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A Model of Fault Gouge with Dissipative Rotational Interactions

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Abstract

Gouge—microstructure near faults—the existence of which is accepted as a compromise between data from laboratory experiments and from earthquakes is modelled under the following assumptions: An internal energy is assumed to be a function of strains referred on the particle frame and rotation differences between rotations referred on the absolute frame and ones on the particle frame. Simultaneously, a dissipation law is formulated from the law of entropy production. From characteristic equations, three branches of solutions are derived:

(i) Non-attenuative and non-dispersive longitudinal waves

(ii) Two types of transversal waves—

A. weakly dispersive and attenuative waves which have a limit to those of perfect elasticity

B. non-causal and dissipative waves which have a limit to those of pure dissipation

Pre-seismic or post-seismic time-dependent deformations and coseismic quasi-elastic deformations are discussed from the proposed model.

1. Introduction

In the so-called crack model of seismic sources (*e.g.*, Kostrov (1966)), the start and stop of fracture motions are controlled by proper boundary conditions on a fracture region, which is usually composed of one fracture plane. In order to obtain more realistic features, the model should have a more complicated structure. For instance, we may consider an increased number of fracture planes. These lines of complications, however, would need the enormous memory of computers and hence there should be alternative ways of complication, retaining high precision in numerical computations. For this reason, models have been simplified under the assumptions that some complicated structures are concentrated on one fracture plane. (*e.g.*, Otsuka (1971), Mikumo and Miyatake (1978))

We discuss here an extension of conventional source models (which we call "*one sheet model*") To overcome some difficulties and restrictions imposed by computer capacity, we introduce a new freedom of motion into the problem. This parameter could be an adequate representation of the nature of fault gouge material for which much attention has been paid in laboratory experiments. (*e.g.*, Byerlee et al. (1978))

A model of fault gouge is formulated under assumptions that the internal energy of the system of an aggregate of deformable particles is composed of two variables, that is, strains on a particle and rigid rotations independent of averaged movements. An intuitive exposition of a model and the necessity of dissipative forces are discussed. Experimental results are cited in support of these formulations. From an observational point of view, there remains some problems to be solved, that is, pre-seismic or post-seismic time dependent deformations and coseismic quasi-elastic deformations. The solution of the above formulated equations gives necessary information for a possible interpretation of these phenomena.

As will be subsequently described in a separate paper, some extension of a model are attempted in a thermodynamical approach (Part II) and in a differential geometrical approach (Part III). The idea of independent-rigid rotations first took rise in Cosserat (1909) whose research had its recaptulation in Mindlin and Tiersten (1962), and independently in Oshima (1953). Our present formulation is more elementary and as explanatory as possible to leave out the ambiguity existing in former researches.

2. Mechanism of friction

Recently much work has been done on the problem of friction in relation to rock mechanics such as Brace and Byerlee (1966), Brace (1972), Scholtz (1968), Brune (1973), Johnson et al. (1975) and others. In frictional sliding experiments, there are several controlling parameters and conditions, *e.g.*, surface roughness, thickness of gouge layers, confining pressure, temperature etc. Brace (1972) has shown that a transition occurs from stick-slip to stable-sliding motions under appropriate P-T conditions. Dieterich (1972) has also shown the time-dependent recovery of fracture strength or static frictions after stick-slip events under some controlled normal pressures. However no complete dynamical experiments wherein the various parameters are controlled have been performed to date, so the above results may reflect only one aspect of the facts.

There has been much discussion about the stress drops estimated from the analysis of seismic waves ranging some ten to one hundred bars, which are one to two orders of magnitude lower than the results from laboratory experiments. When we compare the experimental results with the field data, we must pay attention not only to scaling effects (*e.g.*, Maeda (1977)) but to the effects of compound material including several weak zones. Here we introduce gouge as a possible way of representing these effects. One sheet model with properly controlled surface conditions may be one primitive model along this line.

Recent development in experimental techniques has revealed the nature of gouge in sliding on a microscopic scale. (*e.g.*, Byerlee et al. (1978)) Although the formation process of gouge is not yet clear at present, we have the following concept in constructing our model. After a virgin fracturing in the earth and several repetitions of sliding on or near the same surface, the slipped segments will have been crushed to pieces and lose their elasticity as a whole. An influx of water from the surrounding region may promote further degradation. The region of slippage is called gouge. Although the whole gouge has lost elastic properties, the constituents of the gouge, which we call particles, still have elasticity as in the surrounding region. Even if gouge is substituted by different minerals due to the influx of water, it is usual that each of the particles are still elastic. For this reason, it is now necessary not only to elucidate the nature of friction mechanism of each contacting particle on the gouge material but to model the aggregation of particles.

3. Models of fault gouge

Following discussions in the preceding sections, we model the source of earthquakes as a fault including gouge sandwiched between nearly perfect elastic rocks, unlike that of one dislocation surface. This feature of the source is in harmony with geological field observations of active faults near the earth's surface.

Consider interacting forces between two particles in gouge. The interacting forces between particle A and particle B act only at their contacting point, which is illustrated in Fig. 1, if force at a distance, for instance an electrostatic force, can be neglected. In the case of Coulomb's type of frictions, the tangential component of contacting force is proportional to its normal component. Particle A and particle B rotate keeping in touch with each other, when the relative velocity between the contacting points is zero, i.e., the tangential component of the contacting force is lower than that of static friction or the intrinsic strength. Once slip motion between the two particles initiates, the interacting force will be divided into two distinguishing terms; dissipation force acting due to mutual slippage and restoring force due to breaking of a static equilibrium. After the relative velocity between the particles is generated, eigen rotations distributed to the particles will become uniform or cease immediately, following fault movements. At the initiation of relative motions, the contacting points would collapse in such a way that the relative motion becomes dissipative in itself.

The above stated circumstances are schematically illustrated in Fig. 2, where the stage indicated downward by an arrow is the local motion and not relevant directly to the fault movement. If we make the size of the particles smaller and smaller, the dipole of eigen rotations of the particles in opposite directions will not be essential and would disappear under the assumption of finite eigen rotations. In this case the deformation process reduces to that described by classical continuum mechanics (*e.g.*, Love (1927)), and hence dissipative processes become insignificant. After the initiation of rotations, the system of particles will be placed in a neutral equilibrium. This image of the neutral equilibrium is completed by the introduction of sub-particles as in Fig. 3. But the condition of the convenience of this sort is not always satisfied in the medium composed of aggregated particles.

There could be a different case shown in Fig. 4. At the contacting surfaces between particles and the continuum medium, the dissipation due to rotations without slippage may be negligible, while with internal contacting

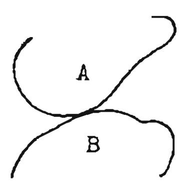

Fig. 1. Two particles which constitute the part of gouge contacting each other without mutual local rotations.

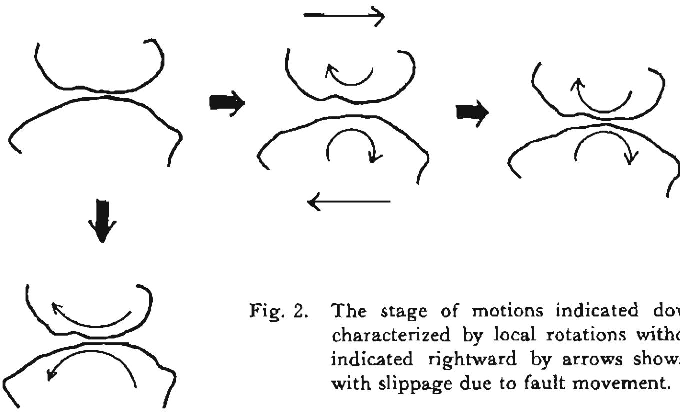

Fig. 2. The stage of motions indicated downward by an arrow is characterized by local rotations without slippage. The stage indicated rightward by arrows shows mutual local rotations with slippage due to fault movement.

points, the dissipation due to slippage is not. (We call rotations without slippage as the motion exerted under static type of friction and rotations with slippage as the motion exerted under dynamic type of friction.) Deformation without dissipation cannot be realized in such a multi-particle system as in Fig. 4. This is contrary to the case of a model of perfect elasticity or a perfect fluid without internal structures. Since we cannot usually expect a favorable configuration of particles such as shown in Fig. 3, we assume here the model of gouge as in Fig. 4.

