

## Optimal Planning of Flood Control Systems Based on Screening, Simulation and Sequential Models

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### Abstract

The aim of this study is to establish the optimal flood control system accounted for by the comprehensive criteria on the whole river basin. Especially, we will examine the effects of flood control projects in the system consisting of multi-sub-basin and multi-defense points against flood inundation probability in time and space.

We define the planning problem of flood control projects as the minimization problem of the construction cost among all alternatives while satisfying the required safety rate for the prevention of flood inundation. First, the better alternatives are extracted by using the random search method. Second, the optimal system on the exact basin model is determined by application of the simulation method for them. Lastly, the optimal construction order of the final system is gained by Dynamic Programming with the criteria that the expectation of the inundation damage under construction is minimized.

So, we call the above three steps i) Screening model, ii) Simulation model and iii) Sequential model in flood control planning, respectively.

### 1. Introduction

The objective of flood control project planning is to decide the most effective system which defends human life and property against the flood. To attain this objective, it is important to know the facts concerning flood and to predict how much damage may be sustained in future. Both the optimal flood control system and the working policy should be planned in harmony with art and economy based on this hydrological information.

In recent flood control planning, the design high water level factor is often calculated only at one terminal point for design rainfall occurring at the river basin. This procedure, however, makes it impossible for the defense points in that basin to have the same safety rate against flooding because of the rapid transfiguration of the basin, such as i) increase of defense points and ii) construction of dam reservoirs. That is to say, it means that the safety rate of the basin must not be estimated at the most important defense point, but the safety rates at all defense points should be made equal to each other taking advantage of interaction of the operational projects and correlation between precipitation in sub-basins.

So, in this paper, we try to compute the flood inundation probability in time

and space introducing the optimal operation for a system consisting of multi-dam reservoirs and multi-defense points and to establish decision procedures for the construction site, scale and time sequence for projects with a required safety rate based on the computed probability of flood inundation.

## 2. Decision procedure for planning flood control projects

### 2.1 The objective of flood control project planning

The objective of flood control is to prevent flood inundation and to reduce flood damage. Concretely, there are i) reduction of flood damage in the whole basin, and ii) reduction of the occurrence frequency of flood inundation. Commonly, the former is represented as the expectation of damage. The control policy is determined through economic evaluation such as benefit-cost analysis. On the other hand, the latter is getting more important because it clarifies the limit of control effect through the inundation probability estimated at each defense point.

Thus, we use the second policy as our control objective and the extraction of the most effective alternative under condition of keeping the inundation probability less than the required rate or the unexceedance probability at any defense point, as planning objective. If the unexceedance probability  $P_*$  is given, the planning objective may be written as the minimizing problem of summing the construction cost as follows:

$$O_b = \sum_{n=1}^N C_0(n) \rightarrow \min \quad (1)$$

subject to

$$\max_{(m)} \{P_{Fm}\} \leq P_* \quad (2)$$

Where  $C_0(n)$  is the construction cost of the project implemented at site  $n$ ,  $P_{Fm}$  is the inundation probability at the defense point  $m$ ,  $N$  is the total number of projects being constructed and  $M$  is the total number of defense points, respectively.

### 2.2 The procedure for planning

Flood control project planning is made of two parts. One of them is the extraction of the optimal system satisfying the control objective from many alternatives. It is called the problem of project site and scale planning. The other is to order in time the projects contained in the extracted alternative most effectively without harming the safety rate under constraint of the limited budget. It is called the problem of schedule planning. For the site and scale planning, many methods have been proposed and 0-1 mixed integer programming is one of the most powerful methods of these. But the more the projects under consideration increase, the more rapidly the execution probability decreases. Nevertheless, mathematical formulation is easy. As the counterplan for this problem, we apply the method proposed by

Ikebuchi in water resources development planning<sup>1)</sup>, where optimal site, scale and time sequence planning is gained through three optimization models. In the first step, some of the better solutions whose construction costs are sheaper and whose inundation probabilities satisfy the required rate, are extracted from the simplified basin model within limits which take cognizance of the reality of flood inundation phenomena. In the second step, final solution is determined by the simulation method based on the more exact basin model from the above better solutions.

The objective of schedule planning in computation is defined as the problem for minimizing the cumulative value of the expected damage over the whole planning period caused by overflowing discharge in excess of the channel capacity at the end of each planning period, as follows.

$$O_{bs} = \sum_{\theta=1}^{\theta \max} \{DA_1^{\theta} - DA_2^{\theta}\} \rightarrow \min \quad (3)$$

Where  $\theta \max$  is the total number of planning periods,  $DA_1^{\theta}$  is the expected damage in the whole basin at planning period  $\theta$ , and  $DA_2^{\theta}$  is the expected damage after all projects are constructed, respectively. The models have been formed in three steps, and are called i) Screening model, ii) Simulation model and iii) Sequential model.

**Fig. 1** shows the relationship between these models.

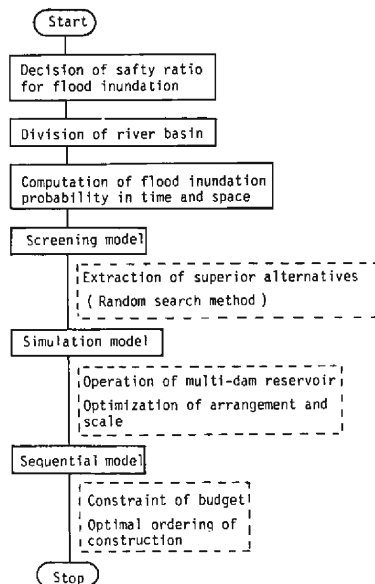


Fig. 1. Total flowchart of flood control planning.

### 3. Formulation of sub-system

#### 3.1 Screening model

In this control project planning, the river basin is divided into several sub-

basins. In each sub-basin, first-order nonhomogeneous Markov chain can be assumed to represent the stochastic process of hourly precipitation in time. And the hourly precipitations between sub-basins can be assumed to be the stochastic process of zero and first-order delayed correlation<sup>2)</sup>. When the flow discharge is defined as having the stochastic process of first-order nonhomogeneous Markov chain in time, the flow discharge that is input to the screening model of each sub-basin can be treated as correlating with the flow discharge on neighbouring sub-basin in space through the runoff analyses applied to the conditional probability sequence of precipitations.

