Scattering of P Waves by Random Heterogeneities with Sizes Comparable to the Wave Length

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Abstract

In order to examine P-wave scattering by random heterogeneities with sizes comparable to the wave length, ultrasonic model experiments are carried out by using two-dimensional models of media with randomly distributed velocity and density heterogeneities. P waves are observed along profiles transversal to the direction of wave propagation. To examine the attenuation of P wave by the scattering, the Q^{-1} value, which is defined as $Q^{-1}=2\alpha_p/k_\alpha$ where α_p and k_α are the total scattering coefficient and the wave number of P wave, respectively, is estimated. On the other hand, to examine the fluctuation characteristics of P wave, the Q_t^{-1} value, which is defined as $Q_t^{-1}=2\alpha_{cp}/k_\alpha$ where α_{cp} is the seismic scattering coefficient defined by Nevsky et al., is estimated. The obtained Q^{-1} values decrease with the increase of $k_\alpha a$ from 4 to 13.5, but the obtained Q_t^{-1} values increase with the increase of $k_\alpha a$ from 4 to 13.5, where a is the size of heterogeneity. When $k_\alpha a=4$, Q^{-1} is more than ten times of Q_t^{-1}, but Q^{-1} and Q_t^{-1} are nearly of the same order when $k_\alpha a=13.5$. This indicates that the order of about 10^{-1} of the total scattered energy contributes to the fluctuation of the wave field when $k_\alpha a=4$, but all of the scattered energy contributes to the fluctuation when $k_\alpha a=13.5$.

1. Introduction

Seismic waves attenuate by the geometrical spreading of the wave front, the absorption of elastic energy and the wave scattering due to various heterogeneities in a medium. Recently, it has been reported by several authors that for seismic S waves in the frequency range of about $1<f<25$ Hz, the value of Q_β^{-1}, which gives the damping of the wave amplitude per unit cycle as $\exp(-\pi Q_\beta^{-1})$, decreases in proportion to $f^{-0.5\sim-0.8}$ [1)~4)]. Further, according to a recent study[5)] of the attenuation of S waves by utilizing the characteristics of seismic coda waves, the wave scattering caused by various heterogeneities in any medium plays a major role in the wave attenuation for the range of frequency from about 5 to 27 Hz. On the other hand, according to Nevsky's et al. studies[6)] of the attenuation of P waves by utilizing the spatial fluctuation characteristics of the wave amplitude and the phase, the attenuation of P waves for $1<f<50$ Hz is strongly affected by the wave scattering due to random velocity heterogeneities.

When the waves are incident on a heterogeneous region, a part of the incident waves are scattered by the heterogeneities and the rest remains unchanged. In the statistical theory of wave scattering, it is considered that the wave field is composed of both coherent and incoherent components. The scattered waves are considered as the incoherent component of a wave field and the remaining incident waves are considered as its coherent component. Accordingly, the coherent component attenuates with the increase of travel distance in the heterogeneous region. Usually, the value of Q^{-1}, which shows the wave attenuation by the wave scattering due to various heterogeneities, is estimated from the energy loss due to the scattering of the coherent component in all directions.

According to the statistical theory of wave scattering caused by random velocity heterogeneities,[7),8)] the value of Q^{-1} is proportional to f^1 in the high-frequency range ($2\pi a/\lambda \gg 1$) where the wave length λ is shorter than the correlation distance a of heterogeneity, and is proportional to f^3 in the low-frequency range ($2\pi a/\lambda \ll 1$) where the wave length is much longer than the correlation distance of heterogeneity, but in the intermediate frequency range (about $0.6 < 2\pi a/\lambda < 10$) where the wave length is comparable to or longer than the correlation distance of heterogeneity, it does not show so simple a frequency dependence as those in the high- and low-frequency ranges. Therefore, Sato[9)] and Wu[10)] indicated that the statistical theory of wave scattering can not explain the decreasing behavior of the observed Q_β^{-1} of S waves with frequency for $1 < f < 25$ Hz, and they discussed the cause of the discrepancy between the theoretical prediction and the frequency dependence of the observed Q_β^{-1} of S waves. Sato's interpretation is as follows. the statistical theory of wave scattering regards forward scattered energy as loss. However, forward scattered S waves, which are strong in the high-frequency range, may arrive nearly simultaneously with the coherent S waves. As S waves are composed of a coherent component and an incoherent one, it should not be considered that the Q_β^{-1} value contains the energy loss due to the forward scattering. Considering the above, Sato proposed a new statistical method for the calculation of Q^{-1} which does not contain the energy loss due to the forward scattering in the high-frequency range, and tried to explain the frequency dependence of the observed Q_β^{-1} by it.

On the other hand, Kopnichev[5)] suggested that the attenuation of short-period body waves is strongly affected by the wave scattering due to numerous cracks in the crust of seismic active region and therefore the frequency dependence of the observed Q_β^{-1} of S waves is inconsistent with that of Q^{-1} estimated theoretically from the scattering loss due to random velocity heterogeneities in a medium. According to the result of Strizhkov's model experiment,[11] when P waves travel in a medium with randomly distributed cracks, the predominant frequency becomes higher with the increase of travel distance. And also according to the results of Shamina's et al. model experiment,[12] when P waves travel through the large inclusion with numerous cracks in the medium, the predominant frequency of P waves observed at the field near the large heterogeneous region is remarkably high compared with a uniform medium. These show that the lower-frequency components of P waves attenuate by the scattering more remarkably than their higher-frequency ones. This supports qualitatively Kopnichev's suggestion. Especially, it is important in their experiments that P waves traveled a sufficiently long distance L as compared with the wave length λ in the heterogeneous region ($2\pi L/\lambda \gg 1$) when the wave length is comparable to or longer than the length l of the cracks ($0.6 < 2\pi l/\lambda < 4$).

