Laboratory Measurements of Elastic Wave Attenuation by Scattering due to Random Heterogeneities

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(Manuscript received November 25, 1987)

Abstract

Scattering attenuation and spatial fluctuation of P waves traveling through a scattering medium were examined experimentally by using an ultrasonic technique. 2-D models of random media used in laboratory experiments are characterized by a triangular correlation function and a 2.4% standard deviation in velocity and density. The range of wave length covered in the experiments corresponds to $2<ka<33$, where k is the wave number and a is the correlation distance of the heterogeneities, i.e., the heterogeneity size. The results obtained are as follows: (1) When the P waves travel sufficiently long compared with the wave length through the models of random media, the travel-time fluctuation, $\sigma(t)$, is small, i.e., about 3–4% of the wave period at intermediate frequencies of $ka=4$–13.5, but at a high frequency of $ka=33$ $\sigma(t)$ is large, i.e., about 11% of the wave period. In contrast to the above, the amplitude fluctuation, $\sigma(\ln A)$, is large, i.e., about 22–25% at $ka=4$–13.5, but at $ka=33$ it is small, i.e., less than 10%. (2) When the forward-scattering effect such as the travel-time fluctuation is neglected, the scattering attenuation, Q^{-1}, increases with ka for $ka<3$, has a peak around $ka=3$–5, then decreases with ka. This frequency dependence of Q^{-1} is in harmony with the observed frequency dependence of $Q_\beta{}^{-1}$ for shear waves. (3) From results (1) and (2) it is concluded that forward scattered energy should not be counted as lost energy when we calculate the scattering attenuation of seismic waves traveling though a scattering medium. (4) Result (1) suggests that initial motions as well as the subsequent phase of direct P or S wavelets are strongly contaminated by ununiform forward-scattered energy; Fourier amplitudes for entire parts of the wavelets should be used to determine the attenuation of the waves as a function of frequency.

1. Introduction

Recent seismological studies[1-2] suggest that the apparent attenuation, $Q_\beta{}^{-1}$, of shear waves for the frequency range of $1<f<25$ Hz may be caused by the loss of energy by scattering due to heterogeneities in the lithosphere. This hypothesis is based on the following factors: Attenuation of coda waves ($Q_c{}^{-1}$) determined by the single-scattering model for the generation of seismic coda nearly agrees with the $Q_\beta{}^{-1}$ determined from local earthquakes and this suggests that the coda consists of scattered S waves (**Fig. 1**)[2-7]; from a comparison of the Q_β determined from long-period surface waves ($f \ll 1$ Hz) with the Q_β determined from S waves of local earthquakes or their coda in the frequency range of $1<f<30$ Hz, it is conjectured that there exists a peak in $Q_\beta{}^{-1}$ in the lithosphere at 0.5–1.0 Hz (**Fig. 2**)[1,2,16-18]; the value of $Q_c{}^{-1}$ at frequencies around 0.5 to 1.0 Hz varies by an order of magnitude from a stable continent to active tectonic areas[2]; the $Q_\beta{}^{-1}$ curves converge to a certain low value at high frequencies, independent of current tectonic activity[2].

From the above, it is suggested that the frequency dependence of $Q_\beta{}^{-1}$, especial-

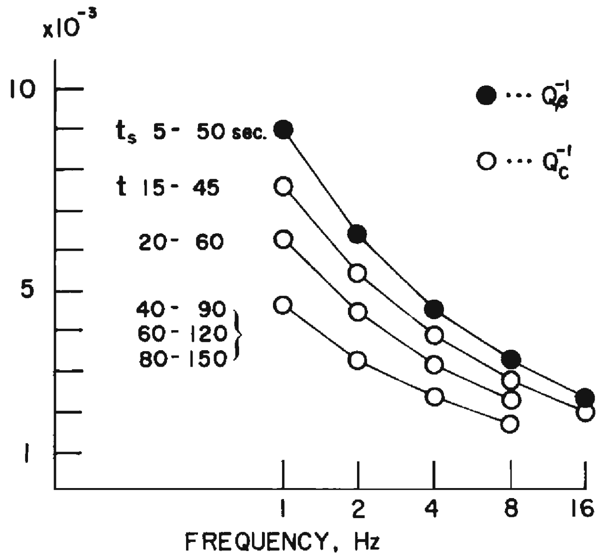

Fig. 1. Q_c^{-1} in various lapse time intervals and Q_β^{-1} for the Kinki district; t, lapse time; t_s, S-wave travel time. Note that Q_c^{-1} converges to Q_β^{-1} with the approach of the lapse time interval to the S travel time. (Reproduced from data listed in Tables 3 and 5 in Akamatsu's paper[6].)

ly the Q_β^{-1} value at 0.5–1.0 Hz, is strongly correlated with the intensity of current tectonic activity closely connected with the degree of heterogeneity in the lithosphere. The heterogeneities in the lithosphere, such as faults, cracks, velocity and/or density anomalies, are capable to scatter seismic waves effectively. Thus, the observations of Q_β^{-1} support the scattering hypothesis for shear-wave attenuation.