(1) Selection of the project site and scale

In order to reduce the amount of required memory and the running time on a digital computer, the random search method can be applied with the random number. First, one alternative having only the project sites is given by generating the uniform random number according to the combination of sites. The scale to be constructed at each project site is given by generating the uniform random number according to the combination of scales. Then, some alternatives are gained using this process repeatedly. If the state variable  $\alpha^{vn}(j)$  indicates whether  $n$ -th project with  $vn$ -th scale in combination number  $j$  is construction or not, then

$$\sum_{j=1}^{\prod V_n} \alpha^{vn}(j) = N \quad (\alpha^{vn}(j) = 1 \text{ or } 0) \quad (4)$$

subject to

$$\sum_{j=jn}^{jun} \alpha^{vn}(j) = 1. \quad (5)$$

Where  $V_n$  is the maximum feasible capacity on project  $n$ ,  $jn$  and  $jun$  are the first and the last number in all combination numbers on project  $n$ . The planning objective  $O_b$  is given as follows.

$$O_b = \sum_{j=1}^{\prod V_n} C_b^{vn}(j) \cdot \alpha^{vn}(j) \rightarrow \min \quad (6)$$

Where  $C_b^{vn}(j)$  indicates the construction cost of  $n$ -th project with  $vn$ -th scale in combination number  $j$ . It is difficult to execute the optimization programming for this problem in terms of computational running time and memory capacity, since the combination number of  $\prod V_n$  alternatives becomes very large. Thus, the original problem (6) will be replaced with a new problem composed of  $u$  alternatives extracted from  $\prod V_n$  alternatives by the uniform random number. The probability that the minimum value  $O'_b$  of the objective function in the new problem is less than the  $x$ -th value  $O'_{bx}$  of the original problem is given as follows<sup>3)</sup>.

$$P(O'_b < O'_{bx}) \sim 1 - \exp \left\{ -u \cdot x / \prod V_n \right\} \quad (7)$$

The necessary number of the extracted alternatives is calculated by equation (7) after assuming the desired probability.

(2) Dam operational rule with correlated input

Dam operational rule should be considered for the correlated input with first-order Markov chain with one dam and one defense point system. The control objective  $P_f$  is defined as minimizing the maximum value among the flood inundation probabilities at all control time steps (they are the function of the storage sequence  $\{S(t)\}$ ) as follows.

$$P_f = \max \{P_f(S(1) | S(0)), P_f(S(2) | S(1)), \dots, P_f(S(T_E) | S(T_E - 1))\} \rightarrow \min \quad (8)$$

Where  $T_E$  is the total number of control time steps. If the probability density function of input is represented as  $g_I(I(t) | I(t-1))$ , then the expectation of flood inundation can be written as

$$P_f(S(t) | S(t-1)) = \int_0^\infty g_I(I(t-1)) \int_{Q_d}^\infty g_I(O(t) + S(t) - S(t-1) | I(t)) \cdot dO(t) dI(t-1) \quad (9)$$

Where  $Q_d$  is the design flood discharge or the channel capacity;  $O(t)$  is the release discharge at control time  $t$ ;  $I(t)$  is the inflow discharge and  $g_I(t-1)$  is the probability density function of the inflow discharge at control time  $t-1$ , respectively.

Setting

$$A(t) = S(t) - S(t-1) \quad (10)$$

and changing the integral order, equation (9) is changed to

$$P_f(S(t) | S(t-1)) = \int_{Q_d + A(t)}^\infty \int_0^\infty g_I(I(t-1)) g_I(I(t) | I(t-1)) dI(t-1) dI(t) \quad (11)$$

With using equation (11), Dynamic Programming can be applied to minimize the objective function and its formulation is written as follows.

$$f_t(S(t)) = \min_{\{S(t-1)\}} [\max \{P_f(S(t) | S(t-1)), f_{t-1}(S(t-1))\}] \quad (12)$$

Though in equation (12) the principle of optimality is not satisfied because of the conditional probability on input and final solution is not always equal to the optimal trajectory<sup>4)</sup>, the computational advantages of this methodology are obvious from the facts that hydrographs of heavy rainfalls have almost only one peak and a great number of the controlled effects by using equation (12) are not too inferior to the effects of the optimal control by considering the all combinations of storage sequences.

(3) Application of shift operation

In changing from the simple control system with one dam and one defense

point to a complex control system with multi-dam reservoirs and multi-defense points, it becomes useful to apply the concept of shift operation<sup>2)</sup> to gain the inundation probabilities at all defense points simultaneously. For example, consider the application to the case shown in **Fig. 2**. In this method, the flow discharge is represented as matrix  $Q$  ( $I \times J$ ) of the conditional probability distribution. Where  $i$  ( $i=1, 2, \dots, I$ ) indicates the  $i$ -th column component of matrix which means the discretized flow discharge at the neighbouring sub-basin,  $j$  ( $j=1, 2, \dots, J$ ) indicates the  $j$ -th row component of matrix which means the discretized flow discharge at the considered basin and  $Q_{ij}$  is the element which means the conditional probability of the flow discharge  $i$  given the flow discharge  $j$ . The term "discretization" mentioned in this paper is used to express that the observed flow discharge or the computed flow

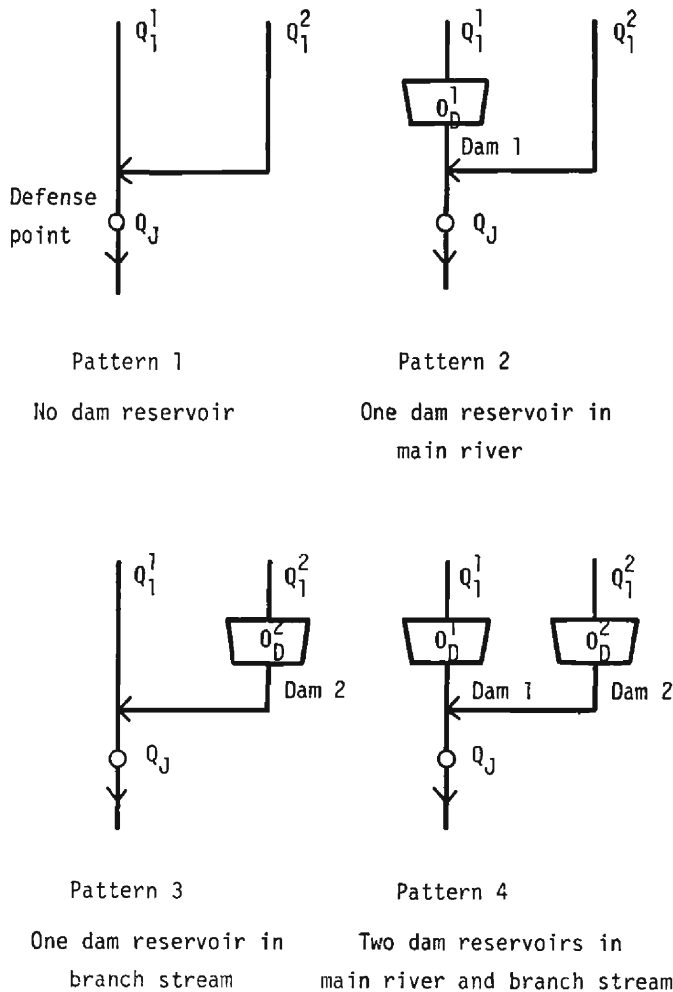


Fig. 2. Basic pattern of river basin and dam reservoirs.