On the other hand, we carried out a preliminary experiment to examine P-wave scattering caused by randomly distributed velocity and density heterogeneities. According to our experimental results, when any P wave travels a long distance in the random medium ($2\pi L/\lambda \gg 1$), the Q^{-1} value, which is estimated from the exponential decay of amplitudes of the coherent component of P waves with the increase of travel distance, roughly shows a decreasing behavior with frequency for the intermediate frequency range and the amplitude fluctuates along profiles transversal to the direction of wave propagation. Though there is a difference in the used scatterers between our experiment and ones carried out by Strizhkov and Shamina et al., it seems that P-wave attenuation properties obtained in our experiment and theirs show the attenuation property by wave scattering for the intermediate frequency range in the case of a sufficiently long travel distance as compared with the wave length in random media.

In view of the above, the attenuation of short-period body waves, which travel a much longer distance than the wave length in the upper lithosphere, is considered to be strongly affected by wave scattering due to various heterogeneities with sizes comparable to or shorter than the wave length. But the kinds of heterogeneities in the upper lithosphere and their fine structures are not well known. Acoustic velocity log and density log usually show fluctuations in the direction of depth.[13),14] This suggests that heterogeneities expressed as spatial fluctuations of wave velocity and density exist extensively in the upper lithosphere. Therefore, we decided to carry out an experiment more exact than the preliminary one to examine the scattering effects by these heterogeneities with sizes comparable to the wave length.

In the experiment, the attenuation property of P waves by scattering and the spatial fluctuation characteristics of the amplitudes and the phases are investigated in detail. From the relation between them, the scattering mechanism is discussed.

2. Experimental Apparatus

A block diagram of the apparatus is shown in **Fig. 1.** The transmitter and receiver used in the experiment consist of PZT ceramics with the same resonance frequency f_0 ($f_0=140\sim150$ KHz). The output pulses of the crystal oscillator with the resonance frequency of 10 MHz are reduced to the pulses of 1 MHz, 100 KHz, 10 KHz and 20 Hz by the frequency divider. The pulses

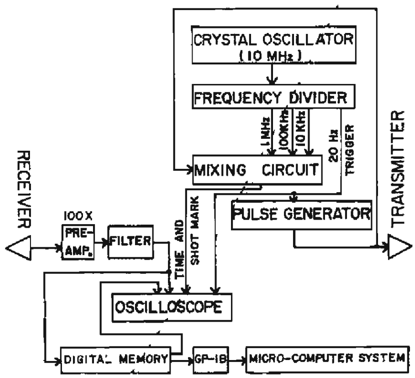

Fig. 1. Block diagram of apparatus.

of 1 MHz, 100 KHz and 10 KHz are used as a time mark and those of 20 Hz as a trigger pulse for the oscilloscope and the pulse generator. When the transmitter is excited by one output pulse of the pulse generator, it emits an ultrasonic wave into the medium of the model. The other output pulse of the pulse generator is used as a shot mark. The ultrasonic wave travels in the medium of the model, and then it is picked up by the receiver. The received signals pass through the pre-amlifier and the low-pass filter, and then they are once stored in the digital memory, whose capacity and resolution are 2 Kwords and 8 bit, respectively, at sampling intervals of 0.2 microsec. The pre-amplifier with input impedance of about 2 MΩ has a gain of 40 dB within the frequency range of 6 KHz–1 MHz. The filter has a cut-off frequency at 150 KHz and an attenuation rate of 48 dB/oct. The filtered digital signals are recorded on the cassette tape by the micro-computer, and are monitored on the oscilloscope screen and photographed by a camera. Travel time of waves is measured on the oscilloscope screen. Wave velocity is determined from the travel time curves. Standard error in estimations of wave velocity is within 1%. Standard error in measurements of wave amplitude is within 6% when the transmitter is fixed during the experiments and 9% when the transmitter is reset in every measurement, respectively. The wave analysis is carried out by a micro-computer system.

3. Models of Medium

We have previously shown[15] that the wave velocity and the density of duralumin plate decrease effectively with the increase of porosity, when the porosity of duralumin plate was given by making small circular holes at the centers and the vertexes of regularly drawn hexagons on the surface of the plate as shown in **Fig. 2.** **Fig. 3** shows the relation between the rate of decrease of wave velocity and the porosity which is equivalent to the rate of decrease of effective density. As may be seen from this figure, the porosity and the rate of decrease of wave velocity are closely correlated. We devised two two-dimensional models, (RA1 and RA2) of medium with randomly distributed heterogeneities of wave velocity and density. The duralumin plates used as the

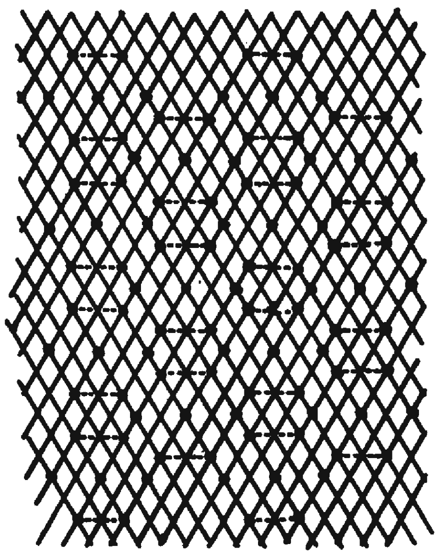

Fig. 2. Distribution of holes perforated in duralumin plates. Holes are made at positions of solid circles by drilling.

