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Solitary Internal Waves in Lake Biwa

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Abstract

This paper describes the behavior of large-amplitude nonlinear internal waves observed in Lake Biwa. A thermistor chain was moored in November of 1992, when the water column in the lake was still well stratified. After a continuous strong north-west wind, a wave train with two distinctive parts was observed traveling from the west to the east. First, a sharp front which caused a temperature decrease of 3.5°C within two minutes appeared. Next, 10 spatially coherent pulses and a lower-amplitude wave train that was less coherent then followed it. We interpret the initial pulses as solitons according to the nonlinear wave theory. The trailing waves can be well-explained by the infinitesimal wave theory. It can be shown by a two layer model (with a thick homogeneous epilimnion, a thin hypolimnion and the thermocline simplified to an interface) that the wave lengths of the solitons are about 51 m to 63 m and their speeds are estimated within a range of 18.92 cm/s to 22.06 cm/s. Thermocline elevations which are predicted by the two-layer model are consistent with the characteristics of the solitons. Subsequent analysis shows that the solitons with periods of 6 to 10 minutes were generated in the western coastal area approximately 5 km away from the sampling station.

1. Introduction

There have been many observations about internal Kelvin waves in Lake Biwa (e. q. Kanari, 1984), but no observation of internal solitary waves has yet been made because of the restrictions imposed by low frequency sampling strategy. There is particular interest in observations of internal solitary waves in Lake Biwa because nonlinear wave theory has been developed to explain the behavior of internal waves in which nonlinear effects play important roles.

A high frequency interrogation sampling program was conducted between the 14th and 25th of November 1992 to catch a sequence of solitary waves in the lake.

It has been shown that when the pressure field is hydrostatic everywhere, the phase velocity of a progressive finite amplitude surface wave is faster at the crest than that at the trough. Therefore, the wave tends to steepen forward, break and consequently form bores. The tendency of a long surface wave to steepen forward, corresponding to a nonlinear effect, may be balanced by an opposite tendency, relative to a dispersive effect, because of the nonhydrostatic pressure field. Thus a solution for a permanent wave may exist when nonlinearity balances against the dispersive effect. Long (1965) has shown that nonlinear long internal waves exist in a field of density-stratified fluid and that both initial internal isopycnal elevation and depression may induce internal solitary waves depending on a different basic density structure. Observations of Ziegenbein (1969) in the strait of Gibraltar showed a detailed picture of wave trains and preceding internal fronts with amplitudes ranging from a few meters to 40 meters, and periods of 6 to 20 minutes. Similar observations have also been made in Loch Ness (Thorpe, 1971, 1972), Massachusetts Bay (Halpern, 1971), Lake Seneca (Hunkins & Fliegel, 1973), coastal area in California (Winant, 1974) and in Babine Lake (Farmer, 1978). Kunkins & Fliegel (1973) showed that the non-stationary solutions of the Korteweg-de Vries (KdV) equation obtained by various authors provide insight into the nonlinear internal waves observed in Lake Seneca. Lee and Beardsley (1974) modeled nonlinear internal waves with arbitrary density distributions using the KdV equation, and found that the numerical nonstationary solutions agree excellently with a laboratory experiment, and could explain the essential features of Halpern's (1971) observation in Massachusetts Bay. Maxworthy (1980), by laboratory experiments, showed that the collapse of a mixed region, with a potential energy greater than its surroundings, results in the generation of a number of large-amplitude internal solitary waves. Such large-amplitude waves have closed internal streamlines in the mixed fluid which is derived mainly from the collapsed region. As the wave propagates it carries this mixed fluid with itself. This process is particularly important for understanding the mechanism of horizontal and vertical transport of materials by solitary waves in Lake Biwa because it is related to the biogeochemical processes operating in this lake.

It was found at the turn of this century that the KdV equation can be applied to nonlinear dispersive waves in shallow-water. Assuming the motion is stationary, Benjamin (1966) found that permanent internal waves are possible in shear flows with stratification. By using a two-parameter perturbation expansion method, Benney (1966) extended Benjamin's work to describe the slow evolution of internal waves by the KdV equation under the condition of fairly small amplitude long wave length in comparison with the total fluid depth. To describe long internal waves in a two layer system, in which one layer is infinitely deep and the other is shallow, the Benjamin-Ono equation was presented (Benjamin 1967; Davis & Acrivos 1967; Ono 1975). Recently, there has been considerable interest in an Intermediate Long-Wave equation (ILW) which is a weakly nonlinear integro-differential equation that governs the evolution of long internal waves in a stratified fluid of finite depth (Joseph, 1977). For two extreme conditions, the ILW equation reduces to the KdV equation and Benjamin-Ono (BO) equation separately. Both the KdV and ILW equations were tested experimentally by Segur & Hammack (1982) through comparing measured and theoretical profiles of solitons. A solitary wave solution of the ILW equation was found by Joseph (1977), and finally after much labor, a periodic solution of the equation was derived by Miloh (1990).

