Dispersion of Bed Load Particles

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Abstract

It is important to clarify the dispersion process of river sediment because it is essential to predict and prevent the damage caused by contaminated sediment, and to detect the source of the contaminants from their spatial and temporal distribution.

In this study, the dispersion process of bed load particles under the condition in which bed level changes with time is discussed.

The distribution of elevation of bed particles, and the pick up rate for each elevation, were measured through flume tests; it was found that

1) The thickness of the active layer for flat bed conditions is about the maximum diameter of the bed material.

2) Coarser particles are apt to be more exposed than finer particles.

3) The pick up rate of the bed particles increases with the degree of exposure.

The occurrence of the following phenomena were determined through flume tests, and explained through stochastic simulation, taking account of the time series of rest periods and step length combined with a conventional analysis of river bed variation.

1) On a bed of non-uniform material under high flow conditions, coarser particles move faster and disperse more widely than finer particles.

2) Bed level variation exhibits two opposite effects. The increase of the degree of exposure accelerates dispersion, but the reduction of shear stress decelerates it.

1. Introduction

It is important to clarify the dispersion process of river sediment because it is essential to predict and prevent the damage caused by contaminated sediment, and to detect the source of the contaminants from their spatial and temporal distribution.

The sediment supplied to river courses generally has a wide range of sizes, and is transported in various modes such as bed load and suspended load, or bed material load, contact load and wash load. As sediments undergoing different transport modes are transported at different speeds, a mass of sediment which is initially concentrated in space widely disperses as time elapses. Moreover, even if the transport mode is the same, nonuniform particles disperse when their travel velocity depends on the particle size. These modes of dispersion are due to nonuniformity of particle sizes and are accompanied by spatial sorting of different grain sizes.

On the other hand, even for nearly uniform sediment, remarkable dispersion often occurs, due to the irregularity of the transport process of sediment particles. The factors which cause irregularities of sediment transport are the flow turbulence and the irregularities of the shape and arrangement of particles.

Moreover, when the hydraulic conditions vary with time or space, sediment particles disperse in time and space.

Generally, these factors are superimposed to cause a complicated dispersion with grain size sorting. At the present stage, however, it is difficult to deal with all of them simultaneously. In this study, only the dispersion of bed load particles under nearly flat bed conditions is dealt with.

There are two major approaches to analyze dispersion problems. The first one is to use diffusion equations and the second one is to use stochastic simulation. As it is difficult to estimate the diffusion coefficient of bed load for complicated conditions, it is effective to use stochastic simulation to analyze the dispersion of bed load particles for those conditions in which bed level changes with time.

It is well known that the movement of bed load particles consists of intermittent steps1), and that at least for an equilibrium state it is possible to express generally the dispersion process of grains in terms of the probability distributions of rest periods, moving periods and step lengths of particles2). As the moving period is usually much shorter than the rest period, the rest period and step length are especially important in order to express the bed load movement as a time series. Although they are both stochastic parameters scattered over a wide range, their distributions depend on the nature of the bed material, bed forms and flow conditions.

So far, several formulations have been proposed to express their probability distributions. The simplest one is a representation in terms of an exponential distribution for both rest period and step length; this is theoretically supported in the case of a uniform field for which the individual rest periods and step lengths are independent. When tracers are carefully and almost uniformly placed using uniform sand, the applicability of this approximation has been experimentally ascertained, at least during the initial stage of several experiments3).

When dunes are formed, the distribution of step length deviates from the exponential4), the reason for which is surmised to be that the step length differs systematically according to particle position on a dune. In this case also, however, it is known that the rest period follows an exponential distribution as long as the bed is in equilibrium4).

Although the rest period follows an exponential distribution both in a flat bed and in a dune bed, the contents differ considerably. Namely, in a flat bed in equilibrium, a particle once located on the bed surface is apt to be exposed for a while, and the rest period corresponds to the duration in which a particle is waiting to be subjected to the random application of a large force by the flow. On the other hand, in the case of a dune bed, a particle which drops down the dune face, or deposits in the eddy region just downstream of it, is easily buried until it is exposed again by the advance of the dune. In this case, the rest period approximately corresponds to the duration during which a particle is buried.

According to Nakagawa & Tsujimoto2), it was pointed out that for the rest period associated with an equilibrium flat bed, the Gamma distribution with a shape parameter of 1/2 fits the data better than the exponential one. They argued that the reason why the rest period in an equilibrium flat bed does not follow the exponential distribution, is that the elevation of bed particles varies over a wide range, and that the mean rest period differs according to this elevation. They have, however, not quantitatively clarified the probabilistic distribution of grain elevation and the relation between this elevation and the mean rest period.

If the rest period and the step length are independent and have a definite probability distribution, respectively, the dispersion process can be analyzed using these probability distributions, so that it is not necessary to discuss the elevation of particles. On the other hand, when the stream bed aggrades or degrades, the degree of exposure of a particular particle changes systematically with time resulting in at least a probability distribution of the rest periods that changes with time. Therefore, in such a case, it is necessary to clarify the time change of the degree of surface exposure of a particular particle and the distribution of the rest period corresponding to each degree of exposure. In the case that the degree of exposure varies within a single rest period, however, the rest period for a given degree of exposure cannot be defined. Therefore, it is necessary to parameterize the problem in terms of the pick up rate of a bed particle in a unit time.

In this study, the author clarifies the distribution of the degree of exposure of bed particles and the pick up rate for each degree of exposure through tracer experiments, and simulates the dispersion process of bed load particles with grain size sorting, including the conditions of degradation and aggradation. Some flume tests are also carried out, and the results are compared.

Some parts of this study which have already been published[5~7] are summarized in this paper.

2. Elements necessary for the tracing of bed load particles

It is possible to simulate the dispersion process of bed load particles by computer simulation, generating time series of rest periods and steps. The probability distribution of rest period, however, cannot be determined from the instantaneous hydraulic and bed conditions in the case the conditions vary with time. Then the rest period can be calculated as the sum of the successive time increments in which the particle under consideration is not picked up. It depends on the flow conditions and the degree of exposure of the particle under consideration whether or not it is picked up in the given time increment. Therefore, it is important to estimate the degree of exposure of this particle, and the pick up rate for each degree of exposure.

Although it is not clear which parameter represents the degree of exposure of a particular particle appropriately, the author defines the degree of exposure of any particle by eq. (1), as a quantitative index of the vertical position of each particle.

$$e \equiv (z-\bar{z})/d \quad \cdots\cdots (1)$$

where z is the level of the top of a tracer particle, $\bar{z}$ is the mean level of the top of the adjacent particles as shown in **Fig. 1**, and d is the diameter of the tracer particle. The value of e varies as both the tracer and adjacent particles move.

