Characteristics of Sediment Transport Process on Duned Beds Analyzed by Stochastic Approach

By Hiroji NAKAGAWA and Tetsuro TSUJIMOTO

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Abstract

When the bed is covered by sand waves such as dunes or ripples, the relationship between bed load transport rate and hydraulic parameters becomes different from the one valid in the case of flat bed conditions. Although the sediment motion in such an undulated bed is much complicated, fortunately it is so connected with the geometrical properties of bed configurations that one can describe the bed load transport process, not only the averaged properties but also the variational characteristics, by making use of a stochastic model for bed load dispersion. In this paper, the characteristics of the rest period and the step length in the case of lower regime bed conditions, which are essential elements of the stochastic model and represent the probabilistic characteristics of behaviors of individual bed load particles, were investigated in relation to the statistical properties of sand waves, and a generalized stochastic model which can be applied to such an undulated bed was derived. The theoretical results and estimations in this study were verified by the previous experimental data.

1. Introduction

A number of reaserchers have been interested in sediment transport and subsequent alluvial phenomena since early days, and some kinds of bed load transport formulas were proposed to comparatively estimate bed load transport rates under equilibrium flat bed conditions. Under some hydraulic conditions, however, various kinds of sand waves are formed and the characteristics of relationship between bed load transport rate and hydraulic parameters such as bed shear stress in such cases are appreciably different from those for equilibrium flat bed conditions as shown in Fig. 1. In this figure the solid line shows the bed load function derived by Einstein[1] and it is recognized that the experimental data obtained for undulated beds where ripples or dunes are formed dominantly are smaller than the estimation given by the solid line with considerably scattering. Hence, if one wants to apply the transport formulas previously derived for equilibrium flat bed conditions to such cases as undulated beds, some modifications should be done such as to divide the total hydraulic resistance into two parts; one is the part to be effective to the movements of sediment particles and the other corresponds to the energy loss due to bed form geometry. As for so called 'partitition of hydraulic resistance' which is based on the above idea, some devices were proposed in the previous reaserches[2),3),4)]. In other words, in such a method one must estimate the frictional resistance effectively acting upon the bed surface which is assumed to act upon a hypothetical flat bed as an 'effective tractive force' before one can apply the previously proposed bed load transport formulas. Unfortunately, however, we cannot judge what method is valid or reasonable, or at least adaptable to estimate the 'effective shear stress', now. Futhermore taking notice of the behaviors of sediment particles within one wave length of sand waves, the sediment transport is not in equilibrium but it increases from the reattachment point of the flow to the crest of the sand wave as shown in Fig. 2, and since the bed shear stress is not constant along the bed surface of upstream slope of the sand wave, the sediment pick-up rate p_s which represents the probability density of sediment particle dislodgement per unit time also varies locally as shown in Fig. 3. Therefore, even though the resistance due to surface friction can be estimated fairly, it is not always to be able to estimate the bed load transport rate exactly. The experimental data were obtained by the measurements with an instrument devised as shown in Fig. 4, in which a series of wooden sand wave models given by a triangular section of 2 cm height and 53.5 cm length were set in the experimental flume. In the experiment[5] the friction velocity was about 3.7~6.1 cm/sec and two kinds of sands of which median diameters were 0.074 cm and 0.092 cm were used. In Fig. 3, the relationship between dimensionless pick-up rate defined by the following equation and the distance x' from the reattachment point of the flow is shown.

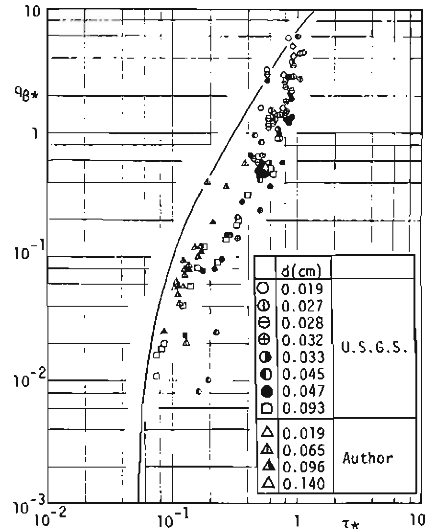

Fig. 1. The relationship between bed load transport rate and bed shear stress in undulated beds.

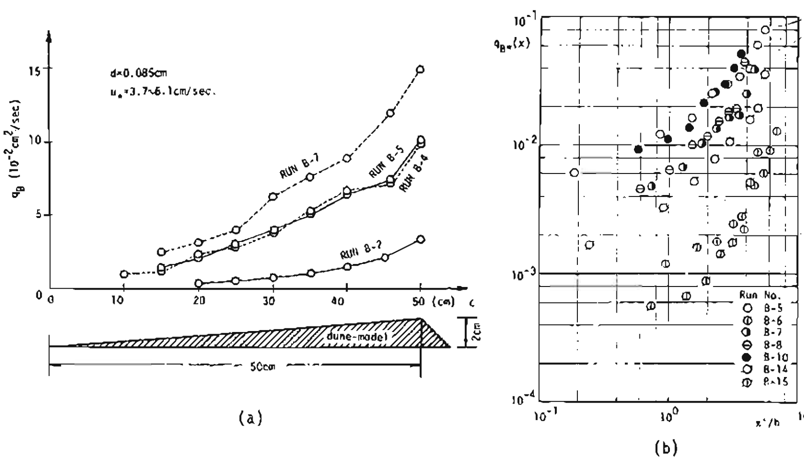

Fig. 2. Spatial variation of bed load transport rate over a simulated dune.

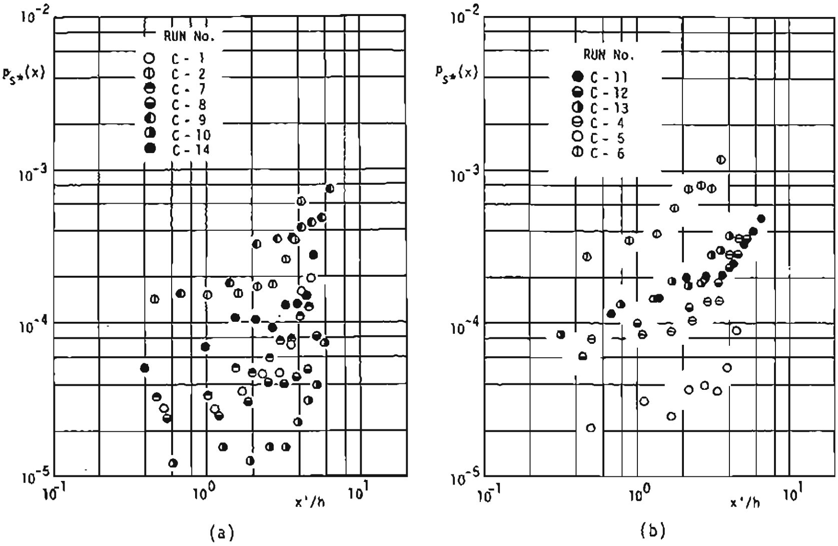

Fig. 3. Spatial variation of sediment pick-up rate over a simulated dune.

$$p_{s*}=p_s\sqrt{d/(\sigma/\rho-1)g} \tag{1}$$

where d is sediment diameter, σ/ρ is the specific weight of sand and g is the acceleration of gravity.