On the other hand, this hypothesis is strongly supported by the approximation theories for scattering attenuation of seismic waves traveling through a medium with random heterogeneities independently proposed by Wu[19–20] and Sato[21–22]. Their theories successfully derive the Q_β^{-1} curve characterized by a peak at frequencies around 0.5–1.0 Hz. In Wu's theory, only the energy that is scattered into the back half space is assumed lost, while in Sato's theory any energy that is scattered in a direction outside a small cone centered about the propagation direction of primary waves is assumed lost. The forward scattering effect, such as the travel-time fluctuation, caused by velocity heterogeneities is neglected in both theories.

Thus, the scattering hypothesis for shear-wave attenuation can more strongly be based on both the observed frequency dependence of Q_β^{-1} varying with the degree of heterogeneity in the lithosphere and the approximation theory proposed for apparent attenuation by scattering. Besides, the frequency dependence characterized by a peak at frequencies around 0.5–1.0 Hz seems to be correct, although it requires more and careful observations. Now, the hypothesis that the attenuation (Q_β^{-1}) is

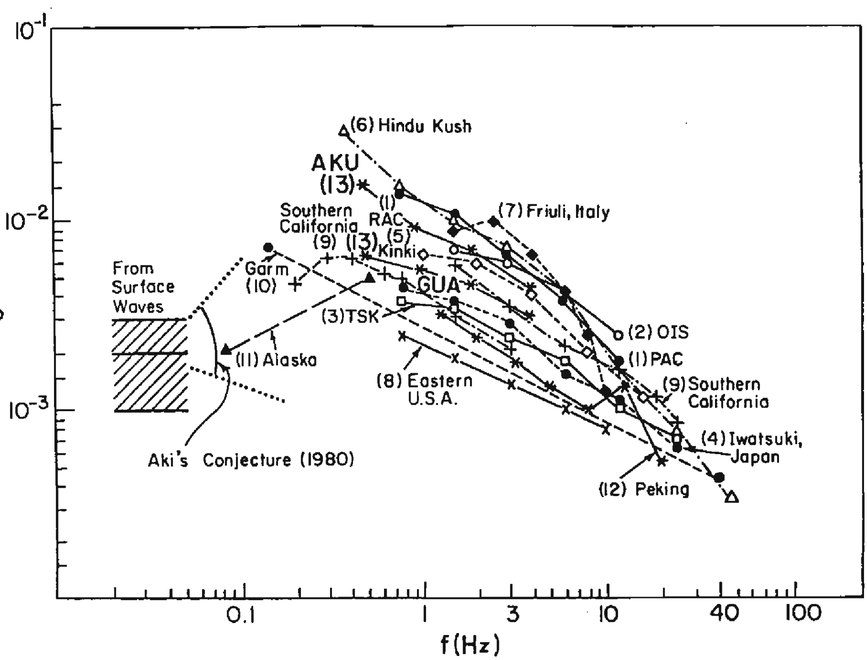

Fig. 2. Q_c^{-1} from local coda analysis: 1, PAC, central California, USA; 2, OIS, western Japan (Aki and Chouet[8]); 3, TSK, Kanto, Japan (Aki and Chouet[8]); 4, Iwatsuki, Japan (Tsujiura[3]); 5, Kinki, Japan (Akamatsu[9]); 6, Hindu Kush, Afghanistan (Roecker et al.[10]); 7, Friuli, Italy (Rovelli[11]); 8, eastern United States (New England area) (Pulli[12]); 9, southern California (Phillips[13]); 10, Garm, central Asia, USSR (Rautian and Khalturin[4]); 11, Alaska (Aki[14]); 12, Peking, China (R. B. Shi et al.[15]); 13, Akureyri (AKU), Iceland and Guam (GUA), Mariana Islands (Jin and Aki[15]). (Reproduced with permission from Jin and Aki[15])

due entirely to scattering, depends very strongly on whether the approximation theory for scattering attenuation is valid or not.

The validity of the proposition of Wu and Sato for the apparent attenuation by scattering has not yet been tested experimentally by the simultaneous examination of the amplitude attenuation and the wave fluctuation. When scattering is strong, i.e., when the scattered energy is not negligible compared with the incident energy, it is very difficult to theoretically synthesize seismograms in which multiple scattering phenomena can be evaluated correctly.

One alternative to the theoretical method is to produce seismograms in laboratory experiments for the models of scattering media using an ultrasonic technique. Seismograms made in this manner include all the multiple-scattered waves. In this study, therefore, I test the proposition for scattering attenuation by laboratory experiments for two-dimensional (2-D) model media with randomly distributed velocity and density heterogeneities using an ultrasonic technique.