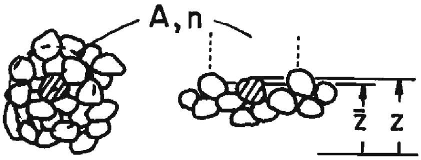

Fig. 1. Concept of the degree of exposure and adjacent region of a tracer particle.

The rate of change of degree of exposure within a rest period is represented by

$$\left(\frac{de}{dt}\right)=-\frac{d\bar{z}}{dt}\Big/d \quad\cdots\cdots(2)$$

Although the distribution of the degree of exposure at the end of each step has not been clarified yet, it is supposed not to be affected strongly by the change in bed level.

In case of equilibrium conditions, the following relation can be expected between the probability density function of the degree of exposure $f_0(e)$ at the end of a step of a tracer particle, and the function $f(e)$ at an arbitrary time;

$$f(e)=\frac{f_0(e)\cdot\bar{t}_r(e)}{\bar{\bar{t}}_r} \quad\cdots\cdots(3)$$

where $\bar{t}_r(e)$ is the mean rest period for a given degree of exposure, and $\bar{\bar{t}}_r$ is the value of $\bar{t}_r$ averaged over all degrees of exposure. For equilibrium conditions, as the pick up rate $p_s(e)$ for a given degree of exposure and the value $\bar{p}_s$ averaged over all degrees of exposure correspond respectively to the reciprocal of $\bar{t}_r(e)$ and $\bar{\bar{t}}_r$ respectively, eq. (3) can be written as

$$f(e)/f_0(e)=\bar{p}_s/p_s(e) \quad\cdots\cdots(4)$$

The relation between $\bar{p}_s$ and hydraulic parameters has been studied previously. For example, Nakagawa & Tsujimoto[8] obtained the result

$$\bar{p}_s/\sqrt{d/(\sigma/\rho-1)g}=\begin{cases}F_0\tau_*\{1-(\tau_{*c}/\tau_*)\}^3 & [\tau_*>\tau_{*c}]\\ 0 & [\tau_*\leqq\tau_{*c}]\end{cases} \quad\cdots\cdots(5)$$

where, σ/ρ is the specific weight of sediment, τ_* is the Shields stress $(\tau_*=u_*^2/((\sigma/\rho-1)gd))$, u_* is the shear velocity and τ_{*c} (about 0.035) is the value of τ_* corresponding to incipient motion, which is smaller than the so called critical tractive force corresponding to considerable sediment transport. F_0 is an experimental constant taking the value of 0.03.

Therefore, if the probabilistic distribution of the degree of exposure of particles in the thin layer at the bed surface (active layer) at equilibrium, and the pick up rate for each degree of exposure are given, the distribution of the degree of exposure at the end of each step can be estimated by eq. (4).

Although the pick up rate and rest period depend on the degree of exposure of the tracer particle, the step length may be determined by the hydraulic conditions and the undulations of the bed surface irrespective of the degree of exposure. Therefore, as long as distinct sand waves are not present, the distribution of step lengths is approximated as equal to that for a flat, equilibrium bed. As mentioned previously, the step length on an equilibrium flat bed is distributed exponentially, with a mean step length approximately equal to one hundred times the particle diameter, namely,

$$f(l)=\frac{1}{\bar{l}}\exp\left(-\frac{l}{\bar{l}}\right) \quad\cdots\cdots(6)$$

$$\bar{l}\fallingdotseq 100d \quad\cdots\cdots(7)$$

So far, the particle diameter has been written simply as *d*. In case of nonuniform material, however, it is necessary to distinguish the moving properties of particles for each particle size class. Among many studies of the effect of size nonuniformity on the critical tractive force, the following equation, derived by Ashida & Michiue[9] based on the work of Egiazaroff[10], is preferred by the author.

$$\tau_{ci}/\tau_{cm}=\begin{cases}\{\log_{10}19/\log_{10}(19d_i/d_m)\}^2 d_i/d_m & (d_i/d_m\geq 0.4)\\ 0.85 & (d_i/d_m<0.4)\end{cases} \quad\cdots\cdots(8)$$

and

$$\tau_{*cm}=\tau_{cm}/(\sigma-\rho)gd_m\fallingdotseq 0.05 \quad\cdots\cdots(9)$$

where the subscript *i* denotes a given size class and *m* denotes the mean diameter. Tsujimoto[11] ascertained that the mean pick up rate for each size class can be estimated from eqs. (5) and (8) for an equilibrium flat bed with nonuniform material.

On the other hand, the step length for each size class of a nonuniform material distributes exponentially, but the mean value becomes smaller than that for the case of uniform material[12].

The time change of the bed level and flow condition themselves can be analyzed by conventional methods, as explained later.

3. Experiments on the degree of exposure of tracer particles, and the pick up rate for each degree of exposure

In order to clarify the distribution of the degree of exposure of bed surface particles $f(e)$, and the pick up rate for each elevation $p_s(e)$, two series of experiments were carried out. In the first series[5], nearly uniform material (d=4.65 mm) was spread along a 10 m long reach of a 20 cm wide flume. Initially, the bed surface was screeded with an inclination of 1/100, and several hundred tracers (colored particles with the same properties as the bed material) were arranged in the surface layer within a 20 cm reach at the upstream end of the erodible bed. The arrangement of the tracers was carefully adjusted so as not to disturb the surface condition. Next, water was supplied from the upstream end, and the discharge was increased gradually until incipient motion. The discharge was then quickly increased to a given value (Q=10 l/s), and sediment with the same properties as the bed material was supplied from upstream (Q_{Bi}=170 g/min) to prevent degradation or aggradation. The Froude number was nearly unity and the bed configuration was almost flat. After several minutes, the water and sediment supply was stopped, and the positions of 50 tracer particles and their adjacent particles were measured in the middle part of the flume. Later, the same procedure was repeated twice, and the distribution of the degree of exposure and

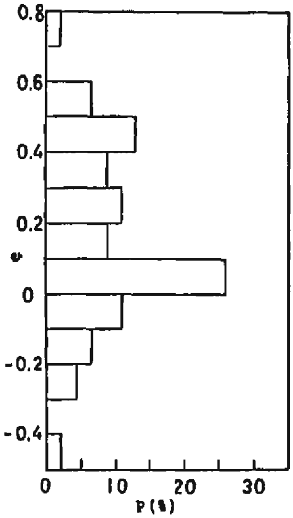

Fig. 2. Distribution of exposure of tracer particles.

the percentage of tracer particles remaining at the origin were surveyed in both cases.

