AN EXAMINATION OF PROFESSOR CASSEL'S QUANTITY THEORY OF MONEY

1. INTRODUCTION

Views pro and con have been published by many scholars regarding Professor Cassel's quantity-theory of money, especially after it was adopted as the theoretical basis of the First Interim Report of the Gold Delegation of the Financial Committee of the League of Nations made in the autumn of 1930. His theory offers important suggestions into the economic laws governing the variations of the general level of prices; hence the universal interest taken in the matter. I shall take the liberty of making a critical survey on his theory in this paper.

First of all, it should be noted that the quantity theory of money may take either of the following two forms: (1) the *a priori* or deductive or speculative quantity theory which contends that the variations of the quantities of money correspond to the variations of the relative value of money; and (2) the *a posteriori* or inductive or positive quantity theory which upholds the theory that the variations of the quantity of money generally tend, in experience, to correspond to the variations of the relative value of money. Now, it is not clear which of the foregoing forms Professor Cassel really upholds when he advances his quantity theory of money. In my present paper, I assume that his theory is an *a posteriori* quantity theory, because the other theory is impossible of logical presentation. I have elsewhere demonstrated the logical weakness of the *a priori* quantity theory which I shall briefly summarise as follows:

The *a priori* quantity-theory will be impossible of logical explanation unless it should take "money" in the sense
either of "stock money" under the condition that the quantity of goods and the velocity of the circulation of money are constant or of "stock of money multiplied by its velocity of circulation." While, in either case if the theory is conditioned in such a way and that is logically necessary for the formation of that theory—it simply assumes that the prices of commodities are already fixed. This assumption is made unwarrantably inasmuch as it is precisely the prices, the general level of which the quantity theory of money attempts to explain. Because the general level of prices is constructed simply by taking a general view of the prices of commodities from a specific standpoint. It should be remembered that the prices of commodities are the foundation of the general level of prices and that in consequence no theory aiming at the explanation of the latter can make an assumption regarding the former. Any theory that makes such an initial assumption falls into the logical error of creating a "vicious circle". Consequently remains only the a posteriori view of Prof Cassel's quantity theory of money to be critically surveyed.

2. VIEWS OF PROFESSORS CASSEL, KITCHIN AND WOYTINSKY

Although Professor Cassel's quantity theory of money is widely known, I shall here briefly summarise his main propositions for the sake of lucidity.

The total stock of gold accumulated in the world (I shall hereafter refer to this simply as "stock of gold") during the years between 1850-1910 has been increased from 10,000 million marks to 52,000 million marks: or in other words, the amount of the stock of gold has been increased by 5.2 times during the past 60 years, at the annual average rate of 2.8 per cent. However, according to Sauerbeck's General Index Number of Whole-sale Prices in England (I shall hereafter refer to this simply as Sauerbeck's L.N.), the general level of prices in 1910 is approximately
the same as in 1850. This fact means that the increase in the stock of gold during the period as a whole has had no influence on the general level of prices, but merely corresponds to what is required in order that the growth of the stock of gold should keep pace with the general economic development. In consequence, had the stock of gold increased precisely at the rate of 2.8 per cent annually during the period (the stock of gold in these circumstances is called "the normal stock of gold"), the variation in the supply of gold would not have given rise to that of the general level of prices. But actually the annual stock of gold has deviated from the normal stock of gold. This deviation is shown by the "relative stock of gold" which is produced by dividing the annual stock of gold by the normal stock of gold. On the other hand, the variations of the relative stock of gold closely correspond to the secular variations of the general price level. (See Diagram 1). It is proved by this that the secular variations of the general price level depend on the contemporary variations of the relative stock of gold.

This explanation, however, is somewhat insufficient even with regard to the secular variation of the general price level.
during the years between 1850-1910, as is apparently shown in diagram 1. Professor Cassel himself admits that "the secular price level in the period 1850-1870 is higher, and in the "nineties rather lower, than it ought to be according to the relative stock of gold"."

On this point Professor Cassel says:

"(One) circumstance is, without doubt, primarily responsible for the depression (in the period, 1875-1890), in so far as it superseded the simultaneous fall in the relative stock of gold. This circumstance is the enormous increase of gold demand in the United States at the end of the seventies and the beginning of the eighties, due to the preparation for and subsequent realisation, of the resumption of cash payments. . . . A factor that acted in the same way on the side of the demand, though to a less extent, was the large imports of gold into India in the early eighties". The foregoing statement, however, cannot explain the fact that the general level of prices during the period, 1850-1870 is higher than the relative stock of gold.

Kitchin, who uses the same method of investigation as Professor Cassel, differs from him in one respect. He made the world's monetary stock of gold (I shall hereafter refer to this merely as "stock of gold money") the starting point of his research. He has succeeded in clarifying the fact that the world's stock of gold money during the period, 1850-1910 increased from 230 million pounds to 1,440 million pounds at the average annual rate of 3.1 per cent. By taking these figures as the basis of his calculation, Kitchin compared the normal stock of gold money with the annual stock of gold money and secured the figures for the relative stock of gold money. He has demonstrated by this calculation that the variation of the relative stock of gold money more closely corresponds to the secular variations of the general level of prices than does Professor Cassel's relative stock of gold; (See Diagram 1).

Both Professors Cassel and Kitchin regard the stocks of gold on of gold money in 1850 and in 1910 to be normal
only because the general levels of prices of these two years 
are roughly the same. Accordingly, they compare the 
stocks of gold or of gold money of these two years and 
calculated the 60th root of the percentage of the increase 
between them, in order to secure the normal rate of increase. 
They then secured the figures for the relative stock of gold 
or of gold money by taking the normal rate of increase as 
the basis of calculation, and then compared them with the 
general level of prices by making them index numbers 
with the figures for 1850 as 100. But many difficulties are 
bound up with such an attempt to find the normal rate."

It needs no demonstration to manifest that the normal 
rate can be best secured by taking into consideration the 
statistical data of all the years. This point is given special 
consideration by W.L. Woytinsky who accepts the fundamental 
idea that runs through the theories of both Professors 
Cassel and Kitchin—the idea of demonstrating their a pos-
teriori quantity theory by taking only the stock of gold or 
of gold money and the general level of prices.

Prof. Woytinsky contends that in order to equate the 
relative stock of gold money—which is produced by dividing 
the annual stock of gold money (m) by the normal stock of 
gold money (I=a.b') and by multiplying the quotient by 
100—with the annual general price level (p), conditions 
should satisfy the following equations:

\[
\frac{100m}{1} = \frac{100M}{a.b^2} = p \quad \text{and therefore} \quad \frac{100M}{p} = 1 = a.b^2
\]

and therefore \( \log \frac{100M}{p} = \log \text{Ig} = \log a + x \log b \).

By means of the method of least squares, the trend line 
(\( \log 1 = \log 2,5662 + \log 0.01343x \)) is computed. And in consequence 
the following computation is made: \( 1 = 232.4 \times 1.03142^x \). Thus, 
Prof. Woytinsky has computed the normal stock of gold 
money as well as the relative stock of gold money which 
has been found to correspond more closely to the secular 
variations of the general price level than does Kitchin's 
calculation of the relative stock of gold.3 (See Diagram 1).
3. EXAMINATION: SECTION (I)

In Section 1 I shall examine what qualifications the data so far given must possess in order to be positive material and to what extent these data possess, if at all, these qualifications. In Section II I shall examine in what sens: these data, supposing they possess such qualifications, are qualified to be positive material: in other words, I shall attempt to learn to what extent these data can assist the demonstration of the a posteriori quantity theory when they are used in the manner in which these scholars did use them. In the last Section, I shall endeavor to investigate how the a posteriori quantity theory can explain the variations of prices since the World War. The main thesis of this paper concerns itself with the problems dealt in the last two Sections: and the first problem with which I shall deal in this Section is to be treated only as a matter of course in preparation for the study of the other more important problems at issue.

In Professor Cassel's theory the following two data are used as the bases of his positive materials: (1) Sauerbeck's index number, representing the variations of the general level of prices of the world and (2) the world's stock of gold.

(1) As is well known, Sauerbeck's index number has been often used in the study of the variations of the general level of prices in the world. Professor Cassel gives the following three reasons for this use: (i) England has been, roughly speaking, a free-trade country (ii) she has been the world's trade market to a greater extent than any other country (iii) and she has maintained the gold standard system, the last named being the decisive factor with regard to the questions at issue.

Now, it should be noted that these reasons given by Professor Cassel may prove the bases of consideration and conjecture, but certainly not that of any necessary assertions. At any rate, their accuracy must be decided in the light of positive evidence. Let us then consider if Sauerbeck's general index number of the wholesale prices of England can represent the variations of the general level of prices of the world, as is supposed by Professor Cassel.

In order to calculate the definite general level of prices, the following two points should first be determined: (a) the composition of goods, with the prices from which the general level of prices are calculated and (b) the method of averaging adopted for computation. These two points, in turn,
are dependent on some definite standpoint. In considering whether or not Sauerbeck's index number can represent the variations of the general level of prices of the world, we face this problem: suppose Sauerbeck's index number is calculated from an international standpoint—in other words, suppose his British standpoint of calculation is cast aside—shall we come upon the same variations of the general level of prices as he did? Now, this is a difficult, if not an impossible, question to solve. We should have to consider the general index numbers of other countries, but we find that each general index number of prices of each nation differs in the composition of goods, with the prices of which they are calculated as well as in the method of averaging adopted in them. Because of this difficulty, such index numbers may have little or no value for our present enquiry. Strictly speaking, the data at our disposal at present may not materially assist us in answering the question confronting us.

On the other hand, a comparative study of the index numbers of the other nations will have some significance, so far as the supposition is permissible that some of them at least have been calculated from a similar standpoint as Sauerbeck's. But here we meet another difficulty, namely, the paucity of index numbers we can secure. I have taken the index numbers of Japan, England, the United States, France and Germany, computed the 9-year moving average of each; and by comparing them I have discovered the following details: the secular movements of the general levels of prices of England and France resemble each other, and those of Germany and the United States also show the same phenomenon, but that no similarity is shown between these two groups. (See Diagram 2).

In consequence, it appears dubious whether Sauerbeck's index number
can be taken as rightly indicating the variations of the general level of
prices of the world. There are several points in this connection that chal-
lenge our attention. Whereas the principal commodities in Sauerbeck's Index
number (for England) are raw materials those in Aldrich's index number
(for the United States) are manufactured goods. Now, the rise of the prices
of manufactured goods is less sharp than that of the prices of raw materials.
If Lenoir's explanation, that the prices of manufactured goods fell during
1850-1860 due to the progress made in the infant industries of the U.S.
may also be applied to the 'seventies; and further, if we can accept as
correct Professor Wagemann's assertion that (the) prices in Germany during
1850-1871 deviate sharply from those of other countries because the cheap
prices in Germany of cereals and potato had affected prices in general in
that country; and again, if one can assert that the secular variations of
the general level of prices in Japan (which greatly resemble those of India),
deviate from those of other countries partly because of the circumstances
attendant on the calculation of the index number, and partly owing to the
limited degree of her participation in the world's general economic activities
—if these conditions are to be accepted, then Sauerbeck's index number may
be held as approximately representing the general level of prices in the
world during the period under consideration. It is questionable, however,
whether so far as the period 1850-1870 is concerned, Sauerbeck's index
number is not somewhat higher than the actual general level of prices in
the world.