When the slippage initiates after the stress exceeds the critical strength, it is to be noted that even when the nature of contacting points that are crushed primarily determines the type of dissipation mechanism, the averaged nature of dissipation would be somewhat different from an elementary process. For instance, in crystallography, viscous medium is inserted within grain boundaries as a model of the mechanism of gliding or attenuation of acoustic vibrations, or in other words, as a model of grain boundaries. (Smoluchowski (1952)) Since grain boundaries should be expressed as an aggregation of dislocation lines (*e.g.*, Cottrell (1953)), this way of modelling has a phenomenological

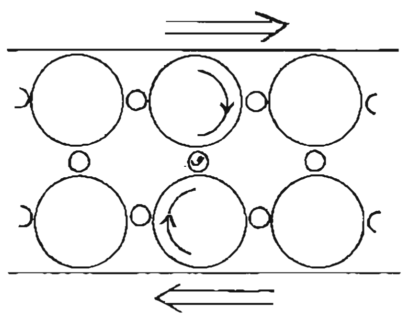

Fig. 3. A multi-size particle system of which motion is generated by mutual rotation without slippage.

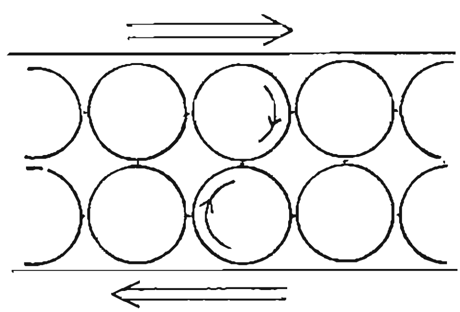

Fig. 4. A schematic exposition of a model of fault gouge: particles rotate in the same direction after being triggered by fault movement.

significance in that the mechanical behavior of the model properly approximates those of the matter.

In this paper we adopt Smoluchowski's model of grain boundaries as a model of dissipation for the system of particles, based on the reasoning stated above, although our particles do not correspond to his crystallographic grains.

4. Formulation of models

We next derive the equations of motion for the system with dissipative forces. As stated in the preceding section, we supposed that gouge is composed of particles with a finite size. This finiteness can be put in another way; inhomogeneities of distribution of relative rotations as modelled below. Each of the particles can deform elastically. Motions of a particle can be decomposed into a relative rotating motion around the center of gravity and a deformation of the system with respect to the center of gravity. This deformation can also be decomposed into strains and rotations within linear transformations.

In the following calculations we apply Einstein's summation convention. And the process of mixing over p indices by constructing all p! isomers obtained by permutations of these indices, addition of these isomers and division of p! is denoted by a round bracket ( ):

$$e.g.,\quad \boldsymbol{A}_{(ij)}=\frac{1}{2}(\boldsymbol{A}_{ij}+\boldsymbol{A}_{ji})$$

A similar process with the only difference that all isomers obtained by odd permutation get a negative sign is denoted by a square bracket [ ]. If indices have to be singled out, the sign | | is used:

$$e.g.,\quad \boldsymbol{B}_{[i|j|k]}=\frac{1}{2}(\boldsymbol{B}_{ijk}-\boldsymbol{B}_{kji})$$

Here we assume that the type of interaction between particles is that described below: Strains on an average of particles are not primary and interacting forces between the particles only act through mutual rotations. This assumption will be debated later with a supplemented extension of a model.

First we assume that dissipative forces act on or near the surface of a particle. Hence the equations of motion for the system of the center of gravity are expressed below,

$$\rho\dot{v}_j=\partial_i\sigma_{ij}+f_j \tag{4-1}$$

$$f_j=\partial_i F_{ij} \tag{4-2}$$

where σ_{ij} is the stress tensor and f_j is the dissipative force. The coordinate system is supposed to be Cartesian hereafter.

Next, the equations of motion for the relative rotating motion of a particle around the center of gravity are expressed in the integrated form,

$$\frac{1}{2}\int\rho I\dot{\omega}_{ki}dV=\int x_{[k}G_{|j|i]}dS_j+\int M_{lki}dS_l \tag{4-3}$$

$$G_{ji}=F_{ji}+\sigma_{ji} \tag{4-4}$$

where M_{lki} is the surface distribution of torque which has an arm in the k-direction and the force exerted in the i-direction on a surface element with a normal in the l-direction. As stated previously, M_{lki} is determined by mutual rotations of neighboring particles. I is the distribution of a moment of inertia per unit mass, which is properly transformed into that in the principal axis, and here assumed to be isotropic for convenience's sake.

After some calculations,

$$\int x_{[k}G_{|j|i]}dS_j=\int(G_{[ki]}+\rho x_{[k}\dot{v}_{i]})dV$$

If we presume that

$$\sigma_{ij}=\sigma_{(ij)} \tag{4-5}$$

and

$$G_{[ij]}=F_{[ij]} \tag{4-6}$$

then

$$\frac{1}{2}\rho I\dot{\omega}_{ki}=F_{[ki]}+\partial_l M_{lki}+\rho x_{[k}\dot{v}_{i]} \tag{4-7}$$

The above expressions (4-7) show that the force is decomposed into the purely conservative part and the dissipative part. Usually the assumption (4-5) is identical to the law of the conservation of angular momentum. Since we introduced the interacting term represented by M_{lki}, our new form of conservation of angular momentum is fulfilled together with the assumptions (4-5) and (4-6). Especially the assumption (4-6) comes from the fact that dissipative forces should be tangent to the surfaces of particles.

These aspects are easily confirmed by formalistic calculations. With the voluminal distribution of torque h_{ij} and that of body force g_i newly introduced, the equations of motion for inertia moment are expressed as,

$$\begin{aligned}[\int x_{[k}\rho v_{j]}dV]' &= \int x_{[k}G_{|i|j]}dS_i+\int M_{ikj}dS_i \\ &\quad +\int x_{[k}g_{j]}dV+\int h_{kj}dV\end{aligned} \tag{4-8}$$

and the translational part of motion is expressed as,

$$\begin{aligned}\int x_{[k}\rho\dot{v}_{j]}dV &= \int x_{[k}\partial_{|i}G_{i|j]}dV+\int x_{[k}g_{j]}dV \\ &= \int x_{[k}G_{|i|j]}dS_i-\int G_{[kj]}dV+\int x_{[k}g_{j]}dV\end{aligned} \tag{4-9}$$

From expressions (4-8) and (4-9), we get

$$\int M_{ikj}dS_i+\int h_{kj}dV+\int G_{[kj]}dV=0 \tag{4-10}$$

Hence, if $M_{ikj}=0$ and $h_{kj}=0$, then $G_{[kj]}=0$ and vice versa.