In the present study, the instrumentation for observation is introduced in section 2. In section 3, the isotherm display and analogue display of temperature are employed to show the characteristics of measured solitary waves in Lake Biwa. Time series of current vectors, which may display the background current in the observation domain, are shown also. Theoretical analyses are extended in section 4.

2. Instrumentation

Lake Biwa is a typical deep lake with a deep oligo-mesotrophic central north basin and an adjacent shallow eutrophic embayment, the south basin. The mean depth of the northern basin is 44 m, and 3.5 m in the southern basins, the maximum depth of each basin is 104 m and 8 m, respectively. During summer and fall the lake is thermally stratified, having a warm surface layer (epilimnion) and a deep layer (hypolimnion). During spring and summer, the eplimnion is gradually warmed, reaching a maximum temperature of about 26°C in August. During fall, the epilimnion begins to cool and to deepen until the lake becomes nearly isothermal in December.

In order to observe traveling internal disturbances, a two-point observation was carried out from the 14th until the 25th of November, 1992, when the thermocline was

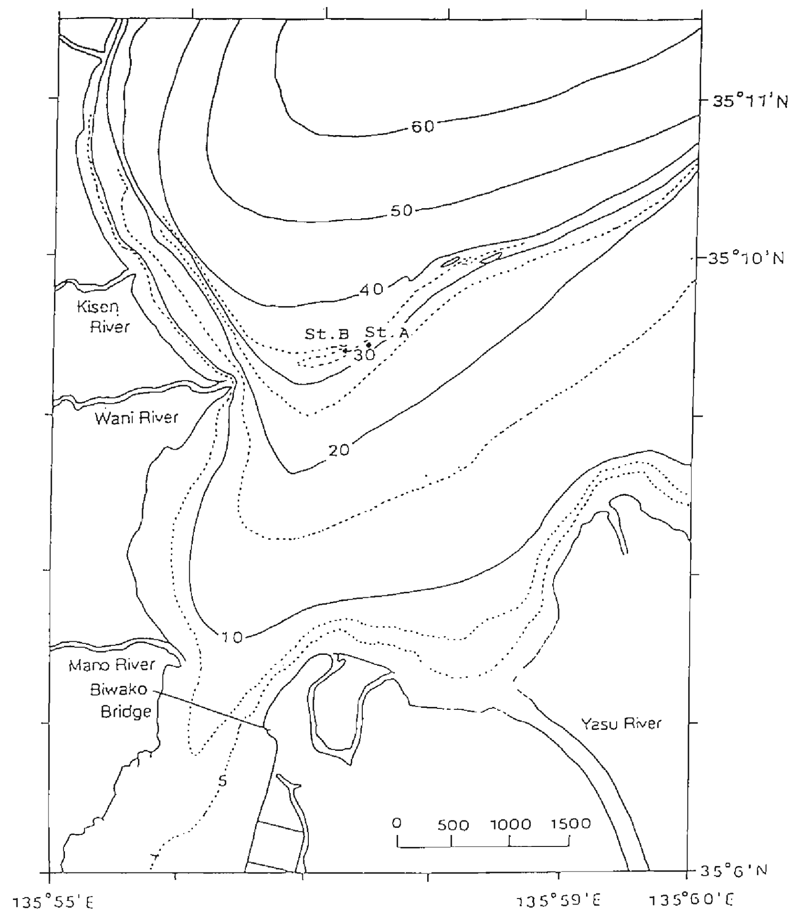

Fig. 1. A map of the transition between north and south basins of Lake Biwa. The contours of 10, 20, 30, 40, 50, 60 meter in depth contours are shown by solid line. St. A carries six electromagnetic current meters, and St. B carries a thermistor chain.

about 25 m deep. The instruments were moored at station-A (35°09′ 24.1″N, 135°57′ 23.9″E) and station-B (35°09′ 24.0″N, 135°57′ 25.1″E), which were 200 m away from each other as shown in **Fig. 1**. At station-A (32.7 m deep) six electro-magnetic current meters, which were equipped with current, temperature and turbidity sensors, were spaced vertically at 2 m interval from the bottom. At station-B (34.5 m deep) a 10 meter thermistor chain with 11 sensors was moored vertically from the bottom (**Fig. 2**).