(2) Many writers have taken the world's annual output of gold as
representing the world's stock of monetary gold; but they have invariably
failed in getting the desired results; nor can it be said to be satisfactory
from the theoretical standpoint. The reason for this failure is clear. There
is a vast difference between the world's annual stock of gold and its annual
output of gold because of its high degree of conservation. Professor Cassel
accordingly has adopted the annual stock of gold in place of the annual
output of gold. His system, however, contains several points that invite
criticism.

In computing the world's annual stock of gold, Professor Cassel has
adopted as the basis of his calculation Professor Lexis' estimate of the total
stock of gold in 1848 which is given as 9,560 million marks. The amount
of the total gold stock in 1860 is estimated at 10,000 million marks. Deducting
one cent per cent of the amount of the stock of gold at the beginning
of every five years from it, for the definite loss of gold through wear, and
adding to the remainder the total output of gold during that lustrum he
computed the total amount of gold stock at the end of each lustrum down
to 1875. After 1876, a two per cent reduction of the amount of stock of
gold at the beginning of each year is made for the same reason and to the
remainder is added the amount of the output of that year, the combined
amount thus representing the stock of gold at the end of that year.

Now as according to Del Mar's estimate, the output of gold in both
1849 and 1850 was much higher than the annual average output of gold during the 'forties', while Soetbeer puts his estimate of the annual average output of gold during the period 1841-1850 at 132.8 million marks, we can easily assume that the increase of gold stock during the period 1848-1850 to be not less than 440 million marks. Accordingly it seems natural, by taking Professor Lexis' estimate for 1848, namely, 9560 million marks, to estimate the amount of stock of gold for 1850 at 10,000 million marks. But as this estimate of Professor Lexis is only concerned with the "Goldvorrat der abendländischen Kultur," though the other countries may have no great importance, it is highly dubious whether such an estimate can justly represent the world's stock of gold. By the way, if we compare the figures given by Soetbeer and by the Minister of the Mint of the United States whom Professor Cassel quotes in his work, concerning the period for which both are given, we see that the figures of the former are generally higher than those of the latter. Del Mar's estimate for 1800-1851 is lower than Soetbeer's. A calculation made according to Professor Cassel's own method has shown the amount for 1800 which is lower than the estimate of the stock of gold by Professor Lexis; a similar calculation for the estimate of the stock of gold for the year 1850 has shown that Professor Cassel's estimate is higher than that of Professor Lexis. This also may be taken as indicating that Soetbeer's estimation of the output of gold is higher than that of Professor Lexis. Again, Soetbeer's estimate for the annual output of gold during the period 1851-1875 is higher than that of Layton. Thus, one may question the accuracy of Soetbeer's estimate which Professor Cassel adopts in his investigation. True, some special study will be required in order to decide which is more correct; the mere fact that many other scholars put the estimate lower than is done by Soetbeer does not necessarily mean that his estimate is wrong. Moreover, some may object to Professor Lexis's policy of deducting from the annual stock of gold a 2 per cent depreciation for the annual definite loss of gold through wear, etc. from the annual stock of gold.

However, the most objectionable point in Professor Cassel's use of gold statistics is not found in the foregoing points; rather it lies in the fact that he deals with the stock of gold and not with the stock of gold money. He has adopted the total stock of gold first because in the case of the gold standard system, there is no clear line of demarcation between stock of money and stock of gold and, in consequence, the stock of gold does not indicate the immensity of its mass and secondly because "it (the stock of gold) is able to give an immediate answer to the question before us—namely, the sufficiency of the annual production of gold." However, to what extent a stock of gold can represent a stock of gold money must be proved empirically. If the amount of the world's stock of gold money is known, it should certainly be used directly. In this sense, a great contribution has been rendered by Kitchin who has made a successful use of the world's annual stock of gold money and by its help, calculated the relative
stock of gold money that better corresponds to the secular variations of the
general level of prices than does Professor Cassel's relative stock of gold.

But how far can we rely on the statistics of this important annual
stock of gold money? Kitchin has gathered statistical materials from all
over the world excepting India, China and Egypt. He says: "It assumes a
stock of £200,000,000 at the end of 1843 and the addition to it each year
of the balance of the world's gold output remaining after deducting the
demands of the industrial arts of Europe and America and the absorption
by India, China and Egypt." Unfortunately, he does not show the statis-
tical sources for this calculation, except those for the gold used by the
industrial arts of Europe and America up to 1913. (I believe he must have
shown them on some other occasion. But at present I am unable to get
access to fuller statistics. As I am quite unacquainted with such matters, I
confess that I am at loss as to what extent I can rely on the information
given by Kitchin. However, in view of the reputation he enjoys in the scholas-
tic world, I shall quote him in subsequent pages."

4. EXAMINATION: SECTION II.

Professor Fisher has once said: "There is—in the
system of general equilibrium—always just one too few
equation to determine the unknown quantities involved.
The equation of exchange is needed in each case to supple-
ment the equations of supply and demand." This view of
Professor Fisher's is due to his confounding the equation
$P = 1$ which is required in the general equilibrium system
besides the various equations indicating supply and demand
functions, etc., with the equation of exchange (for instance,
$MV = TP$). I have demonstrated this elsewhere. It should
be noted that the general equilibrium is not based on "the
equation of exchange"; rather the latter is formed when the
phenomena which are so to speak determined in accordance
with the general equilibrium theory are summarily observed
from some definite standpoint. In other words, one knows
M by deciding what conception of money one should adopt;
and further the equation $MV = \Sigma pQ$ is determined by the
fact that a transaction represented by $\Sigma pQ$ takes place only
so far as its M circulates directly or indirectly. Thus, the
equation $\Sigma pQ = TP \cdot MV = TP$ is obtained by deciding the
following two factors: the descriptions of commodities whose
prices are to be used as the basis and the method of averaging to be adopted, for the calculation of the general level of prices. In consequence, the equation of exchange is not, as is often supposed to be, predicated on the *a priori* quantity theory of money; nor does it prove that theory. And the meanings possessed by all the terms included in the equation of exchange such as M, V, T, and P are essentially determined by the foregoing circumstance.

Now, the equation of exchange is expressed as follows:

$$MT = TP \quad P = \frac{MV}{T} = M \cdot \frac{V}{T}$$

Thus, P is equal to M multiplied by \( \frac{V}{T} \). It is impossible to discover any quantity relation between P and M inasmuch as \( \frac{V}{T} \) is a variable, in so far as we compare them directly. But if the movement of \( \frac{V}{T} \) actually takes place constantly, there must be some quantity relation between P and M on which that constant term is multiplied. To treat \( \frac{V}{T} \) in its entirety, to demonstrate the law governing it, and thereby to demonstrate the quantity theory—all this should constitute the main theme of the study of Professor Cassel's quantity theory.

An attempt at demonstrating the quantity theory from such a standpoint was made as early as 1901 by Aupetit. He took the world's annual output of gold as M and the period of his study was limited to 1878–1900. (Lenoir, on the other hand, has extended the period up to 1910). Professor Cassel took the world's annual stock of gold as M and at first his study covered the period 1850–1900. (He estimated the amount of the world's stock of gold in 1850 at 500 million pounds sterling and that of 1900 at 1,850 million pounds sterling, and gave the normal rate of increment as 26.5 per cent). Later, Hooker extended the period up to 1910. Professor Cassel later made another study over the period 1850–1910 and found a normal rate of
increase of 2.8%. This rate was used as the basis of calculation to discover the relative stock of gold for a period beginning in 1800 and ending in 1925. Kitchin took the world's annual stock of gold money as M and his first study covered the period 1850-1910; and by using as the basis of calculation the normal rate of increment thus found, that is, 3.1%, he discovered the relative stock of gold money for a period beginning in 1807 and ending in 1929. But in all these cases, the increases of $\frac{V}{T}$ is computed with the M of two specific years as the basis of calculation. Unlike Aupetit and Lenoir, Professor Cassel and, in consequence, Kitchin, have accidentally selected the more advantageous years as the result of their desire to select years whose general levels of prices are identical with each other.

Even supposing that the movement of $\frac{V}{T}$ takes place comparatively constantly, it cannot be said that the $\frac{V}{T}$ of any two years necessarily correspond to its normal secular movement simply because their general level of prices shows the same height. It is in this sense that Woytinsky's theory has made a great contribution to the study of the a posteriori quantity theory of money. His theory has proved that $\frac{V}{T}$ was comparatively constant during the period 1850-1910. One may be permitted to assert that Professor Cassel's method of study has been exhausted by Woytinsky's theory.

In order to push forward studies of the a posteriori quantity theory one step further on, one should investigate why the $\frac{V}{T}$ during the period 1850-1910 appears to be comparatively constant; and in order to do this, one should know how T has moved.

I have already pointed out that the calculation of T is dependent on the following two factors definite descriptions of commodities, of which the prices are calculated, and a
definite method of averaging, both of which are determined by a definite standpoint. Inasmuch as T should be included in the equation of exchange, calculation of it must be made from the same standpoint from which P in the equation of exchange is calculated. Consequently, inasmuch as the P we make use of is taken from Sauerbeck’s index number, T must be calculated in our case from the same standpoint from which the P was calculated by Sauerbeck himself. On the recognition of T, Professor Cassel says:

"An approximate idea of the world’s rate of progress may be gained in the following way. In the period 1850–1907, the world’s production of pig-iron increased on an average by 4.2 per cent. Now, the growth of the iron industry may be regarded as characteristic of the whole industrial development of the world. The agricultural development had, of course been much slower and may perhaps be put at the figure of 1.2 per cent per year, which seems to correspond fairly well to the growth of the population and the improvement of its nourishment. If we assume that food represents a third of social income and that the other two-thirds of this income have grown proportionally to the industrial development, we arrive at an average rate of progress of 3.1 per cent. If, on the other hand, we give the rates for food and industrial production the same weight in our calculation of the average progress, this average would be 2.7 per cent. This figure ought to be regarded as the lower limit for our estimate of the world’s average economic progress. As it seems necessary to give the industrial production a somewhat higher weight than the agricultural, we stand on fairly solid ground if we reckon with a figure round 3 per cent as characteristic of the economic development during the period 1850–1910. This corresponds closely to the result of our enquiry into the world’s gold supply. It is possible that the average rate of economic progress for the world as a whole, if it could be calculated exactly, would prove to have been, during the period in question, not greater than 2.8 per cent per year."

How far his calculation of $T$ has been made according to the standpoint from which Sauerbeck's index number was made is open to serious question. However, even if we assume that there is no such doubt, we must question whether or not each of the productions of wheat and iron has increased at the same rate of increase.