By the way, even in the case to estimate the bed load transport rate by making use of 'effective shear stress' which is generally used, it is necessary to determine the geometrical properties of sand waves for estimating the frictional resistance. Bed configurations are, however, too irregular to determine the averaged properties such as mean wave height or mean wave length against the hydraulic conditions without many difficulties.

If the geometrical properties of sand waves are given, a more simplified method is possible. When the mean celerity and the mean wave height are clarified, the bed load transport rate due to any migration of sand waves can be determined. In the case of so called 'equilibrium conditions of sand waves' where the sand waves migrate at a constant speed without any changes of scales nor the shape of sand waves, the apparent bed load transport rate can be expressed by the following equation.

$$q_B=s(1-\rho_0)\bar{U}_W\bar{H} \tag{2}$$

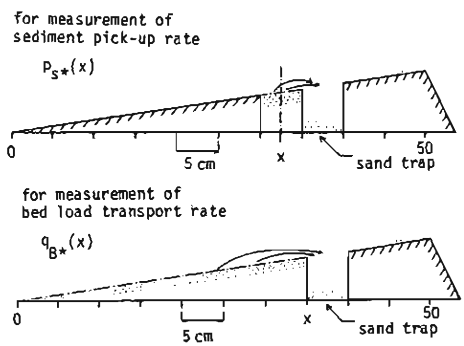

Fig. 4. A simulated dune to measure bed load transport rate and sediment pick-up rate.

where q_B is bed load transport rate expressed by substantial volume per unit width, s is a constant determined by the shape of sand waves (in the case of a triangular type, the value of s is 0.5, and in case of sinusoidal geometry, $2/\pi$), ρ_0 is porosity of sand, $\bar{U}_W$ is mean celerity of sand waves and $\bar{H}$ is mean wave height of sand waves. The quantities representing the geometry of sand waves can be given by the bed profile data which can be easily obtained by improved measuring instruments such as a supersonic bed profile sounder, but also in this case it is difficult to determine the relationship between these quantities and the hydraulic parameters. Eq. (2) has been already verified experimentally by Simons, Richardson & Nordin[6] and others, and the experimental values obtained by U.S.G.S.[7], Stein[8], Grigg[9] and the authors are shown in Fig. 5.

On the other hand, the methods above discussed for estimation of bed load transport rate in duned beds were combined in the previous study by the authors[10], where an estimating method of surface resistance due to friction and its physical meanings were clarified by an energetic consideration. This along with an idea first proposed by Bagnold[11] for equilibrium flat bed conditions that macroscopic energy of flow and the work done by flow against the gravity and the frictional force must be balanced keeps us free from questions on non-equilibrium conditions of sediment transport as well as those on the subsequent variation of the relation between bed shear stress and the sediment behaviors, because this inherent mechanism is contracted in this approach as a black-box-model.

By the way, we can recognize, according to the principle used to derive Eq. (2) and the experimental results shown in Figs. 2 and 3, that bed load transport rate varies locally even if the sand waves of the same scale are arranged on the stream bed, and that in the case where the bed configurations are as irregular as observed the spatial or temporal fluctuations of bed load transport rate are more appreciable. It is considered that such irregularities may play an important role for various kinds of alluvial phenomena which are subsequent to bed load transport, from which point of view it is favourable to describe bed load transport process with some considerations on the irregularities of the sediment particles movements.

Considering the above discussions, a stochastic model for dispersion process of bed load materials, which was first derived by Einstein[12] and improved by some researchers[13),14),15),16], can represent not only the probabilistic characteristics of behaviors of individual sediment particles but also the statistical properties of behavior of the sediment transport process as a whole, and thus this model

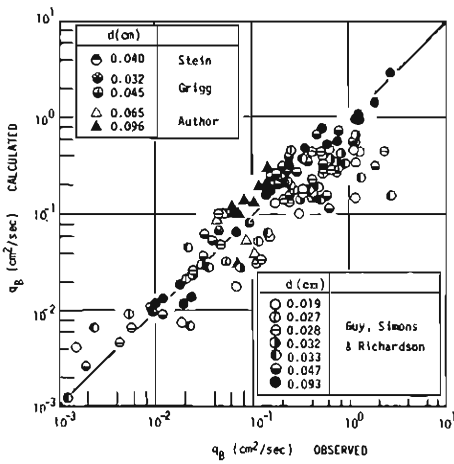

Fig. 5. Bed load transport rate in undulated beds.

may be superior to the others. In this paper, the sediment transport processes in dune regimes are investigated based on the above stochastic approaches. Through such a stochastic model, the sediment transport in dune regimes can be described by a universal frame-work in the same way as the case of flat bed conditions.

In a stochastic model for bed load transport process, the rest period is defined as the time period during which a particle is at rest on bed, and the step length is defined as the distance for a particle to travel from its incipient motion to the next definite stop. These quantities are considered as random variables, which constitute the stochastic model. In equilibrium flat bed conditions, these quantities may be represented by the time scale corresponding to the behaviors of a sand in fluid, which may be related to the falling velocity of a sand particle and may be proportional to $\sqrt{d/(\sigma/\rho-1)g}$ and by the spatial scale corresponding to the irregularity of sand bed surface, which may be proportional to the sand diameter[17]. In the case of dune bed conditions, however, a particle under observation may often be buried beneath the sand bed and may be transported only by a migration of sand waves. Therefore, the probabilistic characteristics of sediment behaviors can be expressed by the statistical properties of bed configurations. Particularly, in the case of lower regimes where dunes or ripples appear dominantly, a sediment particle is dislodged on the upstream slope of a sand wave and deposits on the downstream slope of the same wave as shown in Fig. 6, and thus a unique relationship between the properties of sediment behaviors and those of sand waves is recognized. This fact motivated the derivation of Eq. (2).

Based on the above consideration, here we adopt the definition of the rest period in case of dune bed conditions by Hubbell & Sayre[18] as shown in Fig. 7. In this case the step length can be defined as shown in Fig. 8. The rest period and the step length defined above can be investigated based on the studies of the statistical properties of sand waves and a model of bed configurations proposed by the authors[19], and these discussions will be involved in Chapters 2 and 3.

As for the applicability of a stochastic model, Yang & Sayre[20] and Grigg[9] proposed a model where the step length follows a gamma distribution and the rest period follows an exponential one, and verified it experimentally. In their model, however, inspection for the distributions of step length and rest period and the generalization of the model

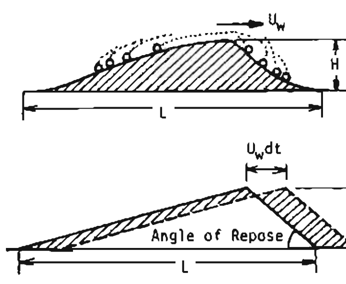

Fig. 6. Behaviors of bed load particles on dune bed.

