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Observations of Crustal Movements by Newly-Designed Horizontal Pendulum and Water-Tube Tiltmeters with Electromagnetic Transducers (2)

—Variations in the Amplitude and Phase of Tidal Tilts Observed with a Water-Tube Tiltmeter at Kamitakara—

By Masaaki Kato

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Abstract

Using the data collected over nearly one year obtained with a water-tube tiltmeter installed at the Kamitakara Station (36°17'N, 137°20'E, H=800 m) in the northwestern Chuhu region, Japan, temporal variations in the amplitude and phase of the M₂ and O₁ constituents have been investigated by means of the least-squares method.

The results show that variations in the amplitude and phase of tidal tilts observed in three of four directions are ±2-4% and ±1-2° for the M₂ constituent, and ±3-6% and ±3.5° for the O₁ constituent, respectively, during the observation period. Considering the error limit in sensitivity measurements, it may be concluded that both the M₂ and O₁ tidal constants have not changed significantly during this period, suggesting no essential change of elastic properties in the shallow crust. This conclusion may be consistent with the fact that seismic activity around the region has not been so high.

Successive analyses with 30-day data have also been performed by shifting the central epoch by a 2-day step, and it has become clear that there is a common periodic oscillation in the amplitudes and phases of both M₂ and O₁, but this can be easily removed by a moving-average method. It might be possible to find out more detailed variations in the tidal constants from the averaged results.

1. Introduction

Using tiltmeters of the horizontal pendulum type, temporal variations in the tidal amplitude and phase of ground tilts have been reported by a few geophysicists. However studies of this kind by the use of water-tube tiltmeters have not been carried out to data.

In this paper, the results of tidal analyses are presented using the data collected from September 1977 to October 1978 obtained with a water-tube tiltmeter with three detectors installed at Kamitakara, and the accuracy of detecting temporal variations in the amplitude and phase of the main lunar semidiurnal M₂ and diurnal O₁ constituents are discussed in detail. As is evident from Table 2 in a previous paper, the major part of the difference between the observed M₂ tidal tilt and the corresponding theoretical one may be interpreted as the oceanic loading effect. Phasor plots of the M₂ tilt tide in four
In the case of analyzing tidal data by means of the least-squares method, we can use a data set with some interruptions and with unequally spaced time intervals, but the obtained results may be slightly influenced by the number of the constituents to be included. Nakagawa and Shiraki showed, however, that the results are not essentially different by the number of constituents as far as four principal ones of gravimetric tides, M2, S2, K1 and O1 are included in the analysis and if both the observed and theoretical tides are analyzed by the same method. In the frequency domain analysis of earth tide records, however, the resolution is severely limited by the time duration of the analyzed data. To avoid the resolution problem, a method of the time domain analysis was proposed by Mikumo and Kato, and applied for the strainmeter records obtained at Kamitakara and Inuyama. The method applied in the analysis of gravimetric tides by Nakagawa and Shiraki, and that of the above time domain analysis are considered to be fully effective if there is a close similarity between the observed and theoretical waveforms.

If the oceanic loading effect is comparatively small, both the methods mentioned above may be well applied to the observed tidal data. However, the estimate by Tanaka shows that the tidal tilts observed at Kamitakara are considerably deformed by the oceanic loading effect, and hence it does not seem appropriate to use these two methods in the present analyses. For these reasons, only the observed data are analyzed by means of the least-squares method, and the obtained M2 and O1 amplitudes are normalized to the mean values in a 18.61-year period.

