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Design of Concrete Structures for Fatigue Reliability

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Abstract

Four different limit states formats for the fatigue design of reinforced concrete and prestressed concrete structures are described and compared within the context of simple second level reliability theory. All four formats lend themselves to reliability treatment but the cycle format appears to be the most useful at the present time, given the limited quantity of design data available on the fatigue properties of the component materials. The problem of choosing appropriate reliability levels for fatigue design is also studied. A comparative calibration method is used to identify a range of acceptable values for the nominal probability of fatigue failure and the associated reliability index β. A design procedure for fatigue is finally proposed which incorporates the results of the present study.

1. Introduction

Results of experimental and theoretical studies of the fatigue properties of plain concrete, reinforcing bars, prestressing tendons, and also of structural concrete members, have been accumulating steadily over the past thirty years[5),6),9),15),16)]. However, the results of this research can be usefully employed in practice only when appropriate fatigue design formats have been developed which provide adequate, but not excessive, margins of safety against fatigue failure. The safety margins and the associated design safety coefficients must allow realistically for inherent variabilities in loads, in the fatigue properties of the materials, and in the performance of members and systems under repeated loading.

In the past, fatigue failure has been primarily regarded as a design problem associated with civil engineering structures such as highway and rail bridges which are subjected to reasonably well defined man-induced repeated loads. However, serious fatigue problems can also arise as the result of complex, naturally occurring, ill-defined patterns of repeated loads. For example, fatigue problems in concrete structures have arisen recently in relation to off-shore and marine construction.

2. Design formats for fatigue

Design procedures for the strength limit states involve the comparison of a design load intensity term S^* with a design resistance R^* of the structure or its component members or materials[14)]. The four fatigue design formats to be discussed here all follow this approach. They differ from each other with regard to the design variable which is used to compare the load intensity with the design resistance. No fundamentally different assumptions are made regarding the underlying processes of deterioration of the materials or of the final mechanism of failure. In the following, the essential elements of each format are first described by reference to a simple tension element subjected to constant cycles of repeated load. The extension to more complex load cycles and to reinforced concrete and prestressed concrete members is in most cases obvious.

2.1 Stress format

Stress level is used as the prime variable in this format. In **Fig. 1** the case of a reinforcing bar, subjected to well defined cycles of load with nominally constant maximum and minimum levels, is considered. **Fig. 1(a)** shows the S–N–P (stress S, number of cycles N, probability of failure P) relation for a constant minimum stress level, as would be obtained from a laboratory test program. In this case S is the maximum stress in the fatigue cycle and will be referred to in this discussion of limit states as f_R. In **Fig. 1(b)** the frequency distributions for two random variables, f_R and σ_S, are constructed at a specific design life N_S^*. In this simple case of constant repeated load cycles the distribution of f_R is obtained directly from the S–N–P relation in **Fig. 1(a)**. It will be noted that fatigue test data are plotted with a logarithmic scale for N, and either a natural or logarithmic scale for S, whichever gives the best fit of data. A normal frequency distribution is therefore obtained either for f_R or log f_R. The second quantity, σ_S, is the maximum load-induced stress and is also a random variable because the upper load level is subject to random variation.

The characteristic strength f_{Rk} is the particular value of f_R for which the probability of fatigue failure, at or before N_S^* cycles, is a predetermined value P_k. The characteristic load term, σ_{Sk}, likewise corresponds to the value of the maximum repeated load which has a probability $(1-P_k)$ of occurring. The design fatigue strength, f_R^*, is obtained by applying a partial safety coefficient, $\gamma_{R\sigma}$, to the characteristic strength:

$$f_R^* = \frac{1}{\gamma_{R\sigma}} f_{Rk} \tag{1}$$

The load-induced design stress is obtained in a similar manner:

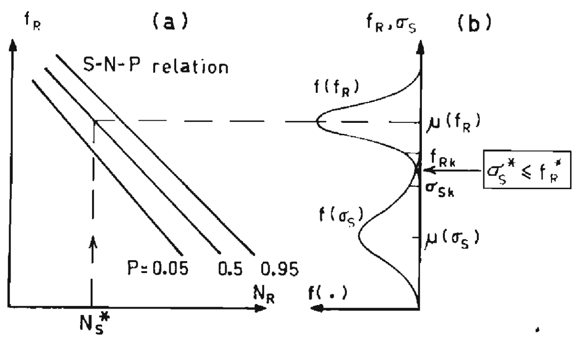


Fig. 1 Stress format

$$\sigma_S^* = \gamma_{S\sigma}\sigma_{Sk} \tag{2}$$

The design requirement for fatigue is then expressed as:

$$f_R^* \geqslant \sigma_S^* \tag{3}$$

The overall level of safety in the design procedure is controlled by means of the safety coefficients $\gamma_{R\sigma}$ and $\gamma_{S\sigma}$, together with the value chosen for P_k. In most limit states formulations, the 0.05 probability level is used for P_k to define characteristic values.