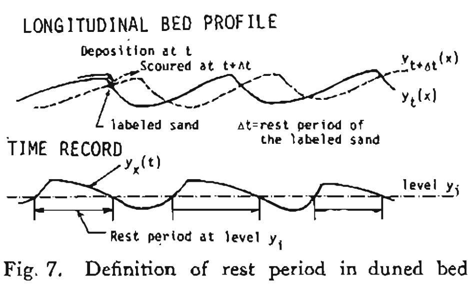

Fig. 7. Definition of rest period in duned bed (after Hubbell & Sayre).

are not sufficient. Therefore, subsequent to the considerations of step length and rest period in Chapters 2 and 3, a generalized stochastic model for sediment dispersion which is applicable to any types of distributions of step length and rest period will be derived in

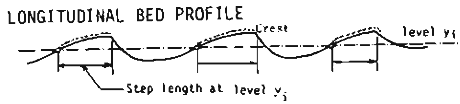

Fig. 8. Definition of step length in duned bed.

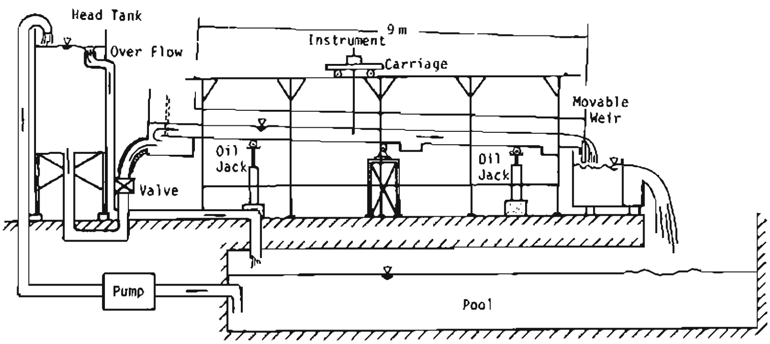

Fig. 9. Experimental flume.

Table 1. Experimental conditions.

RUN NO.	i	d (cm)	τ_*	Re_*	h/d	q_{B*}
A- 1	0.0020	0.065	0.158	26.7	126.2	0.146
A- 2	0.0020	0.065	0.128	23.5	102.3	0.020
A- 6	0.0020	0.065	0.193	28.8	113.9	0.409
A-12	0.0050	0.065	0.384	40.7	123.1	0.667
B- 8	0.0050	0.140	0.170	86.0	55.0	0.107
B- 9	0.0050	0.140	0.209	95.5	67.9	0.189
C- 5	0.0033	0.096	0.167	48.5	81.3	0.120
C- 9	0.0050	0.096	0.232	57.0	75.0	0.276

Chapter 4. Furthermore, in Chapter 5, by combining these results, a description and a forecast of sediment dispersion processes on duned beds will be investigated.

The data used in Chapters 2 and 3 were obtained in experiments conducted in a flume with 33 cm width and 9 m length as shown in Fig. 9. After 90∼120 minutes from introducing a flow to an initially flattened bed, where sand waves may reach their equilibrium

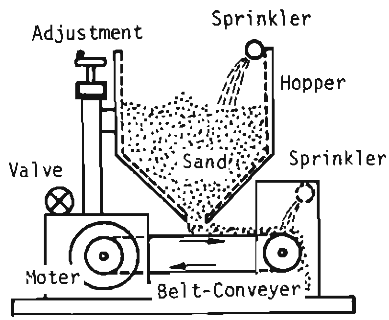

Fig. 10. Automatic sand feeder.

states after full development, the longitudinal bed profiles and the temporal variation of bed elevations at the fixed point were registered by a supersonic sounder. The data were obtained by analyses of them, and the experimental conditions are shown in Table 1. In the experiments, some kinds of uniform sands were used and at the upstream end of the flume the sand was supplied by an automatic sediment feeder, which is a combined type of conveyer belt, hopper and sprinklers as shown in Fig. 10, according to the information obtained by the preliminary experiments.

2. Characteristics of Rest Period in Duned Beds

As the characteristics of the rest period in a duned bed are much influenced by the height at which the labeled particle stays in the bed, as may be expected from its definition shown in Fig. 7, its conditional distribution by the elevation of the particle in the bed, y, is at first considered.

According to the definition of rest period, the conditional mean rest period at y, $\langle T(y)\rangle$ is given by

$$\langle T(y)\rangle=\langle\tau(y)\rangle/\tilde{N}(y) \tag{3}$$

where $\tilde{N}(y)$ is the expected times of up-crossings for the level y per unit time of a stochastic process $y(t)$, and $\langle\tau(y)\rangle$ is the total period when $y(t)\geqq y$ per unit time. Because $y(t)$ is one of Gaussian processes[19], these are obtained by the result of the threshold crossing problem in the theory of stochastic processes and some simple probabilistic considerations respectively.

$$\tilde{N}(y_*)=\tilde{N}_0 \exp(-y_*^2/2) \tag{4}$$

$$\langle\tau(y_*)\rangle=\int_{-\infty}^{y_*} p(y_*)dy_* \tag{5}$$

where $p(y_*)$ is the normalized probability density function of bed elevation, and y_* is the following dimensionless variable using the standard deviation of bed elevation σ_y.

$$y_*=y/\sigma_y \tag{6}$$

As bed elevation follows a Gaussian distribution,

$$p(y_*)=(1/\sqrt{2\pi})\cdot\exp(-y_*^2/2) \tag{7}$$

$$\tilde{N}_0=\tilde{N}(0)=(\tilde{M}_2/\tilde{M}_0)^{1/2} \tag{8}$$

where $\tilde{M}_j$ is the j-th moment of the frequency spectral density of bed elevation.

By the way,

$$\langle T_0\rangle\equiv\langle T(0)\rangle=1/2\tilde{N}_0 \tag{9}$$

and so the following equation can be obtained.

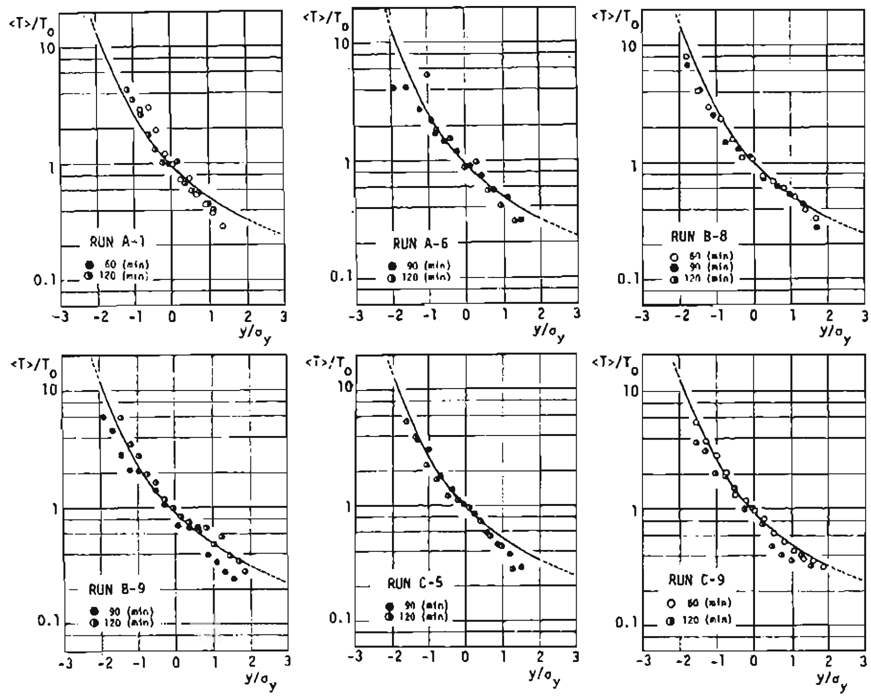

Fig. 11. Variation of conditional mean rest period in y-direction.

$$\langle T(y_*)\rangle/\langle T_0\rangle=\sqrt{2\pi}\exp(y_*^2/2)\int_0^{y_*}\exp(-y_*^2/2)dy_* \tag{10}$$

In Fig. 11 the above equation is compared with the experimental results and we can recognize a good agreement.