discharge is divided into arbitrary blocks to make the computation easier. Shift operation consists of computing the probability of the confluence discharge and shifting the element at column  $i$  and row  $j$  to the position of column  $i+j-1$  and row  $j$  for the product of their probabilities. In the case of pattern 1 type, the probability distribution of the confluence discharge  $Q_J$  is as follows.

$$Q_{con} = Q_1^1 * Q_1^2 \quad (13)$$

$$Q_J = Q_{con} \cdot E \quad (14)$$

Where  $Q_i^u$  ( $u=1, 2$ ) is the conditional probability distribution of the flow discharge  $u$ ,  $*$  indicates the execution of shift operation,  $Q_{con}$  is the conditional probability distribution matrix of the confluence discharge and  $E$  is the unit vector ( $J \times 1$ ;  $J$  equals the number of column in  $Q_{con}$ ) in which all element values are one. If  $O_D^u$  is the transformation matrix of the release discharge for the change of the storage volume through the dam reservoir or etc., the probability distribution are respectively,

$$P\text{-}2 \text{ type: } Q_{con} = (O_D^1 \cdot Q_1^1)' * Q_1^2 \quad (15)$$

$$P\text{-}3 \text{ type: } Q_{con} = Q_1^1 * (Q_1^2 * O_D^2) \quad (16)$$

$$P\text{-}4 \text{ type: } Q_{con} = (O_D^1 \cdot Q_1^1)' * (Q_1^2 \cdot O_D^2) \quad (17)$$

and

$$Q_J = Q_{con} \cdot E$$

Where  $'$  denotes the transposition of matrix. Using equations (13)–(17) repeatedly, the probability distribution of the confluence discharge can be computed at any defense point. The flood inundation probability is calculated by the sum of the probabilities corresponding to the flow discharge in excess of the channel capacity at that defense point.

The flood control system may be evaluated by the maximum value among all inundation probabilities of defense points under the condition of one pattern of transformation matrix at one control time. This matrix is changeable through the gate operation of dam reservoir and is represented as **Fig. 3** by using  $A(t)$  in equation (10). The column component of the matrix means the discretized storage volume and the row component means the discretized release discharge. Substituting this maximum value for  $P_f(\cdot)$  defined as the objective function, equation (12) is applied to the system with multi-dam and multi-defense points only by increasing the number of state variables and decision variables. So, its system may be abandoned if the value of objective function at last control time on DP is greater than the necessary rate.

Then, all alternatives generated by the random numbers are ordered in terms of the construction cost. And several alternatives which are cheaper than the others are selected as the optimal solutions as screening models.

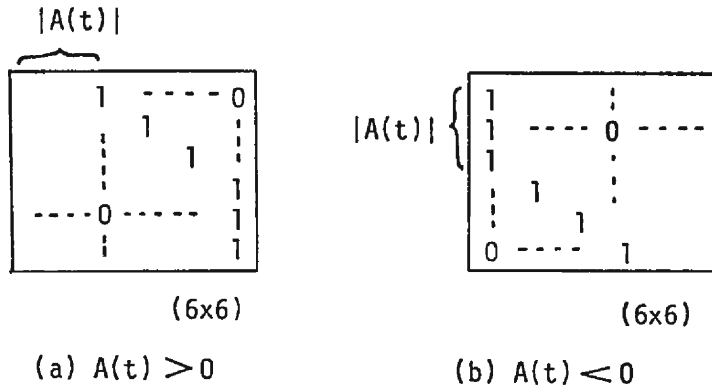


Fig. 3. Transformation matrix of dam reservoir when I is six and  $|A(t)|$  is three.

### 3.2 Simulation model

#### (1) Simulation method

In this model, the hyetograph is simulated by generating random numbers for precipitation probabilities in the previous section. When the gross volume of precipitation is small, it is easy to protect the whole basin from the flood inundation without having any project. And when the gross volume is big, it is important to plan and operate several projects for flood control. The hydrograph is also simulated by applying runoff analyses, such as the Kinematic wave method<sup>5)</sup>, to the generated hyetograph. For each hyetograph, all of flood control projects are optimally operated to estimate the safety against flood inundation, introducing the interaction of projects. After simulating a great number of hyetographs, the evaluation of flood inundation at each defense point can be gained through project operation and flood routing while keeping the statistical characteristics such as the correlation coefficients of flood inundation in time and space between defense points. The evaluation criteria on each alternative may be defined as the maximum value among the ratios of the number of inundations to the number of simulations at all defense points. If its value is greater than the remainder value (the exceedance probability) which is decided by subtracting the required safety rate from 1.0, the alternative has to be abandoned as a simulation model. On the contrary, if the value is equal to and less than the above remainder value, the alternative is regarded as the effective one to be considered in this flood control project planning.

#### (2) Optimal flood control by dam reservoir system

For any given hydrograph, the dam reservoir system has to be operated to gain a well-balanced control among all defense points, that is to say, the optimal control. The control objective which is to prevent flood inundation is written as follows.

$$J_{SM} = \max \left\{ \frac{Q_{1p}}{Q_{1d}}, \frac{Q_{2p}}{Q_{2d}}, \dots, \frac{Q_{Mp}}{Q_{Md}} \right\} \rightarrow \min \tag{18}$$



Where  $Q_{mp}$  ( $m=1, 2, \dots, M$ ) is the peak flow discharge and  $Q_{md}$  is the channel capacity at the  $m$ -th defense point. As the peak flow discharge at the defense point on the downstream from the dam site is influenced by the channel storage, the formulation of DP is changed from the traditional form. Introducing the storage function of river channel into the method in order to represent the flood routing mechanism in the river, the recursive function of DP is as follows.

$$\begin{aligned}
 & f_t(S_1(t), \dots, S_N(t), \bar{S}_1(t+\tau_1), \dots, \bar{S}_W(t+\tau_W)) \\
 &= \min_{\{Q_n(t)\}} \left[ \max \left\{ \frac{Q_1(t+\tau_1)}{Q_{1d}}, \frac{Q_2(t+\tau_2)}{Q_{2d}}, \dots, \frac{Q_M(t+\tau_M)}{Q_{Md}}, \right. \right. \\
 & \left. \left. f_{t-1}(S_1(t-1), \dots, S_N(t-1), \bar{S}_1(t+\tau_1-1), \dots, \bar{S}_W(t+\tau_W-1)) \right\} \right] \quad (19)
 \end{aligned}$$