2. Some Considerations on Sensitivity Measurements

It is to be noted that there exists considerable difficulty in measuring the sensitivity of the water-tube tiltmeter with floats and electromagnetic displacement transducers. For this type of instrument, it is essentially important to measure the output voltage versus the change of water level in each detector as accurately as possible.
Denoting the heights of water level in the $i$-th and $j$-th detectors by $f_i$ and $f_j$, and the distance from the $i$-th to $j$-th detector by $l_{ij}$, the change in tilt is expressed as

$$\Delta \theta = \Delta (f_i - f_j) / l_{ij} \quad (i=1, 2, 3, j=1, 2, 3, \text{ and } i \neq j) \quad (1)$$

Considering the errors involved in the process of calibration, $f_i$ will be rewritten by $(1+\varepsilon_i)f_i$, and $f_j$ by $(1+\varepsilon_j)f_j$, respectively. Then the apparent change in tilt can be written as

$$\Delta \theta' = \Delta [(1+\varepsilon_i)f_i - (1+\varepsilon_j)f_j] / l_{ij} \quad (i=1, 2, 3, j=1, 2, 3, \text{ and } i \neq j) \quad (2)$$

Therefore, the error in the tilt change is given as follows.

$$\Delta \theta_e = \Delta \theta' - \Delta \theta = \Delta (\varepsilon_i f_i - \varepsilon_j f_j) / l_{ij}$$
$$= \varepsilon_i \Delta (f_i - f_j) / l_{ij} - (\varepsilon_j - \varepsilon_i) \Delta f_j / l_{ij} \quad (i=1, 2, 3, j=1, 2, 3, \text{ and } i \neq j) \quad (3)$$

In Eq. (3), the first term of the right-hand side represents the error concerning the actual change in tilt. On the other hand, the second term is concerned with the water-level change of the $j$-th detector. In order to make this term negligibly small, it is necessary to improve the precision of calibration as accurately as possible. For example, the water level of each detector changes concurrently by the same amount responding to short-period fluctuations of the atmospheric pressure with several to some tens of minutes. If the calibration errors are sufficiently small, a greater part of the effect of atmospheric pressure will be canceled out.

As described in the previous paper, absolute sensitivity measurements can be carried out by adding a proper volume of water into one of the three pots. The accuracy of the absolute calibration reaches somewhat better than 1 to 2% at present. On the other hand, relative calibrations are made by giving a displacement with a micrometer to the head of a magnetic-sensor in each detector. Being based on repeated relative calibrations, the temporal variation of sensitivity of each transducer is believed to be less than 0.5% per year, which is well within the present accuracy of calibration.

3. Observations and Data

In order to calculate tidal amplitude and phase, tilt components in the four directions have been transformed from digital output signals from three different detectors. The derived tilts in the northwest and southwest directions are shown in Fig. 2, together with the amount of precipitation. It is to be mentioned that relative calibrations of the three detectors were made on the following dates:

- **WT1**: 1977 May 26, 1978 Feb. 17 and July 1, 1979 Feb. 16

However, all transducers of the three detectors were replaced after the observation
station was disturbed by lightning on June 22, 1978. In Fig. 2, an annual variation of $1.3 \times 10^{-6}$ rad. tilt in double amplitude is clearly seen in the WT21-component. A periodicity with about half a month that appeared in the two components is apparently induced by aliasing through a sampling interval of one day.

In the present tidal analyses, output signals in digital form were basically used. We selected several durations from the whole observation period from September 1977 to October 1978, each of which involves complete data without any interruptions or missing. But in the case when the signal was disturbed by short-period oscillations such as due to earthquake occurrence, we interpolated smoothed curves passing along the centers of such oscillations. Fig. 3 shows examples of the tidal tilt record overlapping on secular tilts, which have been transformed from the hourly values recorded by three detectors. One digit (a minimum unit in digital recording) roughly corresponds to a water-level change of 0.005 μm, and hence the resolution for tilt changes reaches $1.5 \times 10^{-10}$ rad.

Data-sampling of 0.83 Hz has extremely improved the time accuracy of records, in contrast to the accuracy of one minute or so in past analog recording.

4. Expression of the Observed Constituents, $M_2$ and $O_1$, Using Astronomical Variables

The amplitude and argument of an observed constituent vary with a period of 18.61 years. In this paragraph, a method to correct the observed tidal constants for the 18.61-year effect is described.