Fig. 2 shows the distribution of the degree of exposure of tracer particles, where the average crest height of the tracer particle, and of the particles immediately upstream and downstream, was chosen as $\bar{z}$ for eq. (1). The value of e distributes more or less from -0.3 to $+0.7$, with a concentration in the range -0.1 to $+0.5$.

Fig. 3 shows the relation between the initial degree of exposure and the percentage of tracer particles remaining at the origin, in which the tendency for a greater value of e to correspond with a smaller probability to remain can be seen.

Fig. 4 shows the relation between the degree of exposure e and the pick up rate p_s determined from the probability of remaining at the origin. The bold line fitted assuming a value for p_s of 0 at $e=-0.3$ is expressed as

$$p_s=\bar{p}_s\{1+1.25(e-0.5)\} \quad \cdots\cdots\cdots\cdots(10)$$

The value of $\bar{p}_s$ estimated by eq. (5) corresponds approximately to $e=0.5$, indicating

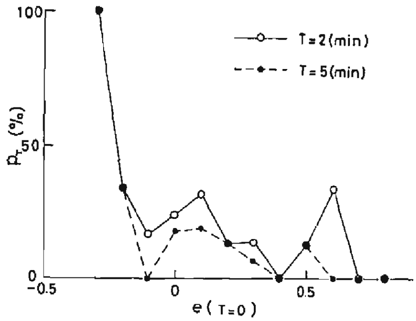

Fig. 3. Percentage of tracer particles remaining in place.

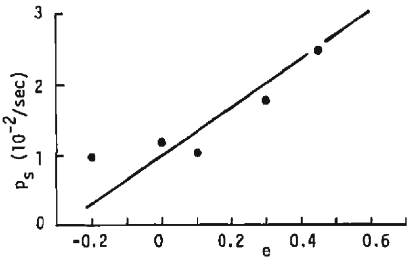

Fig. 4. Relation between the degree of exposure and pick up rate for uniform material ($\tau_*=0.08$).

a considerably exposed condition.

In the second series[7], nonuniform material with the size distribution shown in **Fig. 5** was spread along a 16 m long reach of a 50 cm wide flume with an inclination of 1/125. A given discharge (Q=18.7 l/s) was supplied, without any sediment feed from the upstream end. Three hundred tracer particles (one hundred for each size class shown in **Fig. 5**) were scattered through the water surface at the several points along a 4 m reach located 3 to 7 m from the upstream end. After 5 minutes the water supply was stopped, and the degree of exposure of each tracer was measured. In the case of the tracer particles exposed on the bed surface, the crest level of each tracer, and the mean crest levels of surrounding particles within a radius of about 2 cm measured using a point gauge. For the buried tracer particles, both levels were measured after the particles covering them were removed carefully. In cases when the bed just near the

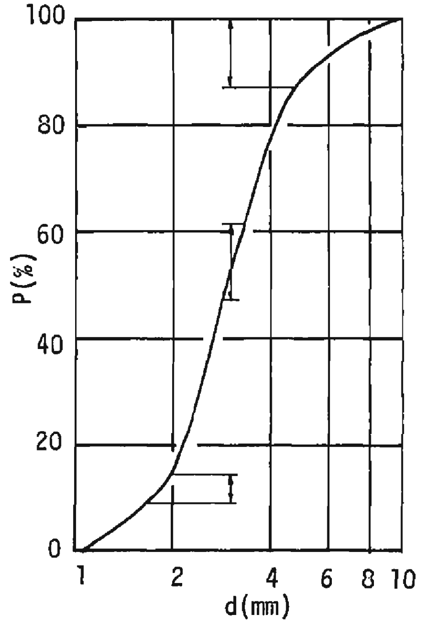

Fig. 5. Grain size distribution of bed material.

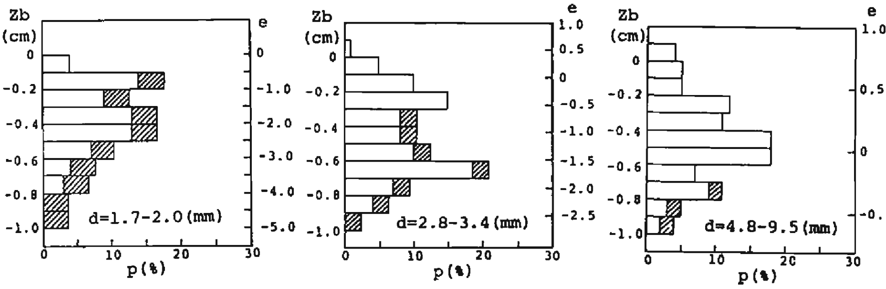

Fig. 6. Distribution of the bottom height and degree of exposure of tracer particles.

tracer was disturbed, the mean bed level was measured at an undisturbed portion slightly downstream.

Fig. 6 shows the distribution of the elevation of tracers for each of three size classes, where z_b is the bottom level of a tracer particle measured relative to the mean level of the crest of the surrounding particles. Some buried tracers were moved before measuring their position. For the want of a better procedure, in **Fig. 6** they have been uniformly distributed within all elevation ranges, except the top (the particles of which are, of course, not buried), and displayed in terms of the hatched bars. The distribution patterns are slightly different for each size class. Recalling that z_b refers to the distribottom elevation of a grain, however, it is seen that the highest values of z_b are essentially identical for all three size classes. Likewise, the same holds for the lowest values of z_b; the difference between the highest and lowest value is seem to be almost identical to the maximum diameter of the bed material. This is considered to be the thickness of the so called active layer, where particles exchange actively with the bedload. From the viewpoint of the degree of exposure defined by eq. (1), the coarser the grain size, the larger becomes the degree of exposure. This may be supposed to be caused by the tendency for finer particles to fall into the pore space between coarser particles.

Next, in order to obtain the pick up rate for each level of exposure, one hundred tracer particles for each size class were embedded at various elevations as acurately as possible. After subjecting the bed to flow for some duration, the probability of remaining at the initial point was determined. The duration of flow was selected so that about 50% of the tracers moved. In case of long runs, sediment was supplied from the upstream end to keep the bed level constant.