And the storage function is represented as follows.

$$\frac{I_w(t-1) + I_w(t)}{2} - \frac{\bar{O}_w(t+\tau_w-1) + \bar{O}_w(t+\tau_w)}{2} = \bar{S}_w(t+\tau_w) - \bar{S}_w(t+\tau_w-1) \quad (20)$$

$$\bar{S}_w(t+\tau_w) = K_w \{\bar{O}_w(t+\tau_w)\}^{P_w} \quad (w = 1, 2, \dots, W) \quad (21)$$

$$Q_m(t+\tau_m) = \bar{O}_m(t+\tau_m) \quad (22)$$

Where  $I_w(t)$  is the inflow discharge to the  $w$ -th channel at the  $t$ -th control time,  $\tau_w$  is the lag time in the storage function,  $\bar{S}_w(t)$  is the channel storage volume,  $\bar{O}_w(t)$  is outflow discharge at the  $w$ -th channel,  $W$  is the total number of channels, and  $K_w$  and  $P_w$  are characteristic parameters of the  $w$ -th channel, respectively. In equation (19), there are two state variables and one decision variable. At the last control time, the optimal objective value is determined by minimizing the estimate function with respect to the channel storage as follows.

$$\begin{aligned}
 f_{T_E}^* = \min \{ & f_{T_E}(S_1^0(T_E), \dots, S_N^0(T_E), \bar{S}_1(T_E+\tau_1), \dots, \bar{S}_W(T_E+\tau_W)) \} \quad (23) \\
 & \{\bar{S}_w(t+\tau_w)\} \quad (w = 1, 2, \dots, W)
 \end{aligned}$$

To deal with the non-linearity of the storage function, we have proposed several approximative methods to discretize the storage variable for the channel storage<sup>6</sup>.

### 3.3 Sequential model

(1) The objective of schedule planning

It is impossible for a great number of projects to be constructed at the same time because of the constraint of the budget. So, it is important to plan construction in the most effective order with reference to the reduction of flood damage except for the damage caused by floods whose occurrence probability is greater than the occurrence probability of design rainfall given for flood control planning. From this point of view, each term of the control objective written in equation (3) is expressed as follows.

$$DA_1^\theta = \sum_{m=1}^M \sum_{t=1}^{T_B} \int_{Q_{md}}^{\infty} f'_{g_m}(Q_m(t)) D_m\{Q_m(t)\} dQ_m(t) \quad (24)$$

$$DA_2^\theta = \sum_{m=1}^M \sum_{t=1}^{T_B} \int_{Q_{md}}^{\infty} f''_{g_m}(Q_m(t)) D_m\{Q_m(t)\} dQ_m(t) \quad (25)$$

Where  $f'_{g_m}(\cdot)$  is the probability density function of the flow discharge  $Q_m(t)$  under construction and computed by the frequency distribution converted from the frequency distribution consisting of the generated hydrographs in 3.2 according to the projects constructed at each control time.  $f''_{g_m}(\cdot)$  is the same function after construction. In equations (24) and (25), it is useful to consider the variation of the probability density function through dam operation because of the accurate evaluation of flood control projects. And the channel capacity and the damage function does not vary in the case of this planning having only dam reservoirs though they will vary depending on the construction order of projects in the case of planning in which the condition of flood inundation is changed, such as the improvement of channel capacity or the construction of a retarding basin.

## (2) Formulation of schedule planning

In planning of the optimal schedule, the following assumptions are introduced for the convenience of the computation:

- i) Each project is constructed one by one in each time step of construction.
- ii) The annual budget and the construction cost increase with the same constant rate.
- iii) The construction period of each project is only the function of the construction cost.
- iv) Total years of construction schedule planning is equal to the sum of the construction years spent on each project.
- v) All input data and computational variables are expressed in unit of one year. Then, with the addition of the variation of damage with time in the process, the control objective (3) is defined as follows.

$$O'_{bs} = \sum_{\theta=1}^{\theta_{\max}} \frac{1}{(1+r_a)^{\theta-1}} (DA_1^\theta - DA_2^\theta) \quad (26)$$

subject to

$$\left. \begin{aligned} \theta \max &= \sum_{i=1}^L \Delta\theta_i \\ \Delta\theta_i &= C_i / C_{year} \end{aligned} \right\} \quad (27)$$

Where  $L$  is the total steps in schedule planning (= the number of projects),  $r_a$  is the annual discount rate,  $C_i$  and  $\Delta\theta_i$  are the construction cost and the term of works of  $i$ -th project,  $C_{year}$  is the annual budget at the initial year and  $\theta$  is the planning period expressed as one year, respectively. Equation (27) does not contain the term  $(1+r_a)^{1-\theta}$ , because the elevation of the construction cost depending on the discount rate is set off against the elevation of the annual budget in another term.

As  $DA_2^0$  is the expected damage after all projects are constructed and constant for all planning periods, its term may be ignored in the computation.

Now, it is possible to regard the annual budget as the function of time, the projects after construction as the state variable  $S'(\cdot)$  and the project to be constructed at the next step as the decision variable  $x$ . Then, the schedule planning is transformed into a multi-stage decision process. Concretely, when  $S'(x_i)$  is the state vector and expresses the construction order,  $\Delta\theta$  is the function of the decision variable and  $f'_l(x_i)$  is the estimate function at step  $l$ , the recursive function of DP is represented as follows.

$$f'_l(x_i) = \min_{(x_{l-1})} \left[ \frac{1}{(1+r_a)^{\sum_{y=1}^{l-1} \Delta\theta(x_y) + \frac{1}{2} \Delta\theta(x_l)}} \{ \Delta\theta(x_l) \cdot DA_1^0(x_l, S'(x_{l-1})) \} + f'_{l-1}(x_{l-1}) \right] \quad (28)$$

In equation (28), provided that the expected damage to the state  $S'(x_i)$  runs for the years  $\Delta\theta(x_i)$  of step  $l$ , the evaluated damage is converted into the present value after multiplying the annual expected damage by the years of its step. The effects of construction do not appear till half time of the construction period has past. Since the effects depend on the state of the constructed projects not only at one step before but also all steps before, this computational process does not satisfy the principle of optimality. Its solution, however, tends to indicate that the greater the project has the damage prevention potential, the faster it will be constructed, with a concomitant decrease in damage even if the control policy differs from the optimal solution. From this point of view, it is useful to use the above formulation to decide the schedule planning.

