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ON THE COEFFICIENTS OF PRODUCTION

1. NATURE OF THE PROBLEM

I think the concept of the coefficients of production first prevailed among the economists of the mathematial school, but at present much importance is attached to it also by scholars not belonging to that particular body of thinkers.

Let us first, consider the question of nomenclature. Different names are often given to one and the same thing. The terms, "coefficients de fabrication" and "technische Koeffiziente" respectively, "coefficients de production," which are sometimes used, mean practically the same thing.[1]

[1] The term "les coefficients de fabrication" is used by Leon Walras (Leon Walras, *Elêments d'économie politique pure* édition definitive, p, 221). Pareto employs the term "les coefficients de production," but presumably he is not the inventor of the term; he adopted it because it was in general use. (Pareto, *Manuel d'économie politique* pp. 304, 607; Zawadzki, *Les mathématiques appliquées à l'économie politique*, 1914; p. 222). Cassel uses the term "technische Koeffiziente," in practically the same sense as Walras's expression (Cassel, *Theoretische Sozialökonomie*, 1923, S. 119). Although there is some slight difference in the senses of the various term employed by these scholars, their chief purport is the same.

It is for the following reasons that I propose to discuss this phase of the matter. If the coefficients of production are absolutely determined by technics,—in other words, if they are inalterably fixed, provided there is no technical change, quite regardless of all social relations—there can be no room for the participation of social power in the determination of the price (at least in the static state). In such case, the prices of productive goods will be determined by the quantities of productive goods (and the demand curve being given) only. If, however, the coefficients of production are not difinitely fixed, the relations of social power can not only influence the price but can be of fundamental importance in determining it. This is my point of view. It is the aim of the present article to make it clear whether the coefficients of production are really fixed by technics exclusively or, if otherwise, how the relations of social power can exert a decisive effect on the price. I must first make clear the meaning of the phrase "coefficients of production."

2. MEANING OF "COEFFICIENTS OF PRODUCTION."

The explanations so far available of the phrase "coefficients of production" are not necessarily in perfect accord, but the parts these explanations have in common my be stated as follows:—

In order to increase the production of one product by the minimum quantity, it is necessary to increase all kinds of productive goods accordingly. The proportions of the former to the latter are called the coefficients of production. Roughly speaking, however, we may also say that the quantum of each kind of productive goods required for producing one new unit of one product is called the coefficient of production. This conception is possible because one unit of the products may, for all practical purposes, be regarded as the minimum quantity.[1]

As a matter of principle, the coefficients of production vary according to the quantity of the goods already produced (when considered from the purely technical point of view). In exceptional cases, however, they remain unaltered, despite the quantity already produced. In such cases, the coefficients of production are called unalterable.

Again, even taking the coefficients of production to mean the quantities of the productive goods required for producing one more unit of a product, it is not equal to the quantities of productive goods per unit of each product. Both are equal only when the coefficients of production are unchangeable. Supposing that a certain tract of land, a certain amount of labour and a certain amount of beancake are needed in producing ten *bu* of rice, and also supposing that for producing another additional *bu* of rice, one-tenth of the land, three days of labour and five *lbs.* of beancake are necessary, the requisite quantities of land, labour and beancake, viz. $\frac{1}{10}$, 3, and 5, may be regarded as the coefficients of production, so long as one *bu* of rice can be regarded as the unit of rice.

As already stated, the coefficients of production, if they are taken as unchangeable and do not vary according to the quantity of the products, are denoted as the quantities of productive goods per unit of product. But they are generally indicated in the form of definite differential co-

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[1] To quote the words employed by Pareto in his explanation of the coefficients of production: "In order to obtain one unit of a product, certain quantities of other products and the service of capital are used. These quantities are what is called the coefficients of production." (Pareto, *Manuel*, p. 304). It must, however, be mentioned that in some other cases, the contents of the coefficients of production which are shown in several formulae take the form of differential coefficients.

Leon Walras adopts the same method of explanation. According to him, $a_t a_p a_k$......, $b_t b_p b_k$......, $c_t c_p c_k$......, $d_t d_p d_k$...... representing the amount of the productive service T, P. and K respectively that enters into the production of each unit of the products A, B, C, and D are called les coefficients de fabrication. (Leon Walras, *Eléments d'économie politique pure*, P. 211).

efficients by those who attach importance to their variability.

Now, let us suppose that the products X, Y, ... are produced out of the productive goods A, B, ..., and that the quantity of the productive goods A, B, ..., which are required for the production of X, Y, ..., is fixed according to the quantity of x, y, ..., which represent the quantity of X, Y, ... respectively. When A=F (x, y, ...) and B= F′ (x, y, ...), the partial derivative function $a_x b_x \ldots a_y \ldots\ldots$ embodies the coefficient of production.

$$a_x=\frac{\partial F}{\partial x},\quad a_y=\frac{\partial F}{\partial y},\quad b_x=\frac{\partial F'}{\partial x},\ldots\ldots$$

There are general costs, which are not affected by the quantity of the products. These costs cannot be reduced, no matter what reduction may be made in the quantity of the products. Nor is there any necessity of increasing them, even if the quantity of the products may be increased. Let A_o indicate the quantity of the productive goods A belonging to such costs, and $a_x dx$ the quantity of A required for producing the amount of dx, when the amount produced is x. Then the size of A (to be more exact, the total quantity consumed for the production of the product A) can be shown in the following formula. (Pareto, *Manuel*, pp. 607–609, Zawadsky, *La mathématique appliquée à l'économie politique*, 1914, p. 209).

$$A=A_o+\int_o^x a_x dx+\int_o^y a_y dy+\ \ldots\ldots$$

In the same way,

$$B=B_o+\int_o^x b_x dx+\int_o^y b_y dy$$

3. RÔLE OF THE COEFFICIENTS OF PRODUCTION IN EQUILIBRIUM

(PART I)

Now, what effect do the coefficients of production in the above sense produce on the price or in what relation do they stand to the price? First, I wish to examine Cassel's theory of price. Cassel discusses the relation between the price in equilibrium and the other economic quantities along the following lines :—

Let as suppose (1) that the supply is fixed, that is to say, the quantity of goods at the disposal of consumers is fixed (and therefore the question of production is put out of all consideration), and (2) that the purchasing power of consumers (the amount of money that can be spent) is fixed.

Now, supposing that the kinds of goods obtainable in a certain society are 1, 2, 3, ... n, and that the total number of kinds is n, and that their respective supplies are A_1 (which means the quantity of goods called 1 that can be supplied. This represents the quantity of goods that must be sold), A_1, A_2, A_3, ... A_n. The suppliers of these goods have their consumers, whose demand for each kind of goods is N_1, N_2, ... N_n. This does not, of course, represent the demand of one consumer, but the total demand of the public. While it is hardly necessary to point out that the demand for each kind of goods fluctuates according to its price, it also fluctuates according to the prices of all other goods. Therefore, the demand for goods 1 fluctuates according to the prices of goods 1, goods 2, ... goods n. In other words, N_1, which represents the demand for goods 1, is the function of p_1 or the price of goods 1, p_2 or the price of goods 2 ...... and p_n or the price of goods n. The same thing may be said of all other goods.

$$
(1) \qquad
\begin{aligned}
N_1 &= F_1\ (p_1,\ p_2 \ldots\ldots p_n) \\
N_2 &= F_2\ (p_1,\ p_2 \ldots\ldots p_n) \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
N_n &= F_n\ (p_1,\ p_2 \ldots\ldots p_n)
\end{aligned}
$$

The supply and the demand accord with each other in the state of equilibrium. In other words, they are equal.

$$N_1 = A_1\ ;\ N_2 = A_2 \ldots\ldots N_n = A_n$$

If this is combined with equation group (1), the following equation group (2) can be obtained, which is valid in the state of equilibrium.

$$F_1\ (p_1,\ p_2 \ldots\ldots p_n) = A_1$$
$$F_2\ (p_1,\ p_2 \ldots\ldots p_n) = A_2$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$F_n\ (p_1,\ p_2 \ldots\ldots p_n) = A_n$$

In equation group (2), let the amount of supply A_1, A_2, ...A_n be given quantities, and the prices p_1, p_2,......p_n unknown. The number of equation group (2) is n, just the same as the number of the unknown quantities p_1, p_2, ... p_n. Thus, the numerical value of p_1, p_2, ...p_n can be definitely determined. If the price is an unknown quantity, the demand and the supply can be determined.