The results were converted to pick up rates; they are shown in **Fig. 7**. According to these data the pick up rate of particles with the same bottom level does not depend on grain size, and the relation between p_s and z_b is approximated by a straight line in a semi-log plot. The value of p_s for $z_b=0$ approximately coincides with p_{sm} estimated by Nakagawa & Tsujimoto's equation. For every size class, the pick up rate rapidly decreases with the lowering of the bottom level, to become only 1/100 of the value at $z=0$ at the bottom of the active layer.

This result was, however, obtained from only one hydraulic condition, for which

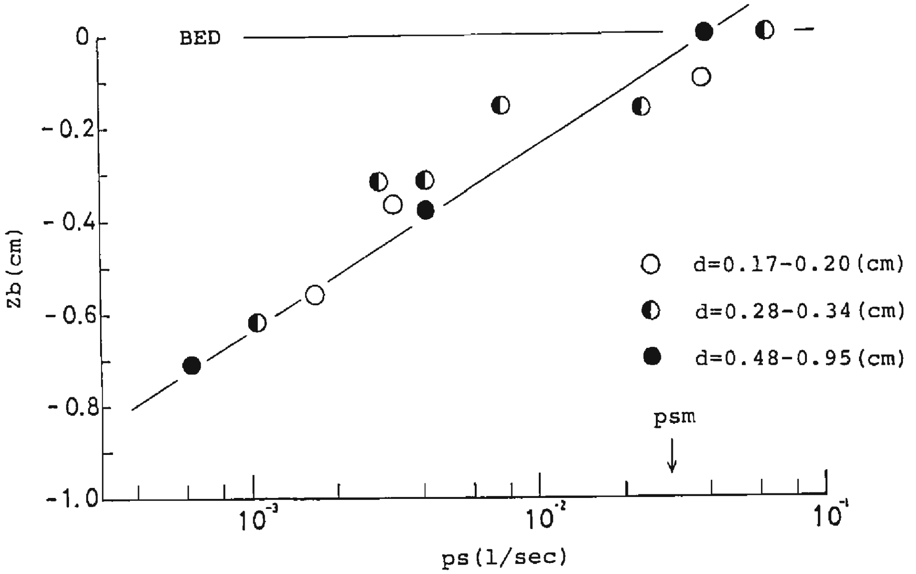

Fig. 7. Relation between pick up rate and bottom height of tracer particles.

all size classes were transported actively. The general applicability of **Fig. 7** has not been ascertained yet.

4. Experiments on the dispersion of bed load particles

In order to clarify the dispersion process of bed load particles experimentally, four series of tests were carried out. In Exps. A to C5), attention was focussed upon the effect of the bed level change, and in Exp. D7) the effect of the flow intensity was investigated. The material used is shown in **Fig. 8**; material A therein was regarded as uniform, and the other two materials were regarded as nonuniform. The colored particles used as tracers were divided into several size classes for each material. For material A, one class could be defined with the same composition as the bed material itself; for material B, two classes (d=1.9–3.4 and 3.4–4.3 mm); and for material C (or D) five classes (d=1.7–2.0, 2.0–2.8, 2.8–3.4, 3.4–4.8 and 4.8–5.5 mm) were defined for use as tracers.

Table 1. Experimental conditions

	d_m (mm)	I	B (cm)	L (m)	Q (l/s)	h (cm)	Q_{Bi} (g/min)	Q_{B0} (g/min)	Re_{*m}	τ_{*m}	Fr
Exp. A	4.65	1/100	20	10	10.0	6.4	0, 175, 780	170	450	0.083	0.99
Exp. B	3.10	1/100	20	10	4.9	4.2	0, 170, 1000	100	170	0.082	0.91
Exp. C	3.20	1/125	50	16	15.1	4.6	0, 920, 4720	850	150	0.070	0.98
Exp. D	3.20	1/125	50	16	9.6 ~ 21.5	3.7 ~ 5.9	0	6 ~ 1405	190 ~ 430	0.054 ~ 0.086	0.86 ~ 0.97

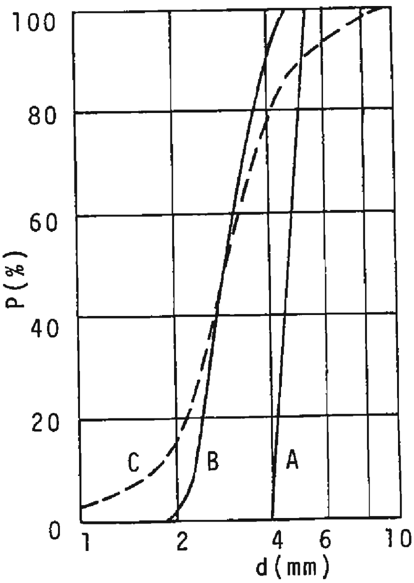

Fig. 8. Grain size distribution of material employed in the experiments.

Hydraulic conditions were set as shown in **Table 1**, so that appropriate sediment transport occured, but no distinct sand waves were formed. During Exps. A to C, the sediment supply was varied to attain, respectively, degrading, equilibrium and aggrading conditions. In Exp. D, the flow discharge was varied from nearly incipient

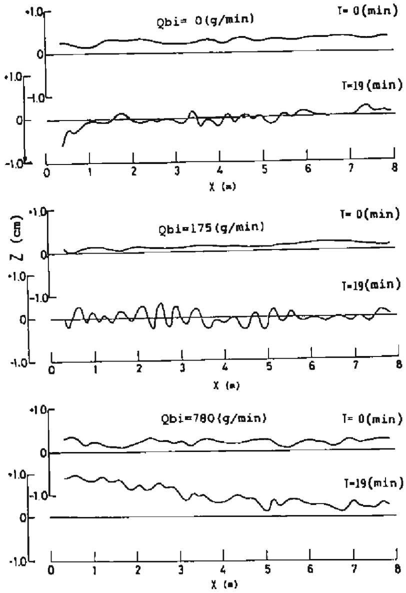

Fig. 9. Bed elevation along channel center line.

motion to a relatively active sediment transport condition without sediment supply. In all test runs, the bed configuration belonged to the upper regime near transition.

Initially, several hundred tracers (one hundred for each size class) were arranged in the surface layer within a 20 cm reach at the upstream end of the erodible bed. After the flow discharge was increased gradually until incipient motion appeared, it was quickly increased to the desired value. After some time, the water supply was stopped and the distribution of the travel distance of tracer particles was measured.

Fig. 9 shows the bed profile measured along the channel center line in Exp. A. Although a flat bed condition was intended, the generation of some antidunes was inevitable. The observed bed form were, however, relatively smooth, and the wave height was not large. In the other cases, no distinct small scale sand waves were detected except for some bars in Exp. D.