The stress format has been adopted for fatigue design in the Tentative Recommendations for the Limit States Design of Concrete Structures of the Japan Society of Civil Engineers[1], with $\gamma_{S\sigma}=1.0$ and $\gamma_{R\sigma}$ equal to the partial safety coefficient for the corresponding static strength limit state. Thus, for fatigue failure of a reinforcing bar in a flexural member, $\gamma_{R\sigma}$ would take a value between unity and 1.15. Although test data will always provide the most reliable means for evaluating f_{Rk} as a function of N_S^*, the following expressions are suggested in the Tentative Recommendations as safe, conservative design values (presumably for $P_k=0.05$):

$$\text{for } \log N_S^* < 6: \quad f_{Rk} = \left(160 - \frac{\sigma_{min}}{3}\right) 10^{-0.2(\log N_S^* - 6)} \tag{4}$$

$$\text{for } \log N_S^* > 6: \quad f_{Rk} = \left(160 - \frac{\sigma_{min}}{3}\right) 10^{-0.1(\log N_S^* - 6)} \tag{5}$$

In the above expressions, stresses are in MPa. In situations where variable repeated load produces a cumulative damage situation, the Tentative Recommendations allow for an equivalent constant load cycle design stress to be determined by means of the Palmgren-Miner linear damage law, which is then used in the design requirement as expressed in Eq. 3 above.

This design procedure lends itself to the simplified second level reliability analysis commonly used in limit states codification[14]. If the frequency distributions for f_R and σ_S are both assumed to be normal, then a margin of safety Z, defined as

$$Z = f_R - \sigma_S \tag{6}$$

is also normally distributed with

$$\mu(Z) = \mu(f_R) - \mu(\sigma_S) \tag{7}$$

$$\sigma^2(Z) = \sigma^2(f_R) + \sigma^2(\sigma_S) \tag{8}$$

where $\mu(.)$ and $\sigma(.)$ represent population means and variances, respectively. The nominal probability of failure

$$P_f = P[Z<0] \tag{9}$$

and the reliability

$$R = 1 - P_f \tag{10}$$

can be determined from the properties of Z. The reliability index β, defined as

$$\beta=\frac{\mu(Z)}{\sigma(Z)}=\frac{\mu(f_R)-\mu(\sigma_S)}{\sqrt{\sigma^2(f_R)+\sigma^2(\sigma_S)}} \tag{11}$$

is the distance between $Z=0$ and the mean $\mu(Z)$, measured in units of standard deviation, and is related to P_f, and hence *R*, by the properties of the normal distribution. Some typical values are contained in **Table 1**.

Table 1 Safety index and probability of failure.

P_f	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}
β	2.3	3.1	3.7	4.3	4.7	5.2

If the fatigue data for the S–N–P relation correlate best on log-log axes, then log-normal frequency distributions become appropriate for f_R and σ_S and the safety margin can be defined as follows:

$$Z=\log f_R-\log \sigma_S \tag{12}$$

The reliability index becomes

$$\beta=\frac{\mu(\log f_R)-\mu(\log \sigma_S)}{\sqrt{\sigma^2(\log f_R)+\sigma^2(\log \sigma_S)}} \tag{13}$$

To obtain quantitative estimates of β and hence P_f and *R*, which correspond to a specific design proposal, such as that in the Tentative Recommendations, it is necessary to use estimates of means and standard deviations of f_R and σ_S for a range of design lives, N_S^*. Approximate calculations are made in Appendix A of this paper to evaluate safety and reliability levels for a range of values of the safety coefficients as used in the JSCE Recommendations.

2.2 Cycle format

The number of cycles to failure, N, can be used instead of stress level as the variable for comparing load intensity with design resistance. For the simple case of constant repeated load cycles, the frequency distribution of the fatigue life at a chosen maximum design stress $\sigma_S^*=f_R$ is constructed from the S–N–P relation as shown in **Fig. 2**. A characteristic fatigue life N_{Rk} is chosen for a specific fatigue failure probability,

$$P[N_R<N_{Rk}]=P_k \tag{14}$$

and a design fatigue life N_R^* is defined by introducing a safety coefficient γ_{RN}:

$$N_R^*=\frac{1}{\gamma_{RN}}N_{Rk} \tag{15}$$

It has been suggested by Grundy[2] that design life should be treated as a random variable, rather than as a design constant. This can be accomplished in the cycle format by replacing the number N_S by a random variable N_S which has a frequency

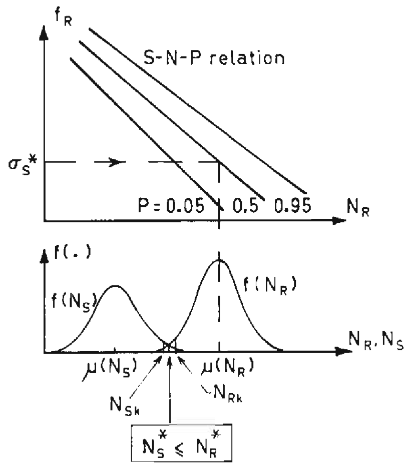


Fig. 2 Cycle format

distribution $f(N_S)$, with characteristic value N_{Sk}, and design value

$$N_S^* = \gamma_{SN} N_{Sk} \tag{16}$$

If design is to be based on a specific design life, then $f(N_S)$ collapses to a spike with value N_S^*. The design requirement is

$$N_S^* \leqslant N_R^* \tag{17}$$

As N is plotted on a logarithmic scale in the S–N–P relation the safety margin is defined as

$$Z = \log N_R - \log N_S \tag{18}$$

and the safety index becomes

$$\beta = \frac{\mu(\log N_R) - \mu(\log N_S)}{\sqrt{\sigma^2(\log N_R) + \sigma^2(\log N_S)}} \tag{19}$$