The expressions (4-7) and (4-10) are identities derived from kinematical relations. Usually since we may presume

$$\frac{1}{2}\rho I\dot{\omega}_{ki}=\rho x_{[k}\dot{v}_{i]},$$

expression (4-10) is identical to (4-7). If the discrete nature of mass distributions is essential, such as of atoms in a crystal, $\rho I\dot{\omega}_{ki}$ may have proper meaning. However, since we assume the smoothness of mass distributions, we get the next expression.

$$F_{[ij]}+\partial_l M_{lij}=0 \tag{4-7$'$}$$

We must assume dynamical relations between kinematics and deformation quantities — constitutive equations. The law of termodynamics — energy conservation and entropy production — must be fulfilled. We express by ϕ_{ij}, the eigen rotations independent of the rotational strains referred at the particle frame. Internal energy u for each of the particles or per unit volume is a function of strains ε_{ij}, relative rotations ζ_{ijk} approximated below and entropy S, that is,

$$u=u(\varepsilon_{ij}, \zeta_{kij}, S) \tag{4-11}$$

where

$$\zeta_{kij}\fallingdotseq\partial_k(\partial_{[i}u_{j]}-\phi_{ij}) \tag{4-12}$$

The ϕ_{ij} are defined on the absolute frame, so that $\partial_{[i}u_{j]}-\phi_{ij}$ have only approximate meanings. Formalistic extension of expressions (4-11) and (4-12) are, of course, conceivable. For instance, as a substitute for ζ_{kij},

$$\zeta_{ij}=\partial_{[i}u_{j]}-\phi_{ij}$$

may be variables of the internal energy. But since we now concentrate our discussions on the model with inhomogeneous distributions of rotations, we do not adopt such models These points will be discussed in Part III.

The law of energy conservation

$$dE=\delta W+\delta Q \tag{4-13}$$

where

dE: the increment of the sum of internal energy U and kinetic energy K.
δW: the work done by the exterior system.
δQ: the inflow of heat (non-mechanical external action)

The expression (4-13) is rewritten as evolutional forms.

$$\frac{dE}{dt}=\frac{dU}{dt}+\frac{dK}{dt}$$

$$\frac{dE}{dt}=\frac{\delta W}{dt}+\frac{\delta Q}{dt}$$

where δ/dt shows the path-dependent derivatives.

If we assume

$$K=K_0=\int \frac{1}{2}\rho\dot{u}_i\dot{u}_i dV,$$

$$\frac{\delta W}{dt}-\frac{dK}{dt}=\int (\sigma_{ij}d_{ij}+F_{ij}l_{ij})dV \tag{4–13′}$$

from expressions (4-1) and (4-2),
where

$$d_{ij}=\partial_{(i}\dot{u}_{j)},\quad l_{ij}=\partial_{[i}\dot{u}_{j]}$$

and

$$\frac{\delta W}{dt}=\int \dot{u}_j\sigma_{ij}dS_i+\int \dot{u}_jF_{ij}dS_i$$

Here we furthermore add the effect of rotational type of interaction, that is, $\dot{\zeta}_{kij}M_{kij}$.

The increment can be written as,

$$\delta W=\int [(\sigma_{kj}+F_{kj})\delta u_j+M_{kij}\delta\zeta_{ij}]dS_k$$
$$=\int [\sigma_{kj}\delta\partial_{(k}u_{j)}+F_{kj}\delta\phi_{kj}+M_{kij}\delta(\partial_k\zeta_{ij})+\rho\ddot{u}_j\delta u_j]dV \tag{4–14}$$

using expressions (4-1), (4-2) and (4-7′). We notice the discrepancies between (4-13′) and (4-14), which originates from newly introduced terms.

The increment of internal energy is

$$dU=\frac{\partial U}{\partial\varepsilon_{ij}}d\varepsilon_{ij}+\frac{\partial U}{\partial\zeta_{kij}}d\zeta_{kij}+\frac{\partial U}{\partial S}dS \tag{4–15}$$

where

$$\frac{\partial U}{\partial S}=T \tag{4–16}$$

The law of entropy production

$$dS\geqq\frac{\delta Q}{T} \tag{4–17}$$

If the entropy per unit volume is s,

$$ds\geqq-\partial_i\frac{Q_i}{T} \tag{4–18}$$

where Q_i is heat flux from a unit surface element with a normal in the i-direction. If we substitute (4-13), (4-14), (4-15), and (4-16) into (4-18), noting

$$\delta Q = -\partial_i Q_i \tag{4-19}$$

then

$$\left(\sigma_{ji} - \frac{\partial u}{\partial \zeta_{ij}}\right) d\varepsilon_{ij} + \left(M_{kij} - \frac{\partial u}{\partial \zeta_{kij}}\right) d\zeta_{kij}$$
$$+ F_{ij} d\phi_{ij} + Q_i \partial_i \frac{1}{T} \geqq 0 \tag{4-20}$$

This inequality must be fulfilled for all variation of ε_{ij}, ζ_{kij}, ϕ_{ij} and Q_i. Hence

$$\sigma_{ij} = \frac{\partial u}{\partial \varepsilon_{ij}} \tag{4-21}$$

$$M_{kij} = \frac{\partial u}{\partial \zeta_{kij}} \tag{4-22}$$

$$F_{ij} = 2\eta(\dot{\phi}_{ij})^{2n+1} \tag{4-23}$$

$$Q_i = -\kappa(\partial_i T)^{2n+1} \tag{4-24}$$

The first relation (4-21) is an extended form of Hooke's law. The relation (4-22) is a dynamical relation between ζ_{kij} and M_{kij} which is defined in this way. The relation (4-24), when $n=0$, is a well-known heat conduction law. The relation (4-23) is a possible form of friction or dissipation law due to the condition of the coexistence with the law of (4-24) and we call η angular viscosity of $(2n+1)$-th order, tentatively. If we set integer n to be zero, (4-23) is analogious to Newtonian type of viscosity.

When u is positive definite quadratic form of ε_{ij} and ζ_{kij}, and the material is assumed to be isotropic, then

$$\sigma_{ij} = \lambda \varepsilon_{kk} \delta_{ij} + 2\mu \varepsilon_{ij} \tag{4-25}$$

$$M_{kij} = C\zeta_{kij} \tag{4-26}$$

where λ and μ are elastic moduli and C is a newly introduced material constant.

From these expressions,

$$(\lambda+\mu)\partial_j(\partial_k u_k) + \mu \Delta u_j = -2\eta \partial_i \dot{\phi}_{ij}{}^{2n+1} + \rho \ddot{u}_j \tag{4-27}$$

$$2\eta \dot{\phi}_{ij}{}^{2n+1} = C\Delta(\phi_{ij} - \partial_{[i} u_{j]}) \tag{4-28}$$

The duality between the image of an aggregation of particles with a finite size and that of properly smoothed but inhomogeneous distributions of relative rotations will not be rectified, but be used accordingly.

5. Estimations of the order of magnitude

The difference in the time derivative with respect to the moving frame (Lagrangean description) and to the absolute frame (Eulerian description) will be negligible when the spatial variations of the physical quantities are small, or when the initial field small.

With sufficient correctness we sometimes identify two types of description of time history.

