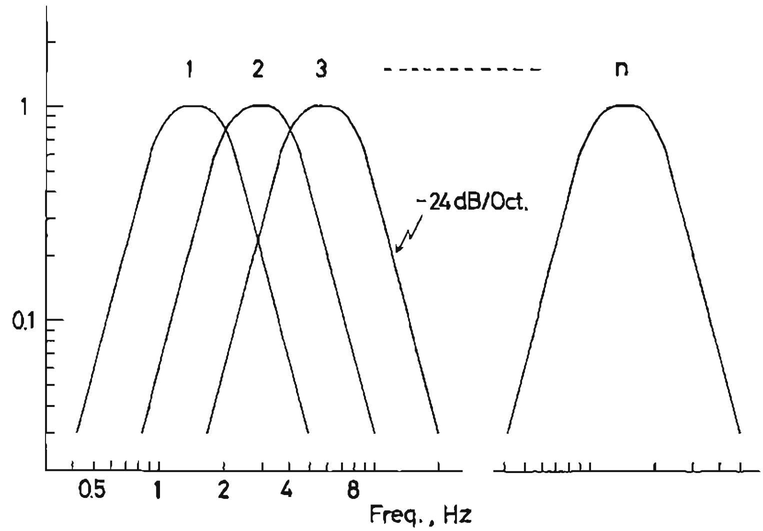

Fig. 1. Amplitude characteristics of the series of band-pass filters for $f_{H,1}=1\text{Hz}$ and $f_{L,1}=2\text{Hz}$.

Eq. (4) indicates that the amplitude of the filtered wave is proportional to the band width of the filter provided that the input wave is impulsive and the filter has the characteristic specified by (2). This feature may also hold approximately for an arbitrary waveform as an input.

3. Measurement of spectral density

Let $f(t)$ be an input wave and $g(t)$ the wave filtered by the band-pass filter, the transfer function of which is $B(\omega)$, then the Fourier transforms $F(\omega)$ for $f(t)$ and $G(\omega)$ for $g(t)$ have the relation

$$G(\omega)=B(\omega)\cdot F(\omega) \tag{5}$$

and the inverse transform of (5) is

$$g(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}B(\omega)\cdot F(\omega)\cdot e^{i\omega t}d\omega \tag{6}$$

The filtered wave should consist of frequency components with a narrow band around ω_0, the peak frequency of the transfer function, which is given by the cutoff frequencies as follows:

$$\omega_0=\frac{1}{2}(\omega_H+\omega_L) \tag{7}$$

In order to evaluate the spectral density at the frequency ω_0 we introduce the following formula for the input, which is considered to be approximately valid for a very narrow frequency band:

$$F(\omega)\approx|\overline{F(\omega_0)}|\cdot e^{-i\omega\cdot t_0(\omega_0)} \quad \text{for } \omega_H\lesssim\omega\lesssim\omega_L \tag{8}$$

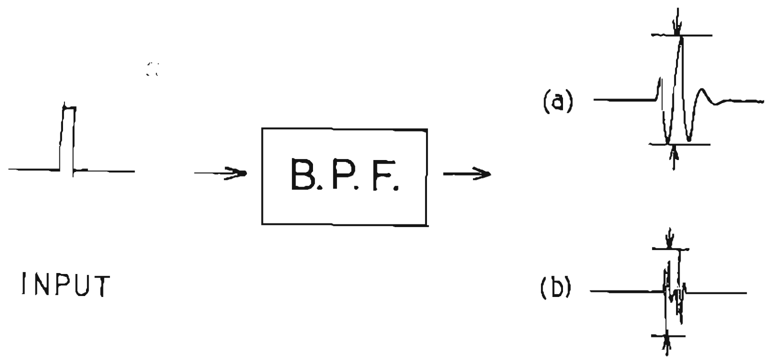

Fig. 2. Example of an input wave and the filtered waves. A rectangular wave is presented.