The cycle format can also be extended to allow the maximum stress level to be treated as a random variable. Assuming a linear equation to relate fatigue life to stress level,

$$\log N_R = a - b \log f_R \tag{20}$$

where a and b are empirical best fit constants for the S–N–P relation, then as safety margin we obtain from Eqs. 18 and 20,

$$Z = a - b \log f_R - \log N_S \tag{21}$$

We also have

$$\mu(Z) = a - b\ \mu(\log f_R) - \mu(\log N_S) \tag{22}$$

$$\sigma^2(Z)=b^2\,\sigma^2(\log f_R)+\sigma^2(\log N_S) \\ +br_{fN}\,\sigma(\log f_R)\,\sigma(\log N_S) \tag{23}$$

in which r_{fN} is the correlation coefficient between $\log f_R$ and $\log N_S$[14]. The safety index is then evaluated as before from Eq. 11.

With design stress now treated also as a random variable it is appropriate to introduce a characteristic value σ_{Sk}, such that

$$P[\sigma_S<\sigma_{Sk}]=P_k \tag{24}$$

say with $P_k=0.95$. The design stress level, σ_S^*, is then defined by means of a partial safety coefficient,

$$\sigma_S^*=\gamma_\sigma\sigma_{Sk} \tag{25}$$

However, given the inadequacies in available data on material properties in fatigue and structural performance under repeated loading, added to the usual lack of design data on magnitude of repeated load cycles, the use of three separate safety coefficients would in most cases be unwarranted, so that a value of unity would usually be appropriate for γ_σ. It should be noted that a linearised S–N relation also allows the stress format of Section 2.1 to be extended to treat N_S^* as a random variable. The development parallels the above.

2.3 Damage accumulation format

Damage accumulation theories are used to predict fatigue life in situations where the repeated load cycles vary in magnitude. Usually, such theories relate the damage caused by a load cycle or a group of load cycles back to the S–N–P relations which are obtained from constant cycle fatigue tests on materials. Some form of cumulative damage treatment must of course be adopted in both the stress and cycle format procedures in order to handle variable repeated loads. However, damage can also be considered to have physical significance. Typically, the assumption is made that each stress increment produces in a fatigue-prone material an increment of damage ΔD, which is dependent on the magnitude of the stress increment and possibly on other quantities such as mean stress level. The increments are summed to produce a quantity of damage D which increases monotonically with the number of cycles applied. Fatigue failure is then assumed to occur when a predefined damage limit is reached. To account for inherent variability in the damage process, the damage increments and hence the total damage D can be treated as random variables. The limit of damage at which fracture occurs can be treated either as a fixed, absolute quantity (e.g. 100 per cent) or as a random variable which reflects the inherent variability in the capacity of apparently similar specimens to resist fatigue. Although various mathematical damage accumulation models have been proposed for specific applications (see, for example Refs. 11 and 12), we shall here be concerned only with general concepts.

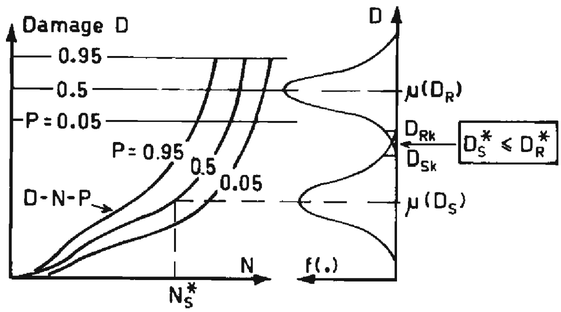
In **Fig. 3(a)** the accumulated load-induced damage D_S is plotted against number of cycles N and probability level P. The damage limit D_R, representing fatigue failure, is also shown as a random variable, which is independent of N. For a specific design life N_S^*, frequency distributions $f(D_S)$ and $f(D_R)$ can be constructed for the load-induced accumulating damage and the limit of damage resistance. By introducing characteristic values D_{Sk} and D_{Rk} and design values:

$$D_S^* = \gamma_{SD} D_{Sk} \tag{26a}$$

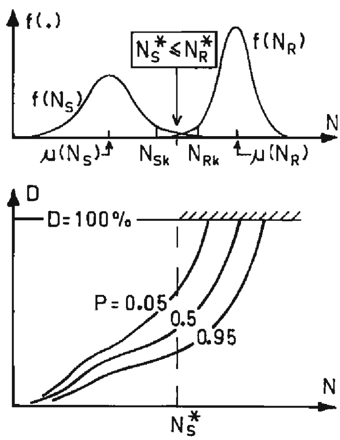
$$D_R^* = \frac{1}{\gamma_{RD}} D_{Rk} \tag{26b}$$

a standard limit states format for fatigue design can be obtained which is analogous to the stress format discussed earlier and expressed in Eq. 3.

In **Fig. 3(b)** the damage limit is treated as a fixed quantity which represents complete (100 per cent) damage. The frequency distribution for the number of cycles required to produce failure, $f(N_R)$, can be constructed and compared with the



(a) Damage format, damage limit D_R variable



(b) Damage format with fracture defined as D = 100%

Fig. 3 Damage accumulation format

design life N_S^*. This formulation is then analogous to the cycle format discussed in Section 2.2.

In both **Figs. 3(a)** and **3(b)** the damage functions are shown as increasing monotonically from an initial value of zero at the commencement of fatigue loading. The damage accumulation format can also be applied in situations where initial damage exists in the material, or the material goes through a crack initiation process followed by crack development, or even to situations in which damage alleviation or damage accentuation take place as the result of special processing of the material.

