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A Numerical Prediction of Semidiurnal Current Patterns in Tanabe Bay

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Abstract

Study of tidal currents in Tanabe Bay has a long history which is related to the fields which cover coastal hazards as well as study on physical oceanography and biological oceanography. In this study, the dynamics of Tanabe Bay is studied by using a model of a finite difference method with comments on the previous observations of tidal currents in Tanabe Bay. In order to get a reasonable solution from the author's model, as has done in the other cases, it is necessary to consider first how to choose the initial conditions of the dynamic factors concerned. Although the effects of the so-called "Kuroshio Current", coastal waters and meteorological effects are occasionally significant in the area of interest, simply the semidiurnal oscillation in the model is considered and compared to the previously observed current velocity of M_2 constituent in this model. The flow patterns at the peak ebb and peak flood in the model are compared to the flow pctterns for the resonant mode with an offshore node of Tanabe Bay as a wide-open bay.

1. Introduction

The dynamic response in Tanabe Bay to a forced offshore semidiurnal oscillation is studied as one step in the study of tidal currents in the bay. As is already well known, the bay is a part of the coastal area located in Wakayama Prefecture, Central Honshu, facing the western North Pacific. So, the basin is strongly affected throughout the year by the activity of the Kuroshio off the bay, by the influences of coastal and brackish waters, by the estuarial discharges and possibly by direct or indirect meteorological impacts caused by atmospheric disturbances, especially by the effects of typhoons and atomospheric cold fronts formed on the sea surface. Nevertheless, the author's focus in this study is the response of the waters in Tanabe Bay to a forced offshore semidiurnal oscillation. The author has studied the responses of water in a wide-open bay in order to determine that a node of the resonant mode is formed when a certain condition is satisfied in the numerical model. For such a case, Tanabe Bay was made a special reference as the model of a wide-open bay (Nakamura, 1987)[1]. So, the present work on basin dynamics of Tanabe Bay can be taken as an extension or as a supplementary note to the author's previous work in relation to the problem of interest.

When we look around for studies on related problems, we find problems of oscillations of narrow-mouthed bays having been sloved by many scientists and engineers. Their contributions are countless and aid us at this time. However, there are too many reports published and the author cannot hope to refer to all of them, so he is forced to cite and refer to only a limited number in the present study. For example, some studies cited are Nakamura and Loomis (1980)[2], and Nakamura (1980a, b,[3][4] 1983a[5], 1984b, c[6][7]) and by Nakamura and Loomis (1980)[8]. On the other hand problems involving a wide-open bay have been scarecely considered except presentation of classic theory for incident waves in a "V-shaped" bay. Considering this lack, a recommendation has been strongly put forth to promote tidal observations in the areas where no data has been collected and where no model has yet agreed well (for example, Melchior, 1984)[9]. In this work, a numerical model of semidiurnal oscillation of the water in Tanabe Bay as an wide-open bay is constructed.

As far as the author can recall, the first oceanographic observation of tidal currents in Tanabe Bay was undertaken by Akikazu Nakamura (1958[10], 1959[11]). He(1958)[10] utilizing a primitive method herein floating ping-pong balls were tracked after releasing them from a small anchored boat. He measured and recorded the length and direction of fine sewing thread which was connected to each ball. The balls were adjusted to move following the flow just at the sea surface or just a few centimeter below the surface in order not to be affected directly by winds on the balls. Their observations were limited to discrete day-time operations. In 1959[11] Akikazu Nakamura reported on tidal currents in Tanabe Bay by using a so-called "Price's Current Meter" suspended from a small anchored boat in order to obtain near maximum current velocities at the flood and ebb tides considering the predicted astronomical tides in Tanabe Bay. These results were utilized in the field of biological oceanography to form a part of the research project "Study of Productivity in Tanabe Bay" which was conducted by Professors Shoitiro Hayami and Denzaburo Miyadi of Kyoto University at that time supported financially by the National Science Fund in Japan.

In 1965, the author[12] developed a current meter which could be used for the easy observation of even a faint tidal current velocity in a small coastal inlet where ocean culture fields were being developed. His observations of tidal currents in the 1960's were the first case employing the newly-built offshore Oceanographic Tower in Japan. However, these observations were yet only for discrete data taken in day-time. The Tower was established in 1965 based on Professor Hayami's concept that any exact and detailed data could only be obtained by using a fixed offshore tower station. Since then, the Tower has been a key station for the fields of coastal disaster prevention and for the coastal oceanography and it is employed actively even at present(1987) for observation. However, the author has wondered whether his research (Nakamura, 1965)[12] was enough to determine the tidal currents in Tanabe Bay. From that time, various observations have been undertaken to promote research in related fields. However the author has yet to clarify the flow patterns of flood and ebb tides in Tanabe Bay numerically with special reference to the observations of others who have employed improved instrumentation (Current Observation Group, 1983)[13].

With the introduction of electronics and improved techniques into oceanographic instrumentation, for the observation of current velocity, there has been a significant innovation using acoustic signals under sea for current velocity measurement at a fixed point determined above the sea floor in a tidal estuary (Current Observation Group, 1983)[13]. Of course, oceanographic observations in Tanabe Bay have been undertaken successively by Professor Hideaki Kunishi et al.[14] after Hayami et al. (1964)[15] arranged and settled the functions of the offshore tower station as the Shirahama Oceanographic Tower. Subsequently, it was supervised by the head of the coastal disaster prevention research section, and has since become a facility of the Shirahama Oceanographic Observatory attached to the Disaster Prevention Research Institute, Kyoto University. The functions of the tower could not have worked well at first without Kunishi's assistance to Hayami (Kunishi, 1962)[16]. However, even with the efforts of Kunishi and his colleagues, the spatial and temporal properties of tidal currents in the bay have not yet been well determined, so the details are not understood at this time.