We evaluate the significance of each term of the system of equations in the case of $n=0$, by estimating the order of their magnitudes (dimensional analysis). If we set particle size or the extent of an approximately homogeneous region as L, and the relaxation time or period as T, then for (4-27),

$$\frac{\rho\ddot{u}}{\mu\Delta u} \simeq \frac{\rho}{\mu}\left(\frac{L}{T}\right)^2 \tag{5-1}$$

and for (4-28),

$$\frac{u}{\phi} \simeq \frac{2\eta}{C}\frac{L^3}{T} \tag{5-2}$$

under the approximation

$$|\phi_{ij}| \ll |\partial_{[i}u_{j]}| \tag{5-3}$$

From these estimates

$$\frac{\mu\Delta u}{2\eta\partial\dot{\phi}} \simeq \frac{\mu}{2\eta}\frac{T}{L}\frac{u}{\phi} \simeq \frac{\mu}{c}L^2 \tag{5-4}$$

For the estimate of (5-1), we can generally assume

$$\frac{L}{T} \gtrsim \sqrt{\frac{\mu}{\rho}} \tag{5-5}$$

Here we define $A=\frac{\mu}{c}L^2$. If $A\simeq 1$, C tends to zero as L approaches zero. In this case the system of equations cannot be reduced to that for perfect elasticity as long as friction terms have not been left out. $\eta=0$ should be substituted for expressions (4-27) and (4-28), if we want to eliminate the effects of friction alone. Then the equations of motion for perfect and isotropic elasticity are derived. In addition, it is necessary that $\partial_{[i}u_{j]}-\phi_{ij}$ are harmonics. Since this part if the equations is time-independent, $\phi_{ij}=\partial_{[i}u_{j]}$ should be kept, if held at an initial time.

There are two possibilities; (i) C tends to zero or (ii) L tends to infinity, if A tends to infinity. In case (ii), as the size of particles increases, we describe a one-particle approximation, that is, on the earth we may let a maximum value of it be a plate size. Of course, this is not directly applicable to the plate motions, because of the assumption (5-3). Under the assumption $|\partial_{[i}u_{j]}| \ll |\phi_{ij}|$, by similar reasoning, we get

$$A=\frac{\mu}{2\eta}\frac{T}{L}\frac{u}{\phi}=\frac{\mu}{2}\frac{L^2}{C}\frac{u}{L\phi}$$

While in the case of $|\partial_{[i}u_{j]}| \simeq |\phi_{ij}|$, we have

$$A=\frac{\mu}{C}L^2 \simeq \frac{\mu T}{2\eta} \simeq \frac{\mu}{2\eta}\frac{T}{L}\frac{u}{\phi} \quad \text{as} \quad L\phi \sim u$$

Hence we can determine the law of similarity of the system of equations (4-27) and (4-28) by parameters;

$$\frac{\rho}{\mu}\left(\frac{L}{T}\right)^2 \tag{5-6}$$

$$\frac{2\eta}{C}\frac{L^2}{T} \tag{5-7}$$

and

$$\frac{\mu}{C}L^2 \quad (=A) \tag{5-8}$$

From the inequality (5-5),

$$\frac{\rho}{\mu}\left(\frac{L}{T}\right)^2 \gtrsim 1$$

Consider, for instance, the difference scheme of equation (4-28) and we get the dissipation condition for ϕ as

$$\frac{2\eta}{C}\frac{L^2}{T} \gtrsim 4 \tag{5-9}$$

In order to give an assurance of terminal stability, we presume this inequality. Similar discussions for the condition (5-5) can be performed as in Courant et al. (1928). However for the quantity of (5-8), we cannot assume definite inequalities. In the case of $A \ll 1$, we get from expression (4-27)

$$2\eta\partial_i\dot{\phi}_{ij} \simeq \rho\ddot{u}_j \tag{5-10}$$

as a limited form, and expression (4-28) does not change at all. The relation (5-10) is rewritten as

$$\left(\int \rho\dot{u}_j dV\right)' = \left(\int F_{ij} dS_i\right)' \tag{5-10$'$}$$

The left hand side of (5-10′) is the time derivative of the linear momentum and the right hand side is that of the purely dissipative forces acting on the surface of particles. Then acted forces are purely dissipative. If $A \ll 1$, there are two possibilities; (i) C tends to infinity or (ii) L tends to zero. In the case (ii) for which the particle size gets smaller and smaller, particle motions reduce to that of microscopic scales, i.e., random motions by thermal agitations. If L tends to zero under the condition of C, ρ, μ and η fixed, similarity of motions will not be maintained due to expressions (5-6) and (5-7) so far as T is fixed. As previously stated, to determine similarity relations, parameter A is directly relevant. Here is given some expositions. If L and C simultaneously tend to zero; for expression (5-7),

$$\frac{2\eta}{C}\frac{L^2}{T} \simeq O\left(\frac{2\eta}{T}\right) \gtrsim O(1)$$

owing to the conditions of diffusivity and for expression (5-6), the same inequality as (5-5) is maintained owing to the conditions of stability of hyperbolic type of equations and for expression (5-8),

Table 1. Relation between particle size or the extent of approximately homogeneous region (L) and newly introduced material constant C. A is defined by the ratio of the terms of restoring forces and the terms of dissipative forces. The state of fault gouge is characterized between two extreme values of A.

A	0	1	∞
C	∞ (?)	Fault	0 elasticity
L	0 Microscopic motion	Gouge	∞ one-particle (plate tectonics)

$$\frac{\mu}{C}L^2 \simeq O(\mu),$$

then we have $A \simeq O(\mu)$.

In case (i), from the analogy with the treatment of rigid matter in the class of elastic matter, $\phi_{ij} \simeq \partial_{[i}u_{j]}$, i.e., the freedom of rotational motion is degenerated. M_{ijk} and F_{ij} are consequently indefinite so that the relation (5-10′) is only an example aptly converged.

We summarize the results from the above stated discussions in Table 1. Remembering that parameter A is the ratio of the restoring forces to the dissipative forces, we propose to use critical A for describing the state of fault gouge. For instance, the cluster size may get smaller as fracturings proceed, as far as L alone is concerned. However, another parameter C controls the formation of fracturing simultaneously.

Then ,how can we observe and determine these parameters?

6. Determination of some types of solution and their characteristic

Particle size L may be determined by means of field observations. We can determine the remaining parameters, after solving equations (4-27) and (4-28) in the case of $n=0$.

For simplicity, we consider plane waves as,

$$u_i = u_i(x, t) \quad \text{and} \quad \phi_{ij} = \phi_{ij}(x, t) \tag{6–1}$$

As stated above, physical parameters λ, μ, η and C are assumed to be homogeneous. Here if we define

$$\phi_i = \varepsilon_{ijk}\phi_{jk}, \quad \phi_i = (\phi_x, \phi_y, \phi_z) \tag{6–2}$$
$$u_i = (u, v, w)$$

where ε_{ijk} is Eddington's symbol, then

$$(\lambda+2\mu)\frac{\partial^2 u}{\partial x^2} = \rho\ddot{u}, \quad 2\eta\dot{\phi}_x = C\frac{\partial^2}{\partial x^2}\phi_x \tag{6–3}$$

$$\mu\frac{\partial^2 v}{\partial x^2} + \eta\frac{\partial}{\partial x}\dot{\phi}_z = \rho\bar{v}, \quad 2\eta\dot{\phi}_z = C\frac{\partial^2}{\partial x^2}\left(-\frac{\partial v}{\partial x} + \phi_z\right) \tag{6–4}$$

$$\mu\frac{\partial^2 w}{\partial x^2}-\eta\frac{\partial}{\partial x}\dot{\phi}_y=\rho\ddot{w}, \quad 2\eta\dot{\phi}_y=C\frac{\partial^2}{\partial x^2}\left(\frac{\partial w}{\partial x}+\phi_y\right) \tag{6-5}$$

The first part of (6-3) shows plane P-wave propagation. This P-wave is characterized by non-dispersive and non-attenuative nature. If we give an initial condition for the second part

$$\phi_z(x, 0)=\delta(x),$$

then we have

$$\phi_z(x, t)=\frac{1}{\sqrt{2\pi\frac{C}{\eta}t}}\exp\left(-\frac{\eta}{2C}\frac{x^2}{t}\right) \tag{6-6}$$