where $|\overline{F(\omega_0)}| \approx |F(\omega_0)|$, and t_0 is dependent only on ω_0. Although $|\overline{F(\omega_0)}|$ and t_0 are not uniquely determined, we assume the above expression. The right hand side of Eq. (8) can be obtained by fitting appropriate linear curves of amplitude and phase characteristics to $F(\omega)$. Substituting (8) into (6) we have

$$g(t) \approx b(t-t_0) \cdot |\overline{F(\omega_0)}| \tag{9}$$

When the impulse response $b(t)$ is known, the approximate value of the spectral amplitude may be evaluated by the formula

$$|\overline{F(\omega_0)}| = \frac{Max.\, g(t)}{Max.\, b(t)} \tag{10}$$

where $Max.\, g(t)$ and $Max.\, b(t)$ represent the maximum amplitudes of the wave $g(t)$ and $b(t)$, which are measured as indicated in Fig. 2. Eq. (10) is considered to define the value $|\overline{F(\omega_0)}|$ in Eq. (8). The more the spectrum is flat over the wide range of frequency, the more the approximation in (8) together with (10) is excellent.

Now let us use the filters defined by (2). Similarly to (9) we have

$$g_n(t) \approx n \cdot b_1(n(t-t_0)) \cdot |\overline{F(\omega_{0,n})}| \tag{11}$$

where $\omega_{0,n} = \frac{1}{2}(\omega_{H,n} + \omega_{L,n}) = \frac{n}{2}(\omega_{H,1} + \omega_{L,1})$. From (11) and as similarly as (10) we have

$$|\overline{F(\omega_{0,n})}| = \frac{a_n}{n \cdot a}, \tag{12}$$

where $a = Max.\, b(t)$ and $a_n = Max.\, g_n(t)$. The value a was determined by an experiment as mentioned below. First we adopted a rectangular wave as an input to the B.P.F.. The amplitude spectrum for the rectangular wave with a duration time τ and a height 1 is given by

$$|F(\omega)| = \frac{\sin(\pi f \tau)}{\pi f} \tag{13}$$

If $f_{L,n} \ll 1/\tau$ we can regard the wave as an impulse. In such a condition we determined the value $n \cdot a$ using the Eq. (12) where $|\overline{F(\omega_{0,n})}| (= |F(\omega_{0,n})|)$ is known from (13) and

Table 1. Schemes of the band-pass filters used in the present study.

	$f_{H,1}$ (Hz)	$f_{L,1}$	n (used)					a
(a)	1	2	1	2	4	8	16	4.7
			5	10	20	40		
(b)	4	5	1	2	4	8		8.0
(c)	5	8	1	2	4			16

a_n is a measured value. Finally we took the average of the several values of a. The results are given in Table 1. In this table cutoff frequencies chosen in this study are also shown.

Next we must examine the applicability of the above procedure to obtain an amplitude spectrum. For this purpose let us compare the measured spectrum with the theoretical one. Fig. 3 describes an example of the comparison for a rectangular wave as an input. We can see that the measured values are in good agreement with the theoretical curve except for the side lobes where the flatness does not hold. It should be noted that the spectral curve obtained by the measurements is not the envelope of the true curve but indicates a smoothed one.