2.4 Degradation of static strength

Fatigue failure finally occurs in a structural element when the static load capacity, degraded by the development of fatigue cracks and possibly by other adverse effects, falls to the level of the maximum applied load. The situation for a particular concrete structure (**Fig. 4(a)**), usually involves an initial increase, followed by a progressive decrease in static load capacity as the result of various factors such as the development of fatigue cracks under repeated loads, corrosion of reinforcement, deterioration of the concrete and possibly even creep under high sustained load.

In a concrete flexural member, fatigue failure will almost certainly be due to fracture of the steel tensile elements rather than failure of the compressive concrete. Final static collapse occurs after a relatively large proportion of the tensile steel in a critical section has fractured in fatigue. The proportion may be 0.5 or even higher.

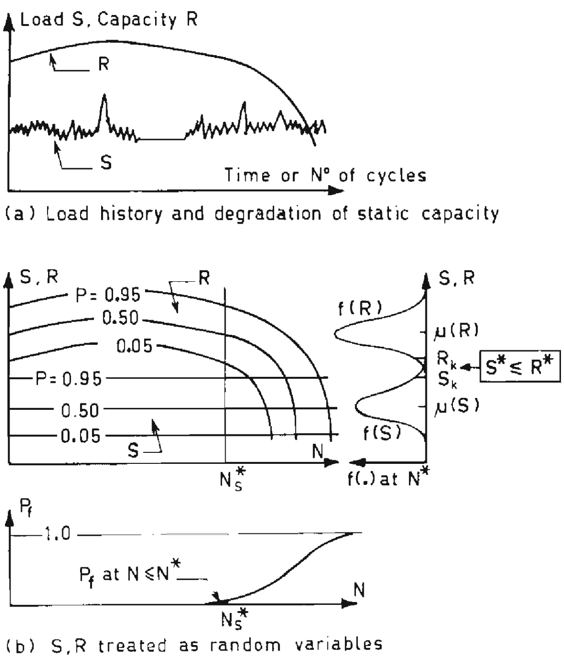


Fig. 4 Format based on degradation of static strength

If the steel is present as a large number of individual bars or tendons, there is a progressive loss of static strength over a relatively large number of load cycles. Indeed the first fracture of a bar or strand has an almost imperceptible effect on structural performance, and in itself is not a significant event from the point of view of design. The use of initial bar fracture for fatigue design stems from the fact that it is the event which lends itself most readily to analysis.

Nevertheless, it sometimes becomes necessary to evaluate the number of cycles to final fatigue collapse, rather than to initial bar fracture, as for example when an estimate is made of the remaining fatigue life of a damaged in-service structure. In such cases it is necessary for the progressive degradation of static strength to be analysed.

The static strength of a particular element after a prior history of loading is a definite quantity which could in principle be determined by test. However, for design purposes it is necessary to treat this quantity as a random variable, so that at a specific design life N_S^* the load capacity and the applied load may be represented in the usual limit states format as shown in **Fig. 4(b)**. This suggests a simple design formulation with design load S^* and design resistance R^* defined in the usual way and used as follows:

$$R^* \geqslant S^* \tag{27}$$

Fig. 4(b) is to some extent misleading because the resistance of the structure depends on the previous load history and may therefore be correlated with the applied load at failure. Furthermore, the structural calculations required to determine strength degradation can become complex. Although there are formidable difficulties in applying the static strength degradation concept in some specific cases, it is the most general of the design formats, and perhaps the most meaningful, physically. It is the most appropriate format for situations involving strength degradation due to mixed causes and for complex situations involving structural repairs carried out during the useful life of the structure.

The concept of degradation of static strength has also been used to handle the problem of creep buckling of concrete columns[16], the static load capacity in this case being progressively reduced by the outward deflection of the column as the result of inelastic, time-dependent strain in the compressive concrete.

2.5 Discussion

All four formats lend themselves to simple second level reliability analysis and in this respect provide a suitable basis for a limit states design procedure for fatigue. However, given the limited quantity and the type of data likely to be available in any specific design situation on the fatigue properties of reinforcing bar and prestressing strand, the stress and cycle formats appear to be potentially the most useful for the development of simple, practical design procedures at the present time.

The damage and static strength degradation approaches nevertheless provide more general and potentially more widely applicable bases for design. Upon careful study, the concept of damage accumulation usually turns out to be a convenient mathematical concept rather than a clearly demonstrable physical process; nevertheless, this format appears to be potentially more versatile than either the stress or cycle formats, particularly for the treatment of random loadings. The static strength degradation format is physically meaningful, and may prove to be the most useful format for treating special design problems such as remaining fatigue life.

In simple design situations the stress and cycle formats are essentially equivalent. Provided the basic S–N–P design data can be linearised, so that the design variables can be assumed to have normal distributions, variabilities in both design load and design life can be handled in a rational manner. It should however be noted that the stress and cycle formats require a different treatment of the S–N–P data, with regression of S on N and N on S, respectively.