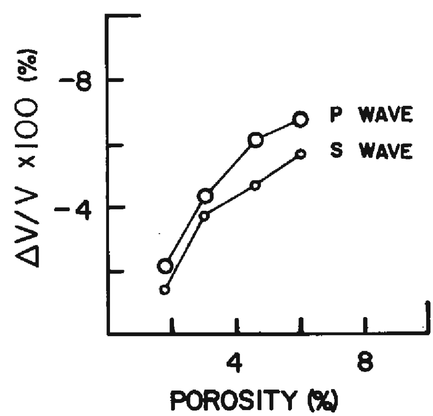

Fig. 3. Relation between rate of decrease of wave velocity, $\Delta V/V$, and porosity.

models are 80 cm in length, 60 cm in width and 0.2 cm in thickness. The method for making RA1 is as follows. First, hexagons of 1 cm in length of side are regularly drawn on the surface of duralumin plate by using a net of rhombuses of 0.5 cm in length of side. Next, as shown in **Fig. 4**, small circular holes, whose diameters are randomly varied from one hexagon to another, are made at each vertex of these hexagons and their centers. On the other hand, the diameters of holes of RA2, as also shown in **Fig. 4**, are randomly varied in each square area of 7 cm in length of side. In the case of RA1, the porosity of the unit surrounded by a circle of about 2.4 cm

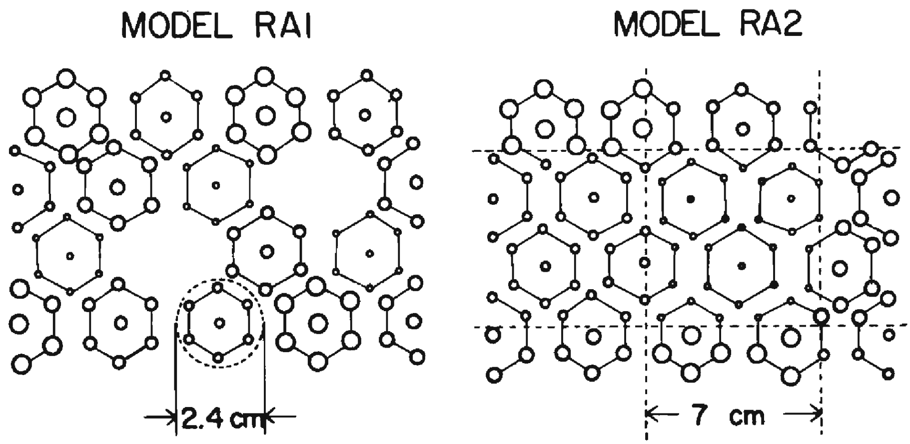

Fig. 4. Distribution of holes in model RA1 and RA2.

in diameter varies randomly from one unit to another, and accordingly the wave velocity and the effective density also vary randomly. Similarly in the case of RA2, the wave velocity and the effective density of the unit surrounded by a square of 7 cm in length of side vary randomly from one unit to another. The range of randomly distributed porosities in both models is from 0 to 6% and the median is 3%. The wave velocity of each unit of both models is determined from the relation shown in **Fig. 3**. In the cases of both models, the mean velocities of P and S waves are 5.21 mm/microsec and 3.03 mm/microsec, respectively, and also the root mean squares (rms) of fractional velocity fluctuation and fractional density fluctuation are about 2.4%. As can be seen from **Fig. 4**, there is a difference in the shape of heterogeneity between RA1 and RA2. However, it is considered that the difference of the shape of heterogeneity is equivalent to that of the dependence of the size of heterogeneity on the direction. In the case of RA1, a unit diameter 2.4 cm can be considered as the size of heterogeneity. On the other hand, in the case of RA2, the length 7 cm of side of the unit can be considered as the size for the direction of sides of the unit, and accordingly for the diagonal directions of the unit, the size is about 10 cm. Thus, in the case of RA2, the size can be considered to be in the range of 7～10 cm.

Thus, the difference between RA1 and RA2 is only the size of heterogeneity. Therefore, when we use both models under the same experimental conditions, we can examine the scattering effects by the heterogeneities, given as spatial fluctuations of wave velocity and density, in a wide range of $k_\alpha a$, where k_α and a are the wave number and the size of heterogeneity, respectively. The rms 2.4% of the velocity fluctuation of our models were devised considering the results of acoustic velocity loggings and density loggings in real media. Therefore, the experimental results are available to examine the scattering effects on short-period body waves which travel a long distance in the upper lithosphere with various heterogeneities.

4. Scattering Effects

According to Chernov,[7] when a P wave travels in a random medium, the P wave attenuates by the wave scattering due to random heterogeneities, and moreover the wave amplitude and phase fluctuate spatially by the interference between the coherent component of the wave field and its scattered incoherent one. In order to discuss the scattering mechanism, the relation between the attenuation property and the fluctuation characteristics must be examined quantitatively. Therefore, we here experimentally examine the attenuation property of P wave and its fluctuation characteristics in the cases of RA1 and RA2.

Fig. 5 shows a schematic diagram of the experiment. P waves are observed along profiles transversal to the direction of wave propagation. The transmitter, whose resonance frequency is about 140–150 KHz, is fixed in the middle of one of the shorter edges of the model. The receiver with the same resonance frequency is moved at intervals of 1 cm along the other shorter edge in which the angle of incidence is within the range of 0 to 15°. Accordingly, it can be considered that the profiles are parallel to the wave front. The models are cut successively to shorten travel distance L along the dotted lines shown in **Fig. 5**, and the measurement of the new profile is carried out. The direction of the oscillation of the observed P waves is nearly perpendicular to the profiles in this case.

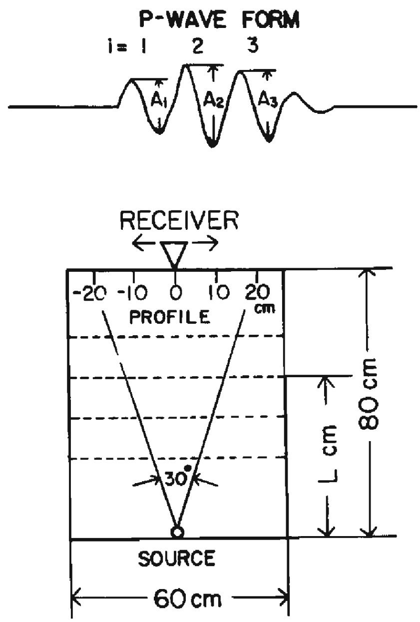

Fig. 5. Schematic diagram of measurements.