In considering the world's production of wheat and pig iron, I shall make an index number by taking the figure of 1913 as 100. Then, in order to eliminate the effects of the business cycle, I calculated a 9-year moving average. By taking the representative figures of every lustrum during 1850-1925 (the figure of 1850 is taken in its original form), the trend line indicating the average development is calculated by the method of least squares. Then, I have divided the above indicated 9-year-moving-averaged index number by this trend line, and secured the results which presents quite uneven lines especially after the Great War. Thus it is impossible to conceive of any even increase of so far as the foregoing considerations go. (See Diagram 3). There is some room for doubt regarding the high rate of deviation for wheat during 1850-1860. The material used may have been of doubtful character.
It is extremely difficult to secure materials over such a long period of time and in such a world-wide matter as we are dealing with. It is questionable how far we can put confidence in those data we can manage to secure; nor can we have any clear notion of their continuity. As I have stated in the foot notes, there is much doubt about the correctness of individual statistics forming the production statistics of pig-iron and wheat. There is much looseness involved in the collection of these data. A mass of incorrect statistics will only give incorrect information. But in order to approach the standpoint of Sauerbeck's index number, it is clear that one will have to gather more statistics and from a wider field. In actual practices, however, statistics which satisfy the desired requirements are very few in number; and out of these few statistics only a still further reduced number of them can be actually used because of the necessity of maintaining a balance in the policy of gathering such statistics.

In order to supplement the foregoing statistics, I have also taken the figures of the world's production of copper, coal and petroleum oil, the world's consumption of raw cotton and the total imports and exports of the principal countries. I have made an index number of each of these with the figures for 1913 as 100. As iron and copper belong to the same industrial group, I have taken their index numbers and secured the square root of their multiplication in order to avoid duplication in statistics. I have multiplied the index number of wheat production by itself in view of the paucity of statistics relating to agricultural productions and to those productions which have a small rate of increase. As oil has come to assume importance comparatively recently, I have taken into consideration only its figures after 1913. I secured the square root of the multiplication of the index number of oil with that of coal. I have taken in the original form the index numbers of other items, namely, the imports and exports of the principal countries, the world's consumption of raw cotton, and the
world's production of coal prior to 1913. I have secured the 6th root of the multiplication of all these figures in order to derive the index number for T. Then I have secured the trend line for T in the same way in which I have secured the trend line in connection with the production statistics of pig-iron and wheat. I have then divided the index number of T by this trend line. The result shows how unequal the increase of T is. (See Diagram 3).

I have included the figures of imports and exports in the above calculation in order to take into consideration the speed of the circulation of commodities. One may, however, object to the correctness of the trade statistics I have employed as well as to the policy of including them in the calculation. By way of meeting such an objection, I have taken the trade statistics entirely out of my calculation, and proceeded to carry on the calculation the same as before, and secured the index number of T' from the trend line of which I found its deviation. The deviation of T' thus found was substantially the same as that of T.

In the case of both T and T' a comparatively equal development has been made during the period 1850-1860. This may be due to errors in the index numbers of the production of wheat. Now, I wish to state that I have gathered statistics, adopted a method of averaging and made the calculation of a synthetic index number, with the intention, throughout my work, of taking up, if possible, the standpoint assumed by Sauerbeck when making his index number. But have I really succeeded in my attempt? Are there not statistical material better than those I have used? These questions must be decided by a future enquiry. Supposing, however, that I have nearly succeeded in occupying Sauerbeck's standpoint in all stages of my investigation, I may be allowed to say that I have come into possession of a strategical point from which future studies into the a posteriori quantity theory may be made, quite advantageously, because in that case I would have secured a index number of T separate from V as a positive datum for the whole
enquiry. So long as \( V \) is constant, \( P \) must be equal to \( \frac{M}{T} \) or the stock of gold money divided by \( T \). (It must be noted that \( P \) in this case corresponds to the relative stock of gold or of gold money of Professor Cassel and others). Now, the real \( P \) deviates from the relative stock of gold money of my calculation. This indicates the fact that the real \( P \) has shifted. And since

\[
MV = PT \quad \therefore V = \frac{PT}{M} = P \cdot \frac{T}{M}
\]

\( V \) is derived by multiplying \( P \) by \( \frac{T}{M} \) which is the reciprocal of \( \frac{M}{T} \), the relative stock of gold of my calculation.

I shall now consider the period 1850–1913 which was under the actual control of the gold standard system. It will be found that the secular variations of the general level of prices in the main correspond to the relative stock of gold of my calculation. Thus, \( V \) has mainly remained constant. Now, it must be noted that \( T \) and \( T' \) in the beginning of the 'fifties are unexpectedly high, seemingly because of the fact that the index number of wheat production (to which much weight has been given in the calcula-

Fig. 4.
tion of the index numbers of $T$ and $T'$) during the same period is exceedingly high; and, in consequence, the relative stock of gold of my calculation is unexpectedly low.

There is another problem that remains unsolved, namely, the fact that $V$ during the period 1885-1905 is generally very low. In this paper I regard it as a sort of abnormal phenomenon born of the special circumstances under which an unusual attachment was shown towards gold and of the inertia of the long period of economic depression. On the other hand, an explanation of opposite nature is conceivable. One may contend, for example, that the phenomenon in question was a normal one under the gold standard system and that the high rate of $V$ during the previous time was an abnormal phenomenon due to the lingering influence of the silver standard system or some other cause; and that Sauerbeck's index number stands mistakenly higher than it really was. At any rate, I shall leave this question to be discussed at a later date.

I have attempted above to make an advance in the treatment of the \textit{a posteriori} quantity theory. Hitherto, the law governing $\frac{V}{T}$ itself under the gold standard system has been dealt with but I have attempted, though in a quite imperfect way to draw up the law governing $V$ under the gold standard system. This seems to provide us with a useful hint for the study of the variations of the general level of prices during the post-bellum years.

5. EXAMINATION: SECTION (3)

The secular variations of the general level of prices before the Great War, as I have demonstrated above, could be explained by the relative stock of gold money in the various theories dealing with the law governing $\frac{V}{T}$ in general. With the commencement of the Great War, $T$ began to show a sharp variation which could never have been seen before the War, so that it is impossible to apply
these various theories in their unmodified forms in an enquiry into the variations of the general level of prices. The relative stock of gold of my calculation has also exceedingly deviated from the variations of the general level of prices. Indeed, as the free market for gold has been kept closed for a long period of time since the Great War, it is a debatable question to what extent the so-called prices in gold represent the real value of gold. However, it is impossible to assume that there has been no great gap between the variations of the general level of prices in gold and the relative stock of gold in my calculation. At any rate, it is clear that the secular variations of the general level of prices since the Great War cannot be adequately explained by the circumstances of monetary gold. Is my enquiry then as so far made useless? Let us see.

Needless is it to state that what I have so far stated concerned itself with the phenomena under the gold standard system. The empirical law which I have observed at that time may have ceased to operate after the gold standard system became suspended; but this does not affect the validity of my study. It should be noted that it was only because the currencies of almost all nations were ultimately connected with the stock of monetary gold that the variations of the stock of monetary gold had close connection with that of the general level of prices in gold; but with the Great War, the currencies of most nations of the world became emancipated from this yoke.

This divorce between the secular variations of the general level of prices and the relative stock of gold in my calculation was due to a change in the special demand for gold which had owed its existence to the connection between currency and gold, and in consequence due to a change in the value of gold itself. On this point Professor Cassel says:

"The general withdrawal of gold from circulation has naturally created a considerable abundance of gold for monetary use, and this abundance has probably in some
degree been increased by the melting of articles of art or luxury caused by patriotism or by distress. Some central banks, as the Austrian, and later on the German, have been forced to part with the greater portion of their gold funds, which have gone to increase the supply in other countries. To the superabundance of gold thus created has been added the considerable production of gold since the beginning of the war. As the consumption of gold in the arts has probably been more restricted during the war than normally, this production must have had a special importance for the monetary supply of the metal. The superfluous gold must of course find some outlet. It poured, during the first years of the war, into the central banks of the neutral countries of Europe; it went in enormous quantities to the United States, and even found its way to South America and Eastern Asia. But still the capacity to receive all this gold proved too small, and the yellow metal suffered a severe loss in value. In this way the masses of paper money created have in fact pressed down the value of gold as against commodities to less than half of what it used to be before the war." And the same phenomenon seems to be seen at the time of the Napoleonic War. Commenting on the condition of that time, Jevons says: "... it cannot be denied that the extraordinary issue of Bank of England notes and private bank notes in England during the suspension of specie payments, drove out a large mass of metallic currency. Other amounts were similarly displaced by the paper currencies of France, Austria, Russia, and some other countries. In the aggregate, a considerable mass of precious metals must have been thrown on European markets, and to this cause we must assign some part of the elevation of prices." The same phenomenon may be seen in the variations of the general level of prices in the United States during the Civil War. According to some scholars, it is only the prices in paper money and not the prices in gold that are affected by the issuance of inconvertible paper money. This contention however not only has no theo-
retical grounds but is hardly harmonious with the facts in the cases indicated in the above mentioned experiences.

At any rate, it is important to note that the prices in gold in these periods were isolated from the relative stock of gold money in my calculation, and exactly therein lies a great significance.

The world War placed the economic intercourse of the world in a crippled state for the time being, but it was clear that such an abnormal condition could not long endure, inasmuch as the world's economic life has been possible because of the international division of labour held in mutual exchange. Accordingly, one of the economic problems that faced the world after the Armistice was how to restore the world's economic intercourse.

But in order to restore general international exchange relations, some sort of international money was required; and to establish an international standard of money regardless of monetary customs in place of the monetary systems of different nations was impossible. Thus, it was imperative to stabilize the money of each nation on some fixed relation to gold, the pre-war international money, or to the money of some country or countries which held to the gold standard. Thus, all proposals for "money free from gold" which arose with the Great War were discouraged and the solution of the monetary problems of the nations turned towards the glittering gold. It is in connection with this fact that the fact that the prices in gold at that time were entirely divorced from the relative stock of gold money in my calculation attains great significance. Professor Cassel who directed the solution of the monetary problems of the world at that time says in his memorandum written for the International Finance Conference held in Brussels in 1920: . . . . gold has lost a considerable part of its pre-war value as against commodities, prices of commodities having risen in a gold standard money like the dollar to about 250 against 100 before the war. In a country where the depreciation of the monetary standard has not
gone much further than this depreciation of gold, it will seem very desirable to uphold the old parity with gold, i.e., to make the notes redeemable in gold at their face value; and this will then be possible by a comparatively small reduction in the supply of the means of payment. A deflation within such reasonable limits is, e.g., doubtless possible in England, where a reduction of the general level of prices by about 20 per cent would probably be sufficient to establish parity between the paper pound sterling and the gold sovereign. . . . .40

The nations also adopted as the basis of their monetary reforms the proportion between paper money and gold at the time of monetary stabilization or the exchange value of their respective currencies vis a vis the currencies of the gold-standard countries. If the value of money was approximately the same as the pre-war par value, it was stabilized at that par, even though the prevailing rate might have been somewhat lower. On the other hand, if the value of money was far below the pre-war par value, its stabilization was made by reducing the value below the pre-war parity, at a point just equal to or a little higher than the rate at the time of stabilization. But such a policy necessarily would cause a big slump in the prices of commodities, unless the world's general level of prices in gold could be held in some measure away from the relative stock of gold money in my calculation which corresponds to the secular variations of the general level of prices during the long period 1850-1913 which was under the actual control of the gold standard system: or the V which was constant during the period 1850-1913 under the gold standard system could expand for many years after the Great War: or the empirical law observed to have operated under the gold standard system during the period 1850-1913 regarding the circulation of money came to be suspended suddenly after the Great War. Professor Cassel says in the same memorandum:

"The withdrawal of gold from circulation and the dis-
appearance of all definite standards of gold have, in a most serious degree, impaired the stability of the value of gold. If gold is to be used henceforth as a monetary standard, it is necessary to take special measures for stabilizing the value of the precious metal. As these measures, which would mainly consist in the establishing of appropriate and stable principles in regard to the gold-holdings of the central banks, naturally must be of an international character, we have to do here with a problem where cooperation between all countries is particularly required.\textsuperscript{40}

Thus, Professor Cassel favors the increase of the velocity of the circulation of gold by reducing the rates of gold reserves through international cooperation, by promoting the gold standard system, by facilitating international loans, by adjusting the industrial demand for gold, etc. This idea is still upheld today by him as well as by the Gold Delegation of the League of Nations' Financial Committee and many scholars and business men. In its interim report the Gold Delegation recommended: "... the concentration of gold in reserves under the direct and immediate administration of those responsible for monetary policy, and its limitation as a means of payment in international transactions; the reduction of the conventional minimum gold cover with international understanding; economy of gold by extending the use of cheques, post-office banking facilities, giro transactions, etc.; and by developing and improving the mechanism of the clearing-house; and the replacement of notes of small denominations by subsidiary coin."\textsuperscript{51}

Thus, after the stability of gold has been brought about, the monetary systems of nations have been changed from the gold standard system of the pre-war years to the gold bullion system or to the gold exchange system; the amount of legal reserves has been reduced; gold was withdrawn from circulation to be concentrated at the centre of every country.