This equation has the same form as that for heat conduction and diffusion equation. If we assume

$$\phi_z=\phi_z^0\cdot\exp\{i(kx-ft)\},$$

$$\phi_z=\phi_z^0\cdot\exp\left(-\sqrt{\frac{\eta f}{C}}\cdot x\right)\exp\left\{i\left(\sqrt{\frac{\eta f}{C}}\cdot x-ft\right)\right\} \tag{6-7}$$

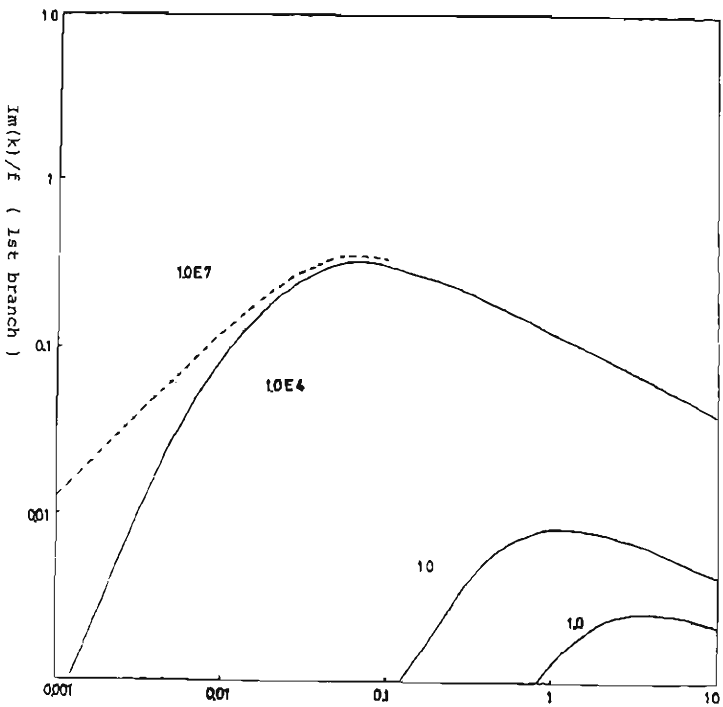

Fig. 5(1). Relation between f and imaginary parts of wave number divided by f ($Im(k)/f$)–first branch of solutions, under $\eta=25$ and C varied.

It is easily shown that equations (6-4) and (6-5) have the same type of solutions. If we assume for (6-4) that

$$v=v^0\cdot\exp\{i(kx-ft)\} \quad \text{and} \quad \phi_z=\phi_z{}^0\cdot\exp\{i(kx-ft)\} \tag{6-8}$$

then we have the next characteristic equation.

$$\begin{vmatrix} \rho f^2-\mu k^2 & -\eta f k \\ -iCk^3 & 2if\eta-Ck^2 \end{vmatrix}=0 \tag{6-9}$$

Dispersion and attenuation relations derived from this equation are shown in Fig. 5. The abscissa of the figures gives the angular frequency (f), and the ordinate gives the magnitude of the real and imaginary parts of the wave number (k) divided by f with parameters C and η varied. The density ρ and rigidity μ are assumed to be unity for simplicity. It is seen from this figure that two branches of solution exist, one or both of which have the properties of a velocity higher than that of S waves. One of the two bracnhes can be properly approximated by the (6-7) in a wide range of combinations of parameters C and η.

There appears a maximum in each of the $Im(k)/f$ *vs.* f curves within the frequency range 0.001 to 10.0 rad/sec, as η increases. In the case when C is relatively larger and η is smaller, the value of f at which each of the $Im(k)/f$ curves of the non-purely dissi-

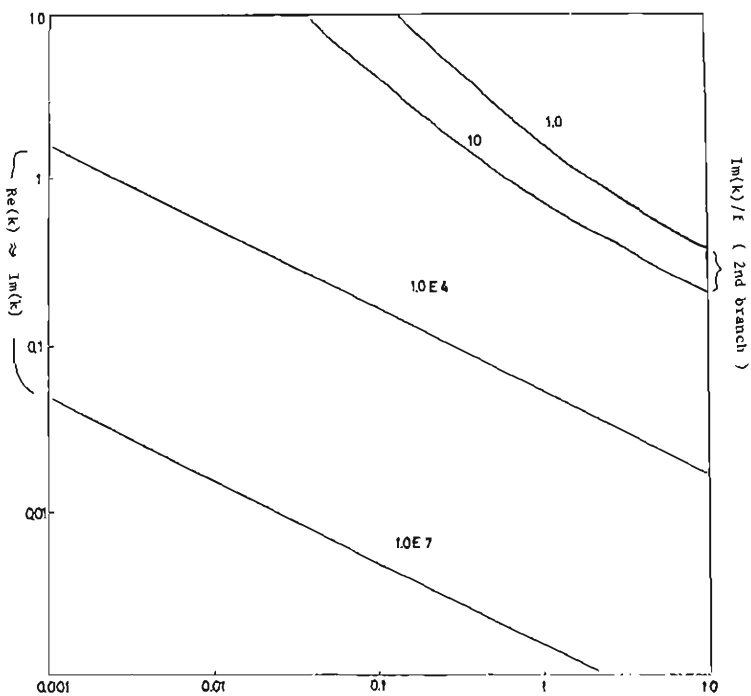

Fig. 5(2). Relation between f and $Im(k)/f$-second branch of solutions, under $\eta=25$ and C varied. For higher values of C (10^4 and 10^7), $Re(k)/f$ is nearly equal to $Im(k)/f$.

pative branch of the solutions takes a maximum is roughly proportional to $1/\eta$. (*e.g.*, $C=10^7$, $\eta=10^2/4, f=0.069$; $C=10^5$, $\eta=10/4, f=0.7$; $C=10^5$, $\eta=1/4, f=6.9$; $C=10^4$, $\eta=1/4$ $f=7.0$; $C=10^3$, $\eta=1/4$, $f=6.9$) If the condition $C\gtrsim\eta$ is satisfied, one of the two branches of solutions shows a close agreement between $Re(k)$ and $Im(k)$. Comparatively speaking under such combinations of C and η as $C\ll\eta$, there appear discrepancies between $Re(k)$ and $Im(k)$. For both circumstances, the velocity necessarily becomes lower than that of S waves at lower frequencies.

As C becomes larger with η left fixed, the velocity of the dissipative branch generally becomes higher, and the motion gradually gains non-attenuative properties. The higher velocity is mainly due to the dissipative forces and results in rigidly coupled motion of the medium. In field observations, we might not detect this type of motion directly through the determination of local distribution of higher propagation velocities, but moderately averaged motions through several instrumental distortions and filtrations. It may be appropriate for models with a sudden start of fracturing in the case of strong interaction between particles. Conversely, as C becomes smaller with η left fixed, lower velocities and highly attenuative branches become predominant, that is, in the case of weak interaction between particles.