In more complex situations, there may be some advantage in using the cycle format. For example, the cycle format lends itself readily to the treatment of system problems in which fatigue failure may take place in several components. In a fatigue-prone cross section which contains p reinforcing bars and q prestressing tendons, where the probability of fatigue fracture at or before N_S^* cycles is P_p for any bar and P_q for any tendon, then the probability of a fatigue fracture in the system can be easily calculated:

$$P\ (\text{System fatigue}) = 1-(1-P_p)^p\ (1-P_q)^q \tag{28}$$

Simple system analyses such as the above can usually by incorporated easily into design procedures which are based on the cycle format. The analysis can become far more complicated if the stress format is used.

The cycle format can also be easily extended to treat varied cycles of loading when a damage accumulation law is required which is more complex than the linear law. In such situations the stress format has to rely on the definition of an equivalent

Table 2 β values for RC beams in flexure (Ref. 8)

Existing Code		Proposed Code		r
Min Value	Max Value	Min Value	Max Value	
3.37	3.85	3.37	4.25	0
3.60	4.25	3.20	4.25	0.25
4.08	5.08	3.70	4.25	0.50
4.00	4.39	3.89	4.08	0.75
3.93	3.93	3.88	3.88	1.00

r = W/(D+L+W)
W = wind load
D = dead load
L = live load

stress to represent non-linear cumulative damage effects.

Although the differences between the stress and cycle formats are not significant in the treatment of simple design cases, there appears to be sufficient reason to prefer the cycle format in Section 4 of this paper, when a practical design procedure for fatigue is formulated.

3. Target reliability levels for fatigue design

Before values for the quantities R, P_f and β are considered for use in fatigue design, it needs to be emphasised that the relative frequency interpretation cannot strictly be applied to the nominal probability of failure P_f. It is more correct to regard P_f and β as convenient but arbitrary relative measures of safety which have little or no physical meaning. Thus, they merely allow comparisons to be made of the relative safety of alternative designs, and do not indicate absolute levels of safety or reliability. Indeed, P_f and β are sometimes interpreted as subjective Bayesean probabilities which indicate a degree of confidence or even belief in the adequacy (safety) of a structural design[14].

An attempt is now made to identify a range of values of P_f for use in fatigue design by making comparisons between the consequences of failure in fatigue and flexure. In this way, the target reliability levels for fatigue failure are related to known (but arbitrary) levels currently in use for flexural design. Even when flexure failure is treated as a calibration standard in this way, the procedure involves guesswork and judgement. This is unavoidable, given the nature of the problem. The implied assumption is that the safety levels in use for flexural design are appropriate. The only justification that can be offered for this assumption is that current flexural design procedures, including partial safety coefficients and implied safety and reliability levels, are based on satisfactory design experience and a low level of failure in the construction industry.

It is necessary to obtain some initial values for reliability levels for current flexural design procedures. In Ref. 8, Leicester has evaluated the safety index β for reinforced concrete beams in bending which have been designed according to the current Australian codes for structural design. Typical values for reinforced concrete beams are listed in **Table 2**. It can be seen from **Tables 1** and **2** that the value of β varies from a minimum of 3.37 to a maximum of 4.08, corresponding to nominal probabilities of failure of between 5×10^{-4} and 10^{-5}. It should be emphasised that these ranges do not derive from extreme, unrealistic design situations.

Differences in the value of β as used in various design calculations (for example for flexure, shear, fatigue, etc) should reflect the corresponding variations in the consequences of failure. In particular, factors such as the type of failure (whether sudden or gradual), the losses due to failure (loss of property, loss of life), cost of replacement or repair, ease of detection of both deterioration and the condition of incipient failure, all need to be considered in the choice of reliability levels and hence in the evaluation of safety coefficients. In principle, the choice of target reliability levels is an optimisation problem, but unfortunately it is one which is too difficult to solve by any other than primitive trial and error means.

The type of failure and the consequences of failure are usually affected by whether the structure is determinate or indeterminate. This is unfortunately rarely recognised in codes of practice. Flexural failure in a section of a statically determinate member is likely to result in complete collapse, with considerable loss of property and, depending on the use of the structure, even loss of life. Collapse is a very serious matter, with total costs likely to be far in excess of the straight cost of replacement of the structure. The same applies to failure of a key element in an indeterminate frame such as a ground floor column or a transfer beam or truss. In contrast, the consequences of flexural fatigue failure in a determinate member are far less serious. The reason for this is that predicted fatigue failure, as determined in the design calculations, corresponds to the fracture of a single bar or wire. This event does not result in collapse; it is followed by progressive fracture of additional steel elements over many additional cycles of load, provided that, as is usually the case, the tensile steel is contained in a large number of separate elements. Once fatigue fracture has occurred either in a bar or prestressing tendon, repair can be a difficult and costly process, so that the cost of fatigue failure is likely to be comparable with full replacement costs.

Flexural failure in a statically indeterminate structure does not imply complete collapse, but rather yielding and hinge formation in a localised region of high moment, with large deflections and a redistribution of moments as further load is applied. The consequences of fatigue failure in an indeterminate structure are also less serious than in a determinate structure. Some loss of stiffness must occur in the section of initial fatigue fracture, and there is a corresponding redistribution of moments with some reduction in moment in the damaged section in subsequent load cycles. The cost of repair following flexural failure in a critical section, with the elimination of permanent local deformations and permanent deflections, might be expected to be somewhat less than the replacement costs. The cost of repair of an indeterminate structure is probably of the same order for both flexure and fatigue failure.