However, V which was greatly expanded as the result
of the emancipation of money from the yoke of gold was suddenly contracted to almost the normal size as soon as the United States took the decisive step in the direction of the gold standard system and all nations had to prepare themselves for a return to that system. The V again expanded during the period when the nations were placed in a state of confusion and doubt, being unable to return to the gold standard system; again, its contraction began at the time when the nations made up their minds to go back to gold. All this indicates how powerful is the custom of monetary circulation under the gold standard system ascertained during the period 1850-1913.

Professor Cassel, who has seen the great slump in the general level of prices after 1920, gives the following view in the memorandum which he presented at the meeting of the Financial Committee of the League of Nations in September, 1921: "Under ordinary circumstances, the endeavour of a country with a gold standard to bring down prices and thus to raise the internal value of its money would, according to the classical doctrine, have caused an influx of gold into that country with a consequent increase in the total volume of the monetary purchasing power within the country. Thus, the depression of prices would have been counteracted and the internal value of the money of the country would have been brought down to that of gold. In the present case, this automatically regulating machinery has not had sufficient opportunity to work effectively."

The unbalanced distribution of gold forming the thesis of the foregoing quotation has again been discussed with much heat recently. However, as may be seen in the statistics of the real rates of gold reserves (see the Supplementary Table), the influx of gold into a country does not necessarily expand the volume of its currency at once. (To give rise to such an effect, the real rates of gold reserves should be constant). The truth of all this is applicable to the pre-War years as well to the post-War years and to all countries without any exception. It has
been true in the past as it is true today. And what are changed are the circumstances which give rise to much outcry over the so-called unbalanced distribution of gold today.

I am conscious of the imperfection of my present enquiry, due to the scarcity of the materials used and to the daring method of treatment and examination, and I hope to remedy the defects of the present article by undertaking further study. But if my enquiry is free from serious shortcomings, it may be said to have provided a sort of key to forecast how long the present fall of prices will continue, so far as it is concerned with the secular movement.

My materials cover the period up to 1930. Now, as may be seen in Diagram (4), the index number of prices in 1930 exceeds the relative stock of gold money given by my calculation only by between 10 and 14 per cent. In 1931, the production of industries does not appear to have been increased; on the contrary, it may have been reduced somewhat. Moreover, no decrease seems to have been made in the output of gold and no increase in its consumption for industrial arts. Accordingly, a considerable increase must have been made in the world's stock of gold and consequently the relative stock of gold money must have somewhat risen. On the other hand the general level of prices continued its downward course. All this points to the conclusion that the relative stock of gold money and the general level of prices must have approached each other to a considerable extent. Rather, it is possible that the former somewhat exceeds the latter. Thus, we may say that the basic circumstances underlying the depreciation of prices have been largely eliminated. Furthermore, it is possible that the $T$ and $T'$ which are made the basis of calculation of the relative stock of gold for the years after 1913 may have been estimated somewhat higher than they really are, due to the fact that oil the increase of which is of high percentage, has been taken into consideration after that particular year. And, in consequence, the re-
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1. INTRODUCTION

Views pro and con have been published by many scholars regarding Professor Cassel’s quantity-theory of money, especially after it was adopted as the theoretical basis of the First Interim Report of the Gold Delegation of the Financial Committee of the League of Nations made in the autumn of 1930. His theory offers important suggestions into the economic laws governing the variations of the general level of prices: hence the universal interest taken in the matter. I shall take the liberty of making a critical survey on his theory in this paper.

First of all, it should be noted that the quantity theory of money may take either of the following two forms: (1) the *a priori* or deductive or speculative quantity theory which contends that the variations of the quantities of money correspond to the variations of the relative value of money: and (2) the *a posteriori* or inductive or positive quantity theory which upholds the theory that the variations of the quantity of money generally tend, in experience, to correspond to the variations of the relative value of money. Now, it is not clear which of the foregoing forms Professor Cassel really upholds when he advances his quantity theory of money. In my present paper, I assume that his theory is an *a posteriori* quantity theory, because the other theory is impossible of logical presentation. I have elsewhere demonstrated the logical weakness of the *a priori* quantity theory which I shall briefly summarise as follows:

The *a priori* quantity-theory will be impossible of logical explanation unless it should take "money" in the sense
either of "stock money" under the condition that the quantity of goods and the velocity of the circulation of money are constant or of "stock of money multiplied by its velocity of circulation." While, in either case if the theory is conditioned in such a way and that is logically necessary for the formation of that theory—it simply assumes that the prices of commodities are already fixed. This assumption is made unwarrantably inasmuch as it is precisely the prices, the general level of which the quantity theory of money attempts to explain. Because the general level of prices is constructed simply by taking a general view of the prices of commodities from a specific stand point). It should be remembered that the prices of commodities are the foundation of the general level of prices and that in consequence no theory aiming at the explanation of the latter can make an assumption regarding the former. Any theory that makes such an initial assumption falls into the logical error of creating a "vicious circle".) Consequently remains only the a posteriori view of Prof Cassel's quantity theory of money to be critically surveyed.

2. VIEWS OF PROFESSORS CASSEL, KITCHIN AND WOYTINSKY

Although Professor Cassel's quantity theory of money is widely known, I shall here briefly summarise his main propositions for the sake of lucidity.

The total stock of gold accumulated in the world (I shall hereafter refer to this simply as "stock of gold") during the years between 1850-1910 has been increased from 10,000 million marks to 52,000 million marks: or in other words, the amount of the stock of gold has been increased by 5.2 times during the past 60 years, at the annual average rate of 2.8 per cent. However, according to Sauerbeck's General Index Number of Wholesale Prices in England (I shall hereafter refer to this simply as Sauerbeck's L.N.), the general level of prices in 1910 is approximately...
the same as in 1850. This fact means that the increase in the stock of gold during the period as a whole has had no influence on the general level of prices, but merely corresponds to what is required in order that the growth of the stock of gold should keep pace with the general economic development. In consequence, had the stock of gold increased precisely at the rate of 2.8 per cent annually during the period (the stock of gold in these circumstances is called "the normal stock of gold"), the variation in the supply of gold would not have given rise to that of the general level of prices. But actually the annual stock of gold has deviated from the normal stock of gold. This deviation is shown by the "relative stock of gold" which is produced by dividing the annual stock of gold by the normal stock of gold.

Fig. 1.

![Diagram 1](image)

On the other hand, the variations of the relative stock of gold closely correspond to the secular variations of the general price level. (See Diagram 1). It is proved by this that the secular variations of the general price level depend on the contemporary variations of the relative stock of gold.²

This explanation, however, is somewhat insufficient even with regard to the secular variation of the general price level
during the years between 1850-1910, as is apparently shown in diagram 1. Professor Cassel himself admits that "the secular price level in the period 1850-1870 is higher, and in the "nineties rather lower, than it ought to be according to the relative stock of gold"."3

On this point Professor Cassel says:

"(One) circumstance is, without doubt, primarily responsible for the depression (in the period, 1875-1890), in so far as it superseded the simultaneous fall in the relative stock of gold. This circumstance is the enormous increase of gold demand in the United States at the end of the seventies and the beginning of the eighties, due to the preparation for and subsequent realisation, of the resumption of cash payments. . . . A factor that acted in the same way on the side of the demand, though to a less extent, was the large imports of gold into India in the early eighties".4 The foregoing statement, however, cannot explain the fact that the general level of prices during the period, 1850-1870 is higher than the relative stock of gold.

Kitchin, who uses the same method of investigation as Professor Cassel, differs from him in one respect. He made the world's monetary stock of gold (I shall hereafter refer to this merely as "stock of gold money") the starting point of his research. He has succeeded in clarifying the fact that the world's stock of gold money during the period, 1850-1910 increased from 230 million pounds to 1,440 million pounds at the average annual rate of 3.1 per cent. By taking these figures as the basis of his calculation, Kitchin compared the normal stock of gold money with the annual stock of gold money and secured the figures for the relative stock of gold money. He has demonstrated by this calculation that the variation of the relative stock of gold money more closely corresponds to the secular variations of the general level of prices than does Professor Cassel's relative stock of gold; (See Diagram 1).5

Both Professors Cassel and Kitchin regard the stocks of gold on of gold money in 1850 and in 1910 to be normal
only because the general levels of prices of these two years are roughly the same. Accordingly, they compare the stocks of gold or of gold money of these two years and calculated the 60th root of the percentage of the increase between them, in order to secure the normal rate of increase. They then secured the figures for the relative stock of gold or of gold money by taking the normal rate of increase as the basis of calculation, and then compared them with the general level of prices by making them index numbers with the figures for 1850 as 100. But many difficulties are bound up with such an attempt to find the normal rate."

It needs no demonstration to manifest that the normal rate can be best secured by taking into consideration the statistical data of all the years. This point is given special consideration by WL Woytinsky who accepts the fundamental idea that runs through the theories of both Professors Cassel and Kitchin—the idea of demonstrating their a posteriori quantity theory by taking only the stock of gold or of gold money and the general level of prices.

Prof. Woytinsky contends that in order to equate the relative stock of gold money—which is produced by dividing the annual stock of gold money \((m)\) by the normal stock of gold money \((I=a.b^t)\) and by multiplying the quotient by 100—with the annual general price level \((p)\), conditions should satisfy the following equations:

\[
\frac{100m}{I} = \frac{100M}{a.b^t} = p \quad \text{and therefore} \quad \frac{100M}{p} = 1 = a.b^t
\]

and therefore \(\log \frac{100M}{p} = \log 1 = \log a + x \log b\).

