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## MARX'S ANALYSIS OF CAPITALISM AND <br> THE GENERAL EQUILIBRIUM THEORY OF THE LAUSANNE SCHOOL

## INTRODUCTION.

The general equilibrium theory enunciated by economists of the mathematical school is regarded as the best economic theory that has so far been advanced. It is true that it has many critics and that, as a matter of fact, it contains points requiring correction, but it cannot be denied that it possesses features which entitle it to the claim that it is the best of all economic theories hitherto evolved. It is, in this sense, gratifying that increasing attention is now being directed to the general equilibrium theory.

It must, however, be admitted that the general equilibrium theory of economists of the mathematical school suffers from its being much too formal. It is ineffectual in making clear systematically either the organisation of present-day capitalistic society or the laws of its development. Some economists are prone to regard this as of little account. But it cannot be denied that political economy has always had for one of its important subjects of study the systematic grasp of the organisation of historical economic society and the laws of its development. Nor can it be denied that anybody who makes a speciality of economics cannot stop short of the systematic grasp of the organisation of the present-day capitalistic society and the laws of its development. For this purpose the general equilibrium theory of the mathematical school is too ineffectual. The dynamic theory dealt with in it has too feeble a relation to the general equilibrium theory elaborately developed by means of intricate high mathematics, with the result that it amounts almost to economics devoid of theorising. It is by no means without reason that the general equilibrium theory is denounced by some critics as a mere sport of
logic devised to conceal entity or a claptrap designed to gloss over the lack of theory.

On the contrary, the Marxian political economy, though it is now shown to contain many defects, sets forth theories which are either intended to enunciate systematically the organisation of present-day capitalistic society and the làws governing its development or have inseparable and necessary bearings on them.

What is it, then, that makes Marxian economics so powerful and the general equilibrium theory of the mathematical school so inert? It is simply this, that whereas in the Marxian economics the organisation of capitalistic production and the laws of its development are analysed in a direct way, in the general equilibrium theory the main attention is directed to the analysis of the mental structure of the individuals who take part in the organisation of capitalistic production. This is, of course, a general description. In its abstractest phase the organisation of capitalistic production is dealt with even in the general equilibrium theory, but the analysis of the organisation of capitalistic production, as we see it in the general equilibrium theory, is quite abstract. What makes the Marxian economics grasp the organisation of capitalistic production and the laws of its development and makes the general equilibrium theory incapable of doing so? For this the direction of concern conditioned by the existence of students may be somewhat responsible, but I do not think this is the sole reason. In my opinion, the structure of the general equilibrium theory, as it stands, renders the analysis of the organisation of capitalistic production and consequently a firm grasp of the laws of its development logically impossible.

What is it, then, which the structure of the general equilibrium theory contains and which makes the analysis of the organisation of capitalistic production and consequently the grasp of the laws of its development logically impossible? And how can it be eliminated? The object of the present article is to study these poinis.

According to the view which I hold at present, by settling these points can the general equilibrium theory be made to attain great potency in analysing both the organisation of capitalistic production and the laws of its development. By their solution also can many important questions which were either looked over or misjudged even by Marx be put in a clear light and be solved. In the present article, I wish to set forth the starting point of my line of thought.

## 1. THE GENERAL EQUILIBRIUM THEORY.

In order to make clear the points which I propose to study, in the present article, I must first briefly explain the general equilibrium theory. The general equilibrim theory, as I here explain, is not, however, the one in its original form, but one in a form somewhat rewritten. One reason for rewriting it is to facilitate the development of the discussion of the subject in the present article (as is evidenced by the addition of the assumption that society is clearly divided into the capitalist and the working classes). The other reason is that the general equilibrium theory, as it stands, has been deemed inadequate. It might be better to explain in detail these points of alteration, but as the present article is not directly concerned with these points, I will here desist from the attempt.

As is usually done in theoretical economic study, it is here assumed that there obtains simple re-production, that capital is all floating, fixed capital being ruled out, that the time of the rotation of capital and the duration of its rotation is equal in all branches of production, that all manufactures are produced capitalistically, that there is perfect capitalistic free competition with the consequent elimination of the participation of the State and other controlling bodies, that property can be differentiated and its demand functions are accordingly continuous, that there is no friction in the process of circulation, and that the given factors undergo
no change. Such assumptions are, of course, permissible in the initial stage of a theoretical economic study. Inasmuch, however, as the present article is not concerned with these assumptions themselves, I may as well refrain from any detailed exposition of these points.

The fundamental productive goods (which are not manufactures in themselves, but are things which are paid for as productive goods) include, first of all, labour-power of all kinds. Given certain conditions, land-power may also be counted among them. But as the present article is concerned with the factors involved before the question of land-power comes in, it is here assumed that the fundamental productive goods consist exclusively of labour-power of all kinds. This assumption is also permissible in the initial stage of a theoretical economic study.