4.1. Attenuation Property

On the assumption that the amplitude $A(x)$ observed at arbitrary point x in the profile of distance L from the source is composed of the coherent component expressed by the average value $\langle A \rangle$ of the amplitudes over the profile and the incoherent one expressed by the fluctuation $\delta A(x)$, we can write the natural logarithmic amplitude $\ln A(x)$ as

$$\ln A(x) = \ln\langle A\rangle + \delta \ln A, \quad \delta \ln A \cong \delta A/\langle A\rangle, \tag{1}$$

where symbol $\langle\ \rangle$ denotes the average value in the profile. According to Chernov's theory,[7] when α_p is the total scattering coefficient estimated from the scattering loss of energy in all directions, the natural logarithmic amplitude $\ln\langle A\rangle$ of the coherent component of P wave attenuates with the increase of distance L in the following form,

$$\ln\langle A\rangle = -\alpha_p L/2. \tag{2}$$

Accordingly, the value of Q^{-1} is derived from

$$Q^{-1} = 2\alpha_p/k_\alpha. \tag{3}$$

Fig. 6 shows the attenuation of natural logarithmic amplitudes corrected for the geometrical spreading of wave front with the increase of distance L.

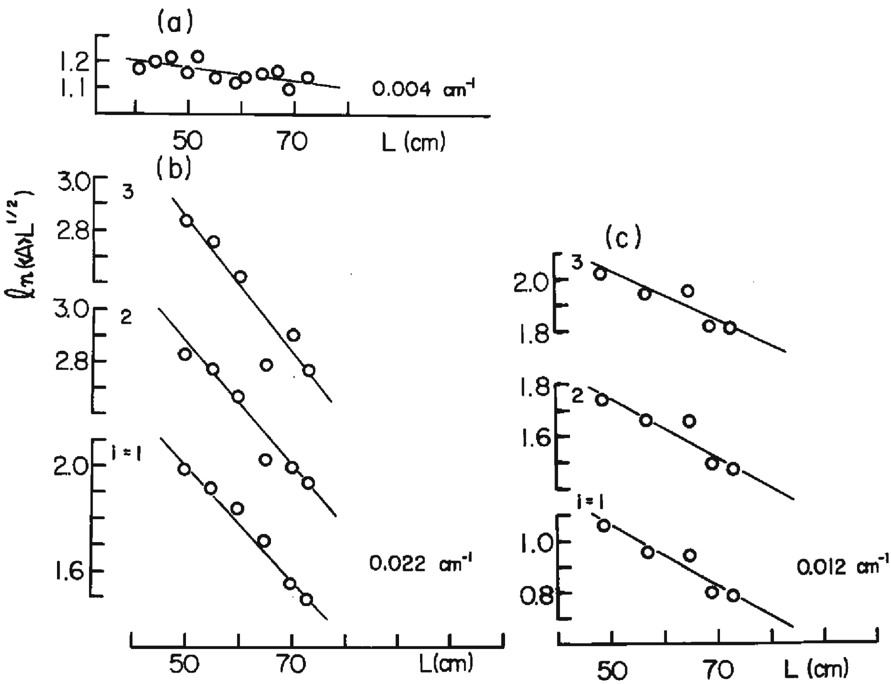

Fig. 6. Attenuation of natural logarithmic amplitudes of P-wave coherent components with distance L. Wave amplitudes were corrected for the geometrical spreading of wave front. (a): homogeneous model with no random heterogeneities, (b): RA1, (c): RA2. i denotes the ith phase of P wave. Attenuation coefficients of amplitudes were determined by the least square method.

Fig. 6-(a) shows the attenuation characteristics of the homogeneous model in which the porosity is uniformly 3%. **Fig. 6-(b)** and **6-(c)** show the attenuation characteristics of RA1 and RA2, respectively. In **Fig. 6-(b)** and **6-(c)**, i denotes the ith phase of P wave. As may be seen from this figure, every attenuation characteristics of the amplitude $A_i (i=1, 2, 3)$ is nearly equal for RA1 and RA2. Further, the attenuation of RA1 and RA2 are remarkable as compared with the homogeneous model. This is clearly the scattering effects caused by randomly distributed heterogeneities. Accordingly, we can consider the difference of the amplitude attenuation coefficient between the random models (RA1 and RA2) and the homogeneous model as the scattering coefficient. When we estimate the attenuation coefficient of amplitude by the least square method as shown in **Fig. 6**, the attenuation coefficient are 0.004 cm^{-1}, 0.022 cm^{-1} and 0.012 cm^{-1} for the homogeneous model, RA1 and RA2, respectively. Accordingly, the scattering coefficient $\alpha_p/2$ of amplitude are 0.018 cm^{-1} and 0.008 cm^{-1} for RA1 and RA2, respectively. From Eq. (3), we can estimate the value of Q^{-1} for RA1 and RA2. The predominant frequency of observed P waves is about 130~140 KHz in both models, and the mean velocity of P waves is 5.21 mm/microsec. Therefore, the wave number k_α is about 1.6 cm^{-1} for both models. Accordingly, the values of Q^{-1} are estimated to be 0.045 and 0.020 for RA1 and RA2, respectively. The standard deviations of these Q^{-1} values are about 20%. As mentioned in Sec. 3, the size a of heterogeneity is about 2.4 cm and 7~10 cm for RA1 and RA2, respectively. Therefore, the value of $k_\alpha a$ is about 4 and 11~16 for RA1 and RA2, respectively. The value of $k_\alpha a$ of RA2 is assumed to be 13.5 ± 2.5 hereafter. **Fig. 9-(B)** shows the relation between Q^{-1} and $k_\alpha a$ in this experiment. In this figure, open circles denote the value of Q^{-1}. As may be seen from this figure, Q^{-1} clearly decreases with the increase of $k_\alpha a$ from about 4 to 13.5.