Eq. (10) is significant only in the range of $-2<y_*<2$, and according to the experimental data shown in Fig. 12 the following relationship between $\langle T_0\rangle$ and the mean rest period $\bar{T}$ is recognized.

$$\bar{T}=\langle T_0\rangle \tag{11}$$

In Fig. 12 are shown the experimental results for the rest period obtained from the registrations of temporal fluctuation of bed elevations after the definition indicated in Fig. 7, accompanied with the experimental data of radioactive tracer tests conducted by Grigg.[9] Both of them show the same tendency, and thus the definition shown in Fig. 7 and the recognition of behaviors of sediment motion as shown in

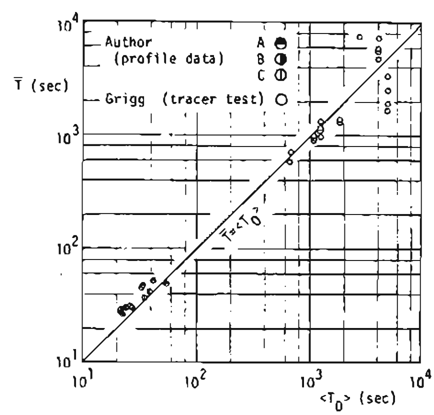

Fig. 12. The relationship between conditional mean rest period at $y=0$ and mean rest period.

Fig. 6 are verified to be valid.

Based on the above considerations, the following approximation for the relation between $\langle T(y)\rangle$ and $\bar{T}$ which was proposed by the authors' experiment[21] is appropriate.

$$\langle T(y_*)\rangle/\bar{T}=\exp(-0.90y_*) \tag{12}$$

On the other hand, as for the y-directional variation of the conditional standard deviation of the rest period, $\sigma_r(y)$, the following experimental equation may be adequate as shown in Fig. 13.

$$\sigma_r(y_*)/\sigma_{r0}=\exp(-0.90y_*) \tag{13}$$

where $\sigma_{r0}=\sigma_r(0)$. From Eqs. (11), (12) and (13), the following equations can be obtained.

$$\sigma_r/\langle T\rangle=\sigma_{r0}/\langle T_0\rangle=\text{const.} \tag{14}$$

This means that the conditional variation coefficient of rest period is constant along the y-direction. According to the experimental results about the rest period shown in Table 2, the conditional variation coefficient of rest period (in Table 2 the value at $y=0$ or α_{T0} is shown) is about unity or slightly smaller than it.

On the contrary, the variation coefficient after releasing the condition, α_T is larger

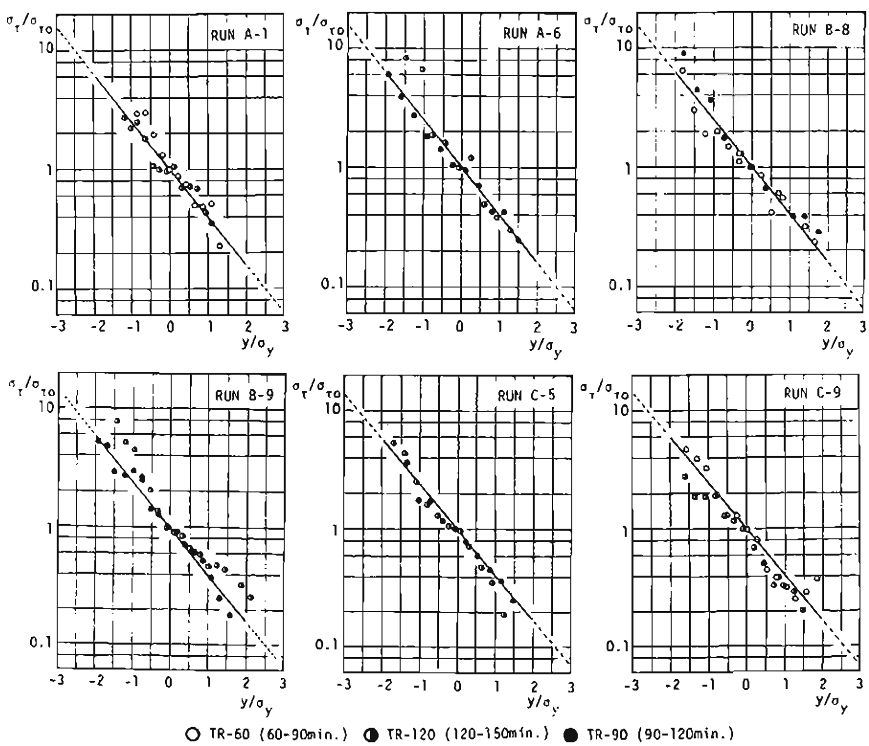

Fig. 13. Variation of conditional standard deviation of rest period in y-direction.

Table 2. Experimental results as for rest period in duned beds.

CASE	$\overline{T}$ (sec)	$\bar{\sigma}_t$ (sec)	α_T	$\langle T_0 \rangle$ (sec)	σ_{t0} (sec)	α_{T0}
A-1-TR 60	48.83	55.36	1.13	45.41	41.52	0.91
90	47.28	49.95	1.06	34.04	35.46	1.04
A-6-TR 90	46.60	51.81	1.11	41.94	40.41	0.96
120	44.20	49.21	1.11	33.15	29.53	0.89
B-8-TR 60	30.63	37.67	1.23	27.57	18.84	0.68
90	26.07	36.61	1.40	22.67	14.64	0.65
B-9-TR 90	28.80	34.16	1.19	21.89	13.66	0.62
120	27.20	33.84	1.24	21.76	13.20	0.61
C-5-TR 90	42.00	53.38	1.27	38.64	42.17	1.09
120	36.80	46.55	1.27	36.31	32.59	0.90
C-9-TR 60	30.19	40.15	1.33	24.15	26.50	1.09
120	30.53	36.86	1.21	26.87	25.80	0.96

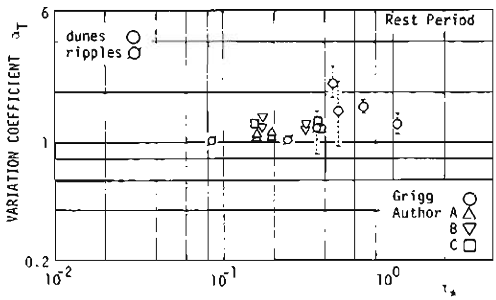

Fig. 14. The relationship between variation coefficient of rest period and bed shear stress in duned beds.

than unity and has about 1.2 as a mean. Such an increase of variation coefficient must be caused by an operation of removing the condition.[22] Meanwhile, the experimental results by radioactive tracer test conducted by Grigg[9] are shown in Fig. 14 which are concerned with dune bed conditions and the mean value of α_T is about 1.2. Consequently, we can estimate that the distribution of rest period after removing the condition may be expressed by a gamma one of which variation coefficient is 1.2 or shape parameter is about 0.7. The adaptability of such type of distribution is recognized to be good as shown in Fig. 15, while the conditional distribution of rest period should be expressed by an exponential one, of which adaptability is also verified in Fig. 16.