In addition, observations for detection of offshore properties of storm surges and tsunamis in recent years in relation to coastal disaster prevention, including coastal warning, protection, evacuation and mitigation of the hazards, are becoming one of the most essential undertaking to establish much more appropriate countermeasures along the coast. From his stand point, the author considers it a necessity to learn first what is the usual state prior to studying any abnormal phenomena caused by storms or the tsunamis.

Recently, Nakamura(1986)[1] has studied the response or water in a wide-open bay with a special reference to Tanabe Bay using a numerical model employing the finite difference method, and found that a node of the resonant mode exists just off the apparent opening of the wide-open bay even in the case of Tanabe Bay. Presently, the author's extending an application of numerical modelling to problems of semidiurnal oscillation in Tanabe Bay as a wide-open bay. This work can be taken as an extension of his work or just a supplementary note to that. And yet this work can also be a key to learn the flow patterns of ebb and flood tides in Tanabe Bay.

In this paper, first presents an overview of the topographical condition of the interested area in Tanabe Bay. Equations used for modelling are briefly introduced in the third section in order to help understand early steps of the dynamic analysis of the model. The fourth section concerns an assumed forcing function and the given condition for a numerical model of a finite difference method. Flow patterns for a semidiurnal oscillation as a result of numerical computation are compared to observed results revealing that the numerical result corresponds well with the previous current observations, especially regarding the properties of the tidal current ellipses for the M_2 constituent at the five mooring stations. Lastly, the author discusses prediction of tidal flow patterns, i. e., the ebb and flood tides in Tanabe Bay comparing the resonant mode with a node which could appear at a significant tsunami accompanying a strong earthquake under sea.

2. Tanabe Bay as a study area

The coastline and bathymetric lines in Tanabe Bay are shown in **Fig. 1.** The area of interest is in the rectangular area encircled by the latitudinal and longitudinal lines

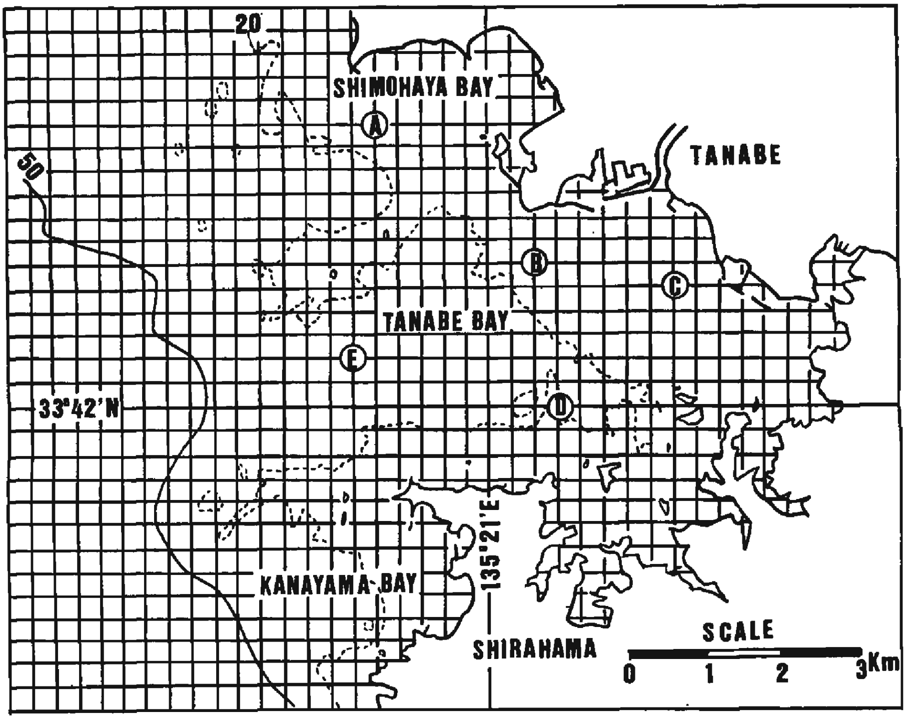

Fig. 1 Modelling area of Tanabe Bay as a wide-open bay.
(1) The sea area is covered by a 300 m spacing mesh.
(2) Five points A, B, C, D and E are the locations where the observed M_2 tide was available.

of 135° 7′ to 25′E and 33°38′ to 45′N, which corresponds to the nautical chart No. 74 published by the Hydrographic Office, Maritime Safety Agency. In **Fig. 1**, the depth contours of 20 m and 50 m are shown by dotted and full lines, respectively. With this, we can see that the geographical and topographical conditions are complicated in Tanabe Bay, even at a glance. For the author's convenience in numerical modeling, the area in **Fig. 1** is covered by a orthogonal Cartesian co-ordinate with a 300 m spacing mesh, and the values for water depth at the mesh points in the area are given after an interpolation of the depths found on the nautical chart. These interpolated values of the water depth are used in this work for numerical computation.