On the other hand, **Fig. 9-(A)** shows the relation between Q^{-1} and $k_\alpha a$ in the preliminary experiment. Also in this figure, open circles denote the value of Q^{-1}. In the preliminary experiment, P waves were observed only

Table 1. Results of preliminary experiment. Predominant frequency of P waves is nearly equal to resonance frequency of source.

Model	Resonance Frequency of Source, f_0 in KHz.	$k_\alpha a$	Total Scattering Coefficient, α_p in $10^{-2}cm^{-1}$.	Q^{-1} in 10^{-2}.	Turbidity Coefficient, g in $10^{-4}cm^{-1}$.	Q_g^{-1} in 10^{-3}.
RA1	110	3.1	2.2±1.1	3.2±1.6	7.1± 4.2	1.0±0.6
RA1	130	4.0	3.6±1.8	4.3±2.1	6.7± 4.0	0.8±0.5
RA1	150	4.3	3.6±1.8	4.0±2.0	8.5± 5.3	0.9±0.5
RA2	110	11.2±2	0.8±0.4	1.2±0.6	9.0± 5.4	1.3±0.8
RA2	120	12.2±2	0.9±0.5	1.3±0.7	16.0±10	2.2±1.3
RA2	130	13.5±2.5	0.8±2.5	1.0±0.5	41.0±25	5.1±3.0

in two profiles with distance $L=60$ cm and $L=74$ cm from the source. Accordingly, the Q^{-1} values have large standard deviations of about 50%. However, the transmitters and receivers with the resonance frequencies of $f_0=110$, 120, 130 and 150 KHz were used for the models (RA1 and RA2) in the experiment. As may be seen from **Fig. 9-(A)**, Q^{-1} roughly decreases with the increase of $k_\alpha a$ from about 3 to 13.5. The results obtained in the preliminary experiment are also listed in **Table 1.**

4.2. Fluctuation Characteristics

Fig. 7 shows the phase fluctuation $\delta\varphi$ of P wave along the profile with distance $L=70$ cm from the source and the fluctuation $\delta \ln A_1$ of natural logarithmic amplitude between the first crest and trough of P wave composed of several crests and troughs. $\delta\varphi$ is obtained from the relation of $\delta\varphi=2\pi f\delta t$,

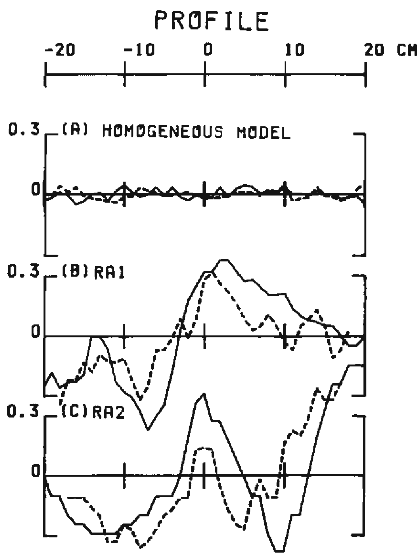

Fig. 7. Phase fluctuation $\delta\varphi$ and amplitude one $\delta \ln A_1$ along a profile with distance $L=70$ cm from the source. Solid and dotted lines denote phase and amplitude fluctuations, respectively.

where δt is the fluctuation of travel time of P wave, i.e., $\delta t=t-\langle t\rangle$. The average travel time $\langle t\rangle$ is estimated from the relation $\langle t\rangle=X/(5.21$ mm/microsec), where X is the distance between the source and receiver, and 5.21 mm/microsec is the mean velocity of P wave. **Fig. 7-(A)** shows the fluctuation characteristics of the homogeneous model in which the porosity is uniformly 3%. As can be seen from this figure, both the phase and amplitude do not remarkably fluctuate along the profile. **Fig. 7-(B)** and **7-(C)** show the fluctuation characteristics of $k_\alpha a=4$ for RA1 and those of $k_\alpha a=13.5$ for RA2, respectively. Both cases show remarkable fluctations along the profile. These are clearly the scattering effects caused by randomly distributed heterogeneities. In both cases of $k_\alpha a=4$ and $k_\alpha a=13.5$, the fluctuating behaviors of

$\delta\varphi$ and $\delta \ln A_1$ along the profile are similar.

According to Nevsky et al.[6], the mean intensity $\langle(\delta U)^2\rangle$ of the fluctuation of wave field in transversal profile can be expressed as

$$\langle(\delta U)^2\rangle = \langle(\delta \ln A)^2\rangle + \langle(\delta\varphi)^2\rangle$$
$$= \delta J/J_0 \tag{4}$$

where δJ is the energy flux of the incoherent component of wave field and J_0 that of its coherent one. They introduced the coefficient α_{cp} defined as

$$\alpha_{cp} = (\delta J/J_0)/L \tag{5}$$

and called it seismic scattering coefficient. Usually, to roughly examine the degree of heterogeneity of a medium, turbidity coefficient g is estimated from the following equation[16],

$$g = \langle(\delta \ln A)^2\rangle/L. \tag{6}$$

Therefore, the relation between α_{cp} and g can be written as

$$\alpha_{cp} = g(1+\xi), \tag{7}$$

where $\xi = \langle(\delta\varphi)^2\rangle/\langle(\delta \ln A)^2\rangle$. We here introduce Q_t^{-1} defined as