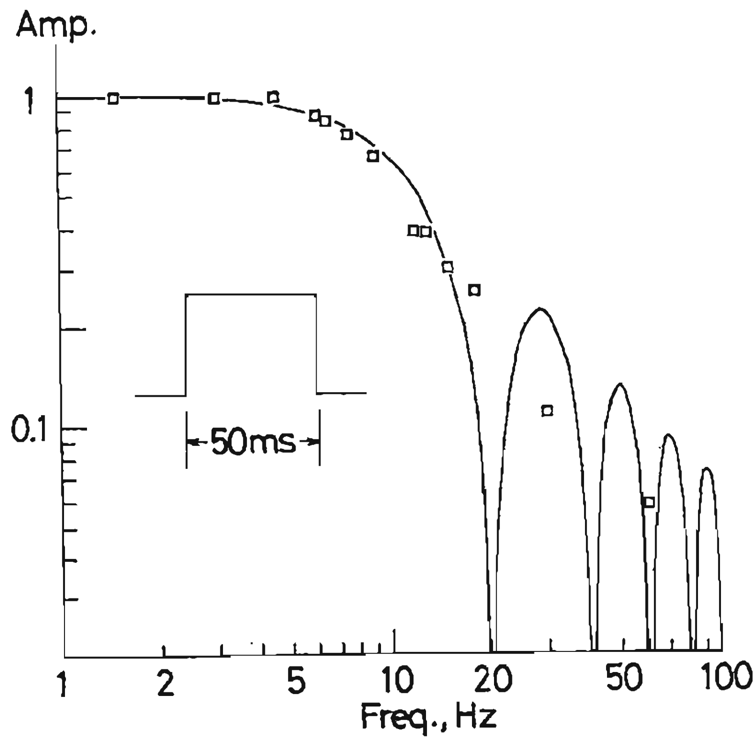

Fig. 3. Comparison of the measured spectrum with the theoretical one for a rectangular wave with duration time of 50 m sec.

4. Application to seismic waves

Eq. (12) in the last section is an approximate formula to evaluate the spectral density. In this section we shall apply this to typical example of seismic waves. We concentrate on the earthquakes which occurred near the Shikano and Yoshioka faults in Tottori Prefecture. The depths of foci range from 5 to 9 km.

The data are taken from the telemetering system of the Tottori Microearthquake Observatory, Disaster Prevention Research Institute, Kyoto University. The overall frequency response of the system is such that the velocity sensitivity is nearly flat for frequencies from 1 to 70 Hz[4]. When the band-pass filter with the frequency band of 40–80 Hz is applied to this system, the spectral value should be multiplied by 1.2 because the band exceeds the cutoff frequency of the system. We shall use of the displacement spectrum instead of the velocity and transform the velocity $v(\omega)$ into the displacement $u(\omega)$ by the formula

$$u(\omega_{0,n})=\frac{v(\omega_{0,n})}{n^2\cdot\omega_{0,1}\cdot a} \tag{14}$$

Fig. 4 shows a P wave which involves relatively higher frequencies. The filtered waves as well as the original wave of the P phase are so simple in their wave forms that it is easy to measure the maximum amplitudes.

In Fig. 5 ,on the other hand, the leading part of the seismic wave has two prominent phases which are marked by P_1 and P_2. It is reasonable to treat these phases differently when obtaining the spectrum because their spectra might show some difference. Really the P_2 wave is more abundant in higher frequencies than P_1 as seen from the figure.

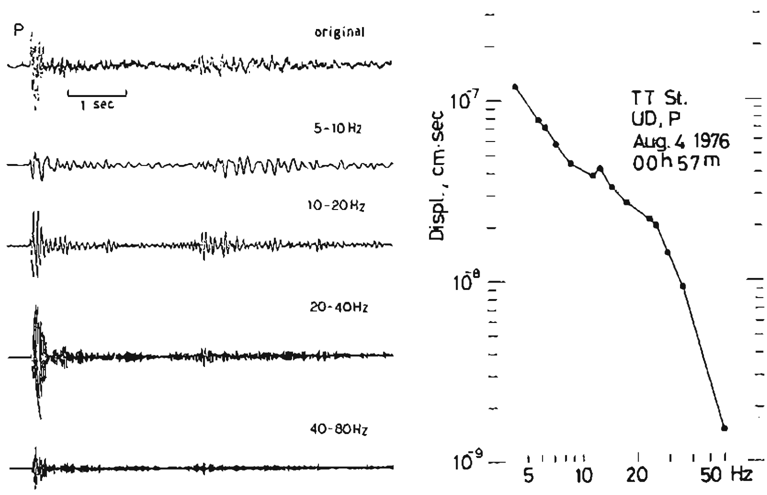

Fig. 4. Seismic waves filtered by the band-pass filters of various frequency-bands and the ground displacement spectrum of the P wave for a microearthquake of Aug. 4, 1976, which occurred near the western part of the Shikano fault, Tottori Prefecture, and observed at the Tottori station of the Tottori Microearthquake Observatory.