Tanabe Bay faces the western North Pacific and the maximum tidal range is about 200 cm, which is the maximum difference of sea level at the time of a new or full moon. For convenience, the first forty constituents of tides at Shirahama are tabulated. As seen in **Table 1**, the most significant constituents in the table are Sa(annual), O_1 and K_1(diurnal, respectively), and M_2 and S_2 (semidiurnal, respectively). M_2 has the largest amplitude of the constituents listed in **Table 1**. This suggests that the main trend of predicted astronomical tides could be found by studying the tide of the M_2 constituent. Though the tidal constituents are utilized for prediction of the tide level on the coast, there is no prediction for tidal currents in Tanabe Bay at this time. If

Table 1 The important tidal constituents at Shirahama

No. : constituent	amplitude (cm)	phase (degree)	remark*
1. Sa	14.23	156.84	S; Elliptic tide of the 1st order to S_0
2. Ssa	1.25	65.70	S; Declination tide to S_0
3. MSm	-	-	M; Evection tide to M_0
4. Mm	0.82	69.64	M; Elliptic tide of the 1st order to M_0
5. MSf	0.34	322.54	M; Variation tide to M_0
6. Mf	0.63	158.35	M; Declination tide to M_0
7. σ_1(sigma-1)	-	-	M; Variation tide to O_1
8. Q_1	3.44	158.36	M; Elliptic tide of the 1st order to O_1
9. ρ_1(rho-1)	0.64	159.46	M; Evection tide to O_1
10. O_1	16.89	170.11	M; Diurnal main lunar tide
11. NO_1	-	-	;
12. MP_1	0.24	173.36	;
13. M_1	0.80	188.29	;
14. π_1	0.38	181.17	;
15. P_1	7.14	188.72	S; Diurnal main solar tide
16. S_1	0.40	43.82	;
17. K_1	21.86	190.88	M, S; Diurnal main declination tide
18. ψ_1(psi-1)	0.55	167.87	;
19. ϕ_1(phi-1)	0.35	193.53	;
20. J_1	1.19	214.02	M; Elliptic tide of the 1st order to K_1
21. SO_1	0.23	279.49	;
22. OO_1	0.70	228.97	M; Diurnal declination tide of the 2nd order
23. $2N_2$	1.12	165.36	M; Elliptic tide of the 2nd order to M_2
24. μ_2	1.23	172.16	M; Gravitational variation tide to M_2
25. N_2	8.52	170.69	M; Gravitational elliptic tide of the 1st order to M_2
26. ν_2	1.77	172.47	M; Gravitational evection tide to M_2
27. OP_2	0.36	126.72	;
29. M_2	47.44	176.14	M; Semidiurnal main lunar tide
29. λ(lambda)	0.44	179.76	;
30. L_2	1.64	186.21	M; Kl. elliptic tide of the 1st order to M_2
31. T_2	1.18	177.23	S; Gravitational elliptic tide of the 1st order to S_2
32. S_2	21.15	200.41	S; Semidiurnal main solar tide
33. R_2	0.29	333.32	;
34. K_2	5.93	195.62	M, S; Semidiurnal declination tides to M_2 & S_2
35. $2SM_2$	0.12	85.08	;
36. MO_3	0.12	314.77	;
37. M_3	0.50	183.42	M; Tertiary-diurnal main lunar tide
38. MK_3	0.10	292.71	;
39. SK_3	0.24	30.81	;
40. M_4	0.19	35.76	;
41. MS_4	0.24	50.99	;
42. M_6	0.11	161.76	;
43. $2S_6$	0.12	188.32	;

* Several constituents are added with the remark referring to Tabelle 56 (p. 352) of "Allgemeine Meereskunde" by G. Dietrich (Gebrüder Bornträger, Berlin) in 1957. In the table Mo and So are the constant lunar and solar tides.

the shape of a bay is simple (for example, a rectangular bay with a constant width and a constant depth), it is easy to predict tidal currents accompanied by the predicted as shown in **Table 1**. In such cases, a simple application of hydrodynamics can be helpful. Nevertheless, Tanabe Bay has a complicated coast line with a significant bathymetric undulation in the area of interest as seen in **Fig. 1**. This is one of the causes for the lack of prediction for the area of Tanabe Bay.

As for Tanabe Bay, we have only a little data on the observed currents. The author feels it most fortunate that the Current Observation Group (1983)[13] has obtained "long-term observations of currents in Tanabe Bay" already been partly published at this time. The result has been analyzed and the data on currents has been obtained at five stations. Each station, where the currents were observed for about a month from November to December 1981, is shown by an encircled notation of A, B, C, D or in **Fig. 1.**

3. Equations

The details of the formulation can be found in the author's previous works, which was based at first on the package program for time-stepping long waves into coastal regions, referring to Loomis' work (1972)[2].

In order to study the tidal currents theoretically, we start to describe the motion for long waves for a homogeneous body of water with variable water depth. The coordinate is taken so that x axis is eastward, the y axis is northward and the z axis is upward positive, respectively. As an approximation, spatial and temporal variations of sea level are considered not to be large compared to the horizontal displacements of the water particles in certain water layer. This is one reason that it is possible for the author to use the equations of motion for long waves.

Now, the author starts to use the equations of motion for current velocities u and v of the x and y components. The equations of motion are as follows ;

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}-fv=-g\frac{\partial \zeta}{\partial x}+\frac{\partial F}{\partial x}+\frac{\partial \tau_x}{\partial z}, \quad \cdots\cdots(1)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}-fu=-g\frac{\partial \zeta}{\partial y}+\frac{\partial F}{\partial y}+\frac{\partial \tau_y}{\partial z}, \quad \cdots\cdots(2)$$

and the equation of continuity is

$$\frac{\partial \zeta}{\partial t}-(\zeta+h)\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right), \quad \cdots\cdots(3)$$

where the still sea surface was taken to be the plane which was formed by the x and y axes. The vertical displacement of the sea surface above the stationary sea level was denoted as ζ. The friction factor was expressed as τ with the suffix x or y for the x or y component. External forcing function was written as F. The Colioris factor f is given by 2Ω $\sin\varphi$, for the angular velocity of the earth's rotation Ω and the latitude φ of the area of interest in this study. The water depth below the stationary sea surface was denoted as h.