By means of the method of least squares, the trend line \((\log 1 = \log 2,3662 + \log 0.01343x)\) is computed. And in consequence the following computation is made: \(1 = 232.4 \times 1.03142^t\). Thus, Prof. Woytinsky has computed the normal stock of gold money as well as the relative stock of gold money which has been found to correspond more closely to the secular variations of the general price level than does Kitchin's calculation of the relative stock of gold.\(^3\) (See Diagram 1).
3. EXAMINATION: SECTION (I)

In Section I I shall examine what qualifications the data so far given must possess in order to be positive material and to what extent these data possess, if at all, these qualifications. In Section II I shall examine in what sense these data, supposing they possess such qualifications, are qualified to be positive material: in other words, I shall attempt to learn to what extent these data can assist the demonstration of the \textit{a posteriori} quantity theory when they are used in the manner in which these scholars did use them. In the last Section, I shall endeavor to investigate how the \textit{a posteriori} quantity theory can explain the variations of prices since the World War. The main thesis of this paper concerns itself with the problems dealt in the last two Sections: and the first problem with which I shall deal in this Section is to be treated only as a matter of course in preparation for the study of the other more important problems at issue.

In Professor Cassel's theory the following two data are used as the bases of his positive materials: (1) Sauerbeck's index number, representing the variations of the general level of prices of the world and (2) the world's stock of gold.

(1) As is well known, Sauerbeck's index number has been often used in the study of the variations of the general level of prices in the world. Professor Cassel gives the following three reasons for this use: (i) England has been, roughly speaking, a free-trade country (ii) she has been the world's trade market to a greater extent than any other country (iii) and she has maintained the gold standard system, the last named being the decisive factor with regard to the questions at issue.\(^\text{a}\)

Now, it should be noted that these reasons given by Professor Cassel may prove the bases of consideration and conjecture, but certainly not that of any necessary assertions. At any rate, their accuracy must be decided in the light of positive evidence. Let us then consider if Sauerbeck's general index number of the wholesale prices of England can represent the variations of the general level of prices of the world, as is supposed by Professor Cassel.

In order to calculate the definite general level of prices, the following two points should first be determined: (a) the composition of goods, with the prices from which the general level of prices are calculated and (b) the method of averaging adopted for computation. These two points, in turn,
are dependent on some definite standpoint. In considering whether or not Sauerbeck's index number can represent the variations of the general level of prices of the world, we face this problem: suppose Sauerbeck's index number is calculated from an international standpoint—in other words, suppose his British standpoint of calculation is cast aside—shall we come upon the same variations of the general level of prices as he did? Now, this is a difficult, if not an impossible, question to solve. We should have to consider the general index numbers of other countries, but we find that each general index number of prices of each nation differs in the compositions of goods, with the prices of which they are calculated as well as in the method of averaging adopted in them. Because of this difficulty, such index numbers may have little or no value for our present enquiry. Strictly speaking, the data at our disposal at present may not materially assist us in answering the question confronting us.

On the other hand, a comparative study of the index numbers of the other nations will have some significance, so far as the supposition is permissible that some of them at least have been calculated from a similar standpoint as Sauerbeck's. But here we meet another difficulty, namely, the paucity of index numbers we can secure. I have taken the index numbers of Japan, England, the United States, France and Germany; computed the 9-year moving average of each; and by comparing them I have discovered the following details: the secular movements of the general levels of prices of England and France resemble each other, and those of Germany and the United States also show the same phenomenon, but that no similarity is shown between these two groups. (See Diagram 2).

In consequence, it appears dubious whether Sauerbeck's index number
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can be taken as rightly indicating the variations of the general level of prices of the world. There are several points in this connection that challenge our attention. Whereas the principal commodities in Sauerbeck's Index number (for England) are raw materials those in Aldrich's Index number (for the United States) are manufactured goods. Now, the rise of the prices of manufactured goods is less sharp than that of the prices of raw materials. If Lenoir's explanation, that the prices of manufactured goods fell during 1850-1860 due to the progress made in the infant industries of the U.S. may also be applied to the 'seventies; and further, if we can accept as correct Professor Wagemann's assertion that (the) prices in Germany during 1850-1871 deviate sharply from those of other countries because the cheap prices in Germany of cereals and potato had affected prices in general in that country; and again, if one can assert that the secular variations of the general level of prices in Japan (which greatly resemble those of India),

deviate from those of other countries partly because of the circumstances attendant on the calculation of the index number, and partly owing to the limited degree of her participation in the world's general economic activities —if these conditions are to be accepted, then Sauerbeck's index number may be held as approximately representing the general level of prices in the world during the period under consideration. It is questionable, however, whether so far as the period 1850-1870 is concerned, Sauerbeck's index number is not somewhat higher than the actual general level of prices in the world.

(2) Many writers have taken the world's annual output of gold as representing the world's stock of monetary gold, but they have invariably failed in getting the desired results; nor can it be said to be satisfactory from the theoretical standpoint. The reason for this failure is clear. There is a vast difference between the world's annual stock of gold and its annual output of gold because of its high degree of conservation. Professor Cassel accordingly has adopted the annual stock of gold in place of the annual output of gold. His system, however, contains several points that invite criticism.

In computing the world's annual stock of gold, Professor Cassel has adopted as the basis of his calculation Professor Lexis' estimate of the total stock of gold in 1848 which is given as 9,560 million marks. The amount of the total gold stock in 1860 is estimated at 10,000 million marks. Deducting one per cent of the amount of the stock of gold at the beginning of every five years from it, for the definite loss of gold through wear, and adding to the remainder the total output of gold during that lustrum he computed the total amount of gold stock at the end of each lustrum down to 1875. After 1876, a two per mill reduction of the amount of stock of gold at the beginning of each year is made for the same reason and to the remainder is added the amount of the output of that year, the combined amount thus representing the stock of gold at the end of that year. Now as according to Del Mar's estimate, the output of gold in both
1849 and 1850 was much higher than the annual average output of gold during the 'forties' while Soetbeer puts his estimate of the annual average output of gold during the period 1841-1850 at 122.8 million marks, we can easily assume that the increase of gold stock during the period 1848-1850 to be not less than 440, million marks. Accordingly it seems natural, by taking Professor Lexis' estimate for 1848, namely, 9560 million marks, to estimate the amount of stock of gold for 1850 at 10,000 million marks. But as this estimate of Professor Lexis is only concerned with the "Goldvorrat der abendländischen Kultur," though the other countries may have no great importance it is highly dubious whether such an estimate can justly represent the world's stock of gold. By the way, if we compare the figures given by Soetbeer and by the Minister of the Mint of the United States whom Professor Cassel quotes in his work, concerning the period for which both are given, we see that the figures of the former are generally higher than those of the latter. Del Mar's estimate for 1800-1851 is lower than Soetbeer's. A calculation made according to Professor Cassel's own method has shown the amount for 1800 which is lower than the estimate of the stock of gold by Professor Lexis; a similar calculation for the estimate of the stock of gold for the year 1850 has shown that Professor Cassel's estimate is higher than that of Professor Lexis. This also may be taken as indicating that Soetbeer's estimation of the output of gold is higher than that of Professor Lexis. Again, Soetbeer's estimate for the annual output of gold during the period 1851-1875 is higher than that of Layton. Thus, one may question the accuracy of Soetbeer's estimate which Professor Cassel adopts in his investigation. True, some special study will be required in order to decide which is more correct; the mere fact that many other scholars put the estimate lower than is done by Soetbeer does not necessarily mean that his estimate is wrong. Moreover, some may object to Professor Lexis's policy of deducting from the annual stock of gold a 2 per cent depreciation for the annual definite loss of gold through wear, etc. from the annual stock of gold.

However, the most objectionable point in Professor Cassel's use of gold statistics is not found in the foregoing points: rather it lies in the fact that he deals with the stock of gold and not with the stock of gold money. He has adopted the total stock of gold first because in the case of the gold standard system, there is no clear line of demarcation between stock of money and stock of gold and, in consequence, the stock of gold does not indicate the immensity of its mass and secondly because "it (the stock of gold) is able to give an immediate answer to the question before us—namely, the sufficiency of the annual production of gold." However, to what extent a stock of gold can represent a stock of gold money must be proved empirically. If the amount of the world's stock of gold money is known, it should certainly be used directly. In this sense, a great contribution has been rendered by Kitchin who has made a successful use of the world's annual stock of gold money and by its help, calculated the relative.
stock of gold money that better corresponds to the secular variations of the
general level of prices than does Professor Cassel's relative stock of gold.

But how far can we rely on the statistics of this important annual
stock of gold money? Kitchin has gathered statistical materials from all
over the world excepting India, China and Egypt. He says: "It assumes a
stock of £200,000,000 at the end of 1843 and the addition to it each year
of the balance of the world's gold output remaining after deducting the
demands of the industrial arts of Europe and America and the absorption
by India, China and Egypt." Unfortunately, he does not show the statis-
tical sources for this calculation, except those for the gold used by
the industrial arts of Europe and America up to 1913. (I believe he must have
shown them on some other occasion. But at present I am unable to get
access to fuller statistics. As I am quite unacquainted with such matters, I
confess that I am at loss as to what extent I can rely on the information
given by Kitchin. However, in view of the repute he enjoys in the scholas-
tic world, I shall quote him in subsequent pages."

4. EXAMINATION: SECTION II.

Professor Fisher has once said: "There is—in the
system of general equilibrium—always just one too few
equation to determine the unknown quantities involved.
The equation of exchange is needed in each case to supple-
ment the equations of supply and demand." This view of
Professor Fisher's is due to his confounding the equation
\[ P_a = 1 \] which is required in the general equilibrium system
besides the various equations indicating supply and demand
functions, etc., with the equation of exchange (for instance,
\[ MV = TP \]). I have demonstrated this elsewhere. It should
be noted that the general equilibrium is not based on "the
equation of exchange"; rather the latter is formed when the
phenomena which are so to speak determined in accordance
with the general equilibrium theory are summarily observed
from some definite standpoint. In other words, one knows
\( M \) by deciding what conception of money one should adopt;
and further the equation \( MV = \Sigma pQ \) is determined by the
fact that a transaction represented by \( \Sigma pQ \) takes place only
so far as its \( M \) circulates directly or indirectly. Thus, the
equation \( \Sigma pQ = TP \). \( MV = TP \) is obtained by deciding the
following two factors: the descriptions of commodities whose
prices are to be used as the basis and the method of averaging to be adopted, for the calculation of the general level of prices. In consequence, the equation of exchange is not, as is often supposed to be, predicated on the *a priori* quantity theory of money; nor does it prove that theory.\(^2\) And the meanings possessed by all the terms included in the equation of exchange such as \(M, V, T,\) and \(P\) are essentially determined by the foregoing circumstance.

Now, the equation of exchange is expressed as follows:

\[
MT = TP \therefore P = \frac{Mv}{T} = M \cdot \frac{V}{T}
\]

Thus, \(P\) is equal to \(M\) multiplied by \(\frac{V}{T}\). It is impossible to discover any quantity relation between \(P\) and \(M\) inasmuch as \(\frac{V}{T}\) is a variable, in so far as we compare them directly. But if the movement of \(\frac{V}{T}\) actually takes place constantly, there must be some quantity relation between \(P\) and \(M\) on which that constant term is multiplied. To treat \(\frac{V}{T}\) in its entirety, to demonstrate the law governing it, and thereby to demonstrate the quantity theory—all this should constitute the main theme of the study of Professor Cassel's quantity theory.