3. Characteristics of Step Length over Duned Beds

The chracteristics of step length of a sediment particle traveling over a duned bed can be obtained by the longitudinal bed profile after the definition shown in Fig. 8. In this case, we can obtain the conditional step length at first as pone in the case of rest

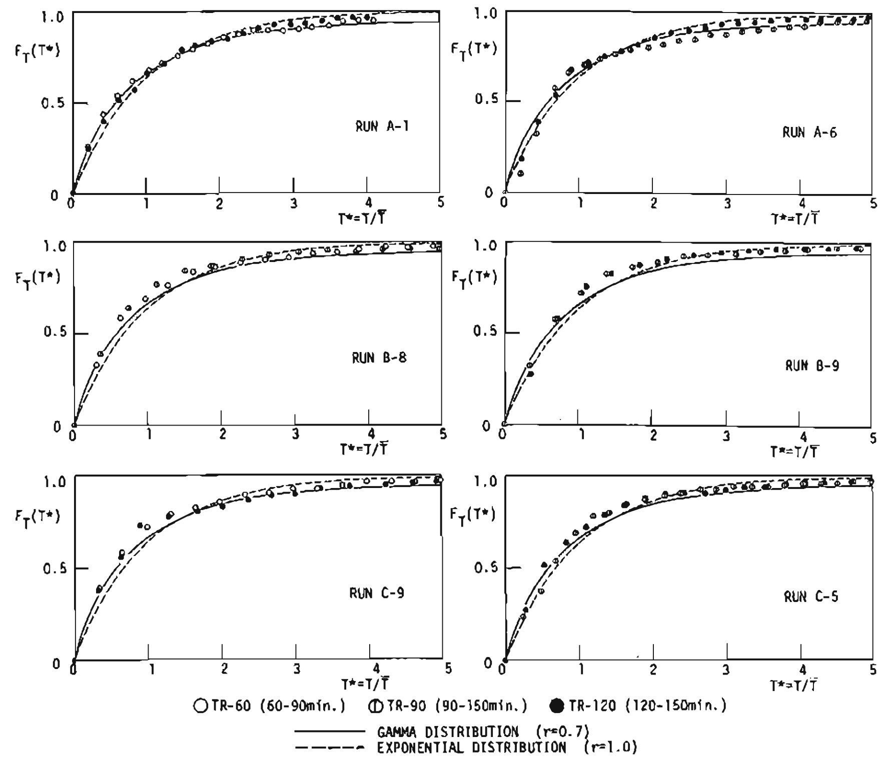

Fig. 15. Distribution of rest period in duned beds.

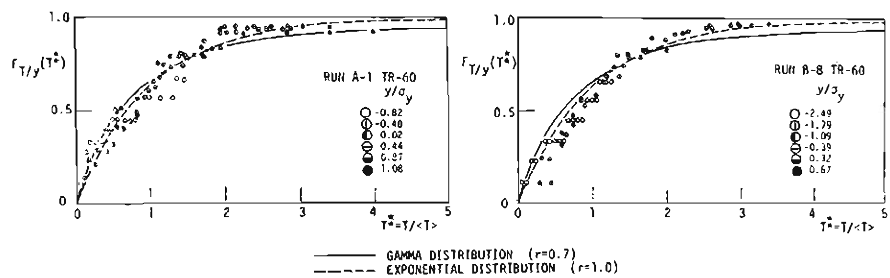

Fig. 16. Conditional distribution of rest period in duned beds.

period, but particles of which step lengths are to be measured are limited to the ones traveling along the bed surface and the distribution of step length after removing the condition is far different from the one of rest period. In other words, the frequency of the conditional step length at y is half of the numbers of y-level crossings of $y(x)$, while the probability of the conditional rest period at y is expressed by one of bed elevations, and the operating methods on removing the condition are different between the step length and the rest period.

As for the step length, the motion of sediment particles is much influenced by the bed configurations as mentioned in Chapter 1, and despite that the bed is much undulated, some regularity is recongized so that the variation coefficient of the wave length of bed forms is about 0.5[10),21)]. Hence the variation coefficient of step length becomes comparatively small.

According to the definition of step length shown in Fig. 8, the distribution of step length is expected to be similar to the one of zero-crossing intervals of $y(x)$. Under such an insight, the mean and the variance of step length which are represented by Λ and σ_X^2 are a half of the mean wave length and a half of its variance, respectively.

Therefore,

$$\Lambda=\bar{L}/2 \tag{15}$$

$$\alpha_X=\sigma_X/\Lambda=\sqrt{2}\,\alpha_L\simeq 0.7 \tag{16}$$

where $\bar{L}$ and α_L are the mean and the variation coefficient of wave length of sand waves respectively, and α_X is the variation coefficient of step length. In Fig. 17, Eq. (15) is verified by the experimental data. In this figure, besides the results obtained from the analyses of longitudinal bed profile registrations, those obtained by tracer tests conducted by Grigg[9)] and the one measured directly over duned beds. In direct measurements, colored particles were used as tracers and released on the bed surface undulating as dunes were formed. The median diameter of bed

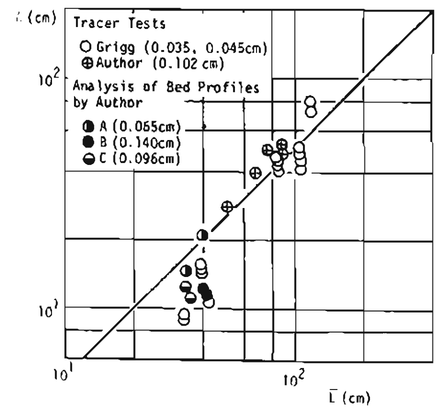

Fig. 17. The relationship between mean step length and mean wave length of sand waves.