The thermistor chain is an AANDERAA instrument, with temperature sensors having a response time of 1 minute. Each thermistor was individually calibrated against standards traceable to a quartz thermometer (Hewlett-Packard 2801A) which was calibrated to within 0.005°C by means of an Equiphase Cell. The precision and accuracy of the final temperature measurements are 0.03°C and 0.05°C, respectively. The data were recorded on a digital magnetic tape, and were analysed after the measurement. Calibration corrections were done to make them suitable for analysis.

All of these instruments were interrogated at 2 minute intervals. These instruments were not operated simultaneously throughout the observation period. Unfor-

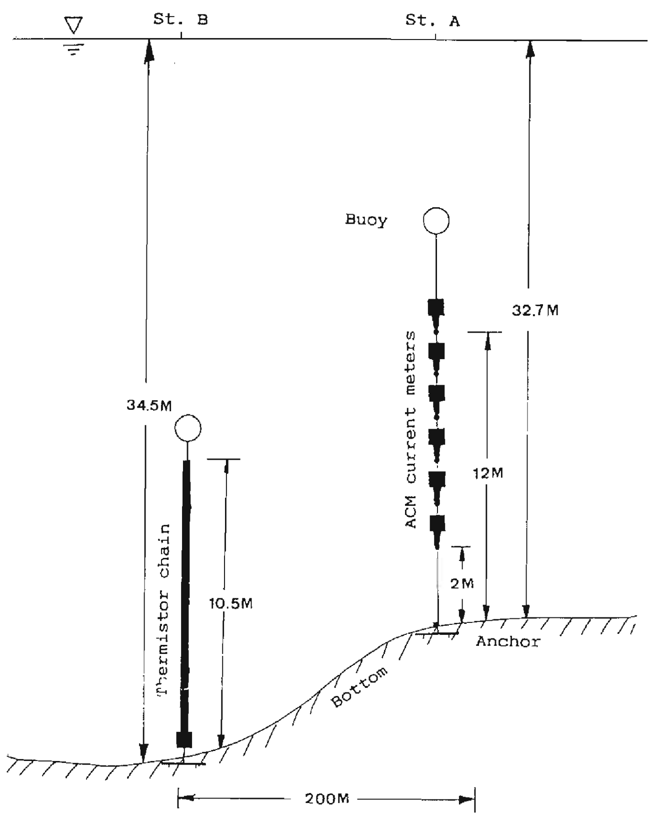

Fig. 2. Schematic diagram of the instrumentation structure. ACM = electromagnetic current meter. Depths of water, heights of the mooring systems from bottom, and distance between the two stations are shown in the figure.

tunately, we could not see the traveling characteristics of internal nonlinear waves in this observation because the memory of the instruments at station-A became full, ceased operating before so that those waves passed station-B.

3. Observation and analysis

Consequently an internal front and related wave trains which were observed at station-B, starting from 16:10 of November 22nd, 1992 can be analysed

There are two possible ways of presenting the data: analogue and isotherm displays. **Figure 3** shows an analogue display of the raw data. Time is passing along the horizontal axis; the vertical axis represents temperature. The scanning rate was 0.5 scans per minute. The eleven traces represent the temperatures measured at each sensor placed at 0.3, 0.8, 1.3, 1.8, 3.8, 5.8, 7.8, 8.3, 8.8, 9.3, 9.8 meters from the bottom respectively. Two fronts are clearly observable. The first one decreases the temperature of the sensor at 0.3 m height by 4°C within 6 minutes and the second makes the temperature at 3.8 m height decrease 3.5°C within 2 minutes. The second front is followed by a highly coherent wave train and then infinitesimal amplitude waves. Although such a display describes quite clearly the temperature changes at the different levels, it does not allow for the actual characterization of amplitude and shape of internal waves, which may only be possible with an isotherm-display.

The heights of the isotherms have been calculated using a linear interpolation between the 11 temperature measurements. In **Fig. 4**, time is passing along the horizontal axis, the vertical axis indicates the heights of the isotherms. Starting from the left of **Fig. 4**, one can first see a well mixed isothermal layer through the measured range of