Considering the 18.61-year movement of the Moon’s node, each constituent of tide-
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1978.07.30, 00H ~ 1978.08.30, 23H 768 HOURS

Fig. 3. Hourly tilts in the northwest and southwest directions during the period from July 30, 00h00m, 1978 to Aug. 30, 23h00m, 1978, (JST).

Generating potential \( T_0 \) is written as

\[
T_0 = fR_0 \cos (V + u)
\]

where \( f \) is the factor varying with the period of 18.61 years (node factor), \( u \) is the angle varying with the period of 18.61 years, \( V \) is the angle varying steadily at the mean speed of the constituent and \( R_0 \) is the mean amplitude of the constituent.

On the other hand, the observed constituent is related to the corresponding constituent of potential by expressing the observed one in the form

\[
T = fR \cos (V + u - \kappa)
\]

where \( R \) is the observed amplitude and \( \kappa \) is the phase lag of the observed constituent behind the phase of the constituent of potential.

For \( M_2 \) and \( O_1 \), \( f \), \( V \) and \( u \) are given by

\[
\begin{align*}
  f(M_2) &= \{ \cos (I/2)/\cos (\omega/2) \cos (i/2) \}^4 \\
  V(M_2) &= 2 \tau \\
  u(M_2) &= 2(\xi - \nu) \\
  f(O_1) &= [\sin I \cos^2(I/2)] / [\sin \omega \cos^2(\omega/2) \cos^4(i/2)] \\
  V(O_1) &= \tau - s + 90^\circ \\
  u(O_1) &= 2\xi - \nu
\end{align*}
\]

where \( \tau \) is the mean lunar time, \( s \) is the mean tropic longitude of the Moon, \( I \) is the
inclination of the Moon's orbit to the equator, $\omega$ is the inclination of the Earth's equator to the ecliptic (23.452° at the time of Jan. 01, 1900), $i$ is the inclination of the Moon's orbit to the ecliptic (5.145°), $\xi$ is the longitude in the Moon's orbit of its ascending intersection with the celestial equator and $\nu$ is the longitude in the celestial equator of its intersection with the Moon's orbit. $I$, $\xi$ and $\nu$ can be calculated as a function of $i$, $\omega$ and $N$, using the next three equations.

$$\cos I = \cos i \cos \omega - \sin i \sin \omega \cos N$$

$$\tan \xi = \frac{\sin i \cot \omega \sin N - \sin^2 (i/2) \sin 2N}{\sin i \cot \omega \cot N - \sin^2 (i/2) \cos^2 N + 1}$$

$$\tan \nu = \sin i \sin N((\cos i \sin \omega + \sin i \cos \omega \cos N))$$

where $N$ is the mean longitude of the node of the Moon's orbit.

The changes of $f$ and $u$ values for $M_2$ and $O_1$ over 20 years from 1968 to 1987, which are derived from Nakano’s tables, are shown in Fig. 4.

In order to correct the observed tidal amplitudes for the nodal effect, the amplitudes obtained should be divided by the $f$ values appropriate to the central epoch of the respective observation periods.

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5. Method of Analysis

Prior to the present tidal analyses, secular tilts and some other drifts involved in the observed data have been removed by applying the Pertzev filter. After doing this, the
least-squares method has been applied to the filtered data to get the amplitudes and phases of the major constituents.

\[ F(t) = A_0 + \sum_{i=1}^{n} H_i \cos{(\omega t - \phi_i)} \]  

where \( A_0 \) is a constant term and \( n \) is the number of the constituents included. Then, each constituent is given by

\[ H(t) = H \cos{(\omega t - \phi)} \]  

Since Eq. (5) exactly corresponds to Eq. (11), we have

\[ H = f R \]

\[ V + u - \kappa = \omega t - \phi \]  