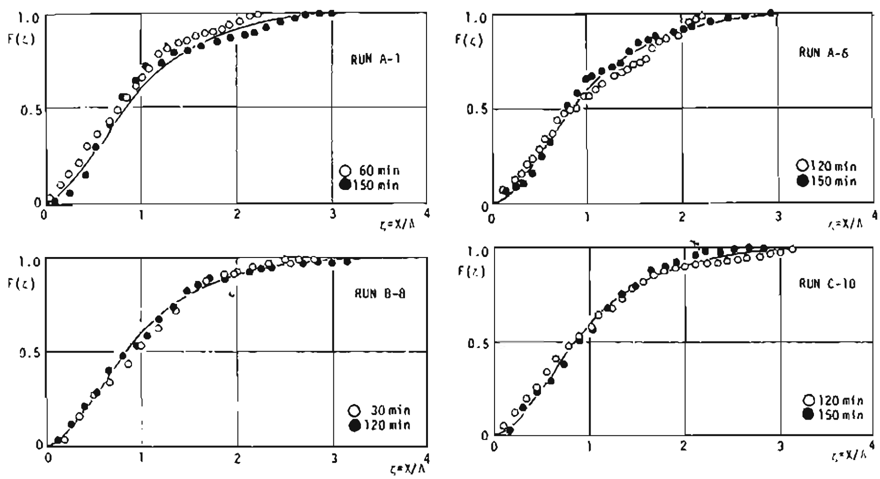

Fig. 18. Distribution of step length in duned beds.

Table 3. Experimental results as for step length in duned beds.

RUN	Λ (cm)	σ_X (cm)	α_X	r
A- 1	14.82	8.86	0.598	2.80
A-12	21.70	14.79	0.682	2.15
B- 8	11.23	8.67	0.772	1.68
B- 9	12.30	8.77	0.713	1.97
C- 5	12.16	8.59	0.706	2.01
C-10	11.14	6.88	0.618	2.62

material was 0.102 cm. According to this figure, the properties of step length obtained from bed profile data show good agreements with those of actual sediment motion clarified by direct measurements and tracer tests, and thus the validity of the definition of step length over duned beds shown in Fig. 8 was verified.

From Eq. (16), the distribution of step length can be expected to be expressed by a gamma distribution, of which shape parameter r should be given by

$$r=1/\alpha_X{}^2\simeq 2.0 \qquad (17)$$

Then the probability density function of the normalized step length $\zeta=X/\Lambda$ which is to be represented by $f_X(\zeta)$ is written as follows.

$$f_X(\zeta)=4\zeta\cdot\exp(-2\zeta) \qquad (18)$$

The above expression corresponds well to the experimental results as shown in Fig. 18.

As an additional remark, the characteristics of the step length obtained from bed profile data are shown in Table 3, and the relationship between the variation coefficient of step length and bed shear stress in Fig. 19.

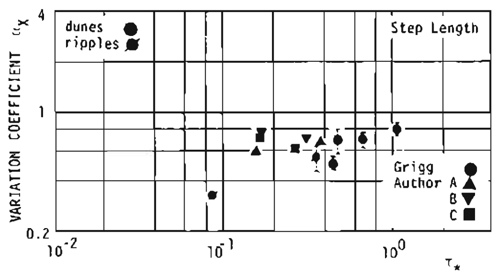

Fig. 19. The relationship between variation coefficient of step length and bed shear stress in duned beds.

4. Stochastic Model for Bed Load Transport and Dispersion Process over Duned Beds

As mentioned in Chapters 2 and 3, in the case where dunes or ripples are formed, rest period and step length do not follow exponential distribution laws and then one cannot deal with the sediment transport process as a compound Poisson process. Even a modified model by Yang & Sayre[20] cannot express its properties because only the distribution of step length was revised in that model. Therefore, here it is necessary to generalize the stochsatic model for bed load dispersion process which is constituted of step length and rest period. In the following, a model for bed load dispersion will be derived without determining the types of the distributions of step length and rest period, of which probability density functions are to be represented by $f_X(\xi)$ and $f_T(\tau)$ respectively.

The probability density $f_t(x)$ of a random variable $\{x(t)\}$ which indicates the distance for a particle noticed to travel in time t expresses the sediment dispersion process well. This can be expressed by the following equation as is recognized easily by consulting Fig. 20[22].

$$f_t(x)=\sum_{n=1}^{\infty} f_X{}^{n*}(x)\cdot P_n(t) \tag{19}$$

where $f_X{}^{n*}(x)$ is the n-fold convolution of $f_X(x)$, and it is equivalent to the probability density function of $Y_n=X_1+X_2+\cdots+X_n$ if individual steps are independent of each other and are members of the identical probabilistic population. And $P_n(t)$ is the following probability when $N(t)$ is defined as the number of steps that appear in time t which is also a random variable.

$$P_n(t)=\text{Prob}\,[N(t)=n] \tag{20}$$

Although Eq. (19) has been already obtained by Yang & Sayre[20], it can be obtained easily by consulting Fig. 20 without any troublesome probabilistic considerations as seen in their paper. Furthermore, it is so difficult to determine $f_t(x)$ generally by using Eq. (19), and from practical point of view it is more important to investigate the moments of $\{x(t)\}$ than $f_t(x)$ itself.

The k-th moment of $\{x(t)\}$ can be expressed in a simplified version by exchanging the order of removing the condition with respect to $\{N(t)\}$ and expected value operation as follows.

$$E[\{x(t)\}^k]=\sum_{n=1}^{\infty}\{E[Y_n{}^k]\cdot P_n(t)\} \tag{21}$$

When $\{X\}$ is independent, the moment of its successive quantity $\{Y_n\}$, or $E[Y_n{}^k]$ is expressed by a polynomial of n with less degrees than or euqal to k and eventually $E[\{x(t)\}^k]$ is expressed by the moments of $\{N(t)\}$ of which degrees are less than k[22]. Particularly, in the cases of $k=1$ and $k=2$, the following equations are obtained[22].

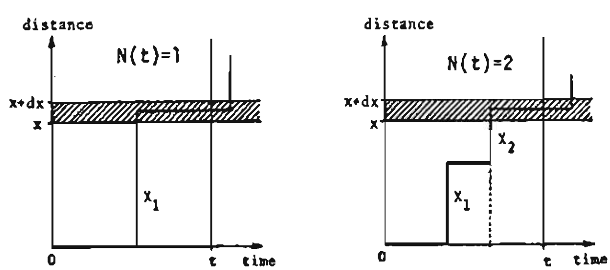

Fig. 20. Definition sketch.

$$E[x(t)]=E[X]E[N(t)] \tag{22}$$

$$E[\{x(t)\}^2]=\mathrm{Var}[X]\cdot E[N(t)]+\{E[X]\}^2\cdot E[\{N(t)\}^2] \tag{23}$$

where Var[·] indicates the variance.

Although the general calculation of the moments of $\{N(t)\}$ is not so easy, it is sufficient to know the behaviors for a situation of $t\to\infty$ in the case that only sediment transport rate and ensemble averaged properties of sediment dispersion process should be clarified under equilibrium conditions of sand waves.