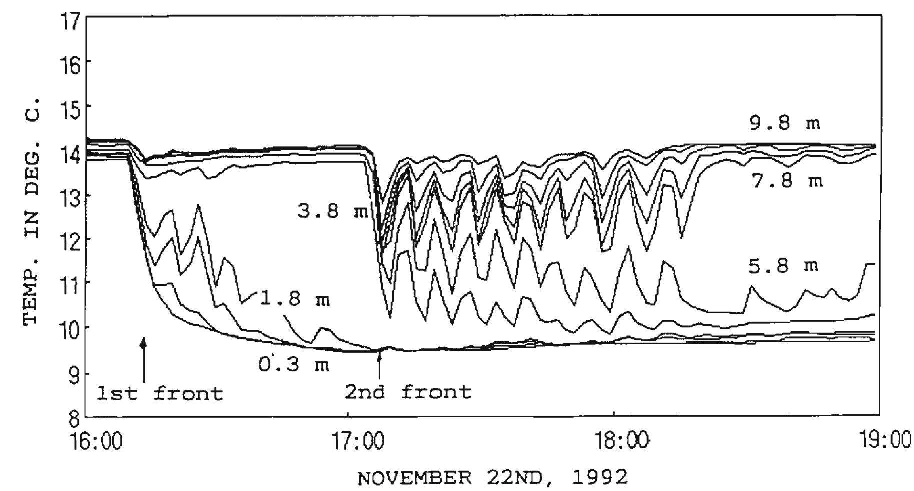

Fig. 3. Temperature record of internal soliton passage on Nov. 22nd, 1992 at St. B. Time is passing along the horizontal axis, and heights of thermistors are 0.3, 0.8, 1.3, 1.8, 3.8, 5.8, 7.8, 8.3, 8.8, 9.3, 9.8 meters from the bottom respectively.

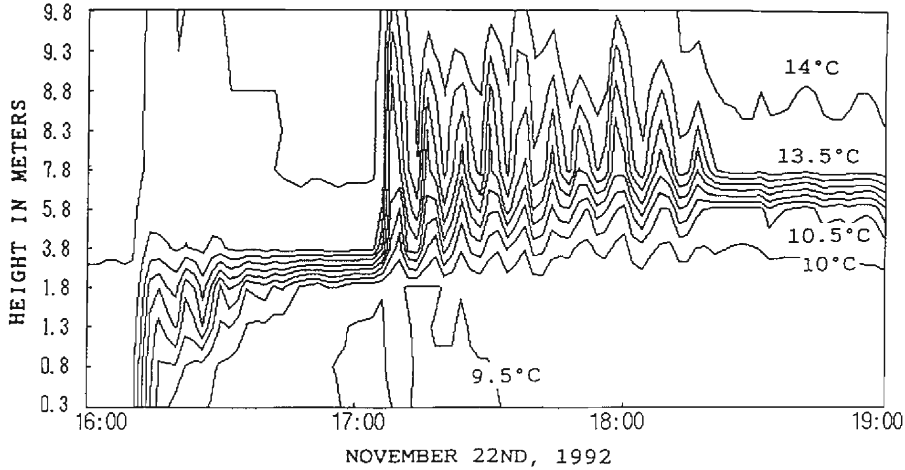

Fig. 4. Heights of isotherms during the passage of the solitons on Nov. 22nd, 1992 at St. B. Time is passing along the horizontal axis. Temperature of the isotherms, from top to bottom, are 14, 13.5, 13, 12.5, 12, 11.5, 11, 10.5, 10, and 9.5 °C. Temperature plot of the same events is shown in Figure 3.

depth. At 16:10 a rapid rise of the isotherms takes place, and the 13.5°C isotherm ascends by 4.5 m within 2 minutes, which indicates the arrival of an internal front. At the same time the temperature structure starts to change in that a distinct sharp gradient appears. The rapid rise of the isotherms indicates the sudden arrival of a cooler water mass. About 55 minutes after the arrival of the first front, another front is observed which makes the 13.5°C isotherms suddenly ascend by about 7 m or more (the upper part can not be shown because of the vertical range limit of the thermistor chain). Following the front is the onset of oscillations, ordered by their amplitudes. The infinitesimal amplitude waves followed the oscillations. The maximum amplitude (7 m) is linked to the 13.5°C isotherm. The major oscillation containing 10 oscillations lasts for 70 minutes. After the high frequency oscillations suddenly disappeared, the thermocline remained at a level higher than it was before the passage of the event, and the gradient of the temperature was maintained with infinitesimal amplitude waves on it.

The oscillations are worth further analysis. The wave train following the second front has two distinctive parts. First there is a train of large-amplitude waves. These oscillations are highly coherent vertically without any significant phase shift among the different heights. The maximum amplitude appears at the front of the oscillation train and the last one is of a smallest amplitude. The periods, which are defined by peak to peak interval, increases gradually from 6 to 10 minutes during the passage of the oscillation. All the waves cause elevations of isotherms above the thermocline, without any depressions. These oscillations may be identified as solitons which are derived by the nonlinear wave theory.