To take the above considerations into account, the ratio α, defined in the following equation

$$P_f(\text{fatigue}) = \alpha\ P_f(\text{flexure}) \tag{29}$$

will be given a value of from 5 to 10 for statically determinate structures. We thus obtain the following ranges for statically determinate structures:

$$5 \times 10^{-3} > P_f(\text{fatigue}) > 5 \times 10^{-5}$$

$$2.75 < \beta\ (\text{fatigue}) < 4.0$$

Values of P_f(fatigue) for indeterminate structures would be considerably higher, but because the effect of indeterminacy on reliability level is largely ignored in flexural design, this point will not be pursued here. Instead, attention will be concentrated on the statically determinate case. It can then be inferred that the safety coefficients so obtained will be conservative, and possibly overconservative, when applied to indeterminate structures. In Appendices A and B, simple reliability analyses are carried out to obtain numerical values for safety coefficients for use in the stress format and cycle format design procedures. The calculations are made for the range of β given above. Results are contained in **Tables A.1** and **B.1**. From **Table A.1** it will be seen that the values of the safety factor $\gamma_{R\sigma}$ suggested for use in the JSCE Tentative Recommendations for fatigue design lead to β values of less than 2.75 for all ranges of the design life N^*. Some consideration might be given to increasing this value.

4. Proposed design procedure

Previous studies of fatigue of concrete structures indicate that fatigue life can be predicted from the fatigue properties of the component materials by transforming the load history of the structure into stress cycle histories for the materials in critical sections and then evaluating fatigue life of each material from relevant, experimentally determined fatigue properties. In the case of flexural members, experimental and theoretical studies suggest that fatigue life in realistic structures will be limited not by fatigue of the compressive concrete but by the fatigue life of the tensile steel elements, i.e. reinforcing bars and prestressing tendons. If relative movement between the tendon and duct or concrete in a prestressed concrete beam can occur, then some downward adjustment of the in-air fatigue properties of the prestressing steel may be necessary.

The following steps comprise a design checking procedure to determine whether there is adequate fatigue reliability in a member or structure which has been already designed for the limit states of strength and serviceability. The cycle format is used, i.e. the required design fatigue life N^*_S is compared with N^*_R, where the latter quantity may be limited either by fatigue in the tensile reinforcing bars or in the prestressing tendons.

To obtain a procedure which is simple enough for practical use, the stress cycles in each material are assumed to occur in blocks which are repeated until the material fractures. The total number of cycles n in any one block is taken to be small in comparison with the total number of stress cycles required to cause fatigue failure. Furthermore, in each block there are assumed to be only k different load cycles, with $k \ll n$. As shown in **Fig. 5**, each stress block contains n_i cycles with maximum and minimum stress levels of σ_{maxi} and σ_{mini}, $i=1, 2, \ldots k$.

It will be realised that this simplifying assumption ignores progressive changes in the response of a structure or member during the fatigue loading. It is therefore important to use the mid-life response of the structure to transform load history into the corresponding stress cycle histories. It should also be noted that the mid-life response may be considerably different to the initial response during the first loading[17].

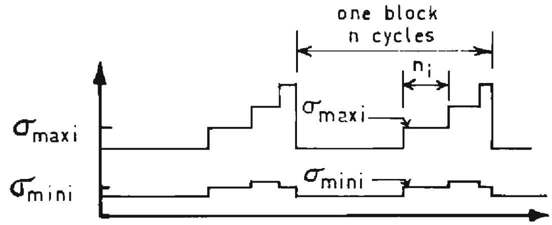


Fig. 5 Equivalent cumulative damage block

It is assumed that a complex, irregular load history can be replaced by an equivalent regular block history such as the one shown in **Fig. 5**. In many situations, such as the design of a bridge girder, available data on loads will enable such a load block, consisting of several representative load levels, to be constructed.

The S–N–P relations for reinforcing bar and prestressing steel are used to obtain the characteristic number of cycles N_{Rk} as a function of the stress term S, which may be cycle amplitude or maximum stress level. Tests on prestressing tendons have shown that the linear cumulative damage hypothesis is at least as reliable as any other theory for predicting mean fatigue life under conditions of variable repeated cycles of load[17]. Applying this hypothesis for the design cycle block, **Fig. 5**, we obtain for the mean number of blocks required to cause fatigue failure

$$m(0.5)=\left[\sum_{i}\frac{n_i}{N(S_i;\ P=0.5)}\right]^{-1} \tag{30}$$

in which $N(S_i;\ P=0.5)$ is the mean fatigue life, given by the S–N–P relation, for the i-th stress cycle S_i being used to represent stress in the S–N–P relation. When Eq. 30 is used in the fatigue reliability check calculations, values of S_i will be obtained from the corresponding load cycles by means of structural analysis and section analysis, all for mid-life response.

Little information is available on the variability to be expected in cases of cumulative damage loading; however, various simple ways can be used to treat probability levels other than 0.5. For example, Eq. 30 can be extrapolated as follows to apply to any probability level P:

$$m(P)=\left[\sum_{i}\frac{n_i}{N(S_i;\ P)}\right]^{-1} \tag{31}$$

If, as is often the case, the standard deviation of log N is assumed to be constant throughout the range of stress treated in the S–N–P relation, there is little point in doing anything other than assume that the same constant value for standard deviation applies also for the cumulative damage condition. The characteristic cycle life N_{Rk} is then obtained from

$$\log N_{Rk}=\log\,[n.m(0.5)]-1.64\sigma\ (\log N_R) \tag{32}$$

and the design cycle life is

$$N_R^* = \frac{1}{\gamma_{RN}} N_{Rk} \tag{33}$$

The design requirement to be checked is that

$$N_R^* \geqslant N_S^* \tag{34}$$

In the proposed design procedure the safety coefficients γ_{RN} for reinforcing steel and prestressing tendon have to be chosen to allow for the following:
(a) variability in the constant cycle fatigue data, over and above the allowance implied by the use of N_{Rk};
(b) variability in the load levels, and hence in the stress levels;
(c) uncertainties and variability in the structural response of the member and structural system, and hence in the transformation of load history into stress history; and
(d) statistical system effects.