From the definition of $P_n(t)$,

$$\sum_{k=0}^{n} P_k(t)=\mathrm{prob}\,[N(t)<n]=\mathrm{prob}\,[S_n>t]=\int_t^\infty f_T{}^{n*}(\tau)d\tau \tag{24}$$

and putting

$$t=nE[T]-\sqrt{n\,\mathrm{Var}\,[T]} \tag{25}$$

the following relation can be obtained.

$$\mathrm{prob}\,[S_n>t]=\mathrm{prob}\,[\{S_n-nE[T]\}/\sqrt{n\,\mathrm{Var}\,[T]}>-\xi] \tag{26}$$

where $S_n=T_1+T_2+\cdots+T_n$. In the case of $n\to\infty$, from the central limiting theorem, the right term of the above equation must converge the following.

$$(1/\sqrt{2\pi})\cdot\int_{-\xi}^\infty \exp\,(-u^2/2)du$$

Meanwhile,

$$\mathrm{prob}\,[N(t)<n]=\mathrm{prob}\left[\frac{N(t)-\{t/E[T]\}}{\sqrt{t\,\mathrm{Var}\,[T]/\{E[T]\}^3}}<\frac{n-\{t/E[T]\}}{\sqrt{t\,\mathrm{Var}\,[T]/\{E[T]\}^3}}\right] \tag{27}$$

And from Eq. (25), the following relation is obtained.

$$\frac{n-\{t/E[T]\}}{\sqrt{t\,\mathrm{Var}\,[T]/\{E[T]\}^3}}\left(=\xi\sqrt{nE[T]/t}\,\right)=\xi(1+\sqrt{n\,\mathrm{Var}\,[T]}\cdot\xi/t) \tag{28}$$

Regarding the above equation as a quadratic equation with respect to $\sqrt{n}\,/t$ the following is obtained.

$$\lim_{t\to\infty}\frac{\sqrt{n}}{t}=\lim_{t\to\infty}\frac{\sqrt{\xi^2\,\mathrm{Var}\,[T]/t^2}\pm\sqrt{\xi^2\,\mathrm{Var}\,[T]/t^2+4E[T]/t}}{2E[T]}=0 \tag{29}$$

$$\lim_{t\to\infty}\frac{n-\{t/E[T]\}}{\sqrt{t\,\mathrm{Var}\,[T]/\{E[T]\}^3}}=\xi \tag{30}$$

Consequently, in the case of $n\to\infty$,

$$\mathrm{prob}\,[N(t)<n]\to\mathrm{prob}\,[\{N(t)-t/E[T]\}/\sqrt{t\,\mathrm{Var}\,[T]/\{E[T]\}^3}<\xi] \tag{31}$$

The above result means that the distribution of $\{N(t)\}$ tends to be a normal one of which mean and variance are $t/E[T]$ and $t\cdot\mathrm{Var}[T]/\{E[T]\}^3$, respectively. Namely,

$$E[N(t)]\rightarrow t/E[T] \tag{32}$$

$$\mathrm{Var}[N(t)]\rightarrow \alpha_T{}^2 t/E[T] \tag{33}$$

Applying the above results to Eqs. (22) and (23), the following equations are obtained as asymptotic expressions.

$$E[x(t)]=\Lambda(/\bar{T})\cdot t \tag{34}$$

$$E[\{x(t)\}^2]=(\Lambda^2/\bar{T})\cdot(\alpha_T{}^2+\alpha_X{}^2+t/\bar{T})\cdot t \tag{35}$$

$$\mathrm{Var}[x(t)]=(\alpha_T{}^2+\alpha_X{}^2)\cdot(\Lambda/\bar{T})\cdot t \tag{36}$$

Therefore, the mean velocity of the group of sediment particles noticed under equilibrium conditions of sand waves which is represented by V_g is expressed by

$$V_g=\frac{\partial}{\partial t}E[x(t)]=\Lambda/\bar{T} \tag{37}$$

And the following bed load transport rate formula can be obtained by considering that the thickness of a moving layer may be equal to the standard deviation of bed elevation σ_y.

$$q_B=(1-\rho_0)\cdot(\Lambda/\bar{T})\cdot\sigma_y \tag{38}$$

As $\Lambda/\bar{T}$ and $2\sigma_y$ may be equal to the mean celerity and mean wave height[19),21)] of sand waves respectively, the above equation is equivalent to Eq. (2).

As shown in the above, the averaged bed load transport rate obtained by stochastic approach is identical to the one given by a simplified consideration based on the averaged geometry of sand waves. On applying a stochastic model, however, it is also possible to estimate the variation or scattering of the traveling distance of sediment particles noticed. This is important to give vital information to the problem to forecast the dispersion process of contaminated sediment in such undulated beds where dunes are formed dominantly. The variational properties cannot be described without involving the probabilistic characteristics of individual sediment motion as an elementary event such as the variation coefficients of rest period and step length, α_T and α_X, and hence they can be expressed only by a stochastic model.

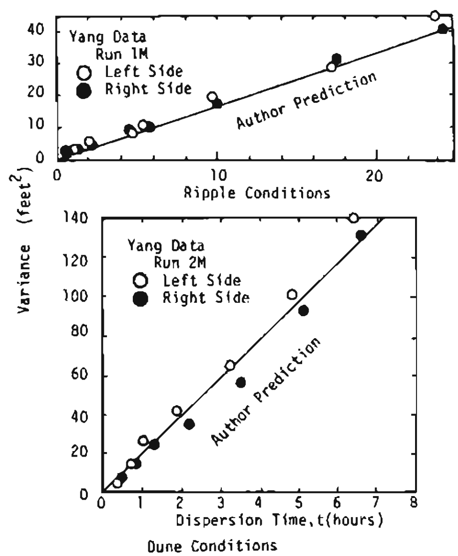

Fig. 21. Dispersion process of group of sediment tracers (after the experimental results by Yang & Sayre).

Into the bargain, such variational characteristics are important because these can give a number of influences to the temporal variation of concentration of contaminated sediment apart from a pollution source, or the amount of contaminants in the channel as a whole.

Substituting $\alpha_X=0.71$ and $\alpha_T=1.2$ into Eq. (36) as obtained in Chapters 2 and 3, the variance of $\{x(t)\}$ is obtained as follows.

$$\mathrm{Var}\,[x(t)]=1.94(\Lambda^2/\bar{T})\cdot t \qquad (39)$$

Differentiation of Eq. (39) yields

$$\frac{\partial}{\partial t}\mathrm{Var}\,[x(t)]=1.94\Lambda^2\bar{T} \qquad (40)$$

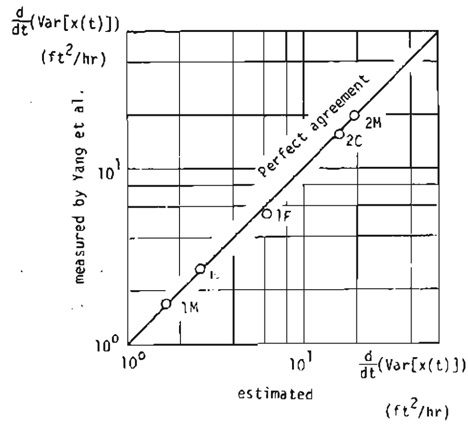

Fig. 22. Experimental verification of forcast of sediment dispersion process by the modified stochastic model.

The validity of Eqs. (39) and (40) are confirmed by the experimental results obtained from radioactive tracer tests conducted by Yang & Sayre[20] as shown in Figs. 21 and 22 respectively.