Since these waves are clearly of finite amplitude, nonlinear effects should be important to their understanding. Wave heights are of the same magnitude or greater than the depth of the cold underlying layer. Small-amplitude linear theory for internal waves

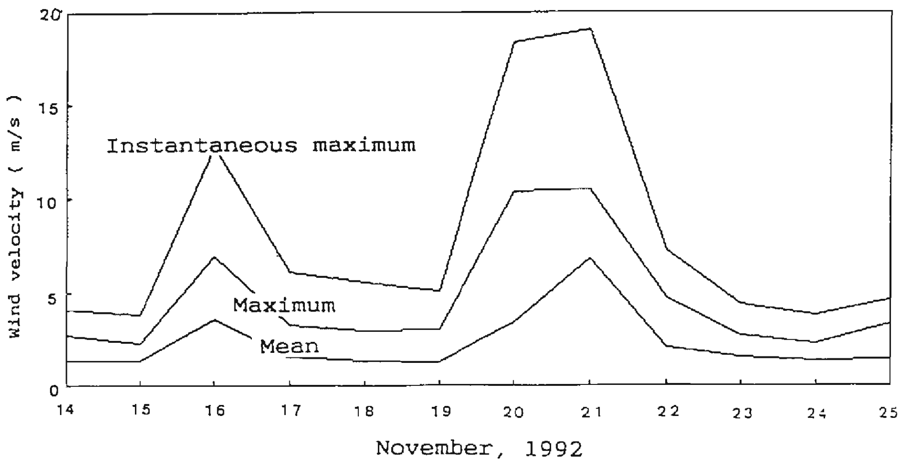

Fig. 5. Wind records by the Hikone Meteorological Observatory from the 14th to the 25th of November, 1992. The solid lines, from top to bottom, are instantaneous maximum value, maximum value, and mean value of wind velocity.

would require that their heights be much less than this characteristic dimension of the cold layer.

Internal wave measurements in Lake Biwa have been abundant (for example, Kanari, 1984). It seems that these measurements missed details of temperature disturbances because low rate sampling filtered out high frequency components, giving observed wave steepness smaller than intrinsic ones. These recordings could thus be considered as linear waves. In oceans, internal tidal waves in the Straits of Gibraltar (Ziegenbein, 1969) and in Massachusetts Bay (Halpern, 1971) resemble the waves observed in Lake Biwa in this study. The internal waves in Seneca Lake also resemble the latter. Contrary to the case in Lake Biwa, the observed solitons in Lake Seneca do not appear in the order of amplitude. The characteristics that solitons are in alignment with their order of amplitude has been predicted by the theoretical model of Hunkins & Fliegel (1973).

Wind records from the Hikone Meteorological Observatory 50 km away from the observation points within the period of 14th to the 25th, November, 1992 are shown in **Fig. 5**. From the figure, one can see that before the solitons appeared at 16:10 on the 22nd a strong northwest wind had been blowing for two days with an instantaneous maximum speed up to 19.1 m/s. Therefore, this strong wind could be responsible for the generation of the solitons.

We failed to observe the solitons simultaneously at the two stations because of the limited memory of the instruments at station-A, but the data before the solitons arrived can be used to show the current background in normal situations. **Figure 6** shows the typical current vector distribution. We can understand the general feature of the current from the figure wherein normal situations, the current goes from the west to the east in the observation region. Since solitary internal waves propagate in the direction of the