Fig. 10 shows the distribution of travel distance of tracer particles for Exps. A to C.

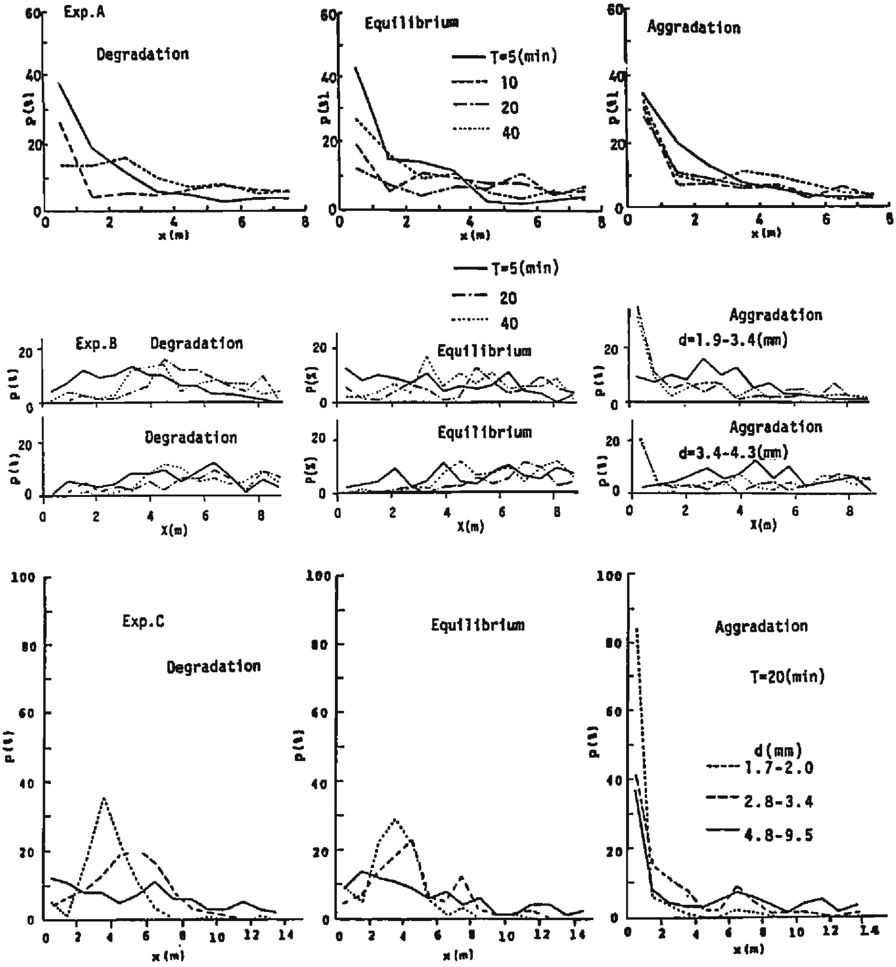

Fig. 10. Distribution of travel distance of tracer particles (Exp. A–C).

Compared with the equilibrium cases, the travel velocity of tracer particles is somewhat faster in the degrading cases and somewhat slower in the aggrading cases. Especially in a short reach near the upstream end, many more tracers remained at or near their

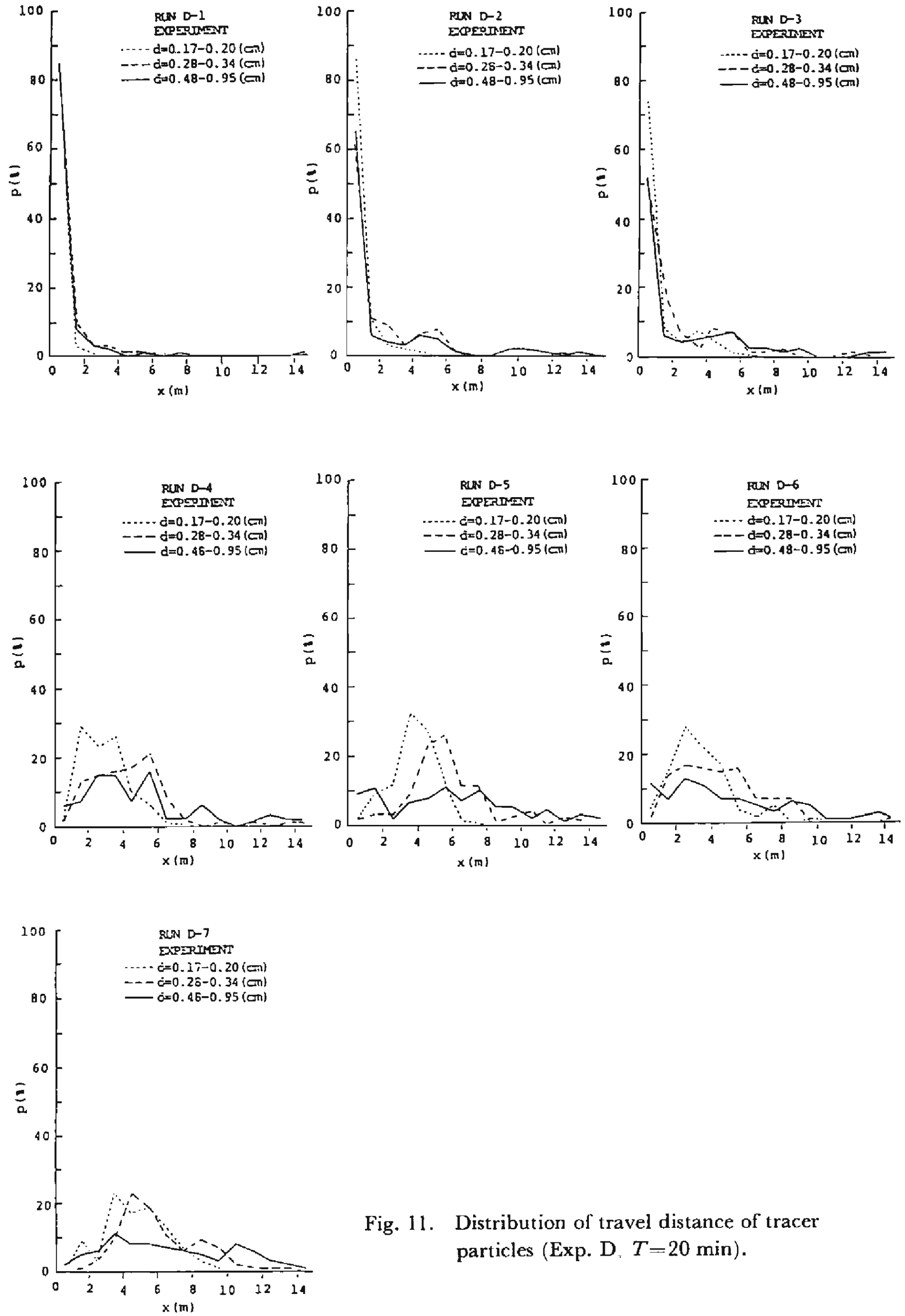

Fig. 11. Distribution of travel distance of tracer particles (Exp. D, $T=20$ min).