In the analysis in **Appendix B** of this paper, it will be seen that the safety coefficients $\gamma_{R\sigma}$ are strongly influenced by the degree of variability in the design load levels. The γ values given in **Table B.1** allow for sources (a) and (b) as listed above. However, the final coefficient to be used in a specific design should be increased somewhat to allow for effects (c) and (d), which have to be estimated for the specific design conditions.

5. Concluding remarks

Four possible formats have been considered for the design of concrete structures subjected to fatigue loading. Although all four can be developed using standard limit states concepts and simple reliability analysis, the stress and cycle formats appear to be the most promising at the present time, mainly because of the limited amount of design data available on fatigue of the component materials.

In obtaining numerical values for the safety coefficients for use with the stress and cycle design formats, it has been necessary to use a good deal of subjective judgement. A comparative procedure has been adopted. The values obtained suggest that the Tentative Recommendations of the JSCE for fatigue design would correspond to use of a safety index of less than 2.75. Values presented in **Table B.1** for the safety coefficient γ_{RN} for use in the cycle design format make allowance for variability in the fatigue load and design life.

It should be emphasised that the safety coefficients have been obtained from a consideration of fatigue data for prestressing tendon. Parallel studies need to be made using fatigue data for reinforcing bar.

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Appendix A

Evaluation of safety coefficient for stress format method

In the evaluation of safety coefficients, realistic fatigue data for reinforcing bars and for prestressing tendon have to be used for the basic S–N–P relations. The following calculations are based on extensive constant cycle data for seven-wire strand and prestressing wire, as reported in Refs. 15 and 17 and shown in **Fig. A.1**. The fatigue tests were carried out in the USA and in Australia. A linear regression analysis of the data was made using logarithms of both fatigue life N_R and maximum stress in the cycle f_R in order to derive the S–N–P relation. The mean resistance, as a function of fatigue life, was found to be:

$$\text{mean}(\log f_R) = 2.814 - 0.282 \log N_R \tag{A.1}$$

Although some decrease in standard deviation appears to occur with decreasing N_R, the trend is not strong enough to warrant special treatment and for the present study a constant standard deviation is used (Ref. 18). We take

$$\text{sd}(\log f_R) = 0.053 \tag{A.2}$$

The characteristic value of $\log f_R$ then becomes

$$\begin{aligned}(\log f_R)_k &= 2.814 - 0.282 \log N_R - 1.64{*}0.053 \\ &= 2.728 - 0.282 \log N_R\end{aligned} \tag{A.3}$$

It should be noted that the above expressions have been derived from data which lie primarily in the region $\log N_R < 6$. It is conservatively assumed here that the data can be extrapolated to higher values of N without the introduction of a

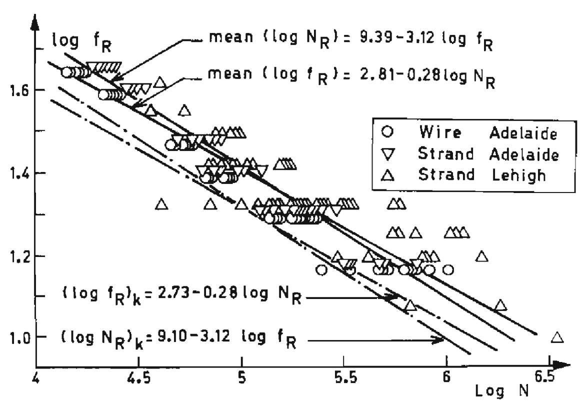

Fig. A.1 Constant cycle fatigue test data fatigue limit or a modified slope of the regression equation, as in Eq. 5.

In the present analysis the term $\gamma_{S\sigma}$ in Eq. 2 is set equal to unity and the single coefficient $\gamma_{R\sigma}$ in Eq. 1 is evaluated for a range of values of the safety index β. By choosing a specific design life N_S^* it is possible to evaluate $(\log f_R)_k$ from Eq. A.3 and, for a specific β value, $\mu(\log \sigma_S)$ from Eq. 13, provided the standard deviation of $\log \sigma_S$, i.e. $\sigma(\log \sigma_S)$, is known or assumed. The characteristic value $(\log \sigma_S)_k$ can then be determined and hence the desired safety coefficient by setting f_R^* equal to σ_S^*. The safety coefficient has been calculated both in terms of the logs of stresses

$$\gamma_{R\sigma}=\frac{(\log f_R)_k}{(\log \sigma_S)_k} \tag{A.4}$$

and in terms of the stress ratio

$$\gamma'_{R\sigma}=\frac{\log^{-1}(\log f_R)_k}{\log^{-1}(\log \sigma_S)_k} \tag{A.5}$$