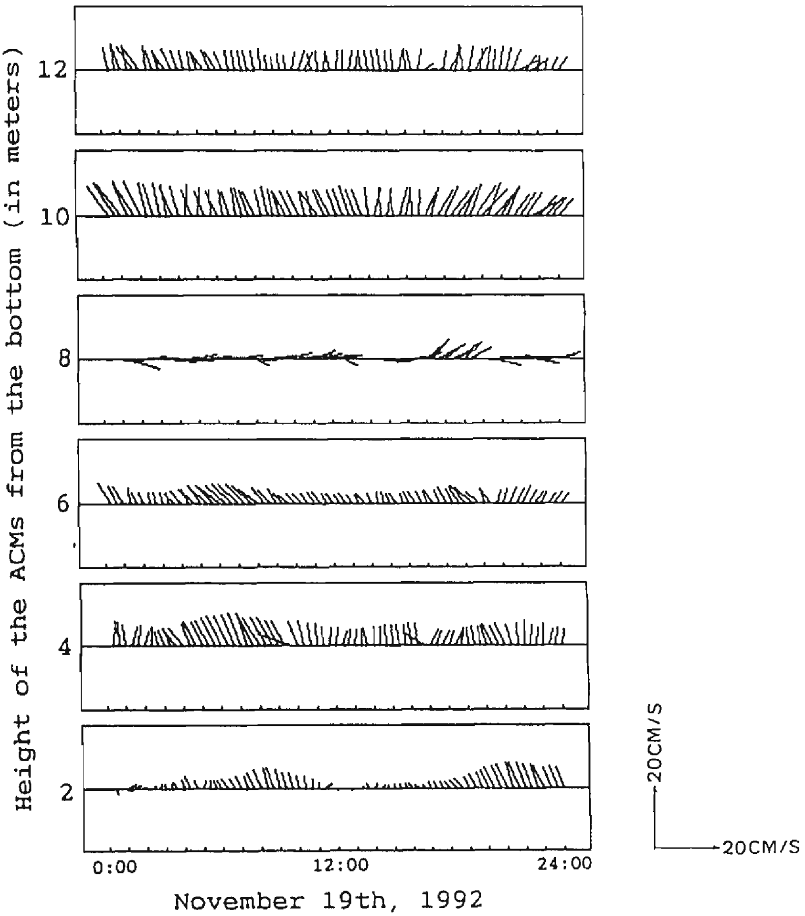

Fig. 6. Current vectors from 00:00 to 24:00 on Nov. 19th, 1992 with the east axis positive upward and the north axis positive to the left. Time is passing along the horizontal axis, and the six boxes show the data on six layers which were situated at 2, 4, 6, 8, 10, 12 meters from the bottom.

currents (Halpern, 1971), it can be expected that the solitons in this area propagate from the west to the east along the isobath line of the lake.

4. Estimation by the nonlinear wave theory

We assume a shallow incompressible inviscid fluid in a two layer system. The temperature structure in **Fig. 4** shows that it is a reasonable first approximation to reduce the thermocline into an interface between a homogeneous epilimnion of density ρ_1 and thickness h_1, and a homogeneous hypolimnion of density ρ_2 and thickness h_2, and expressed in a two dimensional domain for the sake of simplicity as shown in **Fig. 7**. These approximations should not alter the qualitative characteristics of the results.

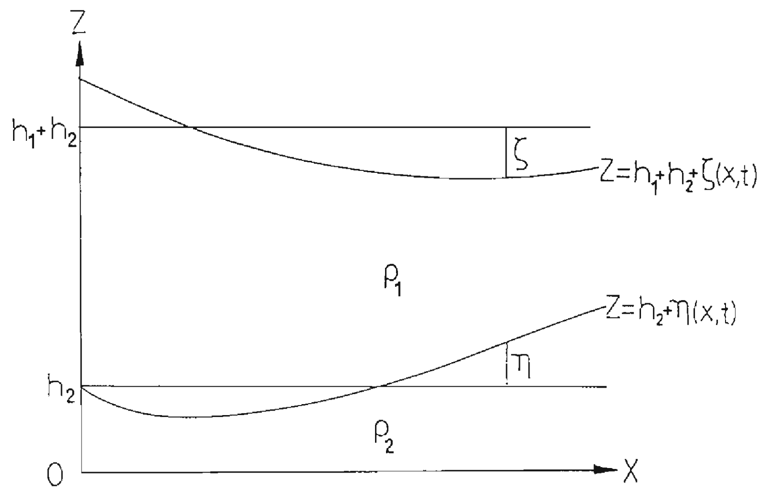

Fig. 7. Schematic structure showing a two layer system. $\zeta(x,t)$ represents the surface displacement and $\eta(x,t)$ the thermocline displacement.

Assuming (1) the waves are long in comparison with the total fluid depth, $K^2(h_1+h_2)^2<<1$, where K^{-1} represents a characteristic horizontal wavelength; (2) the waves are small, $\eta_0/(h_1+h_2)<<1$, where η_0 denotes a characteristic wave amplitude; and (3) the two effects are approximately in balance, $\eta_0/(h_1+h_2)=O(K^2(h_1+h_2)^2)$, and employing a two parameter perturbation asymptotical expansion method, Segur and Hammack (1982) derived a KdV equation with respect to interface displacement η as:

$$C_0^{-1}\frac{\partial\eta}{\partial t}+\frac{\partial\eta}{\partial\chi}+\frac{3}{2}\left(\frac{1}{h_2}-\frac{1}{h_1}\right)\eta\frac{\partial\eta}{\partial\chi}+\frac{1}{6}h_1h_2\frac{\partial^3\eta}{\partial\chi^3}=0 \tag{1}$$

where the vertical axis (Z-axis) is positive upward with the origin on the bottom, and the X-axis horizontal. C_0 is given by

$$C_0=\sqrt{\frac{g\,(\rho_2-\rho_1)\,h_1h_2}{\rho_2\,(h_1+h_2)}} \tag{2}$$

The soliton solution for the KdV equation (1) is

$$\eta=\eta_0\,\mathrm{sech}^2\,[P(\chi-Vt-\chi_0)] \tag{3}$$

where

$$P^2=\frac{3(h_1-h_2)\eta_0}{4h_1^2h_2^2}\ ;\quad V=C_0\left[1+\left(\frac{1}{2}\frac{h_1-h_2}{h_1h_2}\right)\eta_0\right] \tag{4}$$

Because P represents a real number and P^2 is greater than zero, $\eta_0>0$ when $h_1>h_2$ and $\eta_0<0$ when $h_1<h_2$. That means an internal soliton always thickens the thinner layer, which is called the solitons' polarity.