Fig. 5. Temporal variations in the amplitude and phase of \( M_2 \) during the period from Sept. 18, 1977 to Oct. 12, 1978. The amplitude is normalized to the average of those obtained in the six observation periods with the corresponding epochs, 1977 Nov. 18, Dec. 20, 1978 Jan. 21, Apr. 15, Aug. 15 and Sept. 16; the open rectangle represents the probable error; the phase denotes the lag when the time origin is taken at Sept. 02, 00h 00m, 1977, (JST); each dot corresponds to the 14-day moving average value shown in Figs. 7 and 9.
Usually, $\omega = dV/dt$ is taken as the angular velocity of the constituents now in consideration. Therefore the phase $\varphi$ varies slowly with the period of 18.61 years due to the effect of angle $\mu$. To avoid such a problem, $\omega$ is defined as in the following equation both for $M_2$ and $O_1$.

$$\omega = dV/dt + du/dt$$  \hspace{1cm} (13)

Over the period of observations from September 1977 to October 1978, the values of $du/dt$ were assumed to be constant, and we adopted the values, $-0.8 \times 10^{-4}$ and $+0.54 \times 10^{-3}$ (degree/hour) for $M_2$ and $O_1$, respectively (see Fig. 4).

The number of filtered data for each period of analysis was standardized to be 720 (30 days), except one example of 576 (24 days). For the selection of the constituents to be included in the analysis, we examined the following two cases;

Case I: $M_2$, $S_2$, $K_1$, $O_1$, $N_2$, $K_2$, $Q_1$, $P_1$ and $S_1$ (9 constituents)

Case II: $M_2$, $S_2$, $K_1$, $O_1$, $N_2$ and $Q_1$ (6 constituents)

Allowing for the accuracy of the present sensitivity measurements, the correction

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**Fig. 6.** Temporal variations in the amplitude and phase of $O_1$. Each dot corresponds to the 14-day moving average value shown in Figs. 8 and 10 (otherwise presented as in Fig. 5).
factors relating to the filter's response introduced by Nakagawa\textsuperscript{12}) were not taken into consideration.

6. Temporal Variations in Amplitude and Phase of $M_2$ and $O_1$

Figs. 5 and 6 show the obtained amplitudes and phases of $M_2$ and $O_1$ for Case I. Probable errors are shown by open rectangles at the central epoch in each period of the analysis. The amplitudes ($R = H/f$) obtained in each period have been normalized in Figs. 5 and 6 to those averaged over the six observation periods with the corresponding central epochs, namely, 1977 Nov. 18, Dec. 20, 1978 Jan. 21, Apr. 15, Aug. 15 and Sept. 16. The indicated phase implies the lag $\phi$ (refer to Eq. (11)) to the time origin taken at Sept. 02, 00h00m, 1977, (JST).

Amplitudes (before normalization) and phases for $M_2$ and $O_1$, together with their probable errors, respectively, which were obtained by using one-month data for a period with the central epoch of Aug. 15, 1978 are shown in Table 1 as an example. Calculations were made for the two cases, namely, Case I and Case II. The probable errors for Case II are somewhat larger than those for Case I.

As shown in Fig. 5, variations in the $M_2$ amplitude and phase of WT21, WT31 and WTEW are limited to within $\pm 2\sim 4\%$ and $\pm 1\sim 2^\circ$ over the one-year duration, respectively, while for WT32, they reach $\pm 6\%$ and $\pm 4^\circ$, which are about two times as large as those of the other three components. The $M_2$ amplitude of WT32 is the smallest among