$$Q_t^{-1} = 2\alpha_{cp}/k_\alpha. \tag{8}$$

As can be seen from Eq. (5) and Eq. (8), the value of Q_t^{-1} is proportional to the rate $\delta J/J_0$, where δJ is the energy flux of the scattered waves contributing to the fluctuation of the wave field and J_0 is the energy flux of the mean wave field. On the other hand, the value of Q^{-1} is proportional to the rate of the energy flux of the total scattered waves in all directions to the energy flux of the mean wave field. Therefore, we can discuss the scattering mechanism from the relation between Q^{-1} and Q_t^{-1}. We here estimate the value of Q_t^{-1} for $k_\alpha a=4$ and $k_\alpha a=13.5$, and examine the relation between Q^{-1} and Q_t^{-1}. In order to estimate the value of Q_t^{-1}, we must know the values of ξ and g in Eq. (7). From the fluctuations of $\delta\varphi$ and $\delta \ln A_1$ shown in **Fig. 7**, we can estimate the values of ξ for $k_\alpha a=4$ and $k_\alpha a=13.5$. In the case of $k_\alpha a=4$, the variance $\sigma^2(t)$, i.e., $\langle(\delta t)^2\rangle$, of the travel time along the profile is $0.062\times(\text{microsec})^2$, and accordingly $\langle(\delta\varphi)^2\rangle$ is 0.041 from the relation of $\delta\varphi = 2\pi f\delta t$, where f is 130~140 KHz. Further, when $k_\alpha a=4$, $\langle(\delta \ln A_1)^2\rangle$ can be estimated to be 0.020, and therefore the value of ξ is estimated to be about 2 from the relation of $\xi = \langle(\delta\varphi)^2\rangle/\langle(\delta \ln A_1)^2\rangle$. Similarly in the case of $k_\alpha a=13.5$, $\sigma^2(t)$ is $0.089\times(\text{microsec})^2$, and accordingly $\langle(\delta\varphi)^2\rangle$ is 0.059. Further, when $k_\alpha a=13.5$, $\langle(\delta \ln A_1)^2\rangle$ can be estimated to be about 0.048, and therefore the value of ξ is estimated to be about 1.2. However, these values

(2 and 1.2) of ξ have standard deviations of about 45~50%, because the intensities of fluctuations of phase and amplitude, i.e., $\langle(\delta\varphi)^2\rangle$ and $\langle(\delta \ln A_1)^2\rangle$, have standard deviations of about 20%.

In a previous paper,[15] we reported that when a P wave is composed of several crests and troughs as shown in **Fig. 5**, the distortions of the wave form with the increasing travel distance are first generated in the amplitude of the latest phase (composed of the latest crest and trough) by the wave scattering, and shift to the earlier phase, and finally the entire form is distorted. And we showed that this scattering effect could be explained by the characteristic that the growth rate of the variance $\sigma^2(\ln A)$, i.e., $\langle(\delta \ln A)^2\rangle$, per unit travel distance becomes larger for the later phase. Therefore, before determining the turbidity coefficient, we examine the growth rate of $\sigma^2(\ln A)$ per unit travel distance. **Fig. 8** shows the relations between the variance $\sigma^2(\ln A)$ ($\langle(\delta \ln A)^2\rangle$) and the distance L from the source to the profiles for $k_\alpha a=4$ (RA1) and $k_\alpha a=13.5$ (RA2). In this figure, $g_i\,(i=1, 2, 3, 4,)$ denotes the growth rate of $\sigma^2(\ln A_i)$ per unit travel distance for the amplitude A_i of the ith phase of P wave. As may be seen from this figure, when $k_\alpha a=4$, g_i ($i>1$) becomes larger for the later phase, but when $k_\alpha a=13.5$, every g_i is nearly of the same order. Furthermore, every g_i of $k_\alpha a=13.5$ is remarkably large when compared with that of $k_\alpha a=4$. Thus, it is found that the ratio

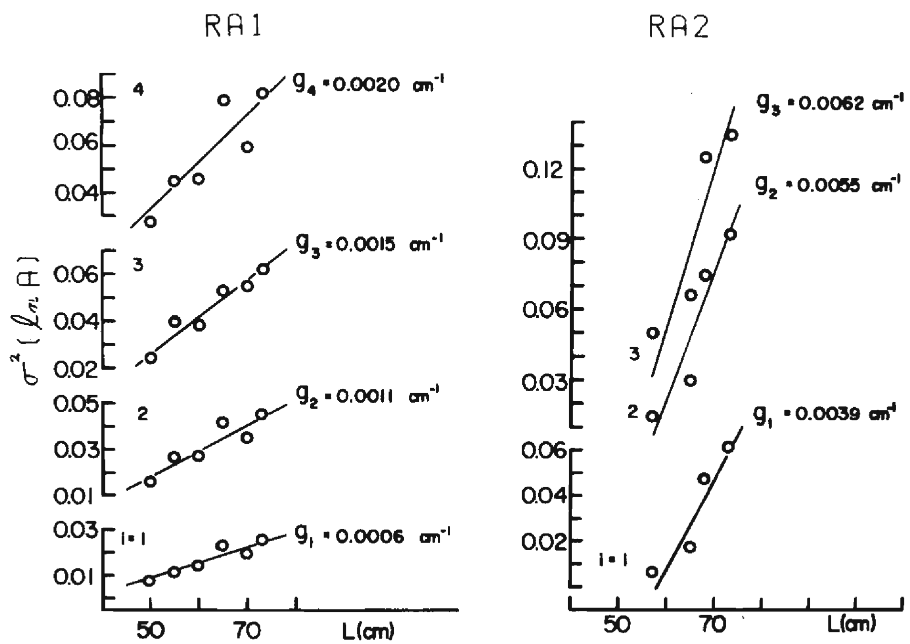

Fig. 8. Relation between variance $\sigma^2(\ln A)$ and distance L. Values of $g_i\,(i=1, 2, 3, 4)$ were obtained by the least square method. i denotes the ith phase of a P wave.