Assuming $h_1=30$ m, $h_2=4.5$ m, and $(\rho_2-\rho_1)/\rho_2=4.6\times10^{-4}$, Eq. (4) gives phase velocities of 22.06 cm/s and 18.92 cm/s for solitons with amplitudes of 7 m and 4.5 m respectively. These amplitudes are those of the first and the last solitons shown in **Fig. 4**. The velocity of linear waves C_0 is calculated as 13.28 cm/s by equation (2).

Since the leading waves evolve into solitary waves and tend to gradually separate from each other (Lee & Beardsley, 1974), the length of the wave train can be defined. When the last soliton arrived at station-B, the first one had passed the station 70 minutes earlier (as shown in **Fig. 4**) with a speed of 22.06 cm/s. This implies that the distance between the first soliton and the last one is 927 m.

The evolution of long internal waves in a stratified fluid of finite depth is governed by the Intermediate Long-Wave (ILW) equation which is a weakly nonlinear integro-differential equation (Joseph, 1977) and given by:

$$\frac{\partial u(\chi,t)}{\partial t}+C_0\frac{\partial u(\chi,t)}{\partial \chi}+Cu(\chi,t)\frac{\partial u(\chi,t)}{\partial \chi} + \frac{C_0 d}{2D}\frac{\partial^2}{\partial \chi^2}\int_{-\infty}^{\infty}\frac{u(\chi',t)\,\mathrm{sgn}(\chi'-\chi)}{\exp(\pi|\chi'-\chi|/D)-1}d\chi'=0 \tag{5}$$

where C is a constant representing nonlinearity and (d, D) are two characteristic length-scales. According to Benjamin (1967), D is the total depth, $d=h_1$, $C=3C_0/2h_1$.

A permanent wave solution of Eq. (5) is derived by Joseph (1977) to describe the velocity of the water,

$$U_S(\xi)=\frac{a}{\cosh(\gamma\xi/D)+\cos\gamma}\ ,\quad a=\frac{d}{D}\frac{C_0}{C}\gamma\sin\gamma \tag{6}$$

where $U(x,t)=U(\xi)$, $\xi=x-Vt$, γ is an arbitrary parameter $0<\gamma<\pi$, and the reference velocity $V(\gamma)$ given by

$$V(\gamma)/C_0=1+\frac{d}{2D(1-\gamma\cot\gamma)} \tag{7}$$

Letting $D\rightarrow 0$ and $\gamma\rightarrow 0$, the shallow depth limit of the intermediate-depth soliton as described by Eq. (6) has been found by Miloh (1990) as:

$$U_S(\xi)=a\,\mathrm{sech}^2(\xi/\lambda)\ ;\quad a=\frac{3(V-C_0)}{C} \tag{8}$$

where λ is an effective wavelength given by

$$\lambda=\pi\left(\frac{1/6\,dD}{V/C_0-1}\right) \tag{9}$$

With the parameters estimated by the observation in November, 1992, ($D=34.5$ m, $d=h_1=30$ m), Eq. (9) gives the maximum effective wavelength of 63 m for the soliton which has an amplitude of 4.5 m and the minimum value of 51 m for the soliton which has an amplitude of 7 m. They correspond to those of the last and first solitons in the observed wave train.

5. Discussion

We have demonstrated the behavior of short nonlinear internal waves observed in Lake Biwa. It may be the first time for a sharp internal front with highly coherent wave train to be observed in the lake.