In a previous study[23], I investigated the effects of strong scattering on elastic waves by similar laboratory experiments. In those experiments, spatial fluctuation of P waves and their apparent attenuation were examined only in the narrow range of $4<ka<13.5$, where k is the wave number and a is the heterogeneity size. In this study, I first examine the forward-scattering effect, such as spatial wave fluctuation, and the apparent attenuation for a wider range of $2<ka<33$ by using additional model media. Next, from the wave fluctuation and the attenuation property observed, I demonstrate the validity of the proposition for scattering attenuation of Wu and Sato.

In the experiment, I used 2-D models of media with randomly distributed velocity and density heterogeneities whose sizes are constant in any location within a model medium. Such a simple model medium can easily be prepared and facilitates the understanding of the physical mechanism of complicated scattering phenomena. The model media were made of duralumin plates using the property that the wave velocity and the density of a perforated plate decrease with the increase of porosity. The porosity necessary for this study was realized by drilling many small circular holes all over the plate. The porosity was varied randomly from one part of the plate to another.

2. Experiments

The apparatus, 2-D models of random media and the procedure are the same as in my papers[23,24] and they have been reported there in detail. For self-consistence of the paper, I describe the outline of the ultrasonic model experiments.

The transmitter and receiver are made of PZT ceramics and have diameters of 4 mm and the same resonance frequency, f (f=86–250 KHz). The received signals are stored in digital memory with a capacity and resolution 4 Kwords and 8 bits. The wave then can be analyzed by a micro-computer system. The standard error in the measurements of wave amplitude was within 6% when the receiver was reset with every measurement but the transmitter was fixed during the experiments, while it was 9% when both the transmitter and receiver were reset with every measurement. The frequency counter was used to measure the travel time. The resolution of the counter was ± 0.1 microsecond. Wave velocity was determined from the travel time curves, the standard error being within 1%.

Since the statistical properties of the model media play important roles in wave propagation in the media, I describe below how to prepare the model media, as well as their statistical properties.

The model media were made of duralumin plates using the property that the wave velocity and the density of a duralumin plate decrease effectively with the increase of porosity. The porosity necessary for the present study was attained by making small circular holes at the centers and the vertexes of regularly drawn hexagons on the plate as shown in **Fig. 3.** I prepared three 2-D models (RA1, RA2 and RA3) of the medium with randomly distributed heterogeneities of wave velocity and density. The dimensions of the duralumin plates used as the models are 80 cm in length, 60 cm in width and 0.2 cm in thickness for RA1, RA2 and RA3. The

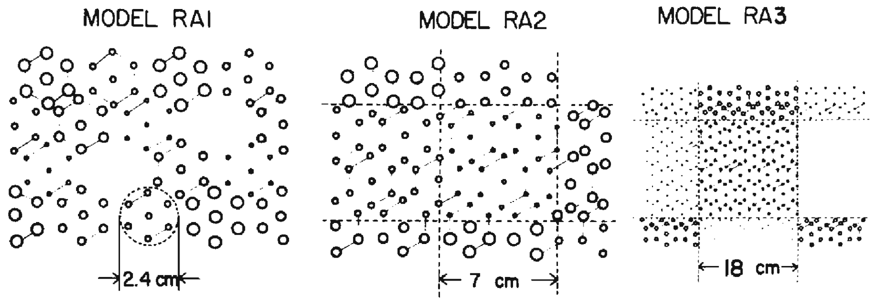

Fig. 3. Distribution of holes in model RA1, RA2 and RA3. Heterogeneity sizes a are 2.4 cm, 7–10 cm and 18–25 cm for RA1, RA2 and RA3, respectively.

process of construction of RA1 is as follows. First, regular hexagons of 1 cm/side are drawn on the surface of duralumin plate using a net of rhombuses of 0.5 cm/side. Next, as shown in **Fig. 3**, small circular holes, whose diameter are randomly varied in the range of 0 to 2.4 mm from one hexagon to another, are made at each vertex of these hexagons and their centers. The diameters of the holes of RA2 and RA3, as shown in **Fig. 3**, are randomly varied in the range of 0 to 2.4 mm in each square of 7 cm and 18 cm/side, respectively. For RA1, the porosity of the circle unit area of about 2.4 cm in diameter randomly varies from one unit to another, and accordingly the wave velocity and the effective density also vary randomly. Similarly for RA2 and RA3, the wave velocity and the effective density of the square unit areas of 7 cm and 18 cm/side vary randomly from one unit to another, respectively. The range of randomly distributed porosities in these models is from 0 to 6% and the median is 3%. The wave velocity of each unit of these models is determined from the relation between the wave velocity and the porosity which has been reported in the previous paper[24]. For these models, the mean velocities of P and S waves are 5.21 mm/microsec and 3.03 mm/microsec, respectively, and the mean density is 2.7 g/cm^3. The root mean squares (rms) of fractional velocity fluctuation ($\Delta V/V$) and fractional density fluctuation ($\Delta\rho/\rho$) are about 2.4%, where V and ρ are the wave velocity and the density, respectively. As seen from **Fig. 3**, there is a difference in the shape of the unit area (heterogeneity) between RA1 and the other two models (RA2, RA3). For RA1, the shape of the unit area is circular, while for RA2 and RA3 it is square. Thus, for RA1, heterogeneity size is a unit diameter 2.4 cm, while for RA2 and RA3 it ranges from 7 cm (lengths of sides) to 10 cm (diagonal lengths) and from 18 cm (lengths of sides) to 25 cm (diagonal lengths), respectively. Therefore, for RA2 and RA3, I took an average of the lengths of sides and diagonals to define the mean value, a, as the heterogeneity size. The values of a are 2.4 cm, 8.5 cm and 21 cm for RA1, RA2 and RA3, respectively.