<table>
<thead>
<tr>
<th>Constituent</th>
<th>Component (Direction)</th>
<th>Amplitude $H$</th>
<th>Probable error</th>
<th>Phase $\phi$</th>
<th>Probable error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_2$</td>
<td>WT21 (S 45°W)</td>
<td>$1.563 \times 10^{-8}$</td>
<td>±0.37% (0.44)</td>
<td>$-47.34^\circ$</td>
<td>±0.22° (0.25)</td>
</tr>
<tr>
<td></td>
<td>(1.560) rad.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>WT31 (N 45°W)</td>
<td>2.531</td>
<td>0.30</td>
<td>$-50.75^\circ$</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>(2.516)</td>
<td>(0.38)</td>
<td></td>
<td>$-50.69^\circ$</td>
<td>(0.22)</td>
</tr>
<tr>
<td></td>
<td>WT32 (N)</td>
<td>0.689</td>
<td>0.84</td>
<td>$-56.22^\circ$</td>
<td>0.48</td>
</tr>
<tr>
<td></td>
<td>(0.681)</td>
<td>(1.19)</td>
<td></td>
<td>$-56.73^\circ$</td>
<td>(0.68)</td>
</tr>
<tr>
<td></td>
<td>WTEW (W)</td>
<td>2.894</td>
<td>0.27</td>
<td>$-49.45^\circ$</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>(2.881)</td>
<td>(0.30)</td>
<td></td>
<td>$-49.27^\circ$</td>
<td>(0.17)</td>
</tr>
</tbody>
</table>

| $O_1$       | WT21 (S 45°W)         | 0.782         | 0.75           | $-124.32^\circ$ | 0.43           |
|             | (0.757)               | (0.91)        |                 | $-126.18^\circ$ | (0.52)         |
|             | WT31 (N 45°W)         | 1.545         | 0.50           | 88.58          | 0.29           |
|             | (1.511)               | (0.64)        |                 | 87.54          | (0.36)         |
|             | WE32 (N)              | 1.585         | 0.37           | 77.66          | 0.21           |
|             | (1.543)               | (0.52)        |                 | 76.43          | (0.30)         |
|             | WTEW (W)              | 0.696         | 1.11           | 114.13         | 0.63           |
|             | (0.690)               | (1.26)        |                 | 113.05         | (0.72)         |

Notes: Period of analysis: July 31, 00h00m-Aug. 29, 23h00m, 1978, (JST). $\phi$: phase (refer to Eq. (11)) when the time origin is taken at July 30, 00h00m, 1977, (JST).

Upper values show the results obtained for Case I with the 9 constituents included, while lower ones in parentheses those obtained for Case II with the 6 constituents included.
the four, as shown in Table 1. This may probably be the main reason for the largest probable errors in the amplitude and phase of WT32.

On the other hand, as shown in Fig. 6, variations in the O1 amplitude and phase of WT21, WT31 and WT32 are $\pm 3\sim 6\%$ and $\pm 3.5^\circ$, respectively, while for WTEW with the smallest amplitude, they are $\pm 8\%$ and $\pm 5.5^\circ$. It is to be noted that the larger fluctuations in the results of O1 than those in M2 are naturally due to the difference in resolution, since the frequency of O1 is about a half of that of M2.

Dotted curves in Figs. 5 and 6 correspond to 14-day moving averages of the amplitudes and phases, which will be discussed below. The discrepancy between the dotted
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curves and solid lines does not seem to be large enough to change the pattern of the temporal variations in the amplitude and phase of each component.

In order to investigate more detailed structure of temporal variations of the observed amplitudes and phases, successive analyses have also been performed by shifting the central epoch by a 2-day step, using the filtered data for two periods from Nov. 03, 00h00m, 1977 to Feb. 12, 23h00m, 1978 and from July 31, 00h00m, 1978 to Oct. 26, 23h00m, 1978, (JST). The results for \( M_2 \) and \( O_1 \) obtained for the two periods are shown in Figs. 7, 8, 9 and 10, respectively. Solid squares and large open circles indicate the results from Cases I and II, respectively. It may be immediately noticed that there exists a common oscillation both in the amplitudes and phases of \( M_2 \) and \( O_1 \), and its prominent period appears to be one month for Case I and about half a month (14.5 days) for Case II. These periodic oscillations might be attributed partly to the effect of
contamination from the other constituents varying with time.