To express the above contention in other words, if the quantities of the goods for supply that must be sold are fixed, the prices of all goods can be fixed in one way only, provided the demands of individuals and accordingly the demands of the public are definitely determined. In this case, the prices are invariably fixed. In fact, it would be difficult for observers to know the exact forms of functions, $F_1\ (p_1,\ p_2 \ldots\ldots p_n)$, $F_2(p_1,\ p_2 \ldots\ldots p_n)$, but it is conceivable that they prevail in fact. If the quantity of supply of any goods is fixed, the price of it must be fixed accordingly.[1]

So far, I have gone on the hypothesis that the quantity of supply is fixed. Let me now abandon this hypothesis, and assume that the quantity of supply can be increased by production, with this proviso that productive goods

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1) This point is dealt with in Cassel, *Theoretische Sozialökonime,* 3. Aufl. 1923, S. 117; Amonn, *Grundzüge der Volkswohlstandslehre* 1. 1926, S. 158–159. What Valk calls the system of equations No. 2, corresponds to the above mentioned relations. This is developed in the following part of Walras: Leon Walras, *Éléments d'économie politique pure.* p. 113 et seq. cf. Valk, *Zur Frage der Grenzproduktivität.* Schmollers Jahrbuch, 51 Jahrgang, fünftes Heft, 1927. S. 14 ff.

(productive means) are fixed in quantity. Thus, it comes about that " the whole matter of production enters into the question of price formation."

Let us suppose that represents the kinds of productive goods available and that the total quantity of these goods that can be utilised in a fixed unit period is R_1, R_2, ...R_n respectively. A finished product is produced by means of all kinds of paoductive goods. Now, a_1 (1) represents the quantity of the productive goods (1), which is required for producing one unit of the finished product 1; a_1 (2) represents the quantity of the productive goods (2) for producing it; and so on. So does a_1 (2) represent the quantity of the productive goods (1) required for producing one unit of the finished product 2, and so on. Lastly, a_n (1), a_n (2)......a_n (r) represent the productive goods (1), (2), ... (n) respectively, which are necessary for producing one unit of the finished product n. The quantities of these are called technical coefficients. These embody the technical conditions of production, but as it is assumed that these conditions are fixed, these coefficients are the given quantities. As the above explanation shows, the whole of the productive goods necessary for producing one unit of the finished product 1 is the sum of a_1 (1), a_1 (2)......a_1 (r). Taking q_1, q_2, ...q_r, to represent the price of the productive goods (1)......(r), the following equation (3) may be produced possible, as the price of the finished product is equal, in equilibrium, to the cost of production :—

$$
\begin{aligned}
&a_{1(1)}q_1+a_{1(2)}q_2+\ldots\ldots+a_{1(r)}q_r=p_1\\
&a_{2(1)}q_1+a_{2(2)}q_2+\ldots\ldots a_{2(r)}q_r=p_2\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\
&a_{n(1)}q_1+a_{n(2)}q_2+\ldots\ldots a_{n(r)}q_r=p_n
\end{aligned}
$$

This group of equations is of the same nature as those equations given by Wieser in his interpretation of the theory of the imputation of value. The only difference is that the latter shows the law of the cost of production rather incompletely. This incompleteness arises from two causes. One is that it is not shawn as a part of the whole connected body of economic equilibrium, and the other is that the law of the cost of production is shown in reference to specific cases of production only, instead of its being indicated in a form of general application.

The following group of equations, which was given before, is indicative of the equilibrium between the demand and the supply :—

(4) $$N_1=F_1(p_1,\ p_2 \ldots\ldots p_n)$$
$$N_2=F_2(p_1,\ p_2 \ldots\ldots p_n)$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$N_n=F_n(p_1,\ p_2 \ldots\ldots p_n)$$

(5) $$N_1=A_1,\ N_2=A_2,\ \ldots\ldots N_n=A_n.$$

Let A_1, A_2, ... represent the respective quantity of the finished products 1, 2, ... n respectively which can be produced within the unit of period. In the state of equilibrium, it is necessary that the existing quantity of productive goods (to be employed during the fixed time) should be completely used up, and thus an equation showing it can be worked out. For the production of A_1 (the amount of the finished product 1 for supply), $a_{1(1)}A_1$, or the coefficient of production of the productive goods (1) multiplied by A_1 is required; likewise also $a_{1(2)}A_1$ of (2), ...$a_{1(n)}A_1$ of (n). The same thing may be said of A_2, and A_3. Thus, the quantity of the productive goods (1), (2) ...... (r), which is required for producing A_1, A_2, ... A_n, can be shown in the following equations :—

(6) The quantity of (1) required
$$=a_{1(1)}A_1+a_{2(1)}A_2+\ldots\ldots+a_{n(1)}A_n.$$
The quantity of (2) required
$$=a_{1(2)}A_1+a_{2(2)}A_2+\ldots\ldots+a_{n(2)}A_n.$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
The quantity of (r) required
$$=a_{1(r)}A_1+a_{2(r)}A_2+\ldots\ldots+a_{n(r)}A_n.$$

In the state of equilibrium, the requisite quantity of each of the productive goods is equal to the existing quantity of R_1, R_2, ...R_r, and consequently the equation group (6) can be rewritten as follows:—

$$
(7) \quad \begin{aligned}
R_1 &= a_{1(1)}A_1 + a_{2(1)}A_2 + \dots\dots + a_{n(1)}A_n. \\
R_2 &= a_{1(2)}A_1 + a_{2(2)}A_2 + \dots\dots a_{n(2)}A_n. \\
&\dots\dots\dots\dots\dots\dots\dots\dots \\
R_r &= a_{1(r)}A_1 + a_{2(r)}A_2 + \dots\dots + a_{n(r)}A_n.
\end{aligned}
$$

The equation group (6), in my opinion, serves no better purpose than to make a little clearer the self-evident fact by throwing a side-light on it. R_1, R_2,......R_r in equation group (7) are the given quantities, and the quantities, $a_{1(1)}$, $a_{2(1)}$...... are also given. A_1, A_2, ... A_n can be rewritten into the function of p by virtue of equation group (4) and group (5), and consequently they are the function of q by virtue of equation group (3). Thus it will be seen that the system of equations (7) contains r, representing the r prices of productive goods (q_1, q_2, ...q_r) as unknown quantities. The number of equations is r, and it is, generally speaking, enough to solve them.

Let me now explain the system of equations (7) from a different angle. R_1, R_2, ...R_n are the given quantities, and so are the technical coefficients $a_{1(1)}$, $a_{2(1)}$ .... The demand functions are also fixed. Under such conditions, q_1, q_2, ... q_r are also definitely determined. When we regard the above contention as indisputable, I think we can take the view that the subjective explanation regarding the price is possible. The expression that marginal utility determines the price may be inadmissible in many respects, but I think it may correctly be said that the price function (which may also be termed the utility function) controls the price or that the value principle is decisive of the price.