of g_i/g_1 and the value of g_i for $k_\alpha a=4$ are different from those of $k_\alpha a=13.5$. The characteristic of the ratio g_i/g_1 $(i=1, 2, 3, 4)$ for $k_\alpha a=4$, i.e., the relation of $1<g_2/g_1<g_3/g_1<g_4/g_1$, can be explained by the following scattering mechanism. When the scattering is nearly isotropic, the amplitude fluctuation of the first phase of a P wave is generated by its own phase scattered forward, and the amplitude fluctuation of its later phase is generated by its own phase scattered forward and by the preceding phases scattered sidewards. In these mechanism, it is predicted that there is the relation of $1<g_2/g_1<g_3/g_1<g_4/g_1$ among the growth rates g_i $(i=1, 2, 3, 4)$. On the other hand, the characteristic of the ratio g_i/g_1 $(i=1, 2, 3)$ for $k_\alpha a=13.5$, i.e., the relation of $g_1 \cong g_2 \cong g_3$, can be explained by the following scattering mechanism. When the directivity of the scattering is strong near the direction of wave propagation, the amplitude fluctuation of each phase of a P wave is generated only by its own phase scattered forward. Therefore, in this mechanism, it is predicted that there is the relation of $g_1 \cong g_2 \cong g_3$ among the growth rates g_i $(i=1, 2, 3)$. Thus, from the difference of the characteristic of the ratio g_i/g_1 $(i>1)$ between $k_\alpha a=4$ and $k_\alpha a=13.5$, it is suggested that the directivity of the scattering near the direction of wave propagation is much stronger for $k_\alpha a=13.5$ than for $k_\alpha a=4$. And also from the difference of the value of g_i $(i\geqq 1)$ between $k_\alpha a=4$ and $k_\alpha a=13.5$, it is suggested that the rate of the scattered energy contributing to the amplitude fluctuation to the energy of its coherent component is larger for $k_\alpha a=13.5$ than for $k_\alpha a=4$.

According to Nikolayev[16], the growth rate g_1 for the amplitude A_1 of the first phase of P wave is equivalent to the turbidity coefficient g. Accordingly, as shown in **Fig. 8**, g is 0.0006 cm^{-1} and 0.0039 cm^{-1} for $k_\alpha a=4$ and $k_\alpha a=13.5$, respectively. By using Eq. (7) and Eq. (8), the value of Q_t^{-1} can be estimated to be 0.0023 and 0.009 for $k_\alpha a=4$ and $k_\alpha a=13.5$, respectively. **Fig. 9-(B)** shows these values of Q_t^{-1} and the values of Q^{-1} estimated in Sec. 4.1. In this figure, solid circles denote Q_t^{-1}. As may be seen from this figure. Q_t^{-1} clearly increases with the increase of $k_\alpha a$ from 4 to 13.5.

On the other hand, in the preliminary experiment, the variances $\sigma^2(\ln A_1)$ for the profiles with distance $L=60$ cm and $L=74$ cm from the source were obtained, but the variance of travel time was not measured. Accordingly, we could not estimate the ξ value from the relation of $\xi=\sigma^2(2\pi ft)/\sigma^2(\ln A_1)$, and therefore we could not estimate the Q_t^{-1} value. In order to roughly examine the dependence of Q_t^{-1} on $k_\alpha a$. we here introduce the value of Q_g^{-1} defined as

$$Q_g^{-1}=2g/k_\alpha. \tag{9}$$

As can be seen from Eq. (4), (6) and (9), Q_g^{-1} is proportional to the rate of the scattered energy contributing to the amplitude fluctuation to the energy

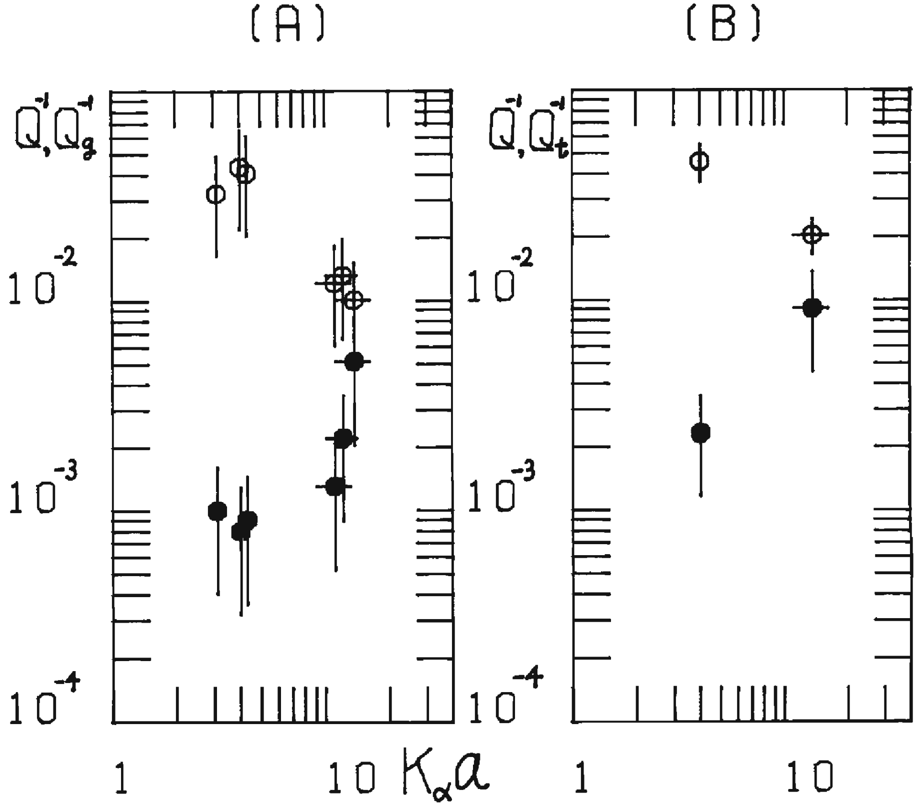

Fig. 9. (A): Dependences of Q^{-1} and Q_g^{-1} on $k_\alpha a$ in the preliminary experiment. Open circles and solid ones denote Q^{-1} and Q_g^{-1}, respectively. (B): Dependences of Q^{-1} and Q_t^{-1} on $k_\alpha a$ in this experiment. Open circles and solid ones denote Q^{-1} and Q_t^{-1}, respectively.