#### 4. A case study

In this chapter, the planning model of a flood control system is applied to a real river basin and the operational characteristics of the dam reservoir control is especially analyzed. The river basin chosen for illustrating the methodology is the Kizu river (the upper area from Nabari city) as shown in **Fig. 4**. Analyzed data are floods for sixty-seven cases which had occurred from 1973 to 1977. We will explain the methodology of shift operation at the application time, the computational procedure of the conditional probability and the control results of the dam reservoir operation.

##### 4.1 Practical use of shift operation

Shift operation will be applied to the complex river basin with multi-sub-basin multi-project as follows.

When the river basin is composed of several sub-basins which has no project

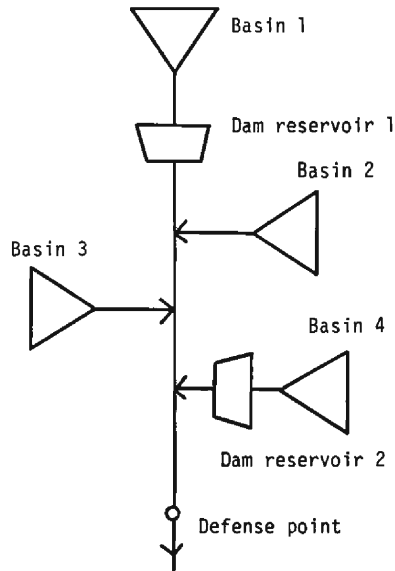


Fig. 4. Typical resp representation of flood control system.

such as pattern 1 shown in **Fig. 2**, the occurrence probability of the confluence discharge is calculated by using equation (13) repeatedly. In the case that the system has one dam reservoir in the upper reach of the main river such as pattern 3, equation (15) is used at the first confluence point and then equation (13) is applied at the other confluence points in the lower reach. When a sub-basin has any project such as pattern 3 or pattern 4, however, it is necessary to discuss the multiplication process of the confluence discharge, because in the former patterns the occurrence probability of the confluence discharge is represented as the form of matrix  $Q_j$  whose column component denotes the discretized confluence discharge and row component denotes the discretized flow discharge of the sub-basin and in the latter patterns the row component denotes the discretized release discharge from the dam reservoir. Consequently, in the product of matrixes there is no row component denoting the flow discharge of the sub-basin. Thus, it is impossible to execute the multiplication between matrixes of the conditional probability and it is necessary that the element of the confluence probability must be put back to the position expressing the flow discharge with reference to row component. That is to say, the term that contains the conditional probability  $P_r = \{P_{uw}\}$  of the flow discharge on the basin  $i$  in the element of the multiplied matrix  $P'_r = \{P_{u'w'}\}$  at the  $i$ -th confluence point is moved to the position at the  $u'$ -th column and the  $w'$ -th row, if the value of suffix  $w$  does not equal to the value of  $w'$  plus 1. Otherwise, the element is not moved. Assuming that  $Q_j^A$  is the occurrence probability at the confluence point,  $Q_j^B$  is the occurrence probability in the branch stream and  $Q_j^C$  is the transformation matrix by the dam reservoir in the branch stream,  $Q_j^D$  which clarifies the elements moved as above mentioned is represented as follows.

$$Q_j^p = Q_j^A(Q_j^C - Q_j^p) \quad (29)$$

Where the negative elements are changed to zero, the moving distance of  $Q_j^p$  depends on  $A(t)$  mentioned in equation (9). If  $A(t) > 0$ , the element is moved the distance of  $|A(t)|$  units to the right and if  $A(t) < 0$ , the element is moved the distance of  $|A(t)|$  units to the left. There is no doubt that this moving distance does not cover the range  $(I \times J)$  of the matrix. Using  $\bar{Q}_j^p$  as the matrix after moving, the probability matrix  $\bar{Q}_j^A$  of the confluence discharge is computed by

$$\bar{Q}_j^A = Q_j^A - Q_j^p + \bar{Q}_j^p \quad (30)$$

When the row component of probability matrix corresponds to the new additional branch stream, the step of computation can go forward to the next confluence point downstream. The auto-computation of the inundation probability has been made possible by this formulation.

#### 4.2 Conditional probability of precipitation and flow discharge

The precipitation data are arranged so that the peaks of hietographs appear at the same time and the total amount of analyzed time is fifteen hours. The computational duration time of precipitation is five hours and the dividing number of discretization is five points, such as 0, 0.1-4.0, 4.1-8.0, 8.1-12.0, 12.1-∞ mm/hour. **Table 1** shows the conditional probabilities at the control time 5 and in basin 2. Several figures in the parentheses of the same table denotes the successive precipitation patterns for five hours up to the control time 5.

The probability distribution of the precipitation are converted to the probability distribution of the flow discharge by using the instantaneous unit-hydrograph method<sup>7)</sup> composed of linear storage model as runoff analysis. As there is only a little observation data for the flow discharge in the divided sub-basins, the identified parameters in sub-basin 4 are applied to the other sub-basins. In **Table 2**, the matrix means the conditional probability distribution of the flow discharge at control time 5 on sub-basin 1 given the flow discharge at control time 4 on the same sub-basin. And in **Table 3** the matrix means the conditional probability distribution of the flow discharge on sub-basin 3 at control time 5 given the flow discharge on sub-basin 2 at the same time. The value of probability which is less than 0.1 is neglected in the computation. The dividing number for the discretized discharge is ten points, such as 0.1-20.0, 20.1-40.0, 40.1-60.0, 60.1-80.0, 80.1-100.0, 100.1-120.0, 120.1-140.0, 140.1-160.0, 160.1-180.0, 180.1-∞ m<sup>3</sup>/sec. Generally speaking the appearance range of the flow discharge tends to be very small. It is supposed that the reasons for this result are the lack of observation data to identify the instantaneous unit-hydrograph and the shortness of duration time such as five hours. Nevertheless, the recession characteristics last for more than ten hours. In future, it is important to improve the identification method of parameters and make the duration time longer.

Table 1. Conditional probability of precipitation whose duration time is five hours.