Next we consider some special configuration of equations (4-27) and (4-28), that is,

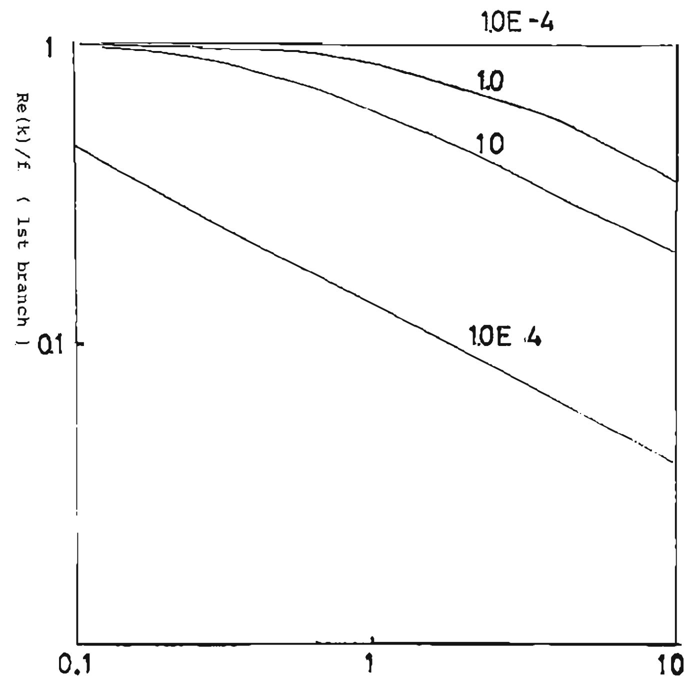

Fig. 5(3). Relation between f and real parts of wave number divided by f ($Re(k)/f$)-first branch of solutions, under $\eta=25$ and C varied.

$u=u(x, y, t)$, $v=v(x, y, t)$, $w=0$ and $\phi_i=\phi_i(x, y, t)$, $1\leqq i\leqq 3$. These equations are reduced to

$$2\eta\dot{\phi}_x=C\Delta\phi_x \qquad (6\text{–}10)$$
$$2\eta\dot{\phi}_y=C\Delta\phi_y$$

$$2\eta\dot{\phi}_z=C(\Delta\phi_z-\partial_x\Delta v+\partial_y\Delta u) \qquad (6\text{–}11)$$

$$(\lambda+\mu)\partial_x(\partial_x u+\partial_y v)+\mu\Delta u-\eta\partial_y\dot{\phi}_z=\rho\ddot{u} \qquad (6\text{–}12)$$

$$(\lambda+\mu)\partial_y(\partial_x u+\partial_y v)+\mu\Delta v+\eta\partial_x\dot{\phi}_z=\rho\ddot{v} \qquad (6\text{–}13)$$

For the condition for initial values, we get from equation (6-10) and the equation for w,

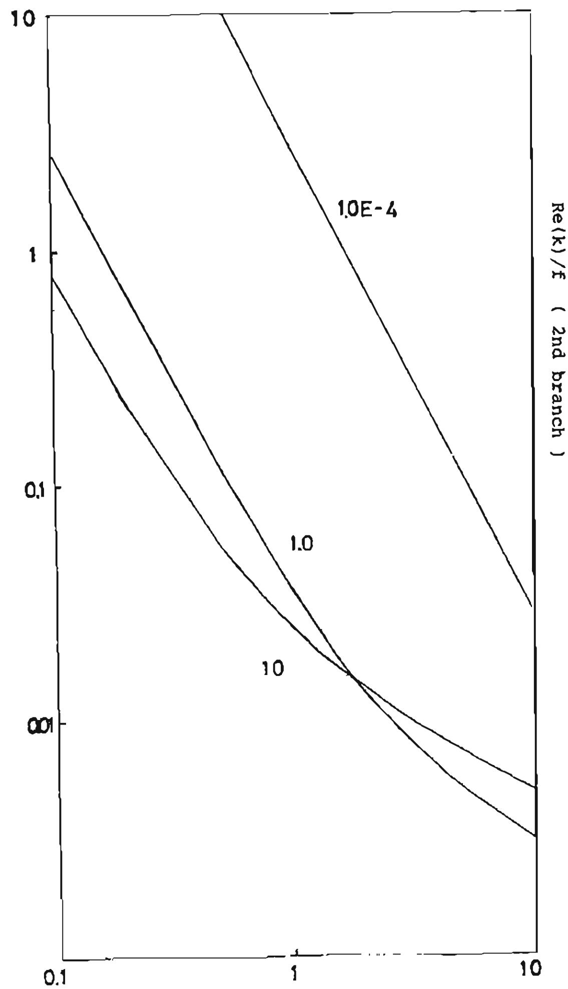

Fig. 5 (4). Relation between f and $Re(k)/f$–second branch of solutions, under $\eta=25$ and C varied.

$$C\Delta(\partial_x\phi_y-\partial_y\phi_x)=0 \tag{6-14}$$

If

$$\partial_x\phi_y=\partial_y\phi_x \tag{6-14'}$$

is fulfilled at infinity or at initial time, then $\partial_x\phi_y=\partial_y\phi_x$ is identically realized at any time. We tentatively call this time-independent identity for ϕ_i the compatibility condition of ϕ_i. Then ϕ_x and ϕ_y field is expressed as a gradient field like $\phi_x=\partial_x\chi$ and $\phi_y=\partial_y\chi$. From the expression (6-10), χ field is purely dissipative. Here if we introduce new functions Ψ and Φ for the sake of convenience,

as
$$\begin{aligned} u&=\partial_x\Phi+\partial_y\Psi \\ v&=\partial_y\Phi-\partial_x\Psi \end{aligned} \tag{6-15}$$

then

$$2\eta\dot{\phi}_z=C(\Delta\Delta\Psi+\Delta\phi_z) \tag{6-16}$$

$$(\lambda+2\mu)\Delta\Phi=\rho\ddot{\Phi} \tag{6-17}$$

$$\mu\Delta\Psi-\eta\dot{\phi}_z=\rho\ddot{\Psi} \tag{6-18}$$

If C tends to infinity with η kept finite, expressions (6-16) and (6-18) are transformed into

$$\mu\Delta\Psi+\eta\Delta\dot{\Psi}=\rho\ddot{\Psi} \tag{6-19}$$

Then the fields Φ and Ψ are decoupled into expression (6-17) and expression (6-19). The field ϕ_z is constrained by the field Ψ in the same form as conventional continuum field, i.e.,

$$\phi_z=\partial_x v-\partial_y u$$

The equation system of (6-17) and (6-19) are a special type of Voigt solid (Voigt (1892), Sezawa (1927), etc) In the scheme of Table 1, we should add the above type of medium.

As in the case of plane waves, P waves are non-dispersive and non-attenuative. It is easily seen that monochromatic 2-dimensional waves

$$\begin{aligned} \Psi&=\Psi^0\cdot\exp\{i(kx+ly-ft)\} \\ \phi_z&=\phi_z{}^0\cdot\exp\{i(kx+ly-ft)\} \end{aligned} \tag{6-20}$$

satisfy the characteristic equation

$$\begin{vmatrix} \mu(k^2+l^2)-\rho f^2 & i\eta f \\ C(k^2+l^2)^2 & C(k^2+l^2)-2i\eta f \end{vmatrix} \tag{6-21}$$

This relation is similar to that of (6-9) if k^2 is substituted for k^2+l^2.

Secondly in the case of $u=0$, $v=0$, $w=w(x, y, t)$ and $\phi_i=\phi_i(x, y, t)$, $1\leqq i\leqq 3$, then we have,

$$2\eta\partial_x\dot{\phi}_z=0,\quad 2\eta\partial_y\dot{\phi}_z=0,\quad \eta\dot{\phi}_z=C\Delta\phi_z \tag{6–22}$$

$$\mu\Delta w+\eta(-\partial_x\dot{\phi}_y+\partial_y\dot{\phi}_x)=\rho\ddot{w} \tag{6–23}$$

$$2\eta\dot{\phi}_x=C(-\partial_y\Delta w+\Delta\phi_x) \tag{6–24}$$

$$2\eta\dot{\phi}_y=C(\partial_x\Delta w+\Delta\phi_y) \tag{6–25}$$

If we put

$$\eta=0 \tag{6–26}$$

equation (6-22) reduces to

$$C\Delta\phi_z=0 \tag{6–27}$$

and from the remaining equations we obtain

$$\begin{aligned} &C(-\partial_y\Delta w+\Delta\phi_x)=0\\ &C(\partial_x\Delta w+\Delta\phi_y)=0 \end{aligned} \tag{6–28}$$

The relations (6-27) and (6-28) can be summarized using vector notation as

$$C\Delta(\mathrm{rot}\ \boldsymbol{U}-\boldsymbol{\Phi})=0 \tag{6–29}$$

As discussed in §. 5, $\boldsymbol{\Phi}-\mathrm{rot}\ \boldsymbol{U}$ are harmonics and

$$\boldsymbol{\Phi}=\mathrm{rot}\ \boldsymbol{U} \tag{6–30}$$

holds if initially holds. Then in this case the system of equations can be reduced to those for perfect and isotropic elasticity.