The numerical computation in this study was undertaken using a finite difference method which was developed first by Loomis (1972)[2]. And its scheme has been improved and adapted repeatedly by the author (Nakamura, 1981 a, b,[17][18] 1983 b,[19] 1984 c, d[20][21] 1985[22]) and by Nakamura and Allison (1983)[23]. For the convenience of numerical computation, the area for the model was taken as shown in **Fig. 1.** This area was covered by a 300 m spacing mesh, so as to be

$$\Delta x = \Delta y = 300\,m.$$

The deepest water depth in the model area was 91 m which was taken as a reference depth for the numerical model. The time stepping interval was taken as $\Delta t = 4.8$ min in order to get a stable solution for the numerical computation after consideration of Neumann's criteria for obtaining stable numerical solutions in the bay.

In addition to the above, the author feels it is necessary to remark upon what is the best way at present to obtain a solution with minimum truncation error. This was discussed previously (Nakamura, 1987)[24] for the problem of rewriting a differential equation in the form of a difference equation as the first approximation. Of course, all possible factors should be eliminated in advance to apply the numerical computation for the model. One of the troublesome factors is remarked on in the appendix. Fortunately, this factor in the appendix has only a trivial effect on the result of the numerical computation.

In practice, Coliori's effect must be taken to be negligible in the case of Tanabe Bay because the size of the area of interest is less than 10 km square which is too small for the effect of the earth's rotation to appear. Now, we have to be aware that no effect outside of the model area can be taken into account of consideration at the numerical computation.

As for evaluation of τ_x and τ_y, the author simply expresses the values as

$$\tau_x = k\rho u^2, \cdots\cdots (4)$$

and

$$\tau_y = k\rho v^2, \cdots\cdots (5)$$

which are similar to the author's previous work. On the other hand, the author feels it hard to find an appropriate approximation of exact water density ρ from the past observation of the area covering Tanabe Bay. Adding to that, it is not so easy to determine an approxmate value of the coefficient k. The value of k could be a function which is determined by spatial and temporal factors. The author took the values of $\rho = 1.03$ and $k = 2 \times 10^{-3}$, respectively. These values give a key to find the values of (4) and (5) which relate to shear stress of the flow acting on the sea floor.

4. Forcing function and conditions

In order to apply numerical computation to the model of Tanabe Bay, it is necessary to give a forcing function and the initial condition and boundary condition.

4. 1 Forcing function

As for the forcing function in the numerical model of Tanabe Bay, we have select a function which has a covenient property for successive analysis within the scope of hydrodynamics. In this case, a simple sinusoidal oscillation of the sea surfcce with a period of 12. 5 hours was taken as the open boundary line at the left end in **Fig. 1** for the author's convenience. As for the open boundaries at the top and at the bottom of **Fig. 1**, those are each to be taken as if they were rigid vertical walls. No remark is given about any effect of these open boundary conditions except some note in the following section.

4. 2 Initial condition

The initial condition for the numerical model of Tanabe Bay is given by assuming that there is no motion of water in the bay at the time $t=0$. That is to say, in an expression,

$$\zeta=0(\mathrm{cm}),\ u=0(\mathrm{cm/s})\ \text{and}\ v=0(\mathrm{cm/s})\ \text{at}\ t=0(\mathrm{sec}).$$

Exactly speaking, a discontinuous start of motion is expected in the model. However, this model is controlled by a selected time increment at time-stepping numerical computation, Δt, so that there must be an effect of mathematical relaxation at the first several steps for the numerical computation. This effect of relaxation is one of the keys to obtain a successful solution numerically even when the numerical model contains a problem of discontinuity.

4. 3 Boundary condition of sea surface

In this model, no external force is considered on the sea surface. So that, for example, no effects of atmospheric pressure and wind stress were taken into account.

4. 4 Boundary condition on the sea floor

In this model, the author assumed the effect of currents in the form of shearing stress on the sea floor. The stress on the sea floor can be expressed by τ_x and τ_y as the x and y component of the stress as indicated in the last section. In actual numerical computation, the effect of the stress on the sea floor is trivial in the deeper part of the model, though it is not negligible if it is shallow.

4. 5 Boundary condition on the coast

In this model, the face of the boundary on the coast is assumed to be vertical so as to completely reflect any incident wave and is assumed to be formed by combinations of the segment face of the boundary parallel to the x axis or to the y axis. This assumption suggests that the model has a discontinuous boundary as far as the mathematics of differential equation is concerned. The author worry that this assumption might be a cause of an erroneous solution for our numerical practice of solving problem by part even when the numerical solution is taken to be exact in a scope of the present numerical mathematics. The author is expecting to be clarified what is evaluation of resultant error in a numelical modelling. This seems not to be pointed ever or clarified in the field of applied mathematics. The author feels it pity to note here that he is not yet have any criteria for evaluation of error in a numerical modelling even after learning various schemes for numerical modelling which can be found as previous contribution by the countless scientists and engineers.