Precipitation pattern on basin 1	Precipitation pattern and conditional probability on basin 2			
(1, 1, 1, 2, 2)	(1, 1, 2, 2, 2) 0.1500			
(1, 1, 1, 2, 3)	(1, 1, 1, 2, 3) 0.0769			
(1, 1, 2, 2, 2)	(1, 2, 2, 2, 3) 0.0588	(1, 2, 3, 2, 2) 0.0074	(2, 2, 2, 2, 2) 0.0588	(2, 2, 3, 2, 2) 0.0074
(1, 1, 2, 2, 3)	(1, 2, 2, 2, 3) 0.0233			
(1, 2, 1, 2, 2)	(1, 1, 1, 2, 2) 0.0270	(1, 2, 1, 2, 2) 0.0541		
(1, 2, 2, 1, 2)	(1, 2, 2, 2, 2) 0.0286	(1, 2, 2, 2, 2) 0.1429		
(1, 2, 2, 2, 2)	(1, 1, 2, 2, 2) 0.0138	(1, 1, 2, 2, 3) 0.0011	(1, 2, 2, 2, 2) 0.0006	(1, 2, 2, 2, 3) 0.1083
	(1, 2, 3, 3) 0.0017	(1, 2, 3, 2, 2) 0.0006	(1, 3, 2, 2, 2) 0.0069	(1, 3, 2, 2, 3) 0.0006
	(2, 2, 2, 2, 3) 0.0040	(2, 2, 2, 3, 2) 0.0011	(2, 2, 3, 2, 2) 0.0011	(2, 3, 2, 2, 2) 0.0069
	(2, 3, 3, 2, 2) 0.0006			
(1, 2, 2, 2, 3)	(1, 1, 2, 2, 2) 0.0018	(1, 1, 2, 2, 4) 0.0018	(1, 1, 2, 3, 2) 0.0018	(1, 2, 2, 2, 2) 0.0128
	(1, 2, 2, 3) 0.0383	(1, 2, 2, 4) 0.0128	(1, 2, 3, 2) 0.0091	(1, 3, 2, 2, 3) 0.0018
	(2, 2, 2, 3) 0.0201	(2, 2, 2, 4) 0.0073	(2, 2, 3, 2) 0.0036	(2, 2, 2, 2, 2) 0.0073
(1, 2, 2, 3, 2)	(1, 1, 2, 3, 2) 0.0152	(1, 2, 2, 2, 2) 0.0152	(1, 2, 2, 3, 2) 0.0758	(1, 2, 2, 3, 3) 0.0152
(1, 2, 2, 3, 3)	(1, 2, 2, 3, 3) 0.1429			
(1, 2, 3, 2, 2)	(1, 2, 3, 2, 2) 0.0323			
(1, 2, 3, 3, 2)	(1, 2, 3, 3, 2) 0.0625			
(2, 1, 1, 2, 2)	(1, 1, 1, 2, 2) 0.1538			
(2, 1, 2, 2, 2)	(2, 2, 2, 2, 2) 0.1778			
(2, 1, 2, 2, 3)	(2, 2, 2, 3) 0.0714			
(2, 2, 1, 1, 1)	(2, 2, 1, 1, 2) 0.3333			
(2, 2, 1, 1, 2)	(2, 2, 1, 1, 2) 0.0625			
(2, 2, 1, 2, 2)	(2, 1, 1, 2, 2) 0.0055			
(2, 2, 2, 1, 2)	(3, 2, 2, 2, 2) 0.0409			
(2, 2, 2, 2, 2)	(2, 1, 2, 2, 2) 0.0028	(2, 2, 1, 2, 2) 0.0006	(2, 2, 2, 1, 2) 0.0009	(2, 2, 2, 2, 2) 0.2902
	(2, 2, 3, 2) 0.0047	(2, 2, 3, 2, 2) 0.0015	(2, 2, 3, 2, 3) 0.0001	(2, 3, 2, 2, 2) 0.0028
	(2, 3, 3, 2, 2) 0.0001	(3, 2, 1, 2, 2) 0.0001	(3, 2, 2, 1, 2) 0.0001	(3, 2, 2, 2, 3) 0.0025
	(3, 2, 2, 3, 3) 0.0006	(3, 2, 3, 2, 2) 0.0002	(3, 3, 2, 2, 2) 0.0014	(3, 3, 2, 2, 2) 0.0001
(2, 2, 2, 2, 3)	(2, 1, 2, 2, 2) 0.0004	(2, 1, 2, 2, 3) 0.0011	(2, 1, 2, 3, 2) 0.0004	(2, 1, 2, 3, 2) 0.0004
	(2, 2, 2, 2, 3) 0.1025	(2, 2, 2, 3, 2) 0.0221	(2, 2, 2, 3, 3) 0.0221	(2, 2, 3, 2, 2) 0.0004
	(2, 3, 2, 2, 3) 0.0011	(2, 3, 2, 2, 4) 0.0004	(2, 3, 2, 3, 2) 0.0004	(2, 3, 2, 2, 2) 0.0041
	(3, 2, 2, 2, 4) 0.0041	(3, 2, 2, 3, 2) 0.0026	(3, 2, 2, 3, 3) 0.0026	(3, 2, 2, 2, 3) 0.0041