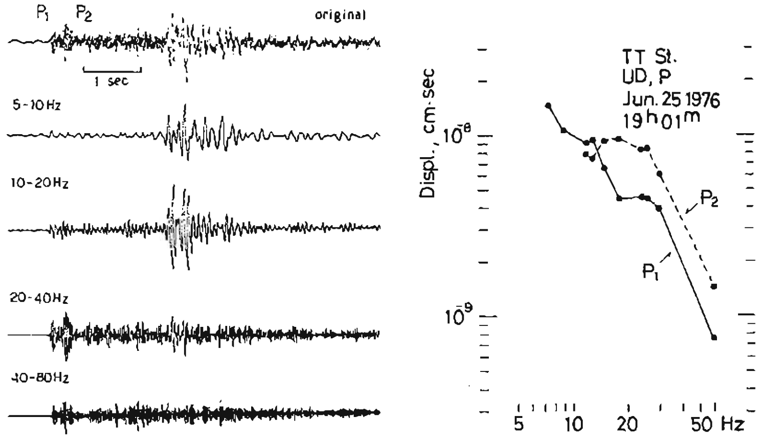

Fig. 5. Filtered waves and the ground displacement spectrum of the P_1 and P_2 waves for the microearthquake of Jun. 25, 1976, which occurred near the central part of the Shikano fault, and observed at the Tottori station.

When a wave commences just after the other wave we cannot obtain the spectra seperately. The earthquakes in Fig. 6 are from the same source region and show large difference in both waveforms and spectra. In the figure (a) can see the onset of a wave at 0.04 sec from the initial onset. This wave is roughly the same as the corresponding phase in (b). However the waveforms just after the initial onsets are largely different from each other. This feature results in the discrepancy in the spectral shapes.

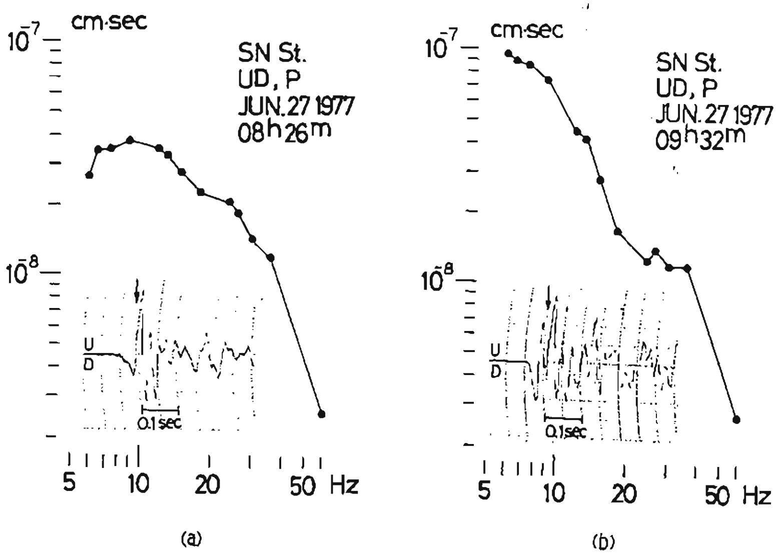

Fig. 6. P waveforms and the ground displacement spectra for the microearthquakes having almost the same focus near the central part of the Shikano fault and observed at the Shikano station of the Tottori Microearthquake Observatory. The waveforms of (a) and (b) resemble each other well except for the leading part from the initial onset to the time indicated by the arrow.

5. Concluding remarks

We have obtained approximately the spectral density by measuring the trace amplitudes of the filtered waves. The measured amplitude spectrum is equivalent to the smoothed spectrum the shape of which is locally flat against frequencies as given by (8). In analog-filtering a series of band-pass filters defined as (2) are available.

Analyses of seismic signals require filtering not only in the frequency domain but also in the time domain. The problem of data window is unavoidable in spectral analyses. In the filtering method we use it implicitly distinguishing between some prominent waves.

In the studies on earthquakes and the structure of the earth it is of importance to make clear the origin of each part of a seismic-wave train which may provide us significant information about the earthquake source and the medium around it. Moreover it is necessary to identify the phases of interest by comparing the siesmogram of an earthquake event with those of the other events. Accordingly we must deal with a number of data of seismic waves. A simplified technique to obtain amplitude spectra as presented in this paper, though it only gives smoothed spectra, is easy to treat and worth applying to the many examples of seismic waves.

References

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