If

$$\eta \neq 0 \tag{6–31}$$

in the equation (6-22) should have a form

$$\dot{\phi}_z=\dot{\phi}_z(t) \tag{6–32}$$

and simultaneously from the equations (6-22) we have

$$\dot{\phi}_z=0 \tag{6–33}$$

The remaining equations are

$$\begin{aligned} &C\ddot{w}=\mu\Delta w+\eta(-\partial_x\dot{\phi}_y+\partial_y\dot{\phi}_x)\\ &2\eta\dot{\phi}_x=C(-\partial_y\Delta w+\Delta\phi_x)\\ &2\eta\dot{\phi}_y=C(\partial_x\Delta w-\Delta\phi_y) \end{aligned} \tag{6–34}$$

It is easily seen that monochromatic 2-dimensional waves

$$\begin{aligned} &w=w^0\cdot \exp\ \{i(kx+ly-ft)\}\\ &\phi_x=\phi_x{}^0\cdot \exp\ \{i(kx+lj-ft)\}\\ &\phi_y=\phi_y{}^0\cdot \exp\ \{i(kx-lt+ft)\} \end{aligned} \tag{6–35}$$

satisfy the characteristic equation

$$\begin{vmatrix} i2\eta f-C(k^2+l^2) & 0 & -iCl(k^2+l^2) \\ 0 & i2\eta f-C(k^2+l^2) & iCk(k^2+l^2) \\ -\eta lf & \eta kf & \rho f^2-\mu(k^2+l^2) \end{vmatrix}=0 \tag{6–36}$$

and that this equation is factorized into a purely dissipative term

$$2i\eta f=C(k^2+l^2) \tag{6–37}$$

and a coupled term as (6-21).

If C tends to infinity with η kept finite, not only the same type of equation as (6-19) is realized, but the same conclusion as the former case under this special condition is easily gained.

If we adopt the conditions that $u_i=u_i(x_1, x_3, t)$ and that the surface element on which a proper boundary condition is assigned is perpendicular to the 3-direction, then displacements, stresses, torques and dissipative forces are decomposed explicitly as follows,

(i) u_2, ϕ_1, ϕ_3,

$$\sigma_{32}=\mu\partial_3 u_2 \tag{6–38}$$

$$\begin{aligned} M_{312}&=\frac{C}{2}\partial_3(\partial_1 u_2-\phi_3) \\ M_{323}&=-\frac{C}{2}\partial_3(\partial_3 u_2+\phi_1) \end{aligned} \tag{6–39}$$

$$F_{32}=\frac{C}{2}\{\partial_1\partial_3(\partial_1 u_2-\phi_3)-\partial_2{}^2\phi_1-\partial_3{}^2(\partial_3 u_2+\phi_1)\} \tag{6–40}$$

(ii) u_1, u_3, ϕ_2,

$$\begin{aligned} \sigma_{31}&=\mu(\partial_3 u_1+\partial_1 u_3) \\ \sigma_{33}&=\lambda(\partial_1 u_1+\partial_3 u_3)+2\mu\partial_3 u_3 \end{aligned} \tag{6–41}$$

$$M_{331}=\frac{C}{2}\partial_3(\partial_3 u_1-\partial_1 u_3-\phi_2) \tag{6–42}$$

$$F_{31}=\frac{C}{2}\{-\partial_1{}^2(\partial_3 u_1-\partial_1 u_3-\phi_2)+\partial_2{}^2\phi_2-\partial_3{}^2(\partial_3 u_1-\partial_1 u_3-\phi_2)\} \tag{6–43}$$

The above decomposition has essentially the same nature as the previous one derived by a rather heuristic manner. Using this 2-dimensional type decomposition, we can determine not only the wave number *vs.* frequency relation, but the absolute amplitudes of displacement and eigen rotation due to the applied external forces. However, the latter approach is not performed in this paper partly because there have not been much data for the estimate of boundary conditions both in the field and in the laboratory except for seismic moment calculated from far field seismograms.

The main reason is that, if a step-wise propagation after the drop of moment is included in calculations, the totality of the nature of the fracturing process would be obscured by the combination of several parameters and would not be obtained from a rather simplified analytic consideration. Our simplified model has two characteristics: relaxation of initial moment distribution can be realized, and delay or deceleration effects which are inherent from the existence of strength are not taken into consideration hereafter.

It is easily inferred from the preceding discussions that type (i) corresponds to an extention of *SH*-type deformation, and that type (ii) corresponds to that of *P* and *SV*-type deformation in conventional continua. The motion of type (ii) can be illustrated as Fig. 4, which is composed of aggregates of cylinders.

Here we notice the coventional definition of the dislocation field. The right hand side of expression (6-39), as an instance, can be transformed as,

$$\int \partial_3(\partial_1 u_2-\phi_3)dx_3\wedge dx_1=[u_2]-\int \phi_3 dx_1 \tag{6–44}$$

The first term of the right hand side of expression (6-44) corresponds to a non-integrability effect (screw-type dislocation) and the second term corresponds to the Volterra type of dislocation. From the relation

$$M_{312}=M_{3[12]} \tag{6–45}$$

and expression (6-44), it is shown that the well-known equivalency between the exertion of double-couples and the occurrence of an infinitesimal dislocation loop in elasticity theory (*e.g.* Maruyama (1963)) is valid in an extended form.

Generally it cannot be expected that the total rotational field disappears in the medium composed of aggregations of coarse particles. The interaction we have adopted in this paper is decomposed into such translational and rotational terms as in expression (6-44). Especially, according to the independence of displacements and eigen rotations, we have formulated dissipative rotational interactions among neighbouring particles with the expression (4-7′) utilized as an intermediary relation.

What differs from other models with porous media (*e.g.*, Biot (1941)) is the assumption that there is no macroscopic flow in a viscous liquid and that the liquid is incompressible. Following this assumption, the propagation of *P* waves is characteristic of nondispersion as well as non-attenuation. The macroscopic flow of liquid becomes important in lower frequencies namely, beyond which the assumption of local relaxation breaks down. (O'Connell and Budianski (1977)) Of cource there remains an upper bound for the frequency, at which the wavelength becomes the size of a pore. As stated previously, it is assumed to be of no consequence that there might exist micro-fluctuations of density distribution, *e.g.*, mixture of bulk water and particles. Hence the effect of the configuration or shape of pore and matrix is replaced only with the distribution of one parameter *C*. This assumption is a simplification for complicated rock structure. Although the influence of the existence of a thin liquid layer between the particles can be partly evaluated by the effect of dissipation in our formulation, there are no effects on the *P* wave propagation from neglecting the effects of compressibility and draining of water.