From the various systems of equations above-mentioned, it may be seen that there are two kinds of bases on which the price is fixed (decisive bases of the price, as Cassel calls it). One is what is called objective, while the other is subjective. Under the former category fall R or the total amount of productive goods, and a embodying the technical coefficients or productive coefficients. To the latter category belongs the demand function ($N_1=F_1$ (p_1, p_2......p_n) etc.). R or the total amount of productive goods and a or the technical coefficients are, as already stated, given quantities. If so, it may be concluded that it is the demand function that is decisive of the price after all. What I mean is this: the price of productive goods is essentially fixed passively. The total quantity of productive goods, R, is combined according to a definite technical coefficient, a, thereby producing finished goods A_1, A_2, ...A_n. Then, according to the system of demand functions, the quantities of A_1, A_2, etc. are determined so that each marginal unit of the purchasing power of every person may have uniform utility in its respective use (for the purchase of A_1, A_2, or A_n), with the result that the prices of the productive goods (1), (2) ...... (r) and the quantities of q_1, q_2, ... q_r are determined. There is no possibility for the prices of products p_1, p_2, to be determined by q_1, q_2, (the prices of productive goods) which are of a self-moving, automatic nature. As already explained by means of equations, the prices of productive goods are assumed not to be liable to automatical change; they are thought to be determined always by the demand function. After such an opinion, the prices of productive goods are absolutely passive; they are entirely devoid of any automatic nature. They are fixed by three factors, namely, the estimate of the utility by each subject (demand function), the technical coefficients (which are fixed by technics) and the total quantity of productive goods. There is no room for the relations of social powers to determine prices.

Of course, it is not the interpretation put by Cassel himself on the above-mentioned system of equations, that prices are fixed by dint of demand function alone. Cassel takes the view that various factors (all basic elements, either subjective or objective) are all important in fixing prices and that consequently neither the objective nor the subjective theory, which respectively ascribes the prices to either the objective or the subjective factors exclusively, will hold water. I shall explain in another part why I can not accept such a view.[2]

[2] In the opinion of a few scholars, there is a close connection between Cassel's theory of price and that of Leon Walras. They think that in a sense, Cassel puts Walras's point of view in a simpler and a more popular form. (Schumpeter, *Cassels Theoretische Sozialökonomie*, Schmollers Jahrbuch, Bd. 51, Zweites Heft, S., 79, Willem Valk, a. a. O. S. 10ff.

Walras treats of the coefficients of production as definitely fixed. Cassel also regards it as fixed according to technics, but in the supplementary principle he admits that with the increase of production, the coefficient of production witnesses a gradual rise or fall. He further admits that in a fixed condition of the price of productive goods, the principle of substitution operates, with the result that it becomes possible for various productive goods to be combined in a variety of ways. But it can hardly be said that the conclusions have been adequately drawn from these two facts or that these conclusions have been thoroughly threshed out, as can be seen by any one who takes the trouble to compare his point of view with that of Pareto. I want to discuss afterwards a marked difference existing between Cassel's treatment of the theory of marginal utility and that of Walras, notwithstanding his adoption of the other's theory of equilibrium.

Again, Valk, commenting on Cassel's system of equations, says that it consists of three parts. One part shows the connection of the prices of finished goods (final products) with the quantities of corresponding supplies. Another part embodies an abstract expression of Wieser's equations indicating the equality of the cost of production to the price. The other part shows that in the state of equilibrium, the existing quantities of productive goods are quite adequate for the maintenance of production. Cassel's equations may be compared with Walras's as follows:—(The first part) Cassel's equations (1) and (2) or (4) and (5) (Cassel, a. a. O. S. 117, 120) are of the same substance with equations (1) and (2) of (Walras, *Elèments d'èconomie politique pure* (édition définitive) p. 211). (The second part) Caseel's equations (3) (Cassel, a. a. O. S. 120) with Walras's equations (4) (Walras, op. cit. p. 212.)

(The third part) Cassel's equations (7) (Cassel, a. a. O. S. 121) with Walras's equations (3) (Walras op. cit. p. 212.)

I think the first part means demand function and accordingly the demand side of the subject; the second part the definiteness of the coefficients of production rather than the law of the cost of production; and the third part the amount of productive means. I am thus inclined to the view that what Cassel calls the subjective determining factors of the price are shown in the first part, and the objective determining factors in the second and third parts. As already mentioned, Wieser's equations of production cover the second part only and fail to show it in the relations of the equilibrium of the whole. (cf. Willem Valk, a. a. O. S. 13-14).

4. RÔLE OF THE COEFFICIENTS OF PRODUCTION IN EQUILIBRIUM

(PART II)

My aim is to make clear how far the coefficients of production are determined by technics and what amount of automatism is possessed by the price of productive goods. Cassel, in his theory of price, lays down that the coefficients of production (or the so-called technical coefficients) are fixed by technics. Notwithstanding the fact that Cassel is personally inclined not only to condemn the theory of marginal utility but to exclude the theory of value from economic theories, his system of equations is framed in such a way that one can not deny the validity of the theory of marginal utility in tracing the equations.

But, to begin with, he manifestly treats the truth with too much carelessness, when he neglects to state that the coefficients of production change with the increase or decrease of the quantities of production in an enterprise or within one management. He presumably confuses particular instances with general instances. As a complementary principle of the price theory, Cassel takes due note of cases where the cost of production either increases or decreases with the increase of the production of finished goods, but this refers to the increase or decrease tkat occurs in the cost of production as enterprises differ, and the contention is that the cost of production increases (and the coefficients of production change accordingly) as, the increase of production being necessary, enterprises which are in an unfavourable state must also participate in production. This fact must, of course, be taken into consideration, but it is not a matter which is existent in a static state at least, nor is it observable in equilibrium. And Cassel (at least in his equation group), like Walras, ignores the existence of general costs. Once general costs are recognised, it can hardly be maintained that the coefficients of production are unchangeable, regardless of the amount of production. Secondly, Cassel disregards some of the factors which determine the coefficients of production, in considering that the coefficients of production reflect a definite technical state, whereto it corresponds. If they are not altogether disregarded, they are not taken into due consideration. In the third complementary principle, it is merely mentioned that in case the price of productive goods is fixed, a choice is made of productive methods, the relation of substitution existing among these goods. But as a matter of fact, the coefficients of production are not unequivocably, so to speak, determined by a definite technical state only. Unlike a combination of chemical elements, productive goods are not combined in an unalterable proportion. Within the limits fixed by technics, their proportions are determined by the prices of productive goods. In other words, the coefficients of production are not the reflex of technical considerations, but are determined by the prices of those productive goods within the limits fixed by technical factors. On the basis of these coefficients of production, the kinds of finished products are chosen, and equilibrium is established among the various economic quantities.

I have observed that Cassel (Walras also, for that matter) treats the coefficients of production as of the given quantities, on the whole, and this theory ignores, in the first place, the fact that the prices of productive goods determine the coefficients of production. Secondly, it disregards the changes that occur in the coefficients of production according to the quantity of production. This, it seems that the various systems of equations are formulated on too many hypotheses in such a way as to allow of the interpretation that the price of productive goods is essentially of passive size. I propose to examine that theory of equilibrium which is devoid of these hypotheses, while stating my views in the course of this examination. I think it well to choose Pareto as typical of the theory of equilibrium free from these hypotheses. For convenience sake, however, I will describe the theory of equilibrium expounded by Zawadzki, who endorses Pareto's interpretation in substance, with as much reference to Pareto as occasion demands.

An explanation of the equilibrium of production must of necessity be preceded by an explanation of the equilibrium of exchange. Suppose that there is utility arising out of a combination of the various finished production, x, y, z, ... Suppose also that the combination of the various products that can produce the total amount of utility designated by 1, is shown in the following form of function. $I=\varphi(x, y, z\ldots\ldots)$. Let us take φ_x, φ_y, φ_z ...... etc., to indicate the derivative functions of x, y, ... respectively; P_y to indicate the price of y in x; p_z to indicate the price of z in x. Then the individual equilibrium for each individual can be shown in the following formula:—

$$\varphi_x=\frac{1}{p_y}\varphi_y=\frac{1}{p_z}\varphi_z$$

This shows that the marginal utility for each kind of goods is in diverse proportion to the price for individuals. To be more exact, it indicates what is called the law of the equality of marginal utility (das Gesetz des Ausgleichs der Grenznutzen). And in the equilibrium of exchange, this applies to all parties to the exchange. Suppose that 1, 2, 3, ... are the number of persons who participate in exchange, and that the above-mentioned function for all is φ_1, φ_2 ...... φ_n.