The results presented in **Table A.1** have been determined for fatigue lives of 10^5, 10^6 and 5×10^6 and for values of r of 1.0 and 2.0, where r is defined by the following equation:

$$\sigma(\log \sigma_S)=r\sigma(\log f_R)$$

Table A.1 Safety coefficients for the stress format.

log N	β	r=1.0		r=2.0	
		$\gamma_{R\sigma}$	$\gamma'_{R\sigma}$	$\gamma_{R\sigma}$	$\gamma'_{R\sigma}$
5.0	2.75	1.03	1.08	1.05	1.16
	3.00	1.04	1.12	1.08	1.24
	3.25	1.06	1.17	1.10	1.33
	3.50	1.07	1.22	1.13	1.42
	3.75	1.09	1.28	1.16	1.52
	4.00	1.10	1.33	1.19	1.63
6.0	2.75	1.03	1.08	1.07	1.16
	3.00	1.05	1.12	1.10	1.24
	3.25	1.07	1.17	1.14	1.33
	3.50	1.09	1.22	1.17	1.42
	3.75	1.11	1.28	1.21	1.52
	4.00	1.14	1.33	1.26	1.63
6.699	2.75	1.04	1.08	1.08	1.16
	3.00	1.06	1.12	1.13	1.24
	3.25	1.09	1.17	1.17	1.33
	3.50	1.12	1.22	1.22	1.42
	3.75	1.15	1.28	1.28	1.52
	4.00	1.18	1.33	1.34	1.63

Appendix B

Evaluation of safety coefficient for cycle format method

Regression of log N_R on log f_R for the same data used to derive Eq. A.1 gives the following, where stresses are in MPa:

$$\text{mean } (\log N_R) = 9.390 - 3.119 \log f_R \quad \text{(B.1)}$$

$$\text{sd } (\log N_R) = 0.175 \quad \text{(B.2)}$$

$$(\log N_R)_k = 9.103 - 3.119 \log f_R \quad \text{(B.3)}$$

It is convenient here to express the S–N–P relation as follows:

$$\log N_R = 9.390 - 3.119 \log f_R + X \quad \text{(B.4)}$$

where X is normally distributed with mean zero and standard deviation of 0.175. To allow for the added variability in log N_R which is introduced by variability in the design stress log σ_S (see **Fig. B.1**), we write

$$\log f_R = Y$$

and hence

$$\log N_R = 9.390 - 3.119Y + X \quad \text{(B.5)}$$

The variance in log N_R is now

$$\sigma^2(\log N_R) = \sigma^2(X) + (3.119 * \sigma(Y))^2 \quad \text{(B.6)}$$

It is assumed that there is no correlation between Y and X because the standard deviation of log N_R is constant.

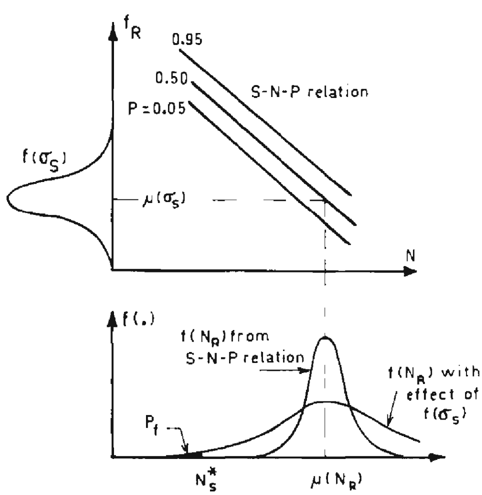

Fig. B.1 Effect of variability in load term

In the present calculations the coefficient γ_{SN} in Eq. 16 is taken to be 1.0 and γ_{RN} in Eq. 15 is evaluated. From Eq. 19 we have

$$\mu(\log N_S) = \log N_S^* = \mu(\log N_R) - \beta\sigma(\log N_R) \tag{B.7}$$

The calculations have been carried out for the following ranges of values (σ_S in MPa):

$\mu(\log \sigma_S)$: 1.0, 1.3
$\sigma(\log \sigma_S)$: 0.053, 0.106
β: 2.75 to 4.00 in increments of 0.25

As in **Appendix A**, safety coefficients are calculated both for the log and natural values.

$$\gamma_{RN} = \frac{(\log N_R)_k}{\log N_S^*} \tag{B.8}$$

$$\gamma'_{RN} = \frac{\log^{-1} (\log N_R)_k}{N_S^*} \tag{B.9}$$

It should be noted that the safety coefficients have here been evaluated for use in a design procedure in which fatigue life is calculated using mean values and not characteristic values for the repeated loads.

Table B.1 Safety coefficients for cycle format.

(log σ_S)	β	σ (log σ_S)			
		0.053		0.106	
		γ_{RN}	γ'_{RN}	γ_{RN}	γ'_{RN}
1.0	2.75	1.05	1.85	1.08	2.58
	3.00	1.06	2.12	1.10	3.20
	3.25	1.07	2.43	1.12	3.96
	3.50	1.08	2.79	1.14	4.91
	3.75	1.09	3.21	1.16	6.07
	4.00	1.11	3.68	1.18	7.52
1.3	2.75	1.06	1.85	1.10	2.58
	3.00	1.07	2.12	1.12	3.20
	3.25	1.09	2.44	1.15	3.96
	3.50	1.10	2.79	1.17	4.91
	3.75	1.11	3.21	1.20	6.07
	4.00	1.13	3.68	1.23	7.52