7. Discussions

The synthesis of seismograms in the regions located near or far from the seismic source has been successfully performed by one sheet model of dislocation. (*e.g.*, Aki (1968)) Furthermore, the synthetic seismograms in the near-field have been constructed not only for direct body waves (*e.g.*, Kanamori (1973)), but for surface waves, surface-associated waves such as *SP* phase and reflected waves at the Moho-discontinuity. (Kawasaki et al. (1972)) If we make use of inversion techniques, the distribution of dislocation could be determined within a statistical allowance. (*e.g.*, Matsu'ura (1977))

Some questions arise here as to what the distribution of dislocations tectonically means and how the movement of faults is developed, if these are controlled by some physical parameters. Some calculations concerning these questions have been performed under approximations by 2-dimensional or 3-dimensional motions of a discrete system composed of spring-mass. (Burridge and Knopoff (1967), Yamashita (1977)) Yamashita (1977) successfully showed the effects of inhomogeneous distribution of the initial stress on the fracture patterns. For the initial stress with spatially linear and quadratic functions, the fracture is more localized in the latter distribution than the former, in the case of uniform distribution of the fracture strength. Such localization of the fracture nucleation for a quadratic distribution will be discussed in Part II, from a somewhat different standpoint.

Tiltmeters and strainmeters have been installed in regions usually far from faults except in the case of the San Andreas Fault, the Yamasaki Fault in Japan etc., where a dense array of these instruments is operated along these faults. (Savage et al. (1976), Oike (1977)) For instance, residual tilts associated with teleseismic earthquakes have been reported by Nishimura (1953) and others, although the elastic strain energy change required to produce the observed tilts is impossibly high. (*e.g.*, Press (1965)) The instruments must be examined for mechanical instability accompanied by ground shaking due to seismic waves such as large-amplitude Rayleigh waves. Even if these instrumental problems remained to be solved, it is not regarded to be unreasonable to assume one sheet model for an earthquake source, especially for shallow earthquakes, as a whole. Probably, it is no exaggeration to say that all facts are hidden in the darkness except these coseismic dynamics which are considered as rather convincing among the observed earthquake phenomena.

It is also an important problem whether laboratory experiments could properly simulate or reproduce earthquake phenomena or not. Some laboratory experiments of fracture—A. E., Kaiser effects etc.—as well as frictional sliding, have been performed with recent advancements in electronics, i.e., extension of frequency band, which will elucidate the problem of dilatancy and some related phenomena. Concerning frictional sliding experiments, it is pointed out that stick-slip motions accompany inhomogeneous distribution of strains and local rotations whithin the gouge and that except in these narrow regions the material does not deform too much. (*e.g.*, Byerlee et al. (1978)) These experiments are considered to give support to our model.

It must be pointed out that no reliable interpretations have been offered particularly with respect to long-period and transient parts of coseismic and associated movements, that is,

(i) aseimic creep motions along faults

(ii) tilt and strain steps

Although concerning (ii), it seems that some consensus, as stated previously, exists under one sheet model, there remains many problems to be solved. It may be said that residual tilts and strains could be investigated more thoroughly by using faster recording speed of signals. Since residual tilts independent of elastic strain change have an important meaning in our model, dynamic behaviors of the model are directly reflected in observation of dynamic behaviors of immediate neighboring regions of faults. It is to be reckoned in our model that the condition under which the fault movement occurs is identical to the condition that $\dot{\phi}_{ij} \neq 0$.

It might be assumed here that there exists a threshold of torque similar to the strength in the case of a discrete system of a mass-spring. The value of matter constant C determines the roughness of an aggregation of particles. If we neglect the deceleration effect due to the threshold of torque at each point, the fracture pattern will be mainly determined by the distribution of matter constants C and η. The above simplified assumption will make the analysis of an initial value problem easier. The assumption will be satisfied in case (i). Then the nucleation or initiation of fracture is characterized by the existence of matter constant C. The derived equations are of a non-causal type due to rigid rotation and dissipation. By means of numerical procedures, we get two branches of solutions of the characteristic equation, i.e., almost purely dissipative motions and coupled motions with a velocity higher than S waves. If the branch of (6-6) or (6-7) is applied to case (i) as

$$v \sim \sqrt{\frac{C}{\eta}}\,\sqrt{f} \tag{7-1}$$

$$L \sim \sqrt{\frac{C}{\eta}}\,\frac{1}{\sqrt{f}} \tag{7-2}$$

where v: the expanding rate of fracture region

L: the magnitude of fractured region,

it follows that $L/v \sim 1/f$. For estimates of $v=1$ to 10 km/day for propagating creep velocity obtained for the San Andreas Fault (Nason (1969)), we get $C/\eta \sim 10^7$ cm²/sec. It is to be noted that similar results can be obtained, except for extreme cases of $C \ll \eta$, by means of the quasi-decoupled branch of solutions as well as the dissipative branch.

From observations of the decay of strain and tilt step amplitudes, one sheet model is a good approximation of the far field data. (*e.g.*, Takemoto (1972)) It may be considered that the propagation velocity of the main part of tilt step movements along faults is of the order of S wave velocities. The nature of propagation for (ii) cannot be regarded as purely dissipative as in the case of (i). Another branch of solution provides the properties of propagation, as

$$\frac{f}{Re(k)} \gtrsim 1 \tag{7-3}$$

$$Im(k) \sim f^2 \sim \frac{1}{L} \qquad \text{(when } f < \eta/1\text{)} \tag{7-4}$$

where the velocity of S waves is assumed to be unity for the convenience of numerical calculations. It can be understood that the attenuation effect shows a strict dependence on frequency as in the type of Voigt model within the above frequency range considered. In the frequency range $f > 1/\eta$, $Im(k) \simeq$ const. A Maxwell type of liquid possesses such an attenuation property. Our model possesses two extreme states in respect to attenuation: At higher frequencies it behaves like a liquid, on the other hand, at lower frequencies it behaves like a solid. The weak dispersion of coupled type waves may be non-detectable and negligible at lower frequencies.

8. Conclusions

We propose a model for gouge which corresponds to microstructures near a fault plane. The existence of gouge may be accepted as an assumption which could reduce the difference in the stress drop by one or two orders of magnitude between laboratory experiments and data from seismic waves. It is frequently observed that clay-like materials are sandwiched between fault surfaces.

There can be two viewpoints for constructing the model of gouge. One is that of compaction and drainage of an ensemble of particles and liquid, of which many examples can be found in soil mechanics. The other is that, as described in our model, which stresses the importance of mutual rigid rotations of interstitial particles and does not emphasize that of pure deformation—strains—of an ensemble of particles. The latter is a possible assumption for simplicity. The rôle of liquid in this model is considered as a mechanism of local dissipations in contrast with the flows responding to the distribution or gradient of pressures in the former model.

The rotation of interstitial particles is often observed in laboratory experiments—brittle fractures of intact rocks, stick-slip on frictional sliding surfaces and so on. Following the introduction of mutual rotations, we assume a newly defined internal energy and physical constants. From some combination of the physical constants and a degree of inhomogeneity or particle-size, we classify various stages of deformation. The formation process of faults is considered to be that of degradation of a particle (a giant particle i.e., a plate). This process would have properties of high non-linearity. In our linearized formulation, the property is expressed only in the form of a division of contributions to the equations of motion--inertial terms, deformations and dissipations—and in the form of combined ratios.

The propagation velocity and attenuation coefficients are also calculated. Because of neglecting macro-strains, longitudinal-type waves are non-attenuative and non-dispersive. Transversal-type waves have two branches of solutions, i.e., weakly dispersive type and non-causal dissipative type. We can apply these results to a unified understanding of creep phenomena—*e.g.*, along the San Andreas Fault—and coseismic strain and tilt steps observed close to several faults.

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