The autocorrelation function of velocity and density fluctuations made in this manner is nearly approximated by a triangular correlation function, $N(r)$, shown in **Fig. 4**, where r is the distance between arbitrary two points in the medium. The

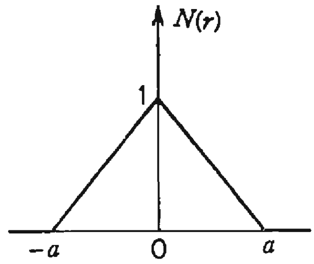

Fig. 4. Autocorrelation function of velocity and density fluctuations for models used in the experiments. The correlation distance is nearly equal to heterogeneity size a.

Fourier transform of $N(r)$, i.e., $P(|\boldsymbol{K}|)$, represents the power spectrum of the velocity and density fluctuations, where $\boldsymbol{K}$ is a wave number vector. Accordingly, fluctuation components with wave-lengths longer than the heterogeneity size, a, are dominant in the model media. When the waves travel through the model media, they are scattered by these fluctuation components that act as scatterers.

The most important difference among the models (RA1, RA2 and RA3) is the heterogeneity size a. When ultrasonic transducers with several different resonance frequencies are used for the same model, the scattering effects by the heterogeneities given as spatial velocity and density fluctuations can be examined in a wider range of dimensionless frequency ka, where k and a are the wave number and the heterogeneity size, respectively.

In the models, there exist many small circular holes which are capable to scatter a wave. This scattering may contaminate the experiments aiming at the scattering due to the velocity and density fluctuations. Accordingly, I examined the scattering effect due to small circular holes in a preliminary experiment[25]. On the basis of those results, for the present study, I employed ultrasonic transducers with resonance frequencies at which the scattering due to the holes is negligible.

Figure 5 shows a schematic diagram of the experiment. P waves were observed along profiles transversal to the direction of wave propagation. Here, the x axis was taken along the profiles, and the distance between the transmitter and the profile was represented by L. The upper part of this figure shows a form of typical P-wave train observed in the experiments. As shown in this figure, the arrival times of the P waves were determined from their onset times indicated by arrows, and the amplitude $A_i (i=1, 2$ and $3)$, which is the amplitude of the ith phase of the P-wave train, was measured. The transmitter was fixed in the middle of one of the shorter edges of the model, the receiver with the same resonance frequency was moved at intervals of 1 cm along the other shorter edge, and the amplitude A_i of the P waves observed and their arrival times were measured. It was shown in the previous work[25] that the radiation pattern of the P waves from the transmitter could be ignored at the observation point when the incidence angle was less than

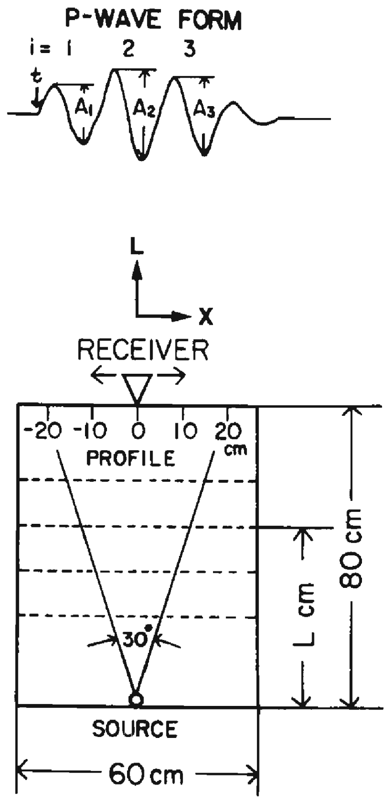

Fig. 5. (Top) Typical P-wave train radiated from the source. A_i ($i=1$, 2 and 3) is the amplitude between the ith phase of the P-wave train.
(Bottom) Schematic diagram of measurements. P-wave trains were observed along profiles transversal to the direction of wave propagation. To vary the travel distance, L, the models were successively cut along the dashed lines.

15°. Accordingly, the observation points, where the incidence angle was less than 15°, were used as shown in **Fig. 5.** Thus, the profiles are considered to be nearly parallel to the wave front, and therefore the direction of the oscillation of the P waves observed is considered to be nearly perpendicular to the profiles. The travel distance, L, was shortened successively by trimming the plate along the dotted lines, and measurements were carried out along new profiles as shown in **Fig. 5.**

3. Spatial fluctuation

In the statistical theory of wave scattering in random media by Chernov[26], the wave field ψ is composed of a mean wave field $\langle\psi\rangle$ (coherent component) and a randomly fluctuating wave field $\delta\psi$ (incoherent component), that is, $\psi=\langle\psi\rangle+\delta\psi$, where $\langle\ \rangle$ denotes an ensemble average. Both random velocity and density heterogeneities are capable to scatter waves sideward and backward. Velocity heterogeneity is also capable to scatter waves forward, while density heterogeity is not. Accordingly, the incoherent component is considered to be waves scattered by random velocity heterogeneities as they travel forward. The observed wave field ψ spatially fluctuates affected by the interference between the coherent component and the forward-scattered incoherent one. The coherent component $\langle\psi\rangle$ apparently attenuates with the increase of travel distance by side- and back-scattering due to random density and velocity heterogeneities.

According to Chernov's theory, amplitude $A(x)$ of the wave field and its phase $\varphi(x)$, observed at an arbitrary observation point, x, in the profile of distance, L, from the source, are random functions stationary for the direction perpendicular to the direction of wave propagation, that is, for the direction of the x-axis. Accordingly, the average amplitude $\bar{A}$ and the average phase $\bar{\varphi}$ over the profile are taken as the amplitude of the mean wave field and its phase, respectively. Thus, the amplitude $\delta A(x)$ of the random wave field and its phase $\delta\varphi(x)$ are obtained from the relations of $\delta A(x)=A(x)-\bar{A}$ and $\delta\varphi(x)=\varphi(x)-\bar{\varphi}$, respectively. According to Nikolayev[27], when $\delta A(x)$ is small compared with $\bar{A}$, the natural logarithmic amplitude $lnA(x)$ can be written as

$$lnA(x) = ln\bar{A}+\delta lnA(x), \quad \delta lnA(x) \simeq \delta A(x)/\bar{A},$$

and therefore

$$\delta lnA(x) = lnA(x)-ln\bar{A}, \tag{1}$$

where the bars over A and φ denote the average value in the profile. $\delta lnA(x)$ is hereafter defined as the amplitude fluctuation along the profile (x-axis).

First, I examined the property of the wave fluctuation along the profile. **Figure 6** shows the phase fluctuation $\delta\varphi(x)$ (denoted by solid lines) of the P waves along the profile at distance $L=70$ cm (about 18 times the wave length) from the source and the amplitude fluctuation $\delta lnA_1(x)$ (denoted by dashed lines) of the first phase of the P-wave train. The phase fluctuation $\delta\varphi(x)$ is obtained from the relation of $\delta\varphi(x)=2\pi f\delta t(x)$, where f is the wave frequency and $\delta t(x)$ is the fluctuation of travel time of the P waves, i.e., $\delta t(x)=t(x)-\bar{t}$. The average travel time $\bar{t}$ is estimated from the relation $\bar{t}=R/(5.21$ mm/microsec), where R is the distance of a direct wave path between the source and the receiver, and 5.21 mm/microsec is the mean velocity of the P waves. **Figure 6-(A)** shows the fluctuation in the homogeneous model in which the porosity is uniformly 3%. **Figures 6-(B)**, **6-(C)** and **6-(D)** show the fluctuations of $ka=33$ (RA3), $ka=13.5$ (RA2) and $ka=4$ (RA1), respectively. From comparisons of the fluctuations for these four cases, the following properties are found: (1) The fluctuation for the homogeneous model is not appreciable compared with the other three cases, i.e., the standard deviation of the amplitude fluctuation, $\sigma(lnA)$, is less than 6% and that of travel time, $\sigma(t)$, is

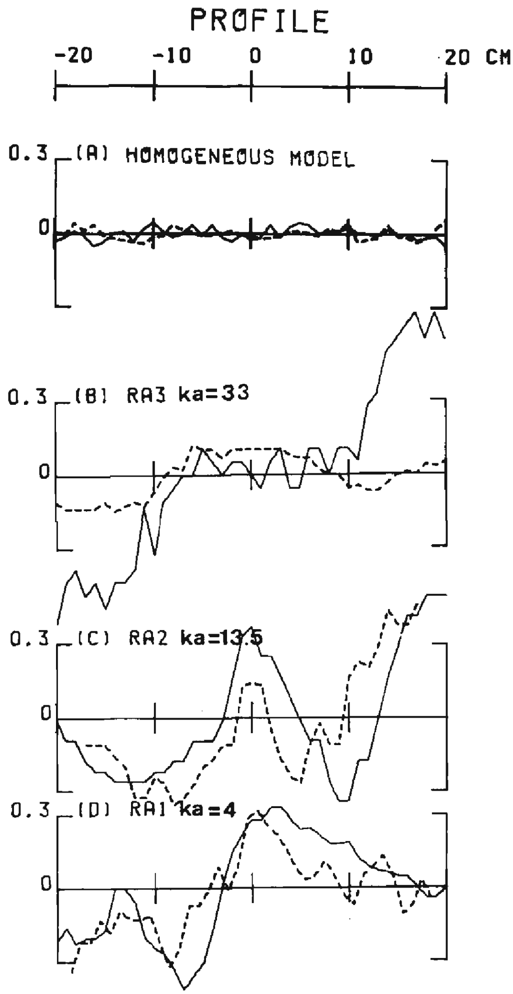

Fig. 6. Wave fluctuations along profiles of distance $L=70$ cm from the source for the homogeneous model ((A)) and heterogeneous models ((B), (C) and (D)). (B): RA3 ($ka=33$), (C): RA2 ($ka=13.5$), (D): RA1 ($ka=4$). Solid and dashed lines represent phase fluctuations, $2\pi f\delta t$, and amplitude fluctuations, $\delta \ln A_1$, respectively; f, wave frequency; δt, travel-time fluctuation; $\delta \ln A_1$, amplitude fluctuation.

less than 0.1 microsec; (2) the amplitude fluctuation of $ka=33$ is not appreciable i.e., $\sigma(lnA)$ is less than 10%, but the phase fluctuation is considerably large, i.e., $\sigma(t)$ is about 0.8 microsec; (3) the fluctuations are notable at $ka=13.5$ and $ka=4$, where $\sigma(lnA)$ are 22–25% and $\sigma(t)$ are 0.2–0.3 microsec. From properties (1) and (3), it is considered that the notable fluctuations in both cases of $ka=13.5$ and $ka=4$ are evidently the forward-scattering effects caused by the velocity heterogeneities.

The wave frequency used, f, is about 130–135 KHz. Accordingly, the wave period, T, is about 7.5 microsec for all cases. The ratio of $\sigma(t)$ to T is about 3–4% for $ka=4$ and $ka=13.5$, while it increases to about 11% with the increase of ka to 33. Thus, the travel-time fluctuation is large at high frequencies of $ka>10$. Therefore, when we take an average of the amplitudes of such P waves with phase shifts and phase lags over the profile, it is necessary to correct for the phase difference of each P wave observed in the profile. The amplitude fluctuation at high frequencies, on the other hand, is not appreciable compared with the phase fluctuation. In addition to property (2) for $ka=33$, it was found that the phase fluctuation roughly agrees with the variation of the travel time calculated along the wave path. These properties for $ka=33$ suggest that the wave behaves ray-theoretically in such a high-frequency range ($ka \gg 1$).

4. Apparent attenuation

As stated in section 3, the waves apparently attenuate with the increase of travel distance by side- and back-scattering due to the density and velocity heterogeneities. Usually, scattering attenuation, Q^{-1}, which shows the wave attenuation by wave scattering due to various heterogeneities, is estimated from the energy loss due to the scattering of the coherent component of the wave field. As shown in section 3, the travel-time fluctuation was marked. In this section, therefore, I took an average of the amplitudes of the P waves corrected for the phase difference over the profile. The average amplitude obtained in such a manner is considered to be the amplitude of the coherent component of the wave field, because any effect of forward scattering was eliminated from the direct P waves by the correction for the phase difference and by averaging the amplitudes over the profile. Q^{-1} for several frequency-bands was estimated from the attenuation of the average amplitude of the P waves with distance traveled.

When α is the scattering coefficient of energy and γ is the absorption coefficient, the natural logarithmic amplitude corrected for geometrical spreading, $ln(L^{0.5}\bar{A})$, of the average amplitude of the P waves observed attenuates with the increase of distance, L, from the source to the profile in the following manner,

$$ln(L^{0.5}\bar{A}) \propto -1/2 \cdot (\alpha+\gamma)\, L. \qquad (3)$$

The value of scattering attenuation, Q^{-1}, was estimated from the relation of $Q^{-1}=\alpha/k$. The value of γ of duralumin plates was previously estimated to be 0.008 cm^{-1} for the frequency range used in the experiment[25]. Accordingly, the value of α was obtained from the diminution of $ln(L^{0.5}\bar{A})$ with the increase of distance, L, as shown in **Fig. 7.** These α values estimated from eq. (3) are listed in **Table 1,** and **Figure 8** shows the ka-dependence of Q^{-1} values estimated (denoted by open circles). These Q^{-1} values have standard deviations of about 50%. As seen from **Fig. 8,** Q^{-1} increases with frequency in the low-frequency range of $ka<3$ where the heterogeneity

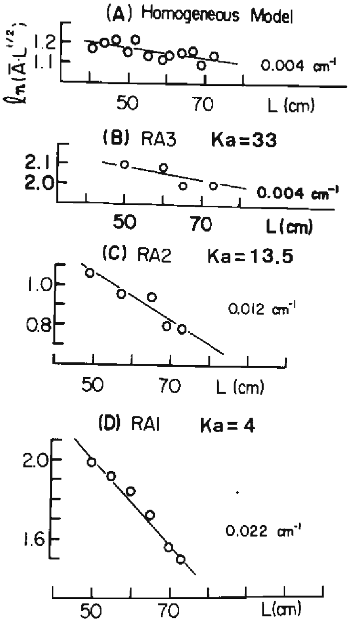

Fig. 7. Attenuation of natural logarithmic amplitude of the P waves with distance, *L*.

Table 1. Values of scattering coefficient

Model	Resonance Frequency of Source, f in KHz.	ka	Scattering Coefficient, α in $10^2\ cm^{-1}$.
RA1	86	2.5	0.6 ± 0.4
RA1	110	3.2	2.2 ± 1.1
RA1	130	4.0	3.6 ± 1.8
RA1	130	4.0	3.5 ± 1.8
RA1	150	4.3	3.6 ± 1.8
RA1	250	4.8	3.8 ± 2.0
RA2	110	11.2 ± 2	0.8 ± 0.4
RA2	120	12.2 ± 2	0.9 ± 0.5
RA2	130	13.2 ± 2.5	0.8 ± 0.4
RA2	135	13.5 ± 2.5	1.6 ± 0.8
RA3	135	33.0 ± 6.6	$\cong 0$

Uncertainties ($\pm$) are the standard errors.

size is smaller than the wave length, has a peak around $ka=3$–5 where the heterogeneity size is comparable to the wave length, and then decreases monotonously with frequency in the high-frequency range of $ka>5$ where the heterogeneity size is larger than the wave length.

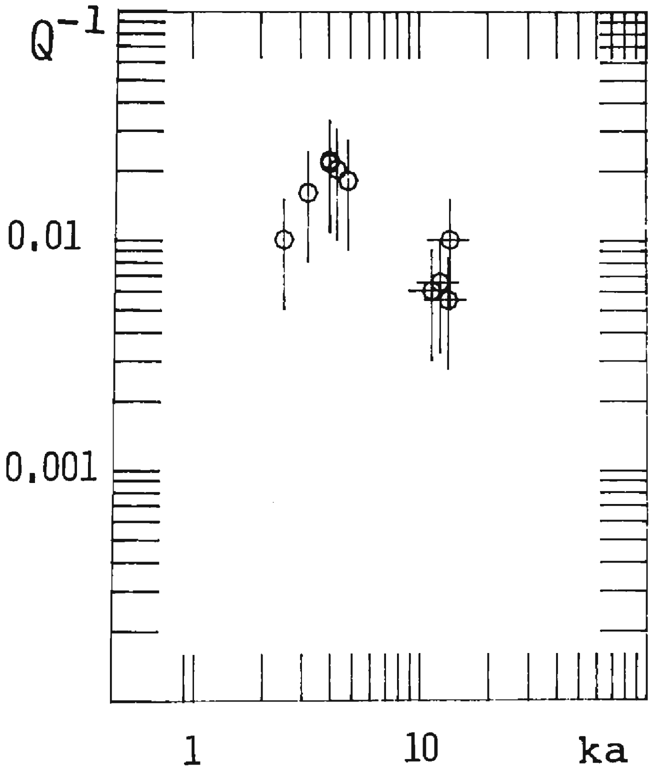

Fig. 8. Measured values of scattering attenuation (Q^{-1}). Bars denote the standard deviation.

5. Discussion

The spatial wave fluctuation and the amplitude attenuation by scattering were examined in the wide range of $2<ka<33$, when the P waves traveled through 2-D models of random media characterized by a triangular correlation function and a 2.4% standard deviation in velocity and density. As shown in section 3, the travel-time fluctuation by forward scattering due to velocity heterogeneities was marked. Especially at the high frequency of $ka=33$, the ratio of the standard deviation of travel time, $\sigma(t)$, to the wave period, T, i.e., $\sigma(t)/T$, was about 11%. Accordingly, before the amplitudes of the P waves observed were averaged over the profile, the P waves were corrected for the phase shift and the phase lag caused by the forward scattering. The scattering attenuation (Q^{-1}) of the coherent component of the wave field obtained in such a manner showed a frequency dependence characterized by a peak at $ka=3$–5.

There have been two fundamental statistical theories in treating the attenuation of seismic waves due to scattering. One is the single-scattering approximation[26]. The other is the binary interaction approximation in the mean wave formalism[28-29].

Both theories predict that Q^{-1} monotonously increases with frequency even in the high frequency range. This prediction disagrees with the experimental result, shown in **Fig. 8,** that Q^{-1} decreases monotonously with frequency in the high-frequency range. Therefore, these statistical theories can not explain the amplitude attenuation of the P waves observed in the models of random media, especially in the high-frequency range.

Wu[19-20] and Sato[21-22] pointed out the reason why Q^{-1} mistakenly increased with frequency in the high-frequency range fiom the point of view of the mean wave formalism. Their explanation is as follows: forward scattered waves, which are strong in the high-frequency range, may reach the receiver at about the same time as the primary waves because they have close propagation directions and close propagation velocities to those of the primary waves; therefore, forward scattered energy should not be counted as lost energy; however, the forward-scattered energy was mistakenly counted as energy loss in the single-scattering and the binary interaction approximations[26,28-29]. From the above consideration, they independently derived the Q^{-1} value that has a peak around the frequency corresponding to the heterogeneity size and decreases with frequency in the high-frequency range.

The frequency dependence of Q^{-1} obtained in this experiment, especially the decreasing behavior of Q^{-1} with frequency at $ka > 5$, is due entirely to the neglect of the forward-scattering effects (that is, due to the correction for the phase difference) which is equivalent to the elimination of energy scattered round the propagation direction of the P waves in calculating energy loss by scattering. This evidently demonstrates that the explanation of Wu and Sato for the reason why Q^{-1} mistakenly increases with frequency in the mean wave formalism in the high-frequency range is valid. Therefore, it follows that we can treat scattering attenuation of seismic waves by using the approximation theories each proposed by Wu and Sato.

I have reported a comparison of the Q^{-1} values obtained in the experiment with those predicted by the approximation theory of Wu in other paper[24]. The principal results reported are (1) that the theoretical Q^{-1} curve roughly matches the observed Q^{-1} values at $\theta_{\min}=15°$ and $\sigma=0.075$, where $\theta_{\min}$ is the minimum scattering angle measured from the propagation direction of the P waves and σ is the rms of fractional velocity fluctuation; (2) that the approximation theory overestimates by about three times the σ value of the model medium used owing to the neglect of multiple scattering.

Usual methods which attempt to determine the attenuation value (e.g., ${Q_\beta}^{-1}$ of S waves) of the direct waves observed as a function of frequency examine the amplitude decay with distance for specific frequency bands. It is well known that body-wave radiation pattern from an earthquake source, local site effect, focusing and defocusing of seismic waves caused by large heterogeneities in the lithosphere produce significant errors in the determination of the attenuation value from the observed amplitudes.

In addition to the above factors, the forward-scattering effects can also cause a significant error in the determination of the attenuation value when the lithosphere is a scattering medium. As shown in section 3, the amplitude fluctuation, $\sigma(lnA)$, is large, i.e., 22–25%, at $ka=4$–13.5, and the ratio of the travel-time fluctuation to the wave period, $\sigma(t)/T$, is large, i.e., about 11%, at $ka=33$. Thus, the forward scattering effects are very marked at intermediate- and high-frequency ranges. This shows that initial motions, as well as the subsequent phases forming direct P or S wavelets are strongly contaminated by forward-scattered waves. Therefore, methods, which use the amplitudes or pulse-widths of specific phases (e.g., initial motions) of the direct wavelets to determine the attenuation value, may result in a significant error.

To reduce the error due to the forward-scattering effects, we should use spectral amplitudes obtained by Fourier analysis of entire parts of the wavelets. The reasons for this proposition are as follows: Forward-scattered energy is ununiformly superposed on the coherent energy in the direct wavelets; therefore, to more accurately estimate the total energy (i.e., the sum of the coherent and forward-scattered energies) of the waves as a function of frequency, we should use Fourier amplitudes of the entire wavelets.

6. Conclusions

In summary, the principal results obtained in this study are

(1) When the P waves travel sufficiently long compared with the wave length through 2-D models of random media approximately characterized by a triangular correlation function and a 2.4% standard deviation in velocity and density, the travel-time fluctuation, $\sigma(t)$, is small, i.e., about 3–4% of the wave period at intermediate frequencies of $ka=4$–13.5, but at the high frequency of $ka=33$ it is large, i.e., about 11% of the wave period. In contrast to the above, the amplitude fluctuation, $\sigma(lnA)$, is large, i.e., about 22–25% at $ka=4$–13.5, but at $ka=33$ it is small, i.e., less than 10%.

(2) When the forward-scattering effects such as the travel-time fluctuation is neglected, the scattering attenuation obtained, Q^{-1}, increases with frequency in the low-frequency range of $ka<3$, has a peak around $ka=3$–5 in the intermediate-frequency range, then decreases monotonously with frequency in the high-frequency range. This frequency dependence of Q^{-1} is in harmony with the observed frequency dependence of $Q_\beta{}^{-1}$ for shear waves.

(3) From results (1) and (2) it is concluded that forward scattered energy should not be counted as lost energy as pointed out by Wu and Sato when we calculate scattering attenuation of seismic waves traveling through a scattering medium.

(4) Result (1) suggests that initial motions, as well as the subsequent phases of direct P or S wavelets are strongly contaminated by ununiform superposition of forward-scattered energy on the coherent energy in the wavelets; therefore, Fourier amplitudes for entire parts of the wavelets should be used to determine the attenuation value, such as $Q_\beta{}^{-1}$, of the waves as a function of frequency.

Acknowledgements

I thank Prof. Soji Yoshikawa, Associate Prof. Kojiro Irikura and Associate Prof. Yoshimasa Kobayashi of Kyoto University for their critical readings of my manuscript and for their valuable suggestions on carrying out my research. I am also indebted to Miss Keiko Amano for her assistance in preparing my manuscript.

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