An attempt at demonstrating the quantity theory from such a standpoint was made as early as 1901 by Aupetit.\(^3\) He took the world's annual output of gold as \(M\) and the period of his study was limited to 1878-1900. (Lenoir, on the other hand, has extended the period up to 1910.\(^4\) Professor Cassel took the world's annual stock of gold as \(M\) and at first his study covered the period 1850-1900. (He estimated the amount of the world's stock of gold in 1850 at 500 million pounds sterling and that of 1900 at 1,850 million pounds sterling, and gave the normal rate of increment as 26.5 per cent.\(^5\) Later, Hooker extended the period up to 1910.\(^6\) Professor Cassel later made another study over the period 1850-1910 and found a normal rate of
increase of 2.8%. This rate was used as the basis of calculation to discover the relative stock of gold for a period beginning in 1800 and ending in 1925. Kitchin took the world's annual stock of gold money as M and his first study covered the period 1850–1910; and by using as the basis of calculation the normal rate of increment thus found, that is, 3.1%, he discovered the relative stock of gold money for a period beginning in 1807 and ending in 1929. But in all these cases, the increases of $\frac{V}{T}$ is computed with the M of two specific years as the basis of calculation. Unlike Aupertit and Lenoir, Professor Cassel and, in consequence, Kitchin, have accidentally selected the more advantageous years as the result of their desire to select years whose general levels of prices are identical with each other.

Even supposing that the movement of $\frac{V}{T}$ takes place comparatively constantly, it cannot be said that the $\frac{V}{T}$ of any two years necessarily correspond to its normal secular movement simply because their general level of prices shows the same height. It is in this sense that Woytinsky's theory has made a great contribution to the study of the a posteriori quantity theory of money. His theory has proved that $\frac{V}{T}$ was comparatively constant during the period 1850–1910. One may be permitted to assert that Professor Cassel's method of study has been exhausted by Woytinsky's theory.

In order to push forward studies of the a posteriori quantity theory one step further on, one should investigate why the $\frac{V}{T}$ during the period 1850–1910 appears to be comparatively constant; and in order to do this, one should know how T has moved.

I have already pointed out that the calculation of T is dependent on the following two factors definite descriptions of commodities, of which the prices are calculated, and a
definite method of averaging, both of which are determined
by a definite standpoint. Inasmuch as T should be included
in the equation of exchange, calculation of it must be made
from the same standpoint from which P in the equation of
exchange is calculated. Consequently, inasmuch as the P
we make use of is taken from Sauerbeck's index number,
T must be calculated in our case from the same standpoint
from which the P was calculated by Sauerbeck himself. On
the recognition of T, Professor Cassel says:

"An approximate idea of the world's rate of progress
may be gained in the following way. In the period 1850–
1907, the world's production of pig-iron increased on an
average by 4.2 per cent. Now, the growth of the iron
industry may be regarded as characteristic of the whole
industrial development of the world. The agricultural
development had, of course been much slower and may per-
haps be put at the figure of 1.2 per cent per year, which
seems to correspond fairly well to the growth of the popu-
lation and the improvement of its nourishment. If we assume
that food represents a third of social income and that the
other two-thirds of this income have grown proportionally
to the industrial development, we arrive at an average rate
of progress of 3.1 per cent. If, on the other hand, we give
the rates for food and industrial production the same weight
in our calculation of the average progress, this average
would be 2.7 per cent. This figure ought to be regarded
as the lower limit for our estimate of the world's average
economic progress. As it seems necessary to give the
industrial production a somewhat higher weight than the
agricultural, we stand on fairly solid ground if we reckon
with a figure round 3 per cent as characteristic of the
economic development during the period 1850–1910. This
corresponds closely to the result of our enquiry into the
world's gold supply. It is possible that the average rate of
economic progress for the world as a whole, if it could be
calculated exactly, would prove to have been, during the
period in question, not greater than 2.8 per cent per year."
How far his calculation of T has been made according to the standpoint from which Sauerbeck's index number was made is open to serious question. However, even if we assume that there is no such doubt, we must question whether or not each of the productions of wheat and iron has increased at the same rate of increase.

In considering the world's production of wheat and pig iron, I shall make an index number by taking the figure of 1913 as 100. Then, in order to eliminate the effects of the business cycle, I calculated a 9-year moving average. By taking the representative figures of every lustrum during 1850-1925 (the figure of 1850 is taken in its original form), the trend line indicating the average development is calculated by the method of least squares. Then, I have divided the above indicated 9-year-moving-averaged index number by this trend line, and secured the results which presents quite uneven lines especially after the Great War. Thus

Fig. 3.

it is impossible to conceive of any even increase of so far as the foregoing considerations go. (See Diagram 3). There is some room for doubt regarding the high rate of deviation for wheat during 1850-1860. The material used may have been of doubtful character.
It is extremely difficult to secure materials over such a long period of time and in such a world-wide matter as we are dealing with. It is questionable how far we can put confidence in those data we can manage to secure; nor can we have any clear notion of their continuity. As I have stated in the foot notes, there is much doubt about the correctness of individual statistics forming the production statistics of pig-iron and wheat. There is much looseness involved in the collection of these data. A mass of incorrect statistics will only give incorrect information. But in order to approach the standpoint of Sauerbeck's index number, it is clear that one will have to gather more statistics and from a wider field. In actual practices, however, statistics which satisfy the desired requirements are very few in number; and out of these few statistics only a still further reduced number of them can be actually used because of the necessity of maintaining a balance in the policy of gathering such statistics.

In order to supplement the foregoing statistics, I have also taken the figures of the world's production of copper, coal and petroleum oil, the world's consumption of raw cotton and the total imports and exports of the principal countries. I have made an index number of each of these with the figures for 1913 as 100. As iron and copper belong to the same industrial group, I have taken their index numbers and secured the square root of their multiplication in order to avoid duplication in statistics. I have multiplied the index number of wheat production by itself in view of the paucity of statistics relating to agricultural productions and to those productions which have a small rate of increase. As oil has come to assume importance comparatively recently, I have taken into consideration only its figures after 1913. I secured the square root of the multiplication of the index number of oil with that of coal. I have taken in the original form the index numbers of other items, namely, the imports and exports of the principal countries, the world's consumption of raw cotton, and the
world's production of coal prior to 1913. I have secured the 6th root of the multiplication of all these figures in order to derive the index number for T. Then I have secured the trend line for T in the same way in which I have secured the trend line in connection with the production statistics of pig-iron and wheat. I have then divided the index number of T by this trend line. The result shows how unequal the increase of T is. (See Diagram 3).

I have included the figures of imports and exports in the above calculation in order to take into consideration the speed of the circulation of commodities. One may, however, object to the correctness of the trade statistics I have employed as well as to the policy of including them in the calculation. By way of meeting such an objection, I have taken the trade statistics entirely out of my calculation, and proceeded to carry on the calculation the same as before, and secured the index number of T' from the trend line of which I found its deviation. The deviation of T' thus found was substantially the same as that of T.

In the case of both T and T' a comparatively equal development has been made during the period 1850–1860. This may be due to errors in the index numbers of the production of wheat. Now, I wish to state that I have gathered statistics, adopted a method of averaging and made the calculation of a synthetic index number, with the intention, throughout my work, of taking up, if possible, the standpoint assumed by Sauerbeck when making his index number. But have I really succeeded in my attempt? Are there not statistical material better than those I have used? These questions must be decided by a future enquiry. Supposing, however, that I have nearly succeeded in occupying Sauerbeck's standpoint in all stages of my investigation, I may be allowed to say that I have come into possession of a strategical point from which future studies into the *a posteriori* quantity theory may be made, quite advantageously, because in that case I would have secured a index number of T separate from V as a positive datum for the whole
enquiry. So long as $V$ is constant, $P$ must be equal to $\frac{M}{T}$ or the stock of gold money divided by $T$. (It must be noted that $P$ in this case corresponds to the relative stock of gold or of gold money of Professor Cassel and others).

Now, the real $P$ deviates from the relative stock of gold money of my calculation. This indicates the fact that the real $P$ has shifted. And since

$$MV = PT \quad \therefore V = \frac{PT}{M} = P \cdot \frac{T}{M}$$

$V$ is derived by multiplying $P$ by $\frac{T}{M}$ which is the reciprocal of $\frac{M}{T}$ the relative stock of gold of my calculation.

I shall now consider the period 1850–1913 which was under the actual control of the gold standard system. It will be found that the secular variations of the general level of prices in the main correspond to the relative stock of gold of my calculation. Thus, $V$ has mainly remained constant. Now, it must be noted that $T$ and $T'$ in the beginning of the 'fifties are unexpectedly high, seemingly because of the fact that the index number of wheat production (to which much weight has been given in the calcula-

Fig. 4.
tion of the index numbers of $T$ and $T'$ during the same period is exceedingly high; and, in consequence, the relative stock of gold of my calculation is unexpectedly low.

There is another problem that remains unsolved, namely, the fact that $V$ during the period 1885-1905 is generally very low. In this paper I regard it as a sort of abnormal phenomenon born of the special circumstances under which an unusual attachment was shown towards gold and of the inertia of the long period of economic depression. On the other hand, an explanation of opposite nature is conceivable. One may contend, for example, that the phenomenon in question was a normal one under the gold standard system and that the high rate of $V$ during the previous time was an abnormal phenomenon due to the lingering influence of the silver standard system or some other cause; and that Sauerbeck's index number stands mistakenly higher than it really was. At any rate, I shall leave this question to be discussed at a later date.

I have attempted above to make an advance in the treatment of the \textit{a posteriori} quantity theory. Hitherto, the law governing $\frac{V}{T}$ itself under the gold standard system has been dealt with but I have attempted, though in a quite imperfect way to draw up the law governing $V$ under the gold standard system. This seems to provide us with a useful hint for the study of the variations of the general level of prices during the post-bellum years.

5. EXAMINATION: SECTION (3)

The secular variations of the general level of prices before the Great War, as I have demonstrated above, could be explained by the relative stock of gold money in the various theories dealing with the law governing $\frac{V}{T}$ in general. With the commencement of the Great War, $T$ began to show a sharp variation which could never have been seen before the War, so that it is impossible to apply
I these various theories in their unmodified forms in an
enquiry into the variations of the general level of prices.
The relative stock of gold of my calculation has also ex-
ceedingly deviated from the variations of the general level
of prices. Indeed, as the free market for gold has been
kept closed for a long period of time since the Great War,
it is a debatable question to what extent the so-called
prices in gold represent the real value of gold. However,
it is impossible to assume that there has been no great gap
between the variations of the general level of prices in gold
and the relative stock of gold in my calculation. At any
rate, it is clear that the secular variations of the general
level of prices since the Great War cannot be adequately
explained by the circumstances of monetary gold. Is my
enquiry then as so far made useless? Let us see.

Needless is it to state that what I have so far stated
concerned itself with the phenomena under the gold standard
system. The empirical law which I have observed at that
time may have ceased to operate after the gold standard
system became suspended; but this does not affect the
validity of my study. It should be noted that it was only
because the currencies of almost all nations were ultimately
connected with the stock of monetary gold that the varia-
tions of the stock of monetary gold had close connection
with that of the general level of prices in gold; but with
the Great War, the currencies of most nations of the world
became emancipated from this yoke.