(2, 2, 2, 3, 2)	(2, 1, 2, 3, 2) 0.0031 (3, 2, 2, 2) 0.0031	(2, 2, 2, 2, 2) 0.0249 (3, 2, 2, 3) 0.0031	(2, 2, 2, 2, 3) 0.0249 (3, 2, 2, 3, 2) 0.0249	(2, 2, 2, 3, 2) 0.2212 (3, 2, 2, 3, 3) 0.0031	(2, 2, 2, 3, 3) 0.0374	(2, 3, 2, 3, 2) 0.0031
(2, 2, 2, 3, 3)	(2, 2, 2, 3, 3) 0.3333	(3, 2, 2, 3, 3) 0.0278				
(2, 2, 3, 2, 2)	(2, 2, 3, 2, 2) 0.0894 (3, 3, 3, 2, 2) 0.0033	(2, 2, 3, 2, 3) 0.0066	(2, 2, 3, 3, 2) 0.0099	(2, 2, 3, 3, 3) 0.0099	(2, 2, 3, 3, 2) 0.0066	(3, 2, 3, 2, 2) 0.0099
(2, 2, 3, 2, 3)	(2, 2, 3, 2, 2) 0.0105	(2, 2, 3, 2, 3) 0.0316	(2, 2, 3, 2, 4) 0.0105	(2, 2, 3, 3, 2) 0.0526	(3, 2, 3, 3, 2) 0.0105	(3, 2, 3, 3, 3) 0.0105
(2, 2, 3, 3, 2)	(2, 2, 2, 3, 2) 0.0513	(2, 2, 2, 3, 3) 0.0128	(2, 2, 3, 3, 2) 0.1154	(2, 2, 3, 3, 3) 0.0256	(2, 3, 3, 3, 2) 0.0128	(3, 2, 3, 3, 2) 0.0128
(2, 2, 3, 3, 3)	(2, 2, 2, 3, 3) 0.1111	(2, 2, 3, 3, 3) 0.2222				
(2, 3, 2, 1, 2)	(2, 3, 2, 2, 2) 0.2000	(3, 3, 2, 2, 2) 0.2000				
(2, 3, 2, 2, 2)	(2, 2, 2, 2, 2) 0.0303 (2, 3, 2, 3, 3) 0.0038	(2, 2, 2, 3) 0.0038 (3, 3, 2, 2, 2) 0.1212	(2, 2, 3, 2, 2) 0.0038 (3, 3, 2, 2, 3) 0.0076	(2, 3, 2, 2, 2) 0.1212 (3, 3, 2, 3, 2) 0.0038	(2, 3, 2, 2, 3) 0.0076 (3, 3, 2, 3, 3) 0.0038	(2, 3, 2, 3, 2) 0.0038
(2, 3, 2, 2, 3)	(2, 2, 2, 2, 3) 0.0120 (3, 3, 2, 2, 2) 0.0120	(2, 3, 2, 2, 2) 0.0120 (3, 3, 2, 2, 3) 0.0361	(2, 3, 2, 2, 3) 0.0361 (3, 3, 2, 2, 4) 0.0120	(2, 3, 2, 2, 4) 0.0120 (3, 3, 2, 2, 2) 0.0120	(2, 3, 2, 3, 2) 0.0120 (3, 3, 2, 3, 3) 0.0120	(2, 3, 2, 3, 3) 0.0120
(2, 3, 2, 3, 2)	(2, 3, 2, 3, 2) 0.1000	(3, 3, 2, 3, 2) 0.1000				
(2, 3, 3, 2, 2)	(2, 3, 3, 2, 2) 0.0909	(3, 3, 3, 2, 2) 0.0909				
(3, 2, 1, 2, 2)	(2, 2, 1, 2, 2) 0.0952					
(3, 2, 2, 1, 2)	(2, 2, 2, 2, 2) 0.2500	(3, 3, 2, 2, 2) 0.0500				
(3, 2, 2, 2, 2)	(2, 2, 2, 1, 2) 0.0010 (3, 3, 2, 2, 2) 0.0241	(2, 2, 2, 2, 2) 0.1896 (3, 3, 2, 2, 3) 0.0020	(2, 2, 2, 2, 3) 0.0010 (3, 3, 3, 2, 2) 0.0010	(2, 2, 2, 3, 2) 0.0030	(2, 2, 2, 3, 3) 0.0030	(2, 2, 3, 2, 2) 0.0010
(3, 2, 2, 2, 3)	(2, 2, 2, 2, 2) 0.224 (3, 3, 2, 2, 3) 0.0096	(2, 2, 2, 2, 3) 0.0671 (3, 3, 2, 2, 4) 0.0032	(2, 2, 2, 2, 4) 0.0224 (3, 3, 2, 3, 2) 0.0032	(2, 2, 2, 3, 2) 0.0160 (3, 3, 2, 3, 3) 0.0032	(2, 2, 2, 3, 3) 0.0160 (3, 3, 2, 3, 3) 0.0032	(3, 3, 2, 2, 2) 0.0032
(3, 2, 2, 3, 2)	(2, 2, 2, 2, 2) 0.0263	(2, 2, 2, 2, 3) 0.0263	(2, 2, 2, 3, 2) 0.1316	(2, 2, 2, 3, 3) 0.0263	(3, 3, 2, 3, 2) 0.0263	
(3, 2, 2, 3, 3)	(2, 2, 2, 3, 3) 0.2500					
(3, 2, 3, 2, 2)	(2, 2, 3, 2, 2) 0.0566	(3, 3, 3, 2, 2) 0.0566				
(3, 2, 3, 3, 2)	(2, 2, 3, 3, 2) 0.1111	(3, 3, 3, 3, 2) 0.1111				
(3, 3, 2, 2, 2)	(2, 3, 2, 2, 2) 0.0758 (3, 3, 2, 2, 3) 0.0047	(2, 3, 2, 2, 3) 0.0047	(3, 2, 2, 2, 2) 0.0758	(3, 2, 2, 2, 3) 0.0047	(3, 2, 2, 2, 2) 0.0047	(3, 3, 2, 2, 2) 0.0758
(3, 3, 2, 2, 3)	(2, 3, 2, 2, 2) 0.0152 (3, 3, 2, 2, 2) 0.0152	(2, 3, 2, 2, 3) 0.0303 (3, 3, 2, 2, 3) 0.0303	(2, 3, 2, 2, 4) 0.0152 (3, 3, 2, 2, 4) 0.0152	(3, 2, 2, 2, 2) 0.0152 (3, 2, 2, 2, 2) 0.0152	(3, 2, 2, 2, 3) 0.0303 (3, 2, 2, 2, 3) 0.0303	(3, 2, 2, 2, 4) 0.0152
(3, 3, 3, 2, 2)	(2, 3, 3, 2, 2) 0.1111	(3, 3, 3, 2, 2) 0.1111				
(4, 3, 2, 2, 2)	(3, 3, 2, 2, 2) 0.3019	(3, 3, 2, 2, 3) 0.0199				
(4, 3, 2, 2, 3)	(3, 3, 2, 2, 2) 0.0588	(3, 3, 2, 2, 3) 0.1861				
(4, 3, 3, 2, 2)	(3, 3, 3, 2, 2) 0.5000					

Table 2. Conditional probability of flow discharge in time on sub-basin 1 at the control time 5.

	Water discharge at time 5 ( $\times 20.0 \text{ m}^3/\text{sec}$ )										
	1	2	3	4	5	6	7	8	9	10	
Water discharge at time 4 ( $\times 20.0 \text{ m}^3/\text{sec}$ )	1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	2	0.0	0.0473	0.8170	0.1356	0.0	0.0	0.0	0.0	0.0	0.0
	3	0.0	0.0226	0.8410	0.0243	0.1121	0.0	0.0	0.0	0.0	0.0
	4	0.0	0.0	0.0144	0.7949	0.1035	0.0871	0.0	0.0	0.0	0.0
	5	0.0	0.0	0.0	0.0687	0.6297	0.2987	0.0028	0.0	0.0	0.0
	6	0.0	0.0	0.0	0.0	0.4737	0.1930	0.3333	0.0	0.0	0.0
	7	0.0	0.0	0.0	0.0	0.0	0.6667	0.3333	0.0	0.0	0.0
	8	0.0	0.0	0.0	0.0	0.0	0.0	1.0000	0.0	0.0	0.0
	9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Table 3. Conditional probability of flow discharge in space, on sub-basin 3 at the control time 5.