Then, the following system of equations in (A) in regard to the market must be accepted:—

$$(\mathrm{A})\begin{cases}\varphi_{1x}=\dfrac{1}{p_y}\varphi_{1y}=\dfrac{1}{p_z}\varphi_{1z}=\ \ldots\ldots\\[2mm] \varphi_{2x}=\dfrac{1}{p_y}\varphi_{2y}=\dfrac{1}{p_z}\varphi_{2z}=\ \ldots\ldots\\[2mm] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\end{cases}$$

If n shows the number of persons in the exchange and m the number of the goods to be exchanged, the number of the equations in this system is $(m-1)n$.

Next, the prices at which goods are sold or purchased by each subject ought to be equal. In other words, the goods to be given in compensation must be equal to the goods to be obtained, in the total sum of prices. Let $x_{1.0}$, $y_{1.0}$ $y_{1.0}$ ...... $x_{2.0}$, $y_{2.0}$ ...... be the amount of the goods possessed by each individual before the exchange and x_1, x_2 ......y_1, y_2 ...... be the amount of those owned by him after it. If x shows the goods that function as money, the following system of equations in (B) can be formed :—

$$\text{(B)}\begin{cases} x_1-x_{1.0}+p_y\,(y_1-y_{1.0})+p_z\,(z_1-z_{1.0})+\ldots\ldots=0 \\ x_2-x_{2.0}+p_y\,(y_2-y_{2.0})+p_z\,(z_2-z_{2.0})+\ldots\ldots=0 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. \end{cases}$$

The number of equations which the system of equations in (B) contains is n.

Lastly, the total amount of all goods before and after the exchange ought to be equal, and therefore the system of equations in (C) is possible. The number of equations which this system contains is m.

$$\text{(C)}\begin{cases} x_1+x_2+x_3+\ldots\ldots=x_{1.0}+x_{2.0}+x_{3.0}+\ldots\ldots \\ y_1+y_2+y_3+\ldots\ldots=y_{1.0}+y_{2.0}+y_{3.0}+\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. \end{cases}$$

One in the equations in (B) and in (C) can be devived from all the others, and so the total number of equations is $(m-1)\ n+m+n-1=(mn+m-1)$. The number of the given quantities is mn, or the quantity definitively secured by each (by the exchange), plus m-1, or the prices of goods other than those functioning as money. Both the number of the unknown quantities and the number of the equations are equally $mn+m-1$, and so the numerical value of the unknown quantities can be definitely determined. This can be explained in another way also. In order that the price may find the equilibrium, each either asks or offers the price according to his own estimate, and the requirements of the equations in (A) and in (B) are satisfied The equilibrium between supply and demand is, however, not yet to be found, and only after a series of trials can the quantity ot demand and supply be brought into accord. The requirements of equations in (C) are then satisfied and the price finds its proper level.[1]

So far I have dealt with the theory of equilibrium regarding the exchange of finished products already in possession. On the basis of this knowledge, the theory of the equilibrium of production must be discussed. The above-mentioned theory of the equilibrium of production by Cassel and Walras presupposes too much. It is formed on the two hypotheses of the non-existence of general costs and the unchangeability of the coefficients of production, but these hypotheses do not evidently accord with the general fact. They must rather be regarded as exceptions to the general rule that there exist general costs and that the coefficients of production are subject to change.

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[1] This part is dealt with in Pareto, *op. cit.*, pp. 591 et seqq. In respect also of the signs used, there is no marked difference. I was not quite correct when I referred to I as the total amount of utility, but as it becomes necessary for Pareto's views of the law of indifference, etc. to be described in detail in order to define it accurately, I confined myself to the above statement for the sake of simplicity. I must here allude to cases where the subject does not personally (directly) recognise the utility of certain goods (as, for instance, in the case of merchants). Some people may be inclined to traverse the theory of marginal utility on this ground. Setting apart the correctness or otherwise of their contention, in cases where some people among those concerned in the exchange do not personally recognise the utility of the goods which they supply, we may suppose that such people supply their goods, regardless of tbe price, and thus the quantity possessed before the exchange makes the quantity supplied. This reduces the unknown factors by one, and the number of equations is correspondingly reduced also. The equation that is thus removed is one concerning persons in the system (A). (Pareto, *op. cit.*, p. 593, Zawadzki, *op. cit.*, p. 204).

Now, let us suppose that the finished products X and Y are produced out of the productive goods A, B, C, ... According to the view which has been prevalent since Walras expounded it, productive goods are, generally speaking, divided into raw materials and service, and service consists of service of three kinds of capital, that is land, workers and movable property or the capital produced. Into a more minute division of these factors I will here refrain from entering. Let a_x, b_x, ... a_y, b_y, ... indicate the coefficients of production. These represent the quantity by which productive goods A or B must be increased in order to increase X or Y by one unit, when X or Y is being produced to the quantity of x or y. Let us now take π_x, π_y, to represent the cost price of production (les prix de revient), and let us also suppose that the product A functions as money. The price is, of course represented by 1.

$$\pi_x\, dx=(a_n+b_x p_b+c_x p_c+\ldots\ldots)\, dx$$
$$\pi_y\, dx=(a_y+b_y p_b+c_y p_c+\ldots\ldots)\, d_y$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

If the prices of productive goods, p_b, p_c, ... , are unchangeable and the coefficients a_x, b_x, ... and a_y, b_y, ... depend solely on the quantities of x, y, ... , less prix de revient (or the cost of production per unit) is the derivative function of the function indicating the total cost of production of X or Y, as shown below :—[2]

$$\pi_x=\pi_{ox}+\int_o^x \pi_y dx$$
$$(1)\quad \pi_y=\pi_{oy}+\int_o^y \pi_y dy, \text{ etc.}$$

[2] This equation bears number (26) in Zawadzki and bears number (105) in Pareto. The equations given below are given such numbers as are suitable for the order of description. Except where it is necessary, the numbers of the equations in Pareto or Zawadzki are not mentioned (Pareto, *op. cit.* p. 609; Zawadzki, *op. cit.* p. 210).

In the economic organisation of free competition (or what Pareto calls the first type), the price is equal to the cost of production, and industrialists neither suffer losses nor gain any profit from their enterprises in such circumstances. Consequently, the following equations can be formed :—

$$(2) \qquad \pi_x = Xp_x$$

There is, however, another circumstance which the organisation of free competition demands. That is that the cost of production of the final unit must be equal to the price.

$$(3) \qquad \pi'_x = p_x.$$

These two conditions and therefore the equations (2) and (3) are, as a rule, incompatible with each other. They are compatible with each other only when the general costs (A_o in the last formula of 2) do not exist and the coefficients of production are unalterably fixed. In all other cases, the two are not necessarily reconcilable with each other. In fact, it is (2) rather than (3) that has a tendency to dominate in the organisation of free competition.

With the above-mentioned explanation as the preface, I will now plunge into a detailed explanation of the theory of the equilibrium of production. Let θ represent the number of the persons who own productive goods. They make transactions with industrialists in the market. The former sell to the latter raw materials and a part of the service of capital, and exchange the rest. The industrialists turn the productive goods which they have obtained into finished products, and pay the suppliers of productive goods. What, then, is the quantity of the various products (to be consumed for productive purposes or to be produced) and what is the amount of the price in the state of equilibrium?

Let the signs be previously fixed as follows (the goods A functions as money, and consequently its price is 1).