5. Conclusion

The sediment movements over undulated beds such as dunes or ripples are strongly intimated to the bed configurations. In this paper, noticing this fact sediment transport process has been described by a stochastic model constituted by step length and rest period which can be defined in relation to undulated bed profiles, particularly the statistical characteristics of bed configurations. At least in the level of coarse graining for the length-scale of the mean wave length of sand waves, a model based on the above consideration can describe the sediment transport process without spoiling the characteristics of motion of individual bed load particles. Moreover, by such a stochastic model the phenomena in dune bed conditions can be explained as well as those in flat bed conditions and this method may be more reasonable or more appropriate for estimation of bed load transport rate in the case of undulated beds than the previous methods such as those using the concept of an effective bed shear stress.

In duned beds, the rest period must be much influenced by the position particularly the elevation of the particle noticed in the bed. Hence, the conditional distribution in y-direction at first and then the distribution after releasing the condition were investigated. As a result, the conditional distribution of rest period can be approximately expressed by an exponential one, but the distribution after condition releasing can be fairly well expressed by a gamma one of which shape parameter is about 0.7 because the variation coefficient is about 1.2. According to them, the variations in y-direction of conditional mean and conditional standard deviation were investigated in relation to the statistical properties of bed configurations.

As for the step length, the object is limited to only a particle in motion and therefore on the bed surface, and its motion is strongly connected with comparatively regular bed undulations as seen from the fact that the variation coefficient of the wave length of sand waves is about 0.5. Hence, the variation coefficient of step length is comparatively small and a gamma distribution of which shape parameter is about 2 is appropriate as the distribution of step length as investigated in relation to the statistical properties of longitudinal bed configurations.

The properties of both rest period and step length investigated based on bed profile analyses showed a good agreement with the results of tracer tests or direct measurements, and this fact convinced us that the parameters representing the individual sediment behaviors can be determined in relation to the statistical properties of bed configurations. Meanwhile, the statistical properties of bed configurations have been vigorously investigated by a number of researchers and the authors derived a stochastic model[19] which can be applied to them conveniently.

After clarifications of the characteristics of individual sediment motion, a stochastic model for bed load transport process was investigated as an integration of them and a general model to unify the explanations of sediment transport process both on flat bed conditions and on undulated beds was presented. By this model, a forecast of the properties of sediment dispersion process over sand waves became possible and its validity was verified by applying this model to the experimental results previously obtained.

In conclusion, the model derived in this paper can explain bed load transport process systematically in dune bed regimes as well as in flat bed conditions, when the statistical properties of bed configurations are clarified. And, the flow properties over sand waves and the mechanism of sediment motion should be investigated to support this study in future.

References

1) Einstein, H. A.: The Bed Load Functions for Sediment Transportation in Open Channel Flows, Technical Bulletin, No. 1026, U.S. Department of Agriculture, Soil Conservation Service, 78 p., 1950.
2) Einstein, H. A. and N. L. Barbarossa: River Channel Roughness, Trans. ASCE, Vol. 117, 1952.
3) Yalin, M. S.: On the Averaged Velocity of Flow over a Movable Bed, La Houille Blanche, No. 1, 1964.
4) Engelund, F.: Hydraulic Resistance of Alluvial Stream, Proc. ASCE, HY 2, pp. 315–326, 1966.
5) Nakagawa, H., T. Tsujimoto and A. Yada: On Estimation of Bed Load Transport Rate over Dune Beds Based on a Stochastic Approach, Preprint of Annual Conference of Kansai Branch, JSCE, II-62, 1978, (in Japanese).
6) Simons, D. B., E. V. Richardson and C. F. Nordin, Jr.: Bed Load Equation for Ripples and Dunes, Geological Survey Professional Paper, 462-H, U.S. Government Printing Office, 9 p., 1965.
7) Guy, H. P., D. B. Simons and E. V. Richardson: Summary of Alluvial Channel Data from Flume Experiments, 1956-61, Geological Survey Professional Paper, 462-I, U.S. Government Printing Office, 96 p., 1965.
8) Stein, R. A.: Laboratory Studies of Total Load and Apparent Bed Load, Journal of Geophysical Research, Vol. 70, No. 8, pp. 1831–1842, 1965.

9) Grigg, N. S.: Motion of Single Particles in Sand Channels, Ph. D. Dissertation, Colorado State University, 142 p., 1969.
10) Nakagawa, H., T. Tsujimoto and K. Honbu: A Consideration on an Energetic Approach to Bed Load Transport over Dune Beds, Preprint, 29th Conference of JSCE, II-107, pp. 237–238, 1974, (in Japanese).
11) Bagnold, R. A.: The Flow of Cohesionless Grains in Fluid, Philosophical Trans., Royal Society of London, A-249, No. 964, pp. 235–297, 1957.
12) Einstein, H. A.: Der Geschiebetrieb als Wahrscheinlichkeitsproblem, Mitteilung der Versuchsanstalt für Wasserbau an der Eidgenössische Technische Hochschule in Zurich, Verlag Rascher & Co., 110 p., 1937.
13) Hubbell, D. W. and W. W. Sayre: Sand Transport Studies with Radioactive Tracers, Proc. ASCE, HY 3, pp. 39–68, 1964.
14) Yano, K., Y. Tsuchiya and M. Michiue: On the Stochastic Characteristics of Transport Mechanism of Sand in a Stream, Annuals, Disaster Prevention Research Institute, Kyoto University, No. 11-B, pp. 61-73, 1968, (in Japanese).
15) Sayre, W. W. and W. J. Conover: General Two Dimensional Stochastic Model for the Transport and Dispersion of Bed Material Sediment Particles, Proc. XIIth Congress, IAHR, Colorado, USA, Vol. 2, pp. 88–95, 1967.
16) Shen, H. W. and P. Todorovic: A General Stochastic Model for the Transport of Sediment Bed Material, Proc. First International Symposium on Stochastic Hydraulics, Pittsburgh, USA, pp. 489–503, 1971.
17) Nakagawa, H. and T. Tsujimoto: Study on Mechanism of Motion of Individual Sediment Particles, Proc. JSCE, No. 244, pp. 71–80, 1975, (in Japanese).
18) Hubbell, D. W. and W. W. Sayre: Closure to Discussion of Sand Transport Studies with Radioactive Tracers, Proc. ASCE, HY 5, pp. 139–149, 1965.
19) Nakagawa, H. and T. Tsujimoto: Study on Statistical Modeling of Bed Configurations in Lower Regimes, Proc. JSCE, No. 290, 1979, (in Japanese).
20) Yang, C. T. and W. W. Sayre: Stochastic Model of Sand Dispersion, Proc. ASCE, HY 2, pp. 265–288, 1971.
21) Nakagawa, H. and T. Tsujimoto: Experimental Study on Evolution of Sand Waves from an Initially Flattened Bed, Annuals, Disaster Prevention Research Institute, Kyoto University, No. 19-B2, pp. 289–309, 1976, (in Japanese).
22) Nakagawa, H. and T. Tsujimoto: On Generalized Stochastic Model for Bed Load Transport and Sediment Dispersion, Proc. JSCE, No. 291, 1979, (in Japanese).