5. Reproduced Tidal Currents

A numerical result for a semidiurnal oscillation as a forcing function with an amplitude of 90 cm and a period of 12.5 hours is shown in **Fig. 2.** At the right end of **Fig. 2,** the sea level variation at the open boundary located at the left end of the

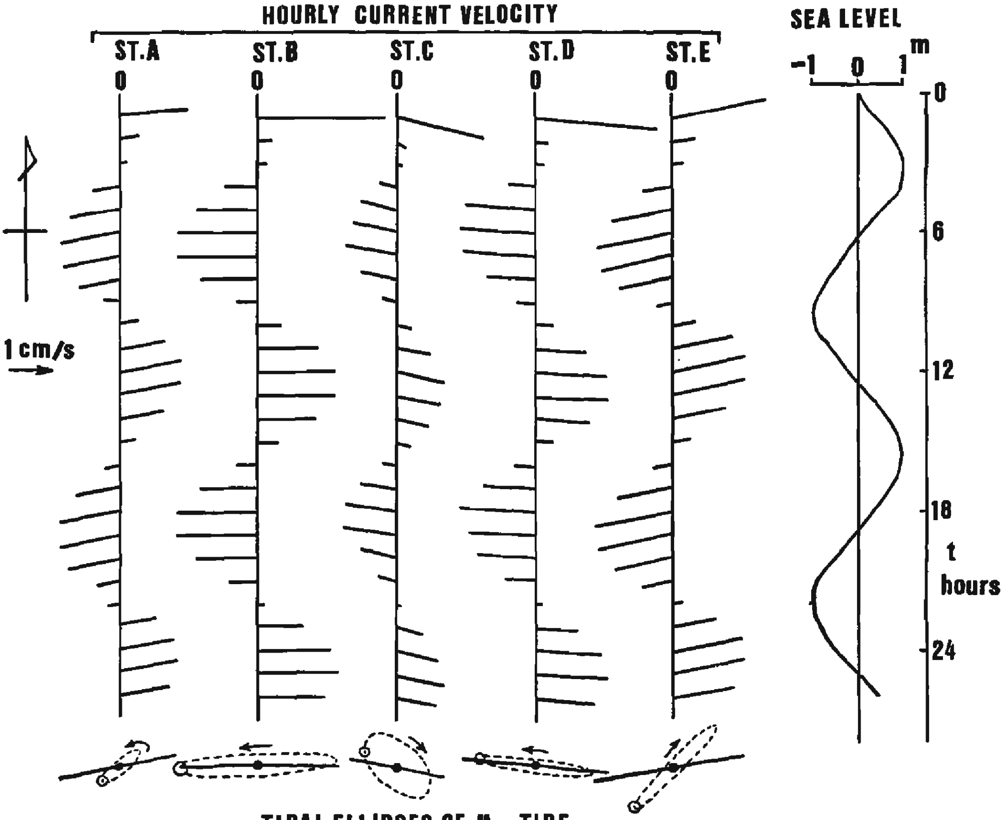

Fig. 2 Semidiurnal oscillation in the numerical model and the observed tidal ellipse of M_2 tide.

(1) Variation of the sea level in Tanabe Bay (in the model).

(2) Variation of hourly current velocity veocity vector at each point of the five corresponding stations.

(3) Comparison of the computed and observed tidal ellipses at each station (shown at the bottom). Solid line and dotted line represent computed results and observed results, respectively.

model area in **Fig. 1** is shown where the axis for time elapse is taken vertically and the displacement of the sea level is shown by the amount of horizontal deviation.

Looking at the result of the numerical computation, the author found that the sea level in the model of Tanabe Bay varies simultaneously with an almost identical amplitude in phase with little differences, which can be taken to be negligible. Such is the case in our experience for a small bay or inlet, so that the numerical result of the sea level variation in the model of Tanabe Bay can be taken reasonably even if some errors must be included mathematically. No illustration is prepared in this manuscript for the sea level variation in the model of Tanabe Bay because the author considered that such a illustration might not give us any new finding about the motion of the water in Tanabe Bay.

As for current velocity, a stick diagram of the current vector at each point corresponding to the respective stations, A, B, C, D or E is shown in **Fig. 2**. This is an expression of a time series of the current vector hourly variation. An effect of relaxation can be found in the initial part of the time series. A little distortion of sinusoidal evolution of the current velocity must be the typical effect of relaxation. Although, on the whole, we can find regular periodic variation of the current velocity except the initial part at each point corresponding to the station where the observation of the current velocity was undertaken. The author expected an ellipse of the current velocity vector obtained by the numerical computation, though the computed current ellipse at each point was very narrow so that it could be taken as an oscillatory current at each point in the model of Tanabe Bay.

At the point A (st. A) in **Fig. 2**, the computed major axis of the current ellipse is about twice that of the observed major axis of the tidal ellipse of M_2 tide. The azimuth of the major axis of the computed ellipse at each of the points B, C and D (st. B, st. C and st. D) agrees well with that of the corresponding observed ellipse.

At each of the points A and E (st. A and E), the azimuth of the computed ellipse is rotated 45° clockwise from the observed ellipse as a reference. The author's purpose was first to obtain a numerical result which reproduced almost all of the current vector field in the bay. Even after the author's repeated trials and errors, a numerical solution that only reproduced a part of the observation fairly well resulted.

Many numerical schemes have been applied successfully to reproduce the tides in a bay or inlet. The present study must be the only case study for the semidiurnal oscillation in Tanabe Bay at present. Therefore, a more exact prediction of tidal currents is requested for any bay of complicated shape.

6. Prediction of Tidal Flow Patternts

With the above numerical result and consideration, the author will attempt a speculative prediction of tidal flow patterns in Tanabe Bay. For the author's convenience, only the tidal flow patterns for M_2 constituent in Tanabe Bay will be focussed on.