The productive goods brought into the market by their owners (1, 2, 3,......)	$a_{10}, a_{20}, a_{30}......, b_{10}, b_{20}, b_{30}......$
The quantities of X, Y, and Z, produced by industrialists ...	$x, y, z,$ ......
The productive goods owned by (1, 2, 3,......) in equilibrium (after the exchange)...............	$a_1, a_2, a_3......, b_1, b_2, b_3......$
The productive goods supplied ; the productive goods converted into finished products ...	$A, B, C......; A', B', C'......$

The conditions of the equilibrium of exchange, namely, (A) and (B) must be fulfilled in this case also. In the first place, the law of the equality of marginal utility applies with as much propriety to productive goods, so far as they can be consumed directly, as to the mutual relations of finished products (there may be plenty of goods which cannot be directly consumed by their owners, and in this case, the entire quantity of the goods in posseseion is supplied, regardless of the price. This fact has already been explained). Secondly, in this case also, there is equality between the sums of prices bought and sold (given and received). The number of the equations indicating the existence of the condition (A) is $(m+n-1)\theta$, while the number of the equations indicating the existence of (B) is θ. The condition (C) to which reference hos already been made does not exist here. Instead, we have a fresh unknown quantity in the shape of A, which represents the balance between the amount of productive goods which individuals possessed before the exchange and that of which they have become possessed after the exchange, or, in other words, the total amount of a, the productive goods supplied by every person. Moreover, X, whieh represents the total of the finished products sold by all industrialists, constitutes another unknown quantity. There does not exist the conditions of (C), or the circumstances such as are shown in the following formula :—

$$\begin{cases} a_1 + a_2 + a_3 + \cdots\cdots = a_{10} + a_{20} + \cdots\cdots \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \end{cases}$$

$$\begin{cases} x_{10} + x_{20} + x_{30} + \cdots\cdots = x_1 + x_2 + x_3 + \cdots\cdots \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \end{cases}$$

Instead, a new system of equations (C') supplants it :—

$$(C') \qquad \begin{cases} a_{10} - a_1 + a_{20} - a_2 + \cdots\cdots\cdots\cdots\cdots\cdots\cdots = A \\ b_{10} - b_1 + b_{20} - b_2 + \cdots\cdots\cdots\cdots\cdots\cdots\cdots = B \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \end{cases}$$

$$\begin{cases} x_1 + x_2 + x_3 + \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots = X \\ y_1 + y_2 + y_3 + \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots = Y \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \end{cases}$$

Supposing that the number of the kinds of the productive goods, a, b,......, is n, and that of the finished products, x, y......is m, the number of these equations is $m+n$. They replace (C) in the equilibrium of exchange forming the new system (C'), but in this case, it does not occur that one equation out of the number can be removed, because it can be obtained as the result of all other equations.

Next, it is a feature of free competition (I will refrain from reference to all cases of monopoly) that the total of the prices is equal to the total of the cost of production. The above-mentioned equations (2) can consequently be formed.

$$(D) \qquad \begin{aligned} Xp_x &= \Pi_x = \pi_{ox} + \int_o^x \pi_x dx \\ Yp_y &= \Pi_y = \pi_{oy} + \int_o^y \pi_y dy. \end{aligned}$$

This system of equations is shown in (D). From (D) the following system of equations (E) can easily be obtained. It shows that the productive goods A', B', ... which have been transformed into finished products (or consumed for purposes of production) are equal to the productive goods A, B, which were purchased by industrialists.[3]

[3] 《2》 The following formula may be formed by summing up both sides of the equations ((2)).

$$Xp_x+Yp_y+\dots\dots\dots=\Pi_x+\Pi_y+\dots\dots$$

The right side of the above formula must be equal to the sum of the right side of (D).

$$\Pi_x+\Pi_y+\dots\dots=\pi_{ox}+\pi_{oy}+\dots\dots+\int_o^x \pi_x dx+\dots\dots$$

Viewed from another angle, the total cost of production is equal to the total amount of the property consumed (transformed) for purposes of production.

$$A'+p_bB'+\dots\dots\dots=\Pi_{ox}+\pi_{oy}+\dots\dots\dots$$

Thus, the following formula is constituted:—

$$Xp_x+Yp_y+\dots\dots\dots=A'+p_bB'+\dots=\dots$$

On the other hand, the following relation can be ascertained from the systems of equations in (B) and in (C').

$$Xp_x+Yp_y+\dots\dots\dots=A+p_xB+\dots\dots\dots$$

From the above two formulae the following result can be derived:—

$$A+p_bB+\dots\dots\dots=A'+p_bB'+\dots\dots\dots$$

Each of A, B, C, ... may be larger than A', B', C' ......, but they cannot be smaller than these, because the industrialist cannot transform more productive goods than he has purchased. If so, the next relation necessarily exists: $A=A'$; $B=B'$; $C=C'$; ...... (cf. Pareto, op. cit. p. 612; Zawadzki, op. cit. p. 217).

Thus far, I have derived (E) from (D). I think we can immediately accept the implications of (E) by assuming that in a statical state the productive goods purchased are essentially equal to the productive goods consumed.

Only where π_{ox} and π_{oy} are zero and the coefficients of production are unalterable, the formula (D) changes to the formula (D') and the formula (E) changes to the formula (E'). Walras (Cassel, also, for that matter) deals with these cases exclusively.

$$(D') \qquad \begin{cases} p_x=a_x+p_bb_x+p_cc_x+\dots\dots\dots \\ p_y=a_y+p_bb_y+p_cc_y+\dots\dots\dots \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \end{cases}$$

$$(E') \qquad \begin{cases} A=a_xX+a_yY+\dots\dots\dots\dots \\ B=b_xX+b_yY+\dots\dots\dots\dots\dots \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \end{cases}$$

(E) $A'=A$; $B'=B$; $C'=C$; ....

One equation out of the systems of equations in (D) and in (E) can be obtained as the natural outcome of all other equations, and consequently it must be obliterated. Thus, the number of the equations will be as follows:—

(1)	$(m+n-1)\theta$	equations in (A)
(2)	θ	equations in (B)
(3)	$m+n$	equations in (C)
(4)	$(m+n-1)$	equations in (D) (E)
total	$(\theta+2)(m+n)-1$	equations.

The numbers of unknown quantities are: m and n kinds of goods conusmed by each of the θ persons, m total quantities of the finished products sold by the industrialists, n total quantities of the productive goods purchased by the industrialist, and $(m+n-1)$ prices of the various goods. Thus the number of the unknown quantities $(m+n)\theta+(m+n)+(m+n-1)=(\theta+2)(m+n-1)$ is equal to the number of the equations, and the problem can be solved. The prices of the productive goods, the quantities consumed, the quantities and prices of the finished products are fundamentally determined.

This is the mechanism of the determination of the price in the state of free competition. Here I abstain from making any reference to how the monopoly price is determined in the state of equilibrium.

So far, I have dealt with the subject on the assumption that the coefficients of production are absolutely determined technically. The coefficients of production are determined not only by technics but by the general economic conditions also—by the prices of productive goods in particular. With due regard to this circumstance, a closer study must be made of the mechanism of the determination of the price. The Pareto theory is formed along these lines.

5. AMBIGUITY OF THE COEFFICIENTS OF PRODUCTION

If the coefficients of production were clearly and fundamentally (in such a way as to exclude all other possibilities) determined, the explanation so far made might suffice to solve the question of equilibrium in production, but it would be against the truth to say that they are fundamentally and consequently definitely determined in this way. Let us suppose that a certain amount of labour, capital goods and land are needed for producing a certain quantity of a commodity. If some reduction is made in the amount of labour, the same quantity of the commodity can yet be produced by increasing to some extent the amount of land and capital goods. Again, the same result can be achieved by increasing the amount of labour and capital goods, when the amount of land is somewhat reduced. This is a fact of general application. This kind of relation existing among productive goods, viz., the relation of mutual compensation for an increase or a decrease in any of the factors for the production of the same amount of products is called the law of compensation among productive goods (la loi de compensation).[1] With regard to specific kinds of productive goods, it is laid down that the coefficients of production are determined by technics. For instance, no more or less than four wheels are needed for making a four-wheeler. Even in this case, however, if we consider the production of a four-wheeler as a whole, it would be impossible to say that the coefficients of production are definitely fixed. Seeing that the coeffi-

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1) In this regard, it is helpful to consult Pareto, op. cit. p. 632, and Zawadzki, op. cit. p. 222. The so-called law of definite proportion or the theory of definite proportion which is supported by Pantaleoni and many others, is traversed by Pareto on the ground of the law of compensation (cf Pantaleoni, *Pure Economics*, translated by Bruce, 1898, pp. 81, 256 et seq.) The only point deserving closer scrutiny is whether the law of definite proportion, as Pantaleoni calls it, means nothing beyond Pareto's interpretation of it.

cients of production in regard to the production of the vehicle are ambiguous and not definite, the coefficients of production concerning the combination of productive goods of the highest order are at least ambiguous, if we consider the entire process of producing a four-wheeler as a whole, though we may have to admit that the coefficients of production at the final stage of production are definite.