origin in the aggrading cases. This is because particles that are not moved are apt to be covered by the excess sediment supply. Except for the short reach near the upstream end, variation of the sediment supply did not produce a notable effect. This is because the following two effects cancel each other. In the case of the aggradation, the degree of exposure of particles remaining at the origin decreases, but the associated increase in the tractive force increases the pick up rate for exposed tracers. Which effect dominates depends on a combination of various factors.

According to **Fig. 10**, traveling distance is not necessarily seen to increase with time locally. This is because after every measurement tracer particles were collected and spread near the upstream end again. The dispersion process is seen to be highly irregular, and shows considerable scatter.

From Exps. B and C, it was found that in the case of a bed of nonuniform material, coarser particles move faster than finer particles. This is because coarser particles are highly exposed and easily eroded, but finer particles are less exposed and easily buried.

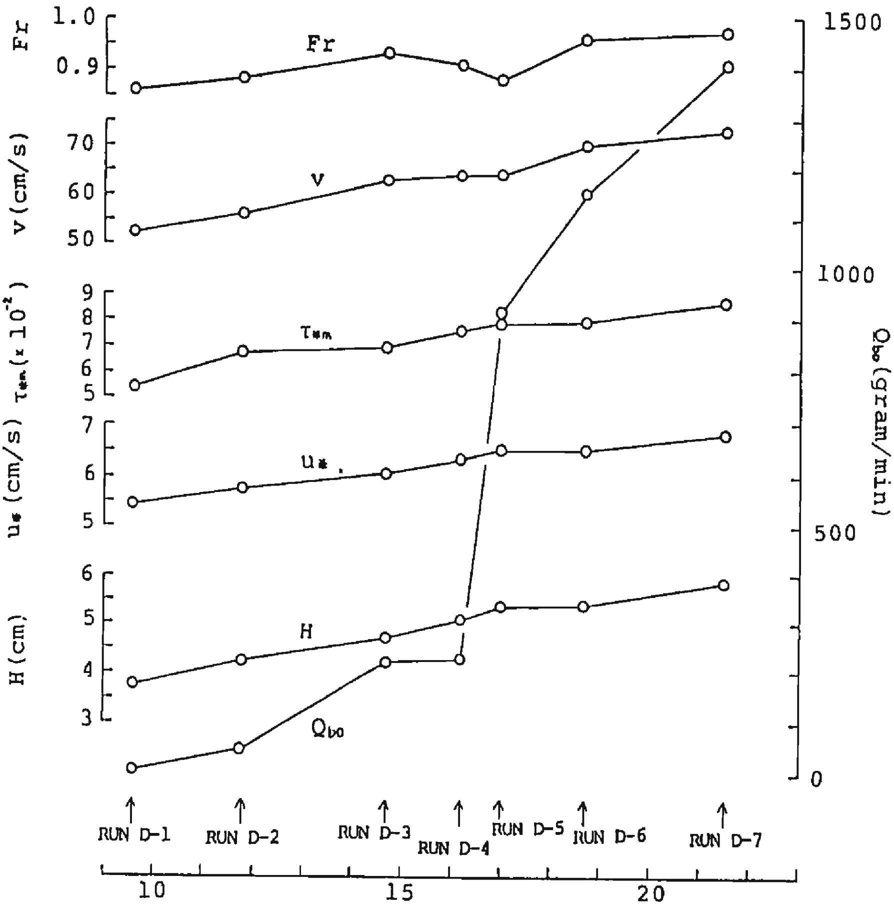

Fig. 12. Variation of several hydraulic parameters with discharge (Exp. D).

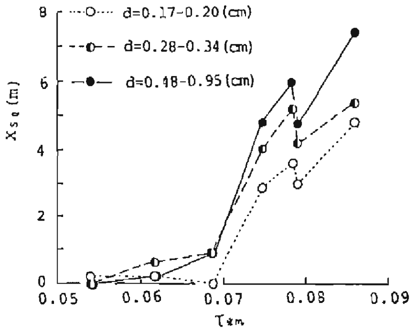

Fig. 13. Relation between the median travel distance and the tractive force (Exp. D, $T=20$ min).

In fact burying of coarser particles was limited to the upstream reach, and only occurred for aggradation; on the other hand many of the finer particles were buried even in the case of degradation. In the case of aggradation, the reach over which fine particles were buried was much longer than that of the coarser particles.

Fig. 11 shows the distribution of travel distance of tracer particles in Exp. D. Hydraulic parameters for each run are plotted in **Fig. 12**. From these figures, it can be seen that the higher the flow intensity, the longer the travel distance is, and the wider is the dispersion range. **Fig. 13** shows the relation between the median values of travel distance and the tractive force for three size classes. In the high shear range, the travel distance of coarse tracer particles is considerably longer than that of fine particles. The travel distance increases rapidly at a shear value of τ_{*m} near 0.07, which corresponds to the critical shear stress for the d_{97} of the material used. According to **Fig. 12**, sediment discharge also increases rapidly at about the same flow condition. This rapid change is considered to correspond to the breaking of the surface armor layer. For flow intensities smaller than this critical condition, some of the coarse particles have little chance to move, so that fine material disperses more widely downstream.

In natural rivers, various flow conditions take place in a time series. Therefore it is necessary to consider unsteady phenomena in order to describe the actual process in a natural river.

5. Stochastic simulation of the dispersion of bed load particles

In this chapter, the dispersion process of bed load particles corresponding to the previous experiments is simulated by a Monte Carlo method, considering the time series of rest period and step length.

As explained in Chap. 2, the duration of the rest period depends on the degree of exposure of tracer particles. When the bed level changes, however, the degree of exposure itself changes with time during the rest period. Therefore, in this simulation, the occurrence of pick up during a short time interval was chosen as a basic stochastic parameter instead of the rest period itself. Namely,

1) A random decision whether the tracer particle is picked up or not during Δt is made according to the probability of the pick up $(p_s \cdot \Delta t)$.