	Water discharge on basin 3 ( $\times 20.0 \text{ m}^3/\text{sec}$ )										
	1	2	3	4	5	6	7	8	9	10	
Water discharge on basin 2 ( $\times 20.0 \text{ m}^3/\text{sec}$ )	1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	2	0.0	0.1048	0.8629	0.0323	0.0	0.0	0.0	0.0	0.0	0.0
	3	0.0	0.0615	0.7352	0.1828	0.0182	0.0023	0.0	0.0	0.0	0.0
	4	0.0	0.0	0.3193	0.4262	0.2184	0.0331	0.0030	0.0	0.0	0.0
	5	0.0	0.0	0.0	0.5214	0.3077	0.1410	0.0299	0.0	0.0	0.0
	6	0.0	0.0	0.0	0.0	0.0	0.5000	0.5000	0.0	0.0	0.0
	7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

### 4.3 Results of dam operation

To illustrate the operational characteristics of dam control for the correlated inputs obtained in the above process, let us consider the flood control system being operated only by dam reservoir 1 without having any dam in other sub-basins.

**Fig. 5** shows the control effects. When the allowable flood discharge (channel capacity) at the defense point is 10 (one unit is  $20.0 \text{ m}^3/\text{sec}$ ), the inundation probabil-



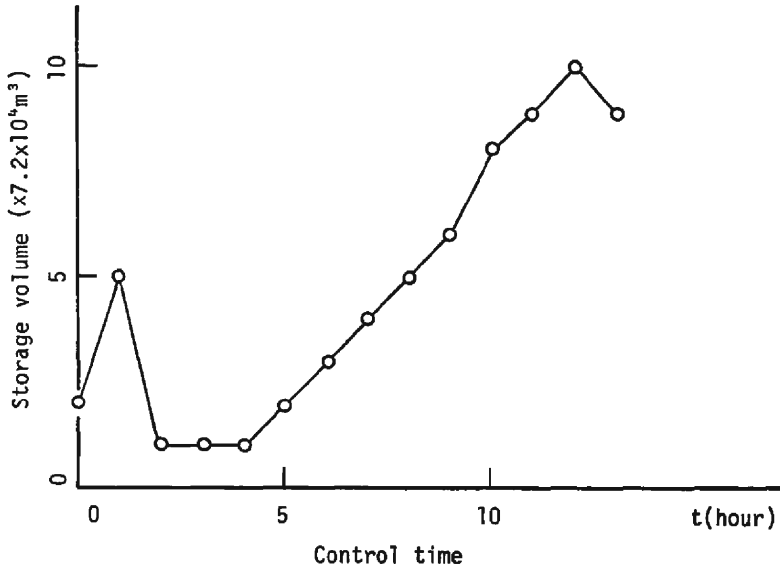


Fig. 5. The sequence of the controlled storage volume.

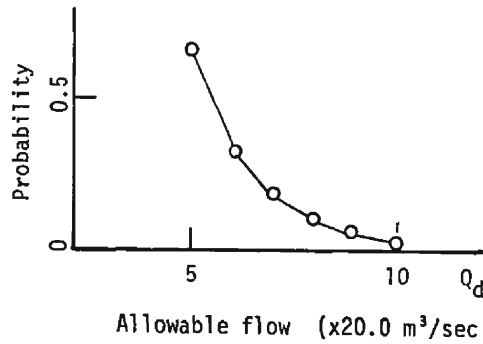


Fig. 6. Variation of the inundation probability by the change of allowable flood discharge.

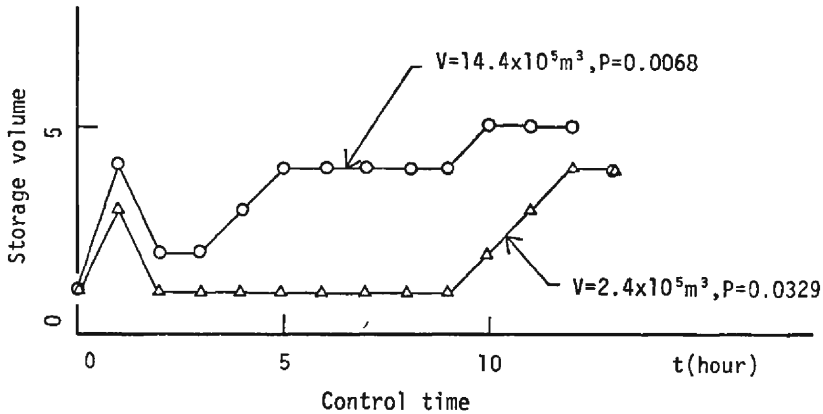


Fig. 7. Comparison of the inundation probabilities between two different storage.

ity becomes 0.0068. **Fig. 6** shows that the inundation probability varies as the allowable flood discharge is changed such as 9, 8, 7, 6, 5, and indicates the importance of flood control based on the spatial and time distribution of the flood inundation. **Fig. 7** shows the control effects for different control capacities of a dam reservoir such as  $2.4 \times 10^5 \text{ m}^3$  (the unit number is six) and  $14.4 \times 10^5 \text{ m}^3$ . The difference of alternatives causes the change of the storage volume sequence and improves the inundation probability from 0.0329 to 0.0069.

In addition to the results of screening models, a future paper will discuss the results of simulation models and sequential models after framing the exact basin model and the evaluation method of the flood damage.

## 5. Conclusion

In this paper, in order to plan flood control systems considering the operational effects on whole river basin, we established the computational methodology of the flood inundation probability in time and space at all defense points and then planned the optimal flood control system based on their probability. To sum up, the following results are gained through the control characteristics of screening models. However, simulation models and sequential models can not be described because of no application.

- i) The procedure for flood control project planning is proposed considering the operational effects on whole river basin with multi-sub-basin and multi-project sites. To estimate flood control value, the maximum value among the flood inundation probabilities at all defense points is used and planning objective is defined as the minimization of its value.
- ii) To decide the optimal flood control project planning, three methodologies are applied. The first step is a screening model in which the site and scale of the project is decided by random search method on the simple basin model. The second step is a simulation model in which the optimal solution is determined among few alternatives on the exact basin model by a simulation method. And the third step is sequential model to make the schedule planning on the extracted projects by Dynamic Programming.
- iii) The optimal operation rule for dam reservoirs was approximately deduced for the conditional input and applied to the real data to inspect its control effects.
- iv) By putting shift operation to practical use for the complex river basin model with multi-sub-basin and multi-project sites, it becomes possible to compute the inundation probability at any defense point.
- v) Lastly, a problem to be solved is the extraction of several patterns of dam operation from the controlled results of the screening model and simulation model, and to develop the real-time operation rule of flood control projects by which any flood would be controlled with the same safety rate as in flood control planning.

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