As we have already seen, tidal current velocity varies from time to time at a fixed point or a fixed station. A set of the current velocities at the mesh points gives infor-

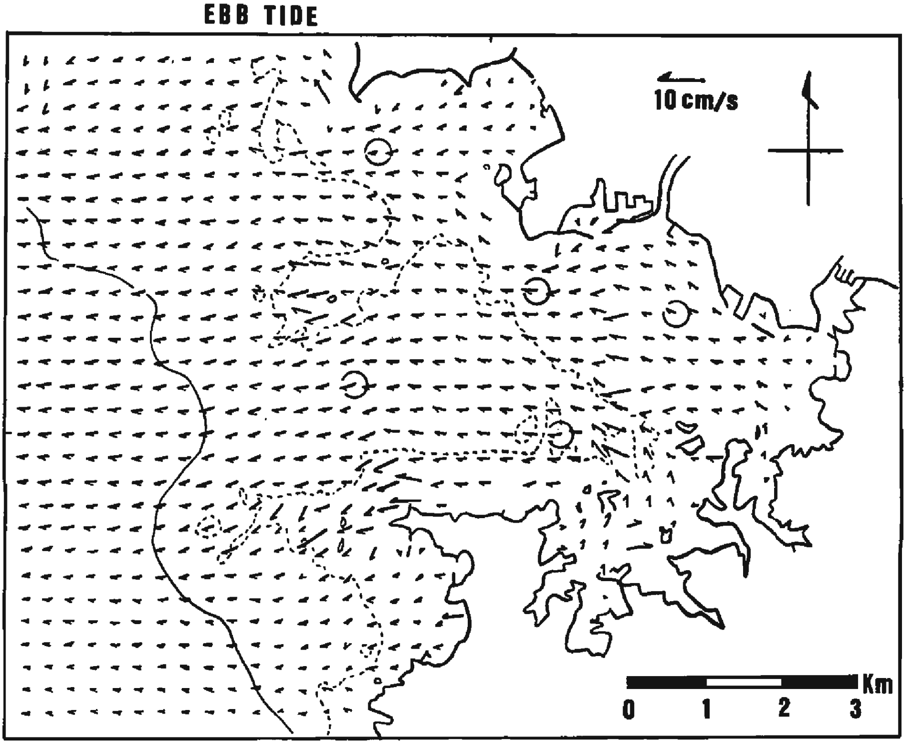

Fig. 3 Flow pattern of peak ebb for the numerical semidiurnal oscillation in the bay.

mation about a current flow pattern or a current velocity field in the bay. Especially in the case of tidal flow patterns, it is more comprehensive and attractive to prepare the flow patterns at the times of the flood and ebb tides. Accordingly, the author will present the flow patterns at the flood and ebb tides of M_2 contsituent in **Fig. 3** and **Fig. 4**, respectively.

The flow patterns shown in **Figs. 3** and **4** have been obtained for one case study. However, these may give a key to consider what is yet needed to ascertain flow patterns in a wide-open bay by using numerical techniques to construct a model.

The flow pattern of the peak ebb at point E(st. E) is shown in **Fig. 3**, where the magnitude of the current velocity vector at each mesh point was shown by the length of the arrow. This enables us to have a glance of the flow pattern at the time of the peak ebb at point E. In **Fig. 3**, the five encircled arrows are the current velocity vectors corresponding to the stations where the observed data of M_2 constituent were collected.

In **Fig. 4.** the flow pattern of the peak flood at point E(st. E) is shown. The flow pattern of the peak flood looks obtainable if every one of the current vector in **Fig. 3**, which are quite similar to those the author discussed in the last section concerning whether tidal currents in the numerical model were well reproduced or not. At a glance, current vectors in **Fig. 3**, if reversed, could be those in **Fig. 4**.

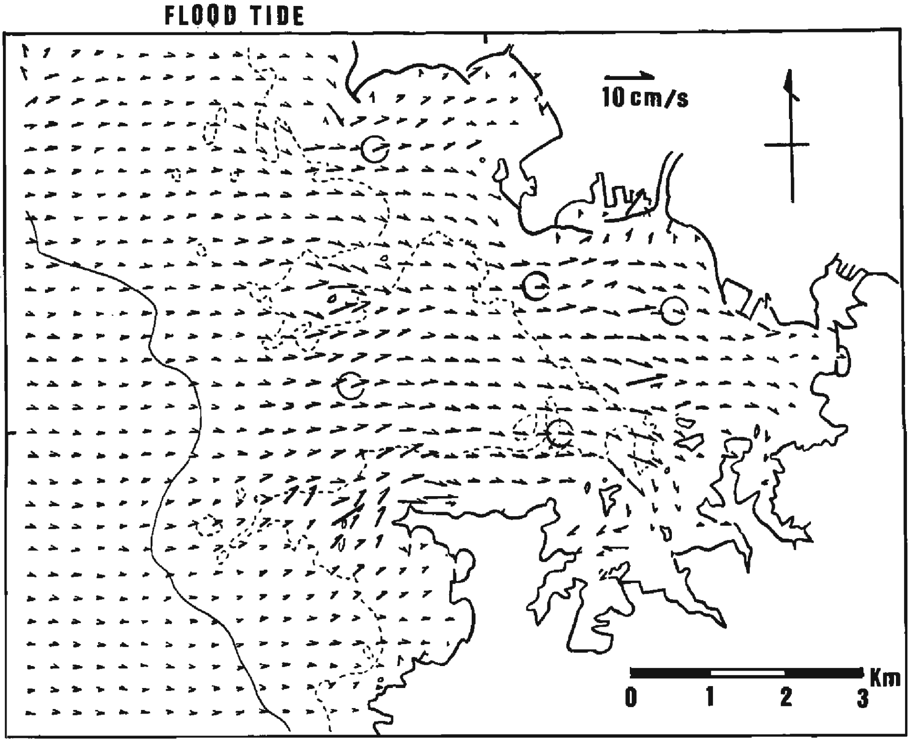

Fig. 4 Flow pattern of peak flood for the numerical semidiurnal oscillation in the bay.