2) In the case that the tracer is not picked up during time Δt, the change of the degree of exposure is calculated as $(de/dt)\cdot\Delta t$ using eq. (2).

3) On the other hand, in case the particle is picked up during Δt, its step length is randomly selected using eq. (6), and the level of exposure at the end of the time increment Δt is randomly selected according to a uniform distribution within the active layer the thickness of which is assumed to be the maximum diameter of the bed material.

4) The pick up rate for each degree of exposure is estimated by eq. (10) for uniform material and eq. (11) for nonuniform material, which is slightly different from **Fig. 7,** in that it expresses the pick up rate as a function of the degree of exposure of the center of the tracer particle.

$$p_{si}(z_m)=\overline{p_{si}}\times 100^{(z_m/0.8a)} \quad (-a+d_i/2<z_m<d_i/2) \qquad \cdots\cdots(11)$$

in which $\overline{p_{si}}$ denotes the pick up rate estimated from eq. (5).

5) Various hydraulic parameters including mean bed level and the size distribution of bed material are calculated using the following equations due to Hirano[13] and Michiue[14].

$$\frac{\partial z}{\partial t}+\frac{1}{(1-\lambda)B}\frac{\partial(q_B B)}{\partial x}=0 \qquad \cdots\cdots(12)$$

$$q_B=\sum_i q_{Bi} \qquad \cdots\cdots(13)$$

$$\frac{q_{Bi}}{i_b u_{*e} d_i}=17\,\tau_{*ei}\left(1-\frac{\tau_{*ci}}{\tau_{*i}}\right)\left(1-\sqrt{\frac{\tau_{*ci}}{\tau_{*i}}}\right) \qquad \cdots\cdots(14)$$

$$\frac{\partial(Bhv)}{\partial x}=0 \qquad \cdots\cdots(15)$$

$$v\frac{\partial v}{\partial x}+g\frac{\partial h}{\partial x}+g\frac{\partial z}{\partial x}=-\frac{u_*^2}{R} \qquad \cdots\cdots(16)$$

$$\frac{v}{u_{*e}}=6.0+5.75\log_{10}\frac{h_0}{d(1+2\tau_*)} \qquad \cdots\cdots(17)$$

$$\frac{\partial i_b}{\partial t}=\begin{cases}\dfrac{1}{a}(i_B-i_b)\dfrac{\partial z}{\partial t}-\dfrac{q_B}{a(1-\lambda)}\dfrac{\partial i_B}{\partial x} & \left(\text{for } \dfrac{\partial z}{\partial t}\geqq 0\right) \qquad \cdots\cdots(18)\\[2ex] \dfrac{1}{a}(i_B-i_{b0})\dfrac{\partial z}{\partial t}-\dfrac{q_B}{a(1-\lambda)}\dfrac{\partial i_B}{\partial x} & \left(\text{for } \dfrac{\partial z}{\partial t}<0\right) \qquad \cdots\cdots(19)\end{cases}$$

where a is the thickness of the active layer and i_b, i_B and i_{b0} are the fractions of size class d_i in the active layer, bed load and original bed, respectively.

To evaluate the flow resistance, the log-law is used;

$$\frac{v}{u_*}=6.0+5.75\log_{10}\frac{h_0}{k_s}, \quad k_s=ad \qquad \cdots\cdots(20)$$

where h_0 is the quasi-uniform flow depth, and the value of a $(=k_s/d_m)$ was determined to be 3.2 for uniform material and 3.0 for nonuniform material by fitting to the experimental data.

The time increment Δt is taken to be 1 sec, and the space increment Δx is taken

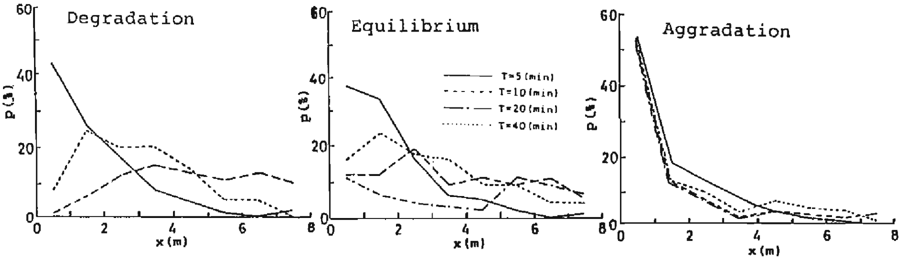

Fig. 14. Distribution of travel distance of tracer particles simulated for Exp. A.

to be 0.5 m for the calculation of bed deformation and 0.1 m for the computation of the water surface according to a "backwater" calculation (the flow may be supercritical). The number of grain size classes is taken to be 10 for nonuniform material. The initial values of the degree of exposure are taken to be zero.

Figs. 14–16 show the simulated distribution of the travel distance of tracer particles corresponding to each experiment. The features found in the experiments are fairly well simulated by this method. In the simulation, however, tracer particles show slightly less dispersion compared to the experiments.

For example, although coarse tracers dispersed widely in the experiments, with a considerable portion transported out of the flume, in the simulation the travel distance of all tracers was less than 14 m. Although the most common travel distance was in the range $x=0$–1 m irrespective of size class for experiments Run D1-3, it clustered between 2 and 3 m for medium and fine sand in the simulation.

These discrepancies may be supposed to be mainly due to the simple assumption of uniform distribution of the degree of exposure at the end of tracer particle step.

Moreover, in this simulation, the change in the degree of exposure is dealt with stochastically only from the viewpoint of the pick up of the tracer itself. Actually,

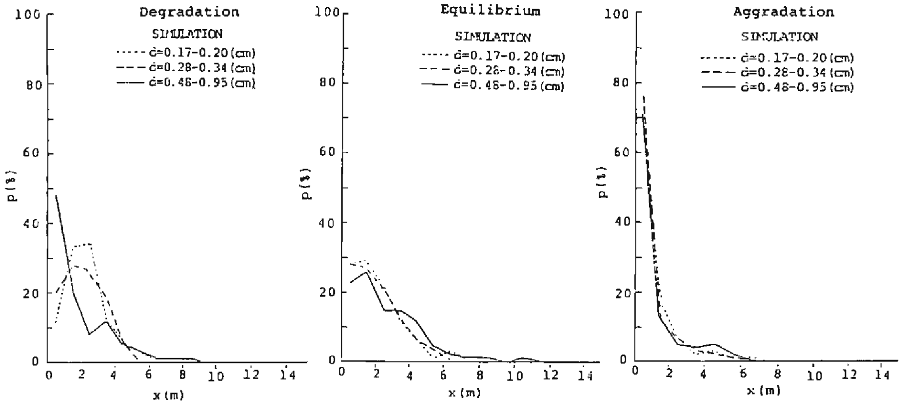

Fig. 15. Distribution of travel distance of tracer particles simulated for Exp. C ($T=20$ min).

however, the change of the degree of exposure within a rest period is also stochastic. This is considered to be another reason why the tracers show less dispersion in the simulation.