Now, let us assume a society which consists exclusively of $m$ capitalists and $\theta$ labourers. Let it be further assumed that the capitalists live on the money which they possess before exchange and on the profit on their capital, and the labourers on their wages only. In this case, according to the subjective value theory, the amount of capital which each of the capitalists invests and the kinds and quantity of goods which he demands, or the amount of labourpower each of the labourers offers and the kinds and quantity of goods he demands depend on the point at which the subjective value which they recognise in each kind of goods finds the equilibrium. Let it be supposed that this society possesses consumable goods of $\mathrm{n}-1$ kinds, and let the prices of these kinds be represented by $\mathrm{p}_{2}, \mathrm{p}_{3}, \ldots \ldots \ldots \mathrm{p}_{\mathrm{n}}$ respectively, and the amount of money possessed by the first of $m$ capitalists after exchange ${ }^{1{ }^{1}}$ be denoted by $\mathrm{N}_{11}$, that of the second capitalist $\mathrm{N}_{21}{ }^{\prime} \ldots \ldots . .$. , and that of the last $m$ th capitalist $\mathrm{N}_{\mathrm{ml}}{ }^{\prime}$. If the quantity of consumable goods of

[^0]the first kind possessed by the first capitalist after exchange is represented by $\mathrm{N}_{\mathrm{Io}^{\prime}}$, that of the second capitalist by $\mathrm{N}_{\circ 2}{ }^{\prime}$, and that of the last $m$ th capitalist by $\mathrm{N}_{\mathrm{m} 2}{ }^{\prime}$; the quantity of consumable goods of the $n$ - 1 th kind held by the first capitalist after exchange by $\mathrm{N}_{\mathrm{fn}}{ }^{\prime}$, that of the second capitalist by $\mathrm{N}_{\mathrm{on}_{1}}{ }^{\prime} \ldots \ldots \ldots$, and that of the last $m$ th capitalist by $\mathrm{N}_{\mathrm{mn}}{ }^{\prime}$, if the average rate of profit is denoted by $\mathrm{p}^{\prime}$, the amount of capital invested by each of $m$ capitalists by $\mathrm{K}_{1}$, $\mathrm{K}_{\mathrm{g}}, \ldots \ldots \ldots \mathrm{K}_{\mathrm{n}}$ respectively; and if the amount of money possessed before exchange is assumed to ${ }^{2}$ be $\mathrm{G}_{\mathrm{i}}, \mathrm{G}_{2} \ldots \ldots \ldots . . \mathrm{G}_{\mathrm{m}}$, we get the first set of equations, containing $m n$ equations which denotes the equilibrium ${ }^{3}$ ) of marginal utility each of the capitalists of $m$ number has in regard to the goods of $\mathrm{n}+1$ kinds (one of which shows the principal and interest accruing in future in the shape of the investments recovered, another is money and the rest the consumable goods of $n-1$ kinds).
(I)

[^1]We also get the second set of equations, containing $m$ equations, which shows the equilibrium of individual revenue and expenditure in respect of the $m$ capitalists:

Again, let it be supposed that the same community has labour-power of $e$ kinds and let the wages for labour-power of these kinds be represented by $\mathrm{q}_{1}, \mathrm{q}_{2} \ldots \ldots \ldots \mathrm{q}_{\mathrm{e}}$. If the amounts of labour-powers of all kinds offered by the first of $\theta$ labourers are denoted by $\mathrm{E}_{11}, \mathrm{E}_{12} \ldots \ldots \ldots \mathrm{E}_{\mathrm{l}}$, that offered by the second labourer by $\mathrm{E}_{21}, \mathrm{E}_{22} \ldots \ldots . . \mathrm{E}_{2}$, and that by the last $\theta_{\text {th }}$ labourer by $\mathrm{E}_{\theta_{1}}, \mathrm{E}_{\theta_{2}} \ldots \ldots \ldots . \mathrm{E}_{\mathrm{gef}}$, and if the amount of money possessed by the first labourer after exchange is denoted by $\mathrm{N}_{11}$, and the quantity of consumable goods of the first kind by $\mathrm{N}_{12} \ldots \ldots \ldots$, and the quantity of the consumable goods of the last $\mathrm{n}-1$ th kind by $\mathrm{N}_{\mathrm{in}}$, those held by the second labourer by $\mathrm{N}_{21}, \mathrm{~N}_{22}, \ldots \ldots . . \mathrm{N}_{2 n}$ respectively and those held by the last "th labourer by $\mathrm{N}_{01}, \mathrm{~N}_{02} \ldots \ldots .$. $\mathrm{N}_{\text {日1 }}$ respectively, then we obtain the third set of equations containing ( $\mathrm{n}+\mathrm{e}-1$ ) $\theta$ equations, which indicates the equilibrium of marginal utility each of the $\theta$ labourers has in regard to the goods of $n+e$ kinds (one of which is money, $\mathrm{n}-1$ represent the consumable goods of $\mathrm{n}-1$ kinds and $e$ embody labour-powers of e kinds):-