Looking at the above two flow patterns shown in **Figs. 3** and **4**, we can see that the maximum semidiurnal oscillation with an amplitude of 90 cm and with a period of 12.5 hours is scarecely more than 10 cm/s in the actual Tanabe Bay including that of M_2 constituent in the model of Tanabe Bay as a wide-oppen bay. There must be other significant disturbance in Tanabe Bay, for example, wind stress on the sea surface, barometric resonance of the sea level or intrusions of offshore waters after any variations of Kuroshio-meandering. Of course, coastal waters and river discharges affect the motion of the waters in Tanabe Bay. Although, it is hard to consider that so-called "residual current" near the coast is significant in the area of Tanabe Bay. In my understanding about the recent activities in the field of coastal oceanography, they are simply concentrating on problems of the "residual current" near coast. This must be taken as if the author were attacking his colleagues. The author here expresses them to clarify what is the exact definition of "residual current". If the definition is based on a daily displacement of water flow driven by only the astronomical tides, the residual current must be included other currents, i. e., wind-induced currents, density currents, gradient currents, circulations of various kind in their scales, and etc. The author feels it strongly at this time considering the problem in a wide-open bay. For, example, one of the significant effects is Kuroshio Current off Tanabe Bay. Of course, the residual current can, however, be important for some specific problem in some areas where such a current exists and is significant. In this study, no consideration was payed for the residual current even though the author modelled a semidiural oscillation in a wide-open bay. Tanabe Bay as a wide-open bay is only a case study competative to the past coastal oceanography.

As the last of the author's note, he wishes to compare the result obtained in this work to the results previously obtained for the resonant mode with an offshore node in a numerical model of Tanabe Bay. Nakamura (1986)[1] Studied the rosponse of the water in Tanabe Bay as a wide-open bay to show that the resonant mode can appear in a wide-open bay as well as in a narrow-mouthed bay. The resonant mode seems to be agitated by an incident tsunami accompanying a significant earthquake under sea. There must, therefore, be essential differences between the flood and ebb flow patterns at the resonant mode (period is about 40 min) and at the semidiurnal oscillation considered in this work.

In these years, problems on "chaos" have been discussed in relation to a solution for differential equations of nonlinear phenomena (for example, Bergé et al., 1984,[25]; Thompson and Stewart, 1986)[26]. Of course, there must be all of the possible factors in the actual bay, for example, in Tanabe Bay. There must be an actual chaotic phenomenon. However, the author has never considered any problems concerning aperiodic phenomena or chaotic phenomena or bifurcation. The focus of this work is only on periodic problems apart of any quasiperiodic or chaotic regime.

7. Conclusions

Semidiurnal oscillation of waters in Tanabe Bay as a wide-open bay was studied by use of a numerical model with a finite difference method. The numerical result of the computed semidiurnal oscillation was compared to the observed M_2 tide in the model area with a fairly successful agreement, thus finding the flow patterns at ebb and flood tides especially for M_2 constituent in Tanabe Bay. This study can be an extension or supplement to the previous work on the response of water in a wide-open bay with a special reference to Tanabe Bay, such as the difference between the flow patterns for M_2 constituent and for the resonant mode which can appear when a tsunami strikes.

Acknowledgements

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Appendix Implicit Instability of Finite Difference Expressions

We express the first derivative of a continuous function u with a variable t in form of a finite difference expression, i. e.,

$$u' = du/dt = \Delta u/\Delta t. \qquad \text{(A-1)}$$

It seems to the author that most of the scientists and engineers trust the above equality are satisfyed to obtain a numerical solution of their model. However, the above expression is the first approximation so that they have to be careful to utilize finite difference expression of a differential equation for solving our problem on hand.

Who trusts $\Delta u/\Delta t=f$ is equivalent to $du/dt=f$ mathematically?

Rewriting (A-1) as

$$\Delta u=u(t+\Delta t)-u(t)=f\,\Delta t, \tag{A-2}$$

we can find an implicit instability of the finite difference expression.

Now, using a Taylor's series expansion of u around t, then,

$$u(t+\Delta t)=u(t)+(1/1!)\Delta t\ u'(t)+\ldots. \tag{A-3}$$

By considering (A-2) and (A-3), we have

$$u(t+\Delta t)-u(t)=\Delta t\cdot u'(t)=f(t)\Delta t, \tag{A-4}$$

or

$$u'(t)+au(t)-au(t+\Delta t)=0,\ \text{with}\ a=1/\Delta t. \tag{A-5}$$

In case of numerical modelling of time-stepping finite difference method, the detecting function of time t, for example, is not known apriori. When we consider a special case of the function $u=u_0\exp(kt)$, we have

$$u(t+\Delta t)=[(k+a)/a]u(t). \tag{A-6}$$

This means that the time-stepping solution can be amplified as much as $(k+a)/a$ at each iterative step in the numerical computation of a model. This property seems to be quite similar to a case of problems concerning a self-exciting system. Hence, we have to be careful when a finite difference expression is introduced for the purpose of solving a differential equation or a set of simultaneous differential equations. In the author's case, there has been no trouble. Althougn, it is possible an implicit instability of a finite difference expression to appear in other case. In such case, a divergent solution can be obtained depending on the selection of the values of k and a, even if any input function f was well defined.