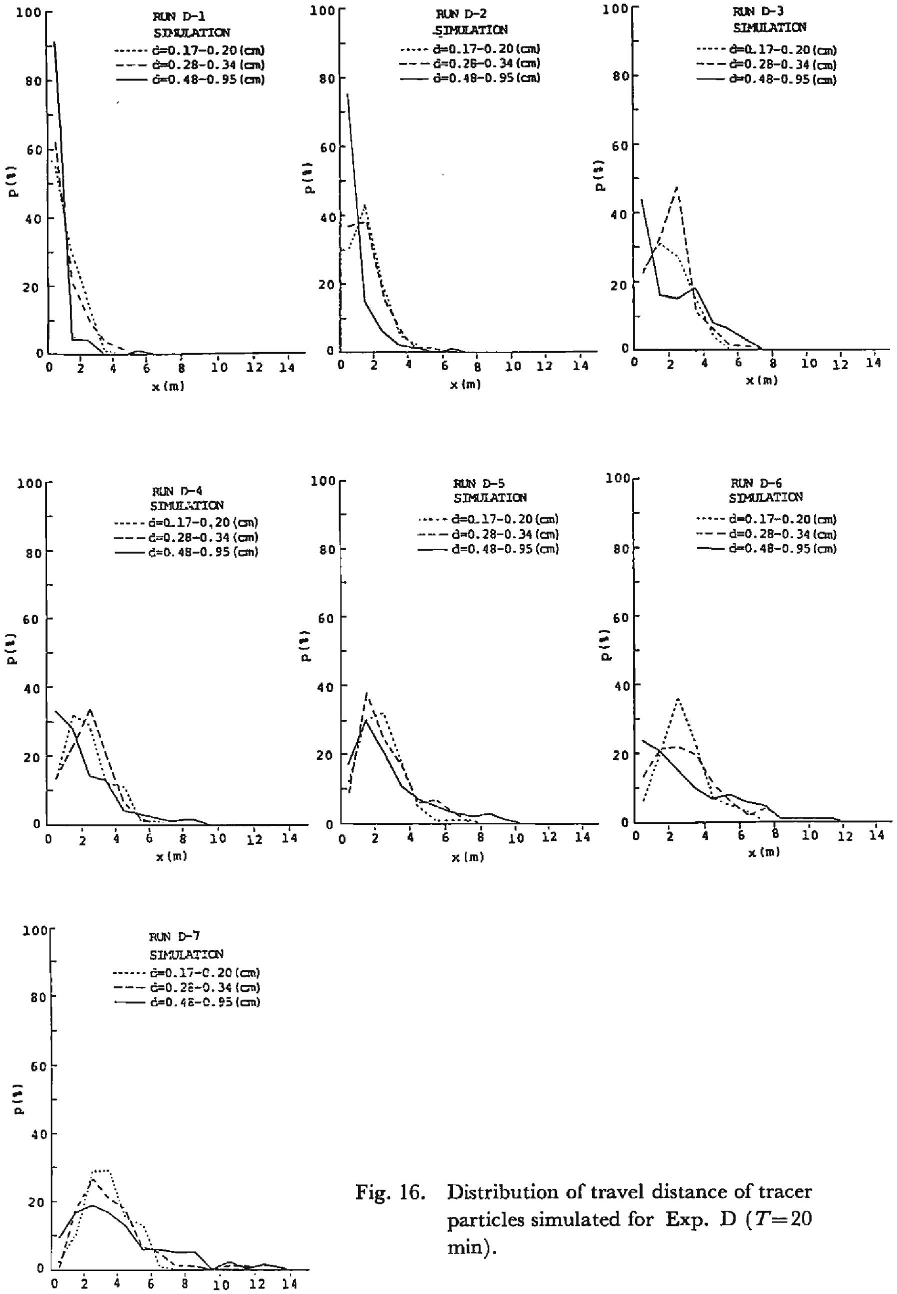

Fig. 16. Distribution of travel distance of tracer particles simulated for Exp. D ($T=20$ min).

6. Conclusion

In this study the dispersion process of bed load particles was analyzed through several flume experiments and a stochastic simulation considering the time series of rest periods and steps.

In the flume tests, measurements were made of the distribution of bed particle elevation, and pick up rate for each elevation. Moreover the dispersion of bed load particles was experimentally studied for nearly flat bed conditions using uniform and nonuniform materials, and including the effect of bed level change.

The results are summarized as follows:

The thickness of the active layer for flat bed conditions is about equal to the maximum diameter of the bed material, irrespective of the size class. In a bed of nonuniform material, a coarser particle is more apt to be exposed and a finer particle is more apt to be buried. Consequently, in the case of nonuniform bed material, coarser particles are more easily picked up and finer particles are less easily picked up than in the case of a uniform bed material.

The pick up rate of the bed particles differs according to particle elevation, i.e. the degree of exposure. The expected value of the pick up rate seems to correspond to a considerably exposed condition, with a degree of exposure of about 0.5. For uniform material, the pick up rate is approximately linearly related to the degree of exposure; the range of variation is not excessively wide. For nonuniform material, the pick up rate varies exponentially with the degree of exposure, and differs by a factor of 100 between the top and the bottom of the active layer.

In the dispersion experiments, the sediment supply from the upstream end was varied so as to attain degrading, equilibrium and aggrading conditions. During degradation, the degree of exposure of uneroded particles increases with time, so that they tend to become more easily picked up; the opposite occurs in the aggrading case. The tractive force itself also changes with bed level change, however, which has an effect opposite to that due to the degree of exposure. Which effect dominates depends on the combination of various factors.

In case of hydraulic conditions for which all size classes of bed particles can move, coarser particles move faster and disperse more widely than finer particles.

These features were fairly well simulated by a stochastic simulation focussing upon the time series of rest periods and step lengths combined with the conventional analysis of bed level variation. In the simulation, tracer particles showed slightly less dispersion than the experiments.

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