This divorce between the secular variations of the
general level of prices and the relative stock of gold in my
calculation was due to a change in the special demand for
gold which had owed its existence to the connection between
currency and gold, and in consequence due to a change in
the value of gold itself. On this point Professor Cassel
says:

"The general withdrawal of gold from circulation has
naturally created a considerable abundance of gold for
monetary use, and this abundance has probably in some
degree been increased by the melting of articles of art or luxury caused by patriotism or by distress. Some central banks, as the Austrian, and later on the German, have been forced to part with the greater portion of their gold funds, which have gone to increase the supply in other countries. To the superabundance of gold thus created has been added the considerable production of gold since the beginning of the war. As the consumption of gold in the arts has probably been more restricted during the war than normally, this production must have had a special importance for the monetary supply of the metal. The superfluous gold must of course find some outlet. It poured, during the first years of the war, into the central banks of the neutral countries of Europe; it went in enormous quantities to the United States, and even found its way to South America and Eastern Asia. But still the capacity to receive all this gold proved too small, and the yellow metal suffered a severe loss in value. In this way the masses of paper money created have in fact pressed down the value of gold as against commodities to less than half of what it used to be before the war. And the same phenomenon seems to be seen at the time of the Napoleonic War. Commenting on the condition of that time, Jevons says: "... it cannot be denied that the extraordinary issue of Bank of England notes and private bank notes in England during the suspension of specie payments, drove out a large mass of metallic currency. Other amounts were similarly displaced by the paper currencies of France, Austria, Russia, and some other countries. In the aggregate, a considerable mass of precious metals must have been thrown on European markets, and to this cause we must assign some part of the elevation of prices." The same phenomenon may be seen in the variations of the general level of prices in the United States during the Civil War. According to some scholars, it is only the prices in paper money and not the prices in gold that are affected by the issuance of inconvertible paper money. This contention however not only has no theo-
retical grounds but is hardly harmonious with the facts in the case as indicated in the above mentioned experiences.

At any rate, it is important to note that the prices in gold in these periods were isolated from the relative stock of gold money in my calculation, and exactly therein lies a great significance.

The world War placed the economic intercourse of the world in a crippled state for the time being, but it was clear that such an abnormal condition could not long endure, inasmuch as the world's economic life has been possible because of the international division of labour held in mutual exchange. Accordingly, one of the economic problems that faced the world after the Armistice was how to restore the world's economic intercourse.

But in order to restore general international exchange relations, some sort of international money was required; and to establish an international standard of money regardless of monetary customs in place of the monetary systems of different nations was impossible. Thus, it was imperative to stabilize the money of each nation on some fixed relation to gold, the pre-war international money, or to the money of some country or countries which held to the gold standard. Thus, all proposals for "money free from gold" which arose with the Great War were discouraged and the solution of the monetary problems of the nations turned towards the glittering gold. It is in connection with this fact that the fact that the prices in gold at that time were entirely divorced from the relative stock of gold money in my calculation attains great significance. Professor Cassel who directed the solution of the monetary problems of the world at that time says in his memorandum written for the International Finance Conference held in Brussels in 1920: . . . . gold has lost a considerable part of its pre-war value as against commodities, prices of commodities having risen in a gold standard money like the dollar to about 250 against 100 before the war. In a country where the depreciation of the monetary standard has not
gone much further than this depreciation of gold, it will seem very desirable to uphold the old parity with gold, i.e., to make the notes redeemable in gold at their face value; and this will then be possible by a comparatively small reduction in the supply of the means of payment. A deflation within such reasonable limits is, e.g., doubtless possible in England, where a reduction of the general level of prices by about 20 per cent would probably be sufficient to establish parity between the paper pound sterling and the gold sovereign.

The nations also adopted as the basis of their monetary reforms the proportion between paper money and gold at the time of monetary stabilization or the exchange value of their respective currencies vis a vis the currencies of the gold-standard countries. If the value of money was approximately the same as the pre-war par value, it was stabilized at that par, even though the prevailing rate might have been somewhat lower. On the other hand, if the value of money was far below the pre-war par value, its stabilization was made by reducing the value below the pre-war parity, at a point just equal to or a little higher than the rate at the time of stabilization. But such a policy necessarily would cause a big slump in the prices of commodities, unless the world's general level of prices in gold could be held in some measure away from the relative stock of gold money in my calculation which corresponds to the secular variations of the general level of prices during the long period 1850-1913 which was under the actual control of the gold standard system: or the V which was constant during the period 1850-1913 under the gold standard system could expand for many years after the Great War; or the empirical law observed to have operated under the gold standard system during the period 1850-1913 regarding the circulation of money came to be suspended suddenly after the Great War. Professor Cassel says in the same memorandum:

"The withdrawal of gold from circulation and the dis-
appearance of all definite standards of gold have, in a most serious degree, impaired the stability of the value of gold. If gold is to be used henceforth as a monetary standard, it is necessary to take special measures for stabilizing the value of the precious metal. As these measures, which would mainly consist in the establishing of appropriate and stable principles in regard to the gold-holdings of the central banks, naturally must be of an international character, we have to do here with a problem where a cooperation between all countries is particularly required.\textsuperscript{190)

Thus, Professor Cassel favors the increase of the velocity of the circulation of gold by reducing the rates of gold reserves through international coöperation, by promoting the gold standard system, by facilitating international loans, by adjusting the industrial demand for gold, etc. This idea is still upheld today by him as well as by the Gold Delegation of the League of Nations' Financial Committee and many scholars and business men. In its interim report the Gold Delegation recommended: "\ldots the concentration of gold in reserves under the direct and immediate administration of those responsible for monetary policy, and its limitation as a means of payment in international transactions; the reduction of the conventional minimum gold cover with international understanding; economy of gold by extending the use of cheques, post-office banking facilities, giro transactions, etc; and by developing and improving the mechanism of the clearing-house; and the replacement of notes of small denominations by subsidiary coin."\textsuperscript{191)

Thus, after the stability of gold has been brought about, the monetary systems of nations have been changed from the gold standard system of the pre-war years to the gold bullion system or to the gold exchange system; the amount of legal reserves has been reduced; gold was withdrawn from circulation to be concentrated at the centre of every country.

However, V which was greatly expanded as the result
of the emancipation of money from the yoke of gold was suddenly contracted to almost the normal size as soon as the United States took the decisive step in the direction of the gold standard system and all nations had to prepare themselves for a return to that system. The V again expanded during the period when the nations were placed in a state of confusion and doubt, being unable to return to the gold standard system; again, its contraction began at the time when the nations made up their minds to go back to gold. All this indicates how powerful is the custom of monetary circulation under the gold standard system ascertained during the period 1850-1913.

Professor Cassel, who has seen the great slump in the general level of prices after 1920, gives the following view in the memorandum which he presented at the meeting of the Financial Committee of the League of Nations in September, 1921: "Under ordinary circumstances, the endeavour of a country with a gold standard to bring down prices and thus to raise the internal value of its money would, according to the classical doctrine, have caused an influx of gold into that country with a consequent increase in the total volume of the monetary purchasing power within the country. Thus, the depression of prices would have been counteracted and the internal value of the money of the country would have been brought down to that of gold. In the present case, this automatically regulating machinery has not had sufficient opportunity to work effectively."

The unbalanced distribution of gold forming the thesis of the foregoing quotation has again been discussed with much heat recently. However, as may be seen in the statistics of the real rates of gold reserves (see Supplementary Table), the influx of gold into a country does not necessarily expand the volume of its currency at once. (To give rise to such an effect, the real rates of gold reserves should be constant). The truth of all this is applicable to the pre-War years as well to the post-War years and to all countries without any exception. It has
been true in the past as it is true today. And what are changed are the circumstances which give rise to much outcry over the so-called unbalanced distribution of gold today.

I am conscious of the imperfection of my present enquiry, due to the scarcity of the materials used and to the daring method of treatment and examination, and I hope to remedy the defects of the present article by undertaking further study. But if my enquiry is free from serious shortcomings, it may be said to have provided a sort of key to forcast how long the present fall of prices will continue, so far as it is concerned with the secular movement.

My materials cover the period up to 1930. Now, as may be seen in Diagram (4), the index number of prices in 1930 exceeds the relative stock of gold money given by my calculation only by between 10 and 14 per cent. In 1931, the production of industries does not appear to have been increased; on the contrary, it may have been reduced somewhat. Moreover, no decrease seems to have been made in the output of gold and no increase in its consumption for industrial arts. Accordingly, a considerable increase must have been made in the world's stock of gold and consequently the relative stock of gold money must have somewhat risen. On the other hand the general level of prices continued its downward course. All this points to the conclusion that the relative stock of gold money and the general level of prices must have approached each other to a considerable extent. Rather, it is possible that the former somewhat exceeds the latter. Thus, we may say that the basic circumstances underlying the depreciation of prices have been largely eliminated. Furthermore, it is possible that the T and T' which are made the basis of calculation of the relative stock of gold for the years after 1913 may have been estimated somewhat higher than they really are, due to the fact that oil the increase of which is of high percentage, has been taken into consideration after that particular year. And, in consequence, the re-
lative stock of gold after 1913 actually may have been somewhat greater than estimated in my own calculation.

I would be, however, jumping to a conclusion, should I venture to draw the inference from the foregoing reasons that the downward trend of prices is bound to come to a full stop. I realize that there is much room for further study and need for gathering more and more relevant materials and for their more adequate treatment. Moreover, as I have already pointed out, during the last half of the period of price depreciation, namely, 1885–1905, the index number of prices is lower than the relative stock of money as given in my calculation. This is partly due to the abolition of the silver standard system by some nations and the resumption of the gold standard by the United States both of which led to a general attachment for gold; and partly to the inertia of the fluctuation of prices for a long period of time.

For the world’s future economic conditions the following two factors seems to have great importance namely, the re-suspension of the gold standard by some of the principal nations of and the devaluation of their currencies. If some of the principal nations of the world should allow their currencies to devalue, that will mean the raising of the relative stock of gold to a corresponding extent; and, in consequence, give rise to some basic factors which will tend to suspend the depreciation of prices or causes their reactionary rise, to the same extent. However, the future production of gold, so far as forecast, will not reach an enormous amount; and, in consequence, no extensive rise in prices such as has been seen in the past is conceivable at present, so long as the gold standard system continues. Of course, the matter would be quite different if the silver standard is adopted side by side with the gold standard system. But the past failure of the bimetallic movement is sufficient to exclude any rash judgment regarding the possibility of such a new factor. On the other hand, should several principal countries of the world keep their conver-
sion systems suspended for many years and stabilize their money when the value of gold has touched bottom, taking the depreciation value as the basis, then the world will inevitably face a general disturbance of its economic intercourse, and a repetition of the post-bellum economic chaos would be unavoidable.

Supplementary Table: Gold Reserve Rates (%)

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6) Nogaro: La question de l'or devant la Société des Nations. (Revue d'économie politique, 140 Année, No. 1) pp. 10-11. Needless to state that Nogaro's criticism is not confined to this point.


9) The general index number of prices in Japan has been derived from the following sources: figures for 1873-1886 are provided by the index number of the wholesale prices in Tokyo investigated by the Monetary System Investigation Commission; figures for 1887-1899 are taken from the Bank of Japan's old index number of prices; figures for years after 1900 are taken from the Bank of Japan's new index number of wholesale prices in Tokyo. In order to form continuous index number, I have tried to multiply the index numbers series concerning the former years by the figure, which will make the index number of the former series of that year, when the index number series concerning the latter years begin, harmonise with the index number of the latter series of the same year. Then I multiplied the index number thus secured by the average exchange rate of the yen on New York in order to approximate it to the price in gold as far as possible. (Concerning the average exchange rate, see Japan Economic Statistics compiled and edited by the Asahi Shimbun, p. 414). Being unable to find the average rate of the yen for 1872, I have used that for 1873 instead. The index numbers thus gained were reduced to 1913=100.