The above point is reailzed by only a small number of scientists, and engineers, and most research workers in related fields surely not aware of this. The above point is essential to ensure whether a numerical solution is exact or not, if the solution is divergent. Therefore, the author believes it necessary to point out the implicit instability of the finite difference method for numerical simulations.

References

1) Nakamura, S. : A response of wide-open bay in a numerical model, Marine Geodesy, Vol. 11, No. 4, 1987, pp. 231-240.
2) Loomis, H. G. : A Package program for time-stepping long waves into coastal regions with application to Haleiwa Harbor, Oahu, NOAA-JTRE-79, HIG-72-21, University of Hawaii, 1972, pp. 1-33.
3) Nakamura, S. : A note on modes of oscillation induced in a Osaka Bay model, Proc. Internat. Conf. on Water Resources Development, May 12-14 1980, Taipei, Taiwan, 1980a, pp. 835-843.
4) Nakamura, S. : Normal modes of oscillation in relation to storm surge and tsunamis in Osaka

Bay, Japan(3), La mer, Tome 18, No. 4, 1980b, pp. 179-183.

5) Nakamura, S. : Tsunami flood control at the opening of a bay or harbur, Proc. 1983 Tsunami Symposium Hamburg (edited by E. Bernard), NOAA, 1983a, pp. 65-81.

6) Nakamura, S. : Tidal torrent control at the entrance of harbour, Proc. 4th Congress of APD of IAHR, Chiang-Mai, Thailand, 11-13 Sept. 1984, 1984b, pp. 567-578.

7) Nakamura, S. : Binuzdenie kolebanya *b* modeli zaliba Osaka, Trudi Darinebostochnogo Ordena Trudbogo Krasnogo Znameni Regionalinogo Nauchno-Issledobateriskogo Instituta, Bipusk 103, Dinamika Dlinnoperiodnih Boln *b* Okeane *i* Issledobanie Tsunami, 1984a, pp. 86-92 (in Russian).

8) Nakamura, S. and H. G. Loomis : Lowest mode of oscillations in a narrow-mouthed bay, Marine Geodesy, Vol. 4, No. 3, 1980, pp. 197-222.

9) Melchior, P. : Resolution of IAPSO, Proc. XVIII General Assembly IUGG Hamburg, Paris, 1984, pp. 371-375.

10) Nakamura, A. : Tidal currents in Tanabe Bay (I)-A study on the productivity of Tanabe Bay, Records of Oceanographic Works in Japan, Special No. 2, 1958, pp. 10-14.

11) Nakamura, A. : Tidal currents in Tanabe Bay(II)-A study of the productivity of Tanabe Bay, Records of Oceanographic Works in Japan, Special No. 3, 1959, pp. 1-16.

12) Nakamura, S. : A study on photoelectric current meters, Bull. Disaster Prevention Res. Inst., Kyoto University, Vol. 15, No. 90, 1965, pp. 63-70.

13) Current Observation Group : Long-term observations of currents in Tanabe Bay, Annuals of Disaster Prevention Res. Inst., Kyoto University, No. 26B-2, 1983, pp. 637-672.

14) Kunishi, H., K. Nishi and Y. Yusa : On observation of water temperature at Shirahama Oceanographic Tower, Annuals of Disaster Prevention Res. Inst., Kyoto University, No. 8, 1965, pp. 677-684.

15) Hayami, S., H. Kunishi and K. Nishi : On instruments and observed records at Shirahama Oceanographic Tower, Annuals of Disaster Prevention Res. Inst., Kyoto University, No. 7, 1964, pp. 434-453.

16) Kunishi, H. : On Oceanographic tower and installed instruments, Bull. Coastal Oceanography, Vol. 2, 1962, pp. 35-41.

17) Nakamura, S. : 1977 Sumbawa tsunami in a scope of numerical experiment, La mer, Tome 18, No. 4, 1980b, pp. 30-37.

18) Nakamura, S. : A numerical tsunami modelling in Osaka Bay and Kii Channel, La mer, Tome 19, No. 3, 1981b, pp. 105-110.

19) Nakamura, S. : Numerical tsunami model in Osaka Bay, Bull. Disaster Prevention Res. Inst., Kyoto University, Vol. 33, pt. 1, No. 295, 1983b, pp. 1-14.

20) Nakamura, S. : On tsunamis in Tanabe and Shirahama, Annuals of Disaster Prevention Res, Inst., Kyoto University, No. 27B-2, 1984c, pp. 591-610.

21) Nakamura, S. : A numerical tracking of the 1883 Krakatoa tsunami, Science of Tsunami Hazards (International Journal of Tsunami Society), Vol. 2, No.1, 1984d, pp. 41-54.

22) Nakamura, S. : On the response of coastal waters to offshore disturbance, La mer, Tome, 23, No. 3, 1985, pp. 111-117.

23) Nakamura, S. and H. Allison : On long period waves on the Western Australian coast, Proc. 28th Conf. on Coastal Engineering in Japan, JSCE, 1983, pp. 44-48.

24) Nakamura, S. : A note on numerical evaluation of tsunami threats by a simple hydrodynamic and stochastic models referring to historical descriptions, Bull. Disaster Prevention Res. Inst., Kyoto University, Vol. 37, pt. 1, 1987, pp. 1-18.

25) Bergé, P., Y. Pomeau and C. Vidal : L'ordre dans le chaos, translated into english by Tuckerman, L. as "Order within Chaos" and published in 1986, Wiley-Intersci. Pub., Herman and John Wiley and Sons, Inc., N. Y., 329p.

26) Thompson, J. M. T. and H. B. Stewart : Nonlinear dynamics and chaos, John Wiley and Sons, Chichester, 1986, 376p.