As shown in **Fig. 4**, it is reasonable to assume that the wave train was produced in the area where the initial perturbation occurred, and then evolved into solitons. By its concept, the solitons with different amplitudes should travel at different speeds and would gradually depart from each other. The first soliton would be expected to travel at the speed of Vmax = 22.06 cm/s and the last soliton at the speed of Vmin = 18.92 cm/s. The length of the soliton wave train, which increases with time, is defined as a measure of the distance traveled from its source. The time that the last soliton took to travel from the source is given by L/Vmin, where L is an unknown distance, and the time of the first one given by L/Vmax. The difference $\Delta t = L/Vmin - L/Vmax$ should be the time delay (70 minutes in **Fig. 4**). In this way, we estimated the distance L to be about 5.5 km. Although the values involve a number of assumptions and approximations, it is considered that the initial perturbation originated in the southwest part of the north basin which is 5.5 km away from station-B where the bottom slope is relatively sharp. The scenario of internal soliton generation is considered to be as follows: (1) Strong northwest winds introduced momentum at the surface of the lake, which made the warmer water mass in the epilimnion move to the southeast part of the central basin of the lake. This surface stress was balanced by both a free surface set up and a tilt in the thermocline (i.e., a depression of the thermocline in the southeast part and an elevation in the northwest part). (2) When the winds declined, the colder water mass with a potential energy greater than its surroundings generated the initial surges by the tendency of the thermocline to recover a level position, which made the colder water mass climb up along the slope of the southwest part. (3) Assuming the initial disturbance was of a finite amplitude, the nonlinear steepening of the fronts of the surges, the third term in Eq. (1), became in force through both the rotational and topographic effects. As the steepening continued, the amplitude increased and the scale of the front became shorter and shorter. Before the breaking of the front occurred, when the length scale of the front became sufficiently short that vertical acceleration could no longer be neglected, the opposite tendency, the dispersion effect which is described by the fourth term in Eq. (1), became in force. The balance between the nonlinear effects and dispersive effects formed finite amplitude, permanent waves which did not break. Thus new waves, the solitons, started to be formed by the interplay of dispersion and nonlinearity. The permanent waves were solitary waves as the ones observed this time. Consequently, it was the strong winds which gave the cold water mass a great potential energy by a steep slope that generated the solitons. The dependence of soliton velocities on their amplitude made them separate spatially, thus producing a train of solitons from a single source as shown in **Fig. 4**.

A definite theory with observations for the generation process does not yet exist. Future investigation may have to concentrate much more on observations in the source area, and to make clear the behavior of traveling internal wave trains. A line of mooring thermistor chains with a high sampling rate will enable information on the history of such an internal wave train to be obtained.

The wave train, with a length order of 1 km as it passed station-B, must play an important role in the horizontal and vertical transport of mass, as suggested by the experiment of Maxworthy (1980). This effect is significant in sedimentological and biogeochemical processes operating in the lake. Simultaneous observation of temperature and turbidity in the wave train may be important.

Acknowledgements

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References

Benjamin, T. B.: Internal waves of finite amplitude and permanent form, J. Fluid Mech., Vol. 25, 1966, pp. 241-270.

Benjamin, T. B.: Internal waves of permanent form in fluid of great depth, J. Fluid Mech., Vol. 29, 1967, pp. 559-592.

Benney, D. J.: Long nonlinear waves in fluid flows, J. Math. Phys., Vol. 45, 1966, pp. 52-63.

Davis, R. E. and A. Acrivos: Solitary internal waves in deep water, J. Fluid Mech., Vol. 29, 1967, pp. 593-607.

Farmer, D. M.: Observation of long nonlinear internal waves in a lake, J. Phys. Oceanogr., Vol. 8, 1978, pp. 63-73.

Halpern, D.: Observation on short period internal waves in Massachusetts Bay, J. Mar. Res., Vol. 29, 1971, pp. 116-132.

Hunkins, K., and M. Fliegel: Internal undular surges in Seneca Lake: A natural occurrence of solitons, J. Geophys. Res., Vol. 78, 1973, pp. 539-548.

Joseph, R. I.: Solitary waves in a finite depth fluid, J. Phys. A: Math. Gen. Vol. 10, 1977, pp. L225-L227.

Kanari, S.: Internal waves and seiches, Lake Biwa, 1984, pp. 185-235.

Lee, C., and R. Beardsley: The generation of long nonlinear internal waves in a weakly stratified shear flow, J. Geophys. Res., Vol. 79, 1974, pp. 453-462.

Long, R. R.: On the Boussinesq approximation and its role in the theory of internal waves, Tellus, Vol. 17, 1965, pp. 46-52.

Maxworthy, T.: On the formation of nonlinear internal waves from the gravitational collapse of mixed regions in two and three dimensions, J. Fluid Mech., Vol. 96, part 1, 1980, pp.47-64.

Miloh, T.: On periodic and solitary wavelike solutions of the intermediate long-wave equation, J. Fluid Mech., Vol. 211, 1990, pp. 617-627.

Ono, H.: Algebraic solitary waves in stratified fluids, J. Phys. Soc. Japan, Vol. 39, 1975, pp. 1082-1091.

Segur, H. and Hammack, T. L.: Soliton models of long internal waves, J. Fluid Mech., Vol. 118, 1982, pp. 285-304.

Thorpe, S. A.: Asymmetry of the internal seiche in Loch Ness, Nature, Vol. 231, 1971, pp. 306-308.

Thorpe, S. A.: Internal surge in Loch Ness, Nature, Vol. 237, 1972, pp. 96-98.

Winant, C.: Internal surges in coastal waters, J. Geophys. Res., Vol. 79, 1974, pp. 4523-4526.

Ziegenbein, J.: Short internal waves in the Strait of Gibraltar, Deep Sea Res., Vol. 16, 1969, pp. 479-487.