$$
\left\{\begin{aligned}
\phi \mathrm{N}_{12} & =\frac{1}{\mathrm{p}_{2}} \phi \mathrm{~N}_{\mathrm{r} 2}=\frac{1}{\mathrm{p}_{3}} \varphi \mathrm{~N}_{13} \ldots \ldots \ldots \\
& =\frac{1}{\mathrm{p}_{\mathrm{n}}} \psi \mathrm{~N}_{1 \mathrm{l}}=\frac{1}{\mathrm{q}_{1}} \psi \mathrm{E}_{11}=\frac{1}{\mathrm{q}_{2}} \psi \mathrm{E}_{\mathrm{i} 2}=\ldots \ldots \ldots=\frac{1}{\mathrm{q}_{\mathrm{g}}} \psi \mathrm{E}_{\mathrm{le}}
\end{aligned}\right.
$$

equal to the marginal value of present-money, and the price of a future money is equal to $\frac{1}{1+p^{\prime}}$. So, it follows that the marginal value of future money, divided by its price $\frac{1}{1+p^{\prime}}$, in other words, multiplied with $\left(1+p^{\prime}\right)$, is equal to the marginal value of present-money.

$$
\begin{align*}
& \begin{aligned}
\psi \mathrm{N}_{22} & =\frac{1}{\mathrm{p}_{2}} \psi \mathrm{~N}_{22}=\frac{1}{\mathrm{p}_{2}} \psi \mathrm{~N}_{23}=\ldots \ldots . . \\
& =\frac{1}{\mathrm{p}_{n}} \psi \mathrm{~N}_{24}=\frac{1}{\mathrm{q}_{1}} \psi \mathrm{E}_{24}=\frac{1}{\mathrm{q}_{2}} \psi \mathrm{E}_{22}=\ldots \ldots \ldots=\frac{1}{\mathrm{q}_{\mathrm{e}}} \psi \mathrm{E}_{2 \mathrm{e}}
\end{aligned}  \tag{III}\\
& \psi \mathrm{~N}_{01}=\frac{1}{\mathrm{p}_{2}} \psi \mathrm{~N}_{92}=\frac{1}{\mathrm{p}_{3}} \psi \mathrm{~N}_{03}= \\
& =\frac{1}{\mathrm{p}_{\mathrm{n}}} \psi \mathrm{~N}_{8 \mathrm{n}}=\frac{1}{\mathrm{q}_{1}} \psi \mathrm{E}_{81}=\frac{1}{\mathrm{q}_{2}} \psi \mathrm{~N}_{82}=\ldots \ldots \ldots=\frac{1}{\mathrm{q}_{\mathrm{e}}} \psi \mathrm{E}_{\theta \mathrm{e}}
\end{align*}
$$

And another fourth set of equations, containing $\theta$ equations, which indicates the equilibrium of individual revenue and expenditure of each of the $\theta$ labourers, is obtained :-

The first set of equations contains ( $\mathrm{mn}+\mathrm{m}+\mathrm{n}$ ) unknown quantities, viz. $n-1$ in regard to the prices of the coosumable goods of $n-1$ kinds, $m$ regarding the amount of capital which each of $m$ kapitalists invests, 1 concerning the average rate of profit, and $m n$ about the quantity of each of $n$ kinds of goods possessed by each of $m$ capitalists after exchange (although, as a matter of fact, there is still another in regard to each capitalist, vis. $K\left(1+p^{\prime}\right)$, which represents the goods held after exchange, it contains no new unknown quantity). As regards the third set of equations, it contains ( $\mathrm{n}+\mathrm{e}$ ) $\theta+\mathrm{e}$ new unknown quantities, viz. $\theta n$ in regard to the quantity of each of the goods of $n$ kinds which each of $\theta$ labourers demands, $\theta e$ in respect of the amount of each of labour-power of $e$ kinds which each of them supplies, and $e$ respecting the price of labour-power of $e$ kinds. Neither the second nor the fourth set of equations contains any new unknown quantities.

Social products can roughly be divided into productive
means and consumptive means. Here it is assumed that all these things are produced capitalistically, and the former is called capital means and the latter consumable goods. In order to turn out products, both capital goods and labourpower are required (land-power is ruled out by a previous assumption). The amount of capital goods and labour-power needed for producing products, multiplied by their respective prices forms the cost of production (as viewed from the capitalist's point of view) necessary for producing these products. As the capitalist tries to produce them in such branches as can secure the largest possible surplus value in excess of this cost of production, the prices of products are, in the ultimate, normally fixed at the cost of production plus the average profit and accordingly at the cost of production multiplied by $\left(1+\mathrm{p}^{\prime}\right)$, provided complete free capitalistic competition prevails. Now, let us suppose that society possesses capital goods of $s$ kinds, and let the prices of goods of these kinds be represented by $k_{1}, k_{2} \ldots \ldots \ldots k_{4}$ respectively. Be it further supposed that the amount of each capital goods and labour power required for the production of each unit of each product is technically given (that is, let the amount of capital goods of the first, the second, ......... and the sth kinds needed for the production of one unit of money be $\alpha_{11}, a_{12} \ldots \ldots \ldots a_{18}$; the amount of labour-power of the first, the seccnd, ........ and the sth kinds be $a_{11}, a_{12} \ldots \ldots \ldots a_{19}$ : the amount of capital goods of the first, the second, ........ and the sth kinds required for the production of one unit of consