Computation of Spectral Coefficients of Vorticity and Divergence from Wind Data for Use in Spectral Atmospheric Models

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(Manuscript received June 26, 1989)

Abstract

A method for computing the spectral coefficients of vorticity and divergence from the wind data for use in spectral models of the atmosphere is described. The procedure for transforming the equations from physical space to spectral space is briefly presented. The transformation of different terms of the equations for vorticity and divergence from physical space to spectral space through Fourier space is derived. It is shown that the transformation has to be carried out in Fourier space and the final equations for spectral coefficients for vorticity and divergence are expressed in terms of Fourier coefficients of wind components.

1. Introduction

In the spectral models, the governing equations are expressed in spectral space (Laplace space) and integration is performed to compute the spectral coefficients at every time step. So the input for these models are the spectral coefficients of the model variables. Observations of meteorological variables are available at different locations in physical space and for use in numerical models these data are interpolated, using some objective analysis procedure, for regular spaced points on the physical grid at intersections of latitude and longitude on the horizontal plane at different vertical levels. To obtain spectral coefficients, this data has to be transformed from physical space to spectral space.

In atmospheric models, which are simplified using the quasigeostrophic assumption following Lorenz[1)], vorticity and divergence are the model variables in place of wind components. So these quasigeostrophic spectral models require the input of spectral coefficients of vorticity and divergence instead of the wind components and the transformation is to be carried out from the spectral coefficients of wind components to maintain the compatibility of the truncation being used. Rochas[2)] has described the method of computation of wind components on the physical grid from spectral coefficients of velocity potential and stream function and emphasised the use of Fourier space in the computation.

In this paper, a method for computing the spectral coefficients of vorticity and divergence (or equivalently, stream function and velocity potential) from spectral coefficients of the zonal and meridional components of wind is described and shown that the transformation takes place in Fourier space. The method, though it might have been implied in many model formulations has not been explicitly described in the literature and is presented here. The expressions derived in this paper will be useful when spectral coefficients of vorticity and divergence are required to be computed to be used as input at the initial time step in spectral models, whereas the expressions given by Rochas[2] will be useful in obtaining the grid point data of wind components from spectral coefficients of stream function and velocity potential.

2. The method

The procedure for transforming a variable from physical space to spectral space is briefly stated as follows. If $A(\lambda, \mu)$ is a variable on a physical grid then the spectral coefficients $A_n{}^m$ are obtained from

$$A_n{}^m = \frac{1}{2}\int_{-1}^{+1}\frac{1}{2\pi}\int_0^{2\pi} A(\lambda, \mu)\, e^{-im\lambda} P_n{}^m \, d\lambda d\mu \tag{1}$$

which is carried out in two steps. First Fourier coefficients are obtained on each latitude circle from

$$A^m(\mu) = \frac{1}{2\pi}\int_0^{2\pi} A(\lambda, \mu)\, e^{-im\lambda} d\lambda \tag{2}$$

which may be computed through the use of Fast Fourier Transformation. The spectral coefficients are computed from Fourier coefficients using Gaussian quadrature from

$$A_n{}^m = \frac{1}{2}\sum_{j=1}^{J} A^m(\mu_j) P_n{}^m(\mu_j) W_j \tag{3}$$

where λ is longitude; $\mu_j = \sin\phi_j$; ϕ_j is the j th latitude; W_j are Gaussian weights at the j th latitude point; and J= the number of latitudinal points from pole to pole; and $P_n{}^m$ are associated Legendre functions.

The equations for vorticity ζ and divergence δ are

$$\zeta = \frac{1}{a(1-\mu^2)}\frac{\partial V}{\partial \lambda} - \frac{1}{a}\frac{\partial U}{\partial \mu} \tag{4a}$$

and

$$\delta = \frac{1}{a(1-\mu^2)}\frac{\partial U}{\partial \lambda} + \frac{1}{a}\frac{\partial V}{\partial \mu} \tag{4b}$$

where $U = u\cos\phi$; $V = v\cos\phi$; u and v are zonal and meridional components of the wind and a is the radius of earth.

To obtain the spectral coefficients of ζ and δ, we need the differentials with respect to λ and μ on the right hand side of equation (4) to be transformed to spectral space. It is convenient to transform the terms into Fourier space and then to express the spectral coefficients of ζ and δ in terms of the Fourier coefficients of U and V as shown below.

In Fourier space, the first terms on the right hand side of equation (4) are written using equation (2) as

$$\left[\frac{1}{a(1-\mu^2)}\frac{\partial}{\partial\lambda}\begin{Bmatrix}U\\V\end{Bmatrix}\right]^m=\frac{1}{2\pi}\int_0^{2\pi}\frac{1}{a(1-\mu^2)}\frac{\partial}{\partial\lambda}\begin{Bmatrix}U(\lambda,\mu)\\V(\lambda,\mu)\end{Bmatrix}e^{-im\lambda}d\lambda \qquad (5)$$

Integration by parts and using circular boundary condition gives

$$\left[\frac{1}{a(1-\mu^2)}\frac{\partial}{\partial\lambda}\begin{Bmatrix}U\\V\end{Bmatrix}\right]^m=\frac{1}{a(1-\mu^2)}\,\mathrm{im}\,\frac{1}{2\pi}\int_0^{2\pi}\begin{Bmatrix}U(\lambda,\mu)\\V(\lambda,\mu)\end{Bmatrix}e^{-im\lambda}d\lambda;$$

that is,

$$\left[\frac{1}{a(1-\mu^2)}\frac{\partial}{\partial\lambda}\begin{Bmatrix}U\\V\end{Bmatrix}\right]^m=\frac{1}{a(1-\mu^2)}\,\mathrm{im}\begin{Bmatrix}U^m(\mu)\\V^m(\mu)\end{Bmatrix} \qquad (6)$$

Using equation (3), the above equation (6) is transformed into spectral space as

$$\left[\frac{1}{a(1-\mu^2)}\frac{\partial}{\partial\lambda}\begin{Bmatrix}U\\V\end{Bmatrix}\right]_n^m=\frac{1}{2}\sum_{j=1}^{J}\mathrm{im}\,\frac{1}{a(1-\mu_j^2)}\begin{Bmatrix}U^m(\mu_j)\\V^m(\mu_j)\end{Bmatrix}P_n^m(\mu_j)W_j \qquad (7)$$

The second term on the right hand side of equation (4) is written as

$$\left[\frac{1}{a}\frac{\partial}{\partial\mu}\begin{Bmatrix}U\\V\end{Bmatrix}\right]_n^m=\frac{1}{2}\int_{-1}^{+1}\frac{1}{a}\frac{1}{2\pi}\int_0^{2\pi}\frac{\partial}{\partial\mu}\begin{Bmatrix}U(\lambda,\mu)\\V(\lambda,\mu)\end{Bmatrix}e^{-im\lambda}d\lambda P_n^m d\mu;$$

that is,

$$\left[\frac{1}{a}\frac{\partial}{\partial\mu}\begin{Bmatrix}U\\V\end{Bmatrix}\right]_n^m=\frac{1}{2}\int_{-1}^{+1}\frac{1}{a}\frac{\partial}{\partial\mu}\begin{Bmatrix}U^m(\mu)\\V^m(\mu)\end{Bmatrix}P_n^m d\mu \qquad (8)$$

Integration by parts and using zero boundary conditions at the poles give

$$\left[\frac{1}{a}\frac{\partial}{\partial\mu}\begin{Bmatrix}U\\V\end{Bmatrix}\right]_n^m=-\frac{1}{2}\int_{-1}^{+1}\frac{1}{a(1-\mu^2)}\begin{Bmatrix}U^m(\mu)\\V^m(\mu)\end{Bmatrix}(1-\mu^2)\frac{\partial}{\partial\mu}P_n^m d\mu;$$

that is,

$$\left[\frac{1}{a}\frac{\partial}{\partial\mu}\begin{Bmatrix}U\\V\end{Bmatrix}\right]_n^m=-\frac{1}{2}\sum_{j=1}^{J}\frac{1}{a(1-\mu_j^2)}\begin{Bmatrix}U^m(\mu_j)\\V^m(\mu_j)\end{Bmatrix}H_n^m(\mu_j)W_j \qquad (9)$$

where

$$H_n^m=(1-\mu^2)\frac{\partial}{\partial\mu}P_n^m$$

Now we can write the equations for the spectral coefficients of vorticity and divergence as

$$\zeta_n^m=\frac{1}{2}\sum_{j=1}^{J}W_j\frac{1}{a(1-\mu_j^2)}[\mathrm{im}\,V^m(\mu_j)P_n^m(\mu_j)+U^m(\mu_i)H_n^m(\mu_i)] \qquad (10a)$$

and

$$\delta_n^m=\frac{1}{2}\sum_{j=1}^{J}W_j\frac{1}{a(1-\mu_j^2)}[\mathrm{im}\,U^m(\mu_j)P_n^m(\mu_j)-V^m(\mu_j)H_n^m(\mu_j)] \qquad (10b)$$

An expression for vorticity similar to (10a) seems to have been given in the report of J.M.A.[3].

3. Conclusions

The method described above shows that the computation of the spectral coefficients of vorticity and divergence from the wind components have to be carried out in Fourier space. The expressions obtained in equation (10) are useful whenever spectral coefficients of vorticity and divergence are to be computed from the coefficients of wind or from wind data on the physical grid.

Acknowledgements

The author gratefully acknowledges the award of a fellowship under the exchange programme of Indian National Science Academy and Japan Society for Promotion of Science and deputation from Andhra University, Waltair, India which enabled him to carry out this work at the Disaster Prevention Research Institute, Kyoto University. The author also expresses his gratitude to Prof. Y. Mitsuta for his kind support and encouragement.

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