In the course of the present successive analyses, it has become clear that the amplitude and period of the oscillation severely depend on the number of the constituents included in the analysis. To remove such a periodic oscillation and to get smoothed values, an attempt was made here to adopt a moving-average method. Since it is desirable that the duration of the period of analysis is as short as possible from the viewpoint of monitoring temporal variations of tidal constants, it is more advantageous to smooth out the periodic fluctuation on the results from Case II by adopting a 14-day average. This method requires 44 days to get one smoothed value. The values of amplitude and phase thus obtained are shown as small open circles in Figs. 7, 8, 9 and 10. Connecting these circles, rather smoothed curves can be obtained. It may be seen that fluctuations

Fig. 9. Results of successive analyses for $M_2$ using the filtered data from July 31, 00h00m, 1978 to Oct. 26, 23h00m, 1978, (JST). The phase denotes the lag when the time origin is taken at July 30, 00h00m, 1978, (JST) (otherwise presented as in Fig. 7).
(the double amplitude of the periodic oscillation) in the $M_2$ amplitude and phase of WT21, WT31 and WTEW are the order of $\pm 1\%$ and $\pm 0.7^\circ$, respectively. For WT32 with the smallest amplitude among the four components, however, the fluctuations are $\pm 2\%$ and $\pm 1^\circ$. For the $O_1$ amplitude and phase of the four components, they are $\pm 5\%$ and $\pm 3^\circ$, respectively, which are about five times as large as those of the three $M_2$ components. The amplitudes of the fluctuations shown in Figs. 9 and 10 are slightly smaller than those in Figs. 7 and 8. This might be explained as the effect of replacement of all transducers of the three detectors, as already described in Paragraph 3.

7. Summary

Nearly one-year data obtained with the water-tube tiltmeter with three detectors at Kamitakara have been analyzed by means of the least-squares method. The results
obtained and the subjects left for future studies are summarized as follows:

(1) Temporal variations in the amplitude and phase of $M_2$ in the main three components over the whole observation period were within $\pm 2\sim 4\%$ and $\pm 1\sim 2^\circ$, respectively. For the $O_1$ constituent, they were $\pm 3\sim 6\%$ and $\pm 3.5^\circ$, respectively.

(2) It was found that a successive analysis for monthly (30 days) data with a shift of 2 days yields spurious periodic oscillations both in the amplitude and phase of $M_2$ and $O_1$, and the amplitude and period of these oscillations severely depend on the assumption of the constituents to be included. Considering from the standpoint of monitoring temporal variations of tidal constants, it is recommended as one of the most practical methods to use a 14-day moving average of the results from the least-squares method under the assumption that the tidal curve consists of the 6 constituents, namely, $M_2$, $S_2$, $K_1$, $O_1$, $N_2$ and $Q_1$.

(3) Allowing for the error limit of calibration in the present observation, it may be concluded that both the $M_2$ and $O_1$ tidal constants have not changed significantly during the whole period, suggesting no intrinsic change of elastic properties there. It is expected, however, that these high precision observations may be useful to detect possible changes in crustal rigidity in relation to large earthquakes if there are significant changes exceeding the error limit.

(4) For further improvement of the precision of results hereafter, it is necessary to increase the accuracy of calibration. The water level in the detectors is now decreasing at a nearly constant rate of 0.5 $\mu$m/day. This makes the output signals of the detectors scale off the range on the recording chart frequently and prevents us from increasing the sensitivities. If the difference between any two outputs is recorded instead of the present recording system, then their drift components cancel each other, and it is possible to increase the sensitivities by about two times. We are now preparing to improve the present recording system into the system above-mentioned.

(5) An analysis of the theoretical tides by the method proposed here is also required and the comparison of the theoretical with observed results will be useful for not only clarifying the nature of the spurious oscillations mentioned above but making their reduction more effectively.

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