In short, as a theory of general application, we must admit that the coefficients of production are always ambiguous and that it is in some special cases only that they are clearly defined. If we treat these exceptional cases as allowing of alterations in the coefficients of production within an extremely limited range, the theory that the coefficients of production are generally ambiguous will present no difficulty. Thus, in the production of goods there does not exist what is called the law of definite proportion (la loi des proportions définies). Unlike chemical combination, in which the proportion of the component elements to one another is always definitely fixed, there does not exist in the relation of productive goods for producing a certain commodity the rule that one component productive goods should invariably be in a certain fixed proportion to other component productive goods.

We must discriminate between the unchangeability or inflexibility of the coefficients of production and its unequivocality. Even where the coefficients of production are unequivocal, it sometimes happens that they change according to the amount of the goods probuced. When they change according to the quantities produced, they do not possess unchangeability. Even where they do not change according to the quantities produced, they are not necessarily unequivocal. When production is carried on by a variety of methods, the coefficients of production are ambiguous. It is, therefore, well to draw a line of demarcation between the unequivocality and the unchangeability of the coefficients of production, and between the ambiguity and the changeability of them. On the whole, it may be said that the coefficients of production regarding practically all goods are changeable and ambiguous.

The coefficients of production are not, as a matter of principle, unequivocal. The methods of production to be adopted and the ways in which productive goods can be put together for producing a commodity may be multifarious. How various and numerous these methods and ways are is decided by technics. In other words, the scope of variety depends on technies. However, as to the final choice of one group of particular coefficients of production out of many, there is the standard of selection. This standard may be the acquisition of the largest utility in case production aims at its utility for the producers, while it may be the acquisition of the largest monopolistic profit, in case of monopolistic production. But where production is undertaken by industrialists under the system of free competition, the standard of choice is always the lowest cost of production. A comparison of the cost of production is impossible, unless the prices of productive goods are taken as the basis of comparison. To be more exact, the coefficients of production are fixed by various items of economic equilibrium or so-called economic quantities (*ökonomische Quantitaten, grandeurs économiques*) —the prices of productive goods in particular—within a certain limited scope.

Thus, it is a more hypothesis to regard the coefficients of production as fixed by technics. As a matter of fact, the coefficients of production are not what is already given by technics; they are of unknown quantities. The conditions requisite for determining these unknown quantities are, as stated below, afforded by the law of compensation, on the one hand, and by the principle that the cost of production should be at the lowest possible level.

Suppose that the productive goods, A, B, etc., are needed for the production of the product X, and that a_x, b_x, c_x,...... represent the coefficients of production of all productive goods which are necessary for producing it. In these coefficients of production, the following law of compensation rules. If any of the component factors is increased, there must occur a decrease in other factors, and vice versa. In this way, the same amount of productive goods is obtained. Such relations are shown in the following equation:—[2]

$$[1] \qquad f(a_x, b_x, c_x \ldots\ldots) = 0.$$

As I have mentioned, the coefficients of production, as a general rule, vary according to the quantity of production, and the total cost of production Π_x in such cases is shown in the following formula.

$$\Pi_x = \pi_{ox} + \int_o^x (a_x + b_x p_b + c_x p_c + \ldots\ldots) dx.$$

And so long as free competition rules, it is necessary for this total cost of production to be minimum.

In free competition (the first type of Pareto), industrialists accept the market prices (the prices of productive goods especially) and determine the coefficients of production accordingly. Their economic actions sometimes influence market prices, though they are not specifically intended for such a purpose. And in such cases, they have to re-estimate costs according to the new market prices. Thus, industrialists shape their course very much along the curve line of pursuit. If the definite coefficients of production, b_x, c_x,......

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[2] This equation appears in equation (121) in Pareto, (Pareto, *op. cit.* p. 632) and in equation (30) in Zawadzki.

Commenting on the theory of definite proportion, Pareto says that the majority of the economists who avail themselves of this theory take the line that there exists a definite proportion in the combination of productive factors, independently of the prices of these factors, but that this view is mistaken. There are many proportions (ratios) that vary according to prices. Not only do the proportions vary according to prices but they vary according to all circumstances of economic equilibrium. (*Manuel*, pp. 372, 637.) The relation indicated in equation (1) owes its existence essentially to technics. It is by no means fixed by the prices of productive goods; technics are decisive factors. All coefficients of production are not necessarily ruled by this relation, however; technical conditions sometimes suffice to determine the coefficients of production, says Zawadzki (Z. op. cit. p. 222).

e_x, are to be altered, the change that comes over the total cost of production will be:—

$$\delta\Pi_x=\int_o^x(\delta a_x+\delta b_x p_b+\delta c_x p_c+\ldots\ldots+\delta p_b b_x+\delta p_c c_x+\ldots\ldots)d\chi$$

If the industrialists unfailingly accept the prices given, regardless of the changes that may occur in prices, the change that occurs in the total cost of production will have to be re-written as follows:—

$$\delta\Pi_x=\int_o^x(\delta a_x+\delta b_x p_b+\delta c_x p_c+\ldots\ldots)dx.$$

In order that the total cost of production, Π_x, may be the minimum, the amount which this formula signifies must be nil. Even if efforts may be made to reduce it to nil, the cost of production will not be reduced to the minimum, if the prices change, and industrialists will have to draw up fresh estimates. There will, however, be no occasion for re-estimation when the above formula signifies nil in the ruling market prices. The equilibrium is maintained in that case, and the following equation [2] co-exists with the other equations (A), (B), (C), (D) and (E) to which reference has already been made.

[2] $$\int_o^x(\delta a_x+\delta b_x p_b+\delta c_x p_c+\ldots\ldots)dx=0.$$

Now, with exclusive reference to equation [1], we can regard one of the coefficients of production to the number of r, say, b_x, as the function of the other coefficients of production, $c_x\ldots\ldots e_x$ (though, in this case, these are independent variables). Then, the following various equations can be derived from equation [2]:—

$$\int_o^x\left(\frac{\partial b_x}{\partial c_x}+p_c\right)\delta c_x dx=0$$

$\delta_{bx}\ldots\ldots\delta_{ex}$ are entirely optional, and consequently these equations cannot be proved in default of the following equations [3].

$$[3] \qquad p_b\frac{\partial b_x}{\partial c_x}+p_c=0, \;\ldots\ldots p_b\frac{\partial b_x}{\partial e_x}+p_e=p_o, \;\ldots\ldots$$

The number of such equations is $r-1$, and from equation [1] the numerical value of b_x, ∂c_x, ∂_{px}, ∂_{ex}...... can be found as its dérivé partiel. Then, the unknown quantities of the coefficients of production, a_x, b_x......e_x to the number of r can be determined by means of the equations to the number of r in [3] and [1], as the functions of p_b, p_c ...... and$_x$.[3]

In the same way, the numerical value of the coefficients of production, a_y, b_y, c_y......, to the number of r' and that of the coefficients of production, a_z, b_z, c_z, ... to the number of r'' can be determined by forming the equations to the number of r' regarding the production of Y and the equations to the number of r'' regarding the production of Z. A group of the equations to the number of r, r' and r''...... namely, $\sum r$ equations, must, in such circumstances, co-exist with the systems of equations (A), (B), (C'), (D), (E), already mentioned, in the state of equalibrium. This is called (F).