10) On the general index number of prices in England. In order to secure the index number for the years 1873-1886 I multiplied Sauerbeck's index number by 1. 768 and took the figure for 1913 as 100. For the years following 1915 I have taken the figures converted into the prices in gold by Ernst Wagemann: Struktur und Rhythmus der Weltwirtschaft, 1931, s. 399. For the years previous to 1845 I multiplied Jevon's index numbers by 1.568.
11) On the general index number of prices in the United States. The general level of prices in gold. (Wagemann: Structur etc., s. 398-9) For 1801-1839, the index number of Professor A.H. Hansen. For 1840-1890, the index number contained in the report made by Mr. Aldrich of the Committee on Finance, March 3, 1833, Senate Document, 52nd, Cong., 2nd Sess., No. 1394, the Wholesale Prices, Wages and Transportation, Part I, Vol. I. For the latter years, the index number compiled by the Board of Labour. According to Kondratieff's Die lange wellen der Konjunktur the index number of prices in the U.S. given by the Annuaire statistique de la France, 1922, is compiled as follows: the figures for 1790-1801 are given by H.V. Roelse's index number—Bulletin of the American Statistical Association, Dec. 1917, for 1802-1824, the index number of C.H. Jouergens—the June No., 1911, of the same bulletin, figures for the subsequent years are regarded as being the same as those given by Wagemann. Thus we see that the index numbers concerning 1802-1824 are ascribed to Hansen by Wagemann as well as by the Annuaire Statistique; but the index numbers concerning 1802-1824 given by Wagemann are different from those given by the Annuaire Statistique of the same period. At present it is impossible for me to ascertain the truth of the matter.

12) On the general index number of prices in France. Wagemann: Structur etc., s. 398-9. Figures for the years after 1913 have been converted into the price in gold on the basis of the gold price in that year.

13) On the general index number of prices in Germany. Wagemann: Structur etc., s. 398-9. The index has been converted into the price in gold.


15) Ernst Wagemann: Structur und Rhythmus der Weltwirtschaft, 1931, s. 398, foot note.


20) Layton: ibid. The diagram given in the appendix.


22) vgl. Karl Helfferich: Das Geld, 1923, s. 95.

23) Layton: ibid.
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24) Cassel: ibid. s. 417.

25) Layton: ibid. pp. 159-160. He seems to owe this information to Bartholomew's Atlas of the World's Commerce. (See the diagram in the appendix). I have been unable to secure an original copy.

26) Kitchin: ibid.


29) Kitchin: The Supply of Gold, etc., ibid., p. 79.

30) Stock of gold money for 1930 was secured in the following way: the amount of increase made in the amount of gold used for currency purposes during 1929 was added to the total amount of gold money in the same year; to this sum was further added the sum, which was calculated by deducting the gold output of the world in 1929 from that for 1930 (according to the Statistical Year-book of the League of Nations, 1930-31, p. 144).


38) Statistical data for the wheat output of the world: For the period 1890-1923, The Yearbook of the United States Department of Agriculture, 1920, p. 549, 1924, p. 469. For 1924-1929, Commerce Yearbook, 1930, 11, p. 623. For 1930, League of Nations: Statistical Yearbook, 1930-31, p. 77. (I compared the Commerce Yearbook with the Statistical Yearbook of the League of Nations for the period 1927-1929, and secured the comparative rate of 1.2268 and divided the figure of 1930 by this rate). For 1878-1889, the rate of 1.9827 (percentage between the world's wheat output of 1890-1894 and the total wheat output of Great Britain, the United States, Germany and France in the same period) was multiplied on the wheat output of Great Britain the United States, Germany and France. For 1870-1877, the rate of 2.1955 (percentage between 1.9827 times of the total wheat output of Great Britain, the United States, Germany and France during 1878-1882 and the total wheat output of Great Britain, the United States and France...
during the same period) is multiplied on the total wheat output of Great Britain, the United States and France. For 1850-1860, the rate of 3.714 (the ratio between the total wheat output of Great Britain, the United States and France during 1870-1874 multiplied by 2.1955 and the total wheat output of Great Britain and France during the same period) was multiplied by the total wheat output of Great Britain and France. The annual average rate of increase of the foregoing index numbers is 1.45 per cent.

39) Statistical data for the pig-iron output of the world: I have tried to use two methods of composition. (A) For 1850, Buchanin: Imperialismus und Weltwirtschaft, s. 32. For 1865-1910, Cassel: Theoretische Sozialökonomie, s. 641. For 1911-1920, the Japan Economic Statistics compiled and edited by the Asahi Shimbun, p. 1259. For 1921-1930, The Statistical Yearbook of the League of Nations, 1930-31, p. 128. Where statistics were not available, I inserted figures secured on the basis of the 15th root of the rates between the output of 1850 and that of 1865. The annual average rate of increase of index numbers thus calculated is 3.56 per cent. (B) For 1853-1843, Woytinsky: Die Welt in Zahlen, IV. s. 169. For 1850-1917, Statistical Abstract of the United States, 1922, p. 737. For 1918-1927, the Japan Economic Statistics compiled and edited by the Asahi Shimbun, p. 1259. For 1921-1930, the Statistical Yearbook of the League of Nations, p. 128. For the year the material for which is lacking, I inserted figures calculated on the basis of the number-of-lacking-years-plus-1 root of the ratio between the preceding and following years of that lacking year. Its annual average rate of increase is 3.88 per cent. I used in this paper only (B), but the result attained by using (A) does not materially differ from what is shown in this paper.

40) Statistics of the world's copper output: For 1850-1924, Annuaire Statistique de la France, 1926, p. 275. For 1925-29, The Commerce Yearbook, 1930, II., p. 652. For the years the material for which is lacking, I repeated the previous method of calculation given in 39) (B).


43) Statistics of the world's consumption of raw cotton: For 1887-1924, Annuaire Statistique de la France, 1926, pp. 380-1. Consumption statistics of Great Britain, Austria, Hungary, Switzerland, Germany, Holland, Belgium, France, Spain, Portugal, Italy, Japan, Canada and the United States. For 1871-1886, the total consumption of Great Britain, France, the
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United States and Germany taken from the same document is multiplied by the rate of 1.1956 (which is the ratio between the total consumption of Great Britain, France, the United States and Germany during 1887-1891 and the total consumption of the foregoing 14 countries during the same period of time). For 1850-1870, I inserted figures calculated on the basis of an average increase rate computed by applying the straight line by means of the least squares method on the logarithm of the total consumption of Great Britain, the United States, France and Germany in each year during the period of 1871-1886, viz., in order to get the figure for 1870 I divided by this increase rate, the total consumption of Great Britain, the United States, France and Germany during 1871, and in order to get that for 1869 I divided by the same rate the figure for 1870, and so on. For 1925-1930, I multiplied the figures given in The Statistical Yearbook of the League of Nations, 1930-31, p. 109 by the rate of 0.78573 (which is the ratio between the figures given by this Yearbook for 1921-1925 and the total consumption of the foregoing 14 countries during the same period of time).

44) Foreign trade statistics of Great Britain, the United States, France, Germany and Japan. (A) British foreign trade statistics: for 1850-1912, computation was made from W. Page: Commerce and Industry (Statistical Tables, 1919) pp. 70-72. For 1913-1929, Statistical Abstract of the United Kingdom, 1930. For 1930, The Statesman's Yearbook, 1931, p. 83. I divided this figure by Sauerbeck's index number of prices in paper money when it differed from prices in gold reduced to 1913 = 100, and converted into the mark unit by the legal par of 1913. (B) American foreign trade statistics: For 1850-1928, The Statistical Abstract of the United States. For 1920-1930, The Statesman's Yearbook, 1931, p. 458. The foregoing figures were divided by the general index number of prices (as given in Foot-note 2, except for 1863-1879 in which case the general index number of prices in paper money was applied), and the quotient was converted into marks by means of the legal par of 1913. (C) French foreign trade statistics: For 1850-1924, Annuaire Statistique de la France, 1926, p. 94-95. For 1925-1927, The Commerce Yearbook, 1930, II, p. 234. For 1928-1930, The Statesman's Yearbook, 1931, 853. The foregoing figures were divided by the general index number of prices (as given in Foot-note 12, except for years after 1913 in which case the index number of the Statistique Général reduced to 1913 = 100 is taken) and the quotient were converted into marks by means of the legal par of 1913. (D) German foreign trade statistics: For 1850-1924, Ernst Wagemann: inleitungen in die Konjunkturlehre, 1930, appendix. I multiplied the amount of foreign trade per capita by the number of the German population as given in Statistisches Jahrbuch f. d. Deutsche Reich. For 1925-1929, Statistisches Jahrbuch f. d. Deutsche Reich, 1930, s. 190. For 1930, I first secured the ratio between the figures of 1928-1929 as given above and the figures given
by Wagemann reported in the International Pamphlet Agency, No. 428, and then multiplied it by the statistics of 1930 as given in the latter source. (E) Japanese foreign trade statistics: For 1868-1929, *The Japan Economic Statics* compiled and edited by the Asahi Shimbun, p. 238. For 1930, *Principal Statistics* compiled and edited by the Commercial University of Kobe. The foregoing figures were divided by the index number of prices with 1913 as 100 (As given in Foot·note 9, but, on the contrary, not yet multiplied by the yen's exchange rate on New York), and the quotient was converted into marks by means of the legal par of 1913. For 1890-1867, the annual average increase rate was first computed by applying the least squares method on the logarithm of the figures of 1868-1882. I inserted figures of years for which statistics are lacking, by dividing by this rate first the figure for 1868, then the figure thus gained, and so on.

48) Karl Marx: *Das Kapital*, Bd. I. Volksausgabe, s. 85.
50) Cassel: *World’s Monetary Problems*, p. 82.
53) (A) British gold reserve rates: For 1856-1928, I divided the average amount of gold bullion at a certain date of each of the four quarters of the year by the average amount of bank notes in circulation at a certain date of each of the four quarters of the same year. For years after 1914, currency notes are included in the amount of the dividend. Materials are taken from the Statistical Abstract of the United Kingdom, No. 18, pp. 115-119: No. 26 pp. 123-127: No. 40, pp. 213-217: No. 51, pp. 265-269: No. 70, pp. 379-381: No. 73, pp. 190-1. For 1922-1930, the Statistical Yearbook of the League of Nations, 1930-31. (B) French gold reserve rate: For 1881-1921, *Annuaire Statistique de la France*, 1926, p. 82. For 1922-1930, Statistical Yearbook of the League of Nations, 1930-31, p. 221. I divided the total domestic gold reserves of the Bank of France by the total amount of notes in circulation. The immense increase after 1922 is due to the devaluation of the franc. (C) American gold reserve rates: For 1880-1928, the Statistical Abstract of the U.S. No. 45, p. 512: No. 51, p. 246. For 1922-1930, the Statistical Abstract of the League of Nations.