of the mean wave field only. From Eq. (7), (8) and (9), the relation between Q_t^{-1} and Q_g^{-1} can be written as

$$Q_t^{-1} = Q_g^{-1}\ (1+\xi). \tag{10}$$

Thus, though we can not estimate the absolute value of Q_t^{-1}, we can roughly know the dependence of Q_t^{-1} on $k_\alpha a$ from the relation between Q_g^{-1} and $k_\alpha a$. The obtained Q_g^{-1} values in the preliminary experiment are listed in **Table 1.** And also **Fig. 9-(A)** shows the relation between Q_g^{-1} and $k_\alpha a$. In this figure, solid circles denote Q_g^{-1}. As may be seen in **Fig. 9-(A)**, the Q_g^{-1} values have large standard deviations of about 60%, but Q_g^{-1} increases with the increase of $k_\alpha a$. Thus, from the relation between Q_t^{-1} and $k_\alpha a$ shown in **Fig. 9-(B)** and from the relation between Q_g^{-1} and $k_\alpha a$ shown in **Fig. 9-(A)**, it is considered that Q_t^{-1} increases with the increase of $k_\alpha a$ from 4 to 13.5. This suggests that the forward scattering becomes stronger with the increase of $k_\alpha a$. In the next section, we will discuss P-wave scattering mechanism from the relation between Q_t^{-1} and the Q^{-1} obtained in Sec. 4.1.

5. A Consideration of Scattering Mechanism

The value of Q^{-1} was estimated from the amplitude attenuation of the coherent component of wave field. Accordingly, Q^{-1} is proportional to the rate of the total scattered energy to the energy of the coherent component of wave field. On the other hand, Q_t^{-1} is proportional to the rate of the scattered energy contributing to the fluctuation of the wave field to the energy of its coherent component. As shown in **Fig. 9-(B)**, Q^{-1} clearly decreases with the increase of $k_\alpha a$ from 4 to 13.5, but Q_t^{-1} clearly increases inversely to the decrease of Q^{-1}. When $k_\alpha a=4$, the value of Q^{-1} is more than ten times the value of Q_t^{-1}, but when $k_\alpha a=13.5$, both values are nearly of the same order. This indicates that the scattered energy contributing to the fluctuation of the wave field when $k_\alpha a=4$ is less than 10^{-1} of the total scattered energy, but when $k_\alpha a=13.5$, the total scattered energy contributes to the fluctuation.

According to Chernov's theory[7] for a strongly fluctuating wave field in the range of $k_\alpha a \gg 1$ where the Fresnel approximation is valid, the value of Q^{-1} is equal to the value of Q_t^{-1} and increases in proportion to $(k_\alpha a)^1$. When the value of $k_\alpha a$ approaches 13.5 from 4 in our experiment, the obtained Q^{-1} and Q_t^{-1} values are nearly of the same order. Therefore, the value 13.5 of $k_\alpha a$ is within the range of $k_\alpha a$ where the Fresnel approximation is valid. This is also supported by our experimental result that every g_t is nearly of the same order for the sharp directivity of the scattering near the direction of wave propagation when $k_\alpha a=13.5$.

6. Conclusion

We experimentally examined the scattering effects on P wave, when a P wave travels a long distance in the medium with randomly distributed velocity and density heterogeneities whose sizes are comparable to the wave length. Though the experiment was carried out only for two models, it is considered that the experimental results are suitable for the examination of the scattering effects on short-period body waves. The main experimental results are as follows.

(1) When a P wave travels in two-dimensional models of random media in which the velocity fluctuation nearly correlates with the effective density fluctuation, the value of Q^{-1}, which shows wave attenuation by the wave

scattering due to the heterogeneities, decreases with increase of $k_\alpha a$ from 4 to 13.5.

(2) When a P wave is composed of several crests and troughs, the value of $g_i(i>1)$, which is the growth rate of the variance $\sigma^2(\ln A_i)$ for the amplitude A_i between the ith crest and trough per unit travel distance, becomes larger for the amplitude between its later crest and trough when $k_\alpha a=4$, but every g_i is nearly of the same order when $k_\alpha a=13.5$. Furthermore, every g_i for $k_\alpha a=13.5$ is remarkably large when compared with that of $k_\alpha a=4$.

(3) When the value of Q_t^{-1} of P wave is defined as $2\alpha_{cp}/k_\alpha$, where α_{cp} is the seismic scattering coefficient, Q^{-1} for $k_\alpha a=4$ is more than ten times of Q_t^{-1}, but for $k_\alpha a=13.5$, the Q^{-1} and Q_t^{-1} values are nearly of the same order. This indicates that the order of about 10^{-1} of the total scattered energy contributes to the fluctuation of the wave field when $k_\alpha a=4$, but all of the scattered energy contributes to the fluctuation when $k_\alpha a=13.5$.

In future, the detailed behavior of Q^{-1} with the increase of $k_\alpha a$ from 0.6 to 10 and also above 10 must be examined experimentally to make clear the attenuation property of short-period body waves by wave scattering.

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