Thus, Ω or the whole of the systems of equations that determine the equilibrium of production can be obtained.

6. INTERPRETATIONS OF THE MATHEMATICAL SCHOOL.

I will desist from any further description of the so-called economic equilibrium, that is, the phase of productive equilibrium within it. How, then, do the economists of the mathematical school explain price on the basis of the concept of the equilibrium?

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3) I owe the mathematical proof for this part to the supplements (103), (104) and (105) of Pareto (*op. cit.* pp. 632–634), and chap. V. 4 of Zawadzki (*op, cit*, pp. 222–224).

Their contention may be summed up as follows:—(1) There exists among various economic quantities a complex relation of mutual conditioning, or the relation of interdependence. (2) It is a mistake to attribute price to a single cause, for the price as one economic factor stands in the relation of interdependence to other economic quantities, and it is impossible to conceive that it takes any of these economic quantities as its sole cause for determining it.

Of course, it took long for this final conclusion to be reached. Leon Walras, the first systematic developer of this concept of economic equilibrium, while regarding the so-called *rareté*, or the marginal degree of utility in substance, as the cause of the price (the exchange value in the terms of Walras), on the one hand, contends that all unknown numbers in economic problems depend on all equations of the economic equilibrium. These two contentions are manifestly contradictory, at least, in the view of the scholars of the mathematical school, who came after him. In the opinion of Pareto and other scholars of the mathematical school, it was because he was influenced by the traditional habit of thinking in political economy that there was one sole cause of price that Walras pointed to *rareté* as the cause of price, and they contend that he ought to have adhered to the view that all unknown quantities are necessarily determined by all equations and accordingly by all other economic quantities.[1]

Thus, scholars of the mathematical school (I refer chiefly to the scholars of the Lausanne school) take the

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1) L. Walras. *Elément*, 289; Pareto, *Manuel*, p. 246. While admitting, on the one hand, that there exists among the various economic quantities the relations of interdependence, Walras contends that the exchange value is equal to the proportion of marginal utility and that some of the unknown quantities can be taken as specifically dependent on certain equations. On the whole, it is noticeable that he kept up his endeavours to seek for the cause. It may also be mentioned that I am not quite correct in defining the *rvreté* of Walras as marginal utility. It ought to be called Grendnutzen, final degree of utility (Jevons) or *ophélimité élémentaire* (Pareto).

line that it is irrational to try to find the cause of price or value. There exists a close relation of interdependence among economic quantities, that is, various unknown quantities. Each of them is conditioned by the others, and they are accordingly determined at the same time. None of them forms the exclusive cause of another. So it follows that there is no sole cause of price. Herein lies the indispensability of mathematical knowledge in the study of economic phenomena. "Ordinary logic" may be helpful in studying the relation of cause and effect, but it is of no help in the study of the relation of interdependence. So long as it aims at finding the cause of price, when it is impossible to find it, in view of the existence of the relation of interdependence, either the theory of the cost of production or that of marginal utility is misplaced.[2]

If so, is there no room whatever for the justification of the theory of marginal utility, so long as the theory of economic equilibrium (mechanism of the determination of price) is accepted? To the best of my knowledge, Schumpeter advocates the theory of marginal utility while acceptting the equilibrium concept. In his opinion, the economic cosmos is an organisation of quantities which are interdependent on and condition one another. It is, in such

[2] Pareto, *op. cit.* p. 247; Zawadzki, *op. cit.* p. 256–258. It may not be correct to regard Cassel as a scholar belonging to the Pareto school, but the fact is generally recognised that he owes his theory of price to Walras. Setting apart the question of the futility of the theory of value, his repudiation of the theory of the cost of production and the theory of marginal utility, on the ground of the theory of equilibrium, puts his opinion in accord with that of the Lausanne school. Cassel does not attempt to deny the concept of the cause of price because of the interdepence of economic phenomena. In other words, notwithstanding his admission of the fact that ecenomic quantities condition one another, he does not deny that price has its causes. He repudiates the theory of the sole cause of price. As the decisive factors, he mentions the demand function or price function (subjective), technics and methods of production (objective), and as distant causes, he makes mention of many things. I propose to discuss this point in another part (Cassel, a.a. O. S. 134 ff.).

circumstances, impossible to say that one thing among these is the cause of another, and so the controversy of the theory of the cost of production and that of utility is meaningless. This, however, is not the case, when viewed from another angle. This organisation is controlled in all other parts by the principle of value, or, to put it more correctly, the principle of marginal utility (vom Wertprinzip, richtiger vom Grenznutzenprinzip), for the marginal utility concept includes in itself the objective relation of goods. For instance, any one element in the astronomical cosmos forms the cause of the motion of any other element. The only thing that rules is the general mutual action. The general demonstrative principle rules in this universe, and the moment which operates according to this principle is called the cause of process. In the same sense, the principle of marginal utility can be viewed in the light as the formulation of unified causes. Needless to say, each individual concrete marginal utility does not constitute a cause in itself, though it is provisionally treated as though it constituted the cause, when each component of the whole system is explained separately.[3)]

I do not think, however, that all parts of the system of economic organisations or the whole body of economic equilibrium is controlled by the principle of marginal utility. This opinion I would uphold even if I were to accept the theory of marginal utility in its entirety. Look at the equations forming a part of the equilibrium of production, for instance. In neither of the following two equations, taken from among the various equations already mentioned,—one showing the equality of the price with the cost of production and the other indicating the law of compensation among

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3) Joseph Schumpeter, Cassel *Theoretische Sozialökonomik.* Schmollers Jahrbuch, 51, Jahrgang, Zweites Heft 1927 S. 79 ff. I am afraid that my summary of the view of Schumpeter contains some ambiguous points. If so, it is due to my lack of understanding. For particulars, I must refer the readers to the original.

the coefficients of production—can one notice the operation of the principle of marginal utility.

(2) $\Pi_x = Xp_x$

[1] $f(a_x, b_x, \ldots\ldots) = 0.$

It seems that the system of equations (A) is the only one which is directly ruled by the principle of marginal utility. The contention can never be accepted that marginal utility should be taken as the cause of price, because it is the general explanatory principle, or the principle that controls the whole of the organition of economic dimensions.

7. MY POINT OF VIEW—BASIS OF THE THEORY OF POWER

I take the following view of the system of equations illustrating the economic equilibrium.

If the Cassel (or Walras. we may also call it) system of equations, which lays it down that the coefficients of production are determined (unequivocably) by technics is to be accepted, it may be concluded that price is entirely fixed by marginal utility, and that there is no possibility of the price of one and the same goods being changed through the operation of social power. I do not think, however, that it is proper to deny the so-called cause of price on the basis of this system of equations. Let me now offer some explanation of thr last part.

The task of economic theory is at once to find laws which are of help in explaining matters and to promote understanding. Its principal aim is to lay hold of comprehensible (verstehbarė) laws, or laws by which what is determind by motives can be traced. Take price, for instance, from amonġ many phenomena which are dealt with by economists. There may be many circumstances responsible for its fluctuations or what are called determining factors, but they can roughly be divided into two categories. One category includes factors which, constituting themselves motives, dictate the formation of the price, viz., subjective factors, while the other category consists of factors which, not forming motives themselves, influence the formation of the price by directing the motives in an indirect way, viz., objective factors. To those who emphasise understanding (verstehen), I think, the former, or subjective factors, may properly suggest themselves as the cause, or the determining ground, if the term "cause" is somewhat irrelevant here. As to the other determining factors, they simply operate through this cause, instead of determining the price directly.

Next, let me consider interdependence and mutual conditioning. There exists among all social phenomena the relation of mutual conditioning or that of mutual determination (these two terms are used in the same sence), but it cannot be said that among them functional relations (and accordingly the relations shown by equations) only are noticeable. Nor is it correct to say that all that can be said is that "one of these things is determined by all others and consequently they are all unknown quantities to be put on an equality with one another, which are determined simultaneously." It is, of course, to be granted that where the principle of mutual conditioning prevails all represent unknown quantities that are determined simultaneously, but the relation of cause and effect between individual factors within the group can surely be made clear. It may further be ascertained which among these factors, is the fundamentally determining one or the predominant one. If this is impossible, then there can be no laws regarding social phenomena. The following point, I think, constitutes the standard of judgment as to the basic or non-basic nature of conditions. A and B condition each other, but while B cannot be considered, independently of the influence of A, A can be considered, apart from the influence of B. That is to say, although B is conditional on A for its very existence, A does not necessarily stand in such a relation to B. In such a case, A's condition is called fundamental.

With this preface, I proceed to set forth my views. If this preface is not generally accepted, I wish to make it clear that my argument is based on this discrimination.

Cassel points out on the basis of his systems of equations that there are subjective and objective determining factors of price. This point has already been touched upon. By subjective determining factors are meant " the coefficients of equations (4) indicating the dependence of the demand on the price." In my opinion, they may properly be regarded as the form itself of the demand function. The coefficients of production and the quantity of productive goods, which are mentioned as objective determining factors, do not represent the determining factors which operate as what I call motives. They may determine the quantity of finished products and accordingly determine the final demand that can be supplied, but the do not constitute any motives in themselves. In my view, it is the subjective factors only that are really decisive. Objective factors merely act on subjective factors. Thus, Cassel's equations must be interpreted as follows:—The decisive reasons (causes) of price are to be sought in the demand function constituting subjective determining factors. As to the dimensions which a price may assume on account of the demand function, objective determining factors operate as contributory conditions.

To state the same argument in my own words, so long as Cassel's equations are accepted, the price is fixed in accordance with the demand function, for the coefficients of production, a, and the quantity of productive goods, R, may be regarded as the quantity already fixed in a certain society. To push this argument a little further, the price is fixed by marginal utility, for, regardless of the refusal of Cassel to push his analysis to that extent, the demand function is determind by marginal utility. Indeed, Cassel himself seeks the principle of determining the price in *rareness*, and there can be no marked difference between Cassel's *rareness* and Walras's *rareté*. Walras's *rareté* evidently means marginal utility.

Thus, if Cassel's system of equations are to be accepted, it seems fair to coclude that the price is fixed by marginal utility. This system of equations, however, is based on the theory that the coefficients of production are definite, as is clear from equations (6), (7) and (3), and it is against the truth to regard the coefficients of production as defined by technics. The coefficients of production itself are unknown quantities.

What, then, determines the coefficients of production? Technics define the scope of their possibilities, and the price of productive goods determines them within this scope. The final judge or the factor that unequivocably determines them is the price of productive goods. The price of productive goods means, in effect, whatever the supplier of productive goods can get. What determines it is the power which he thinks he can exert in obtaining such and such a price. This power determines the price of productive goods as the motive. It will thus be seen that behind the coefficients of production, which were called an objective determining factor of the price, operates the power or the resistance of the economic subject. Needless to say, this resistance, like demand and utility, signifies one phase of the attitude of the subject.

So-called determining factors of price
- (I) Subjective factors (the attitude of the subject)
  - (1) Marginal utility.
  - (2) Resistance (to be more exact, the resistanee of those who occupy the marginal position.)
- (II) Objective factors
  - (1) Coefficient of production
    - Price of productive goods
    - Technics
  - (2) Quantity of productive goods

So long as there operates the so-called law of compensation among productive goods, there can be no means of determining the coefficients of production, apart from the price of productive goods. If production can be carried on, leaving it (price of productive goods) indeterminate, the coefficients of production can assume various forms. The quantities and the prices of products will be determined according to the forms which they take. There can be no equilibrium, if only the systems of equations (A), (B), (C),

(D), and (E) exist, and the system of equations (F) be lacking, and if, moreover, a_x, b_x, c_x, ... are left indeterminate. Even if an equilibrium can be maintained in such circumstances, it is an ambiguous one. It is the resistance of the economic subject that definitely determines the coefficients of production and completes the objective determining factors. This is especially true of the resistance of marginal suppliers in the market of productive goods, namely, marginal resistance.

The causes or the determining reasons of price are what make one understand the formation of price, or, in other words, subjective factors. From this point of view, marginal utility and resistance may be taken as much causes. Do these causes operate quite independently of each other in determining price? My answer is in the negative.

There is no possibility of marginal utility (demand function also) being fixed except on the basis of objective factors. Even if the amount of the purchasing power and the extent of demand may be known, the marginal utility of the goods to be obtained cannot be ascertained, unless the kinds and quantity of the goods available are made known. Where there is neither the coefficient of production, nor the price of products, nor resistance, there can be no marginal utility. Marginal utility is, therefore, inconceivable except on the premises of resistance, so long, at least, as the matter concerns products. Resistance invariably reflects the relation of power. It is of a nature that can exist independently of marginal utility. Of course, the extent to which resistance can exert its influence is restricted by the price of products, and consequently it is amenable to modification by this restriction, but it is one thing for it to exist and it is quite another for it to be subject to modification. The fact that it is amenable to modification and that it is restrained or promoted so that it can maintain its position in the whole body of equilibrium until it attains a definite size does not disprove its independent and spontaneous existence. Resistance is neither modified nor changed, when its size, spontaneously fixed, enables the whole body of the equilibrium of production to be formed. It is altered only when it is of such a quantity as to make it impossible for equilibrium to be brought about. Needless to says, due regard is had for the original size, which is spontaneously fixed, in altering it; it is not fundamentally changed.

There is no marginal utility without resistance, while resistance can exist without marginal utility. Thus viewed, it may be said that of these two subjective determining factors, one is subservient to the other. To a certain extent, resistance is the decider, while marginal utility is what is to be decided. From this point of view, I am inclined to say that resistance is the cause of price. Resistance means social power, so the conclusion is inevitable that social power constitutes the cause of price. In the whole mechanism of the determination of the price, it is social power that plays the predominant part. The role played by marginal utility and demand function is passive and negative in that they operate to decide the point at which the action of the predominant factor should be stopped.

The theories which admit the correlative existence of utility and costs have been prone to regard their relation in the same light as the relation of each part of a pair of scissors to the other, but I see some difference between them in the degree of importance. I put costs in the position of superiority to that of utility. I do not, however, attempt to seek the contents of costs in pains; I seek them in resistance and accordingly in social power. From this point of view, mine is the theory of power rather than the theory of costs.

8. SUMMARY

In conclusion, I with to summarise my views.

If the coefficients of production are determined by technics, the prices of productive goods cannot but be determined by the prices of products. The cost principle of the price is evolved as the result of the determination of the price by marginal utility. Consequently, it is absolutely impossible to admit that the relation of power dictates the price, independently of the action of utility. A different view may be taken of the matter, if the changeability of the demand and the infinite possibilities of the creation of new goods are taken into consideration, but it is not my present purpose to discuss the subject on such a premise.

In fact, the coefficients of production are determined by the price of productive goods, instead of by technics. This price depends on the resistance of the supplier of productive goods and accordingly his power. On the basis of the coefficients of production, productive goods are combined in a variety of ways. And the quantities of various kinds of products are regulated so that price of products may be made equal to the cost of production. If this is found difficult, a reaction comes over the price of the productive goods, and resistance is either weakened or strengthened. Thus, the whole body of equilibrium is formed. There is no price of productive goods, if there be no relation of power, and if there be no price of productive goods, the coefficients of production cannot be fixed. Again, if there be no determined coefficients of production, production is impossible. The equilibrium of production must start from the relation of power. The price represents the range of the bullet discharged with power as the spontaneous spring, even so are the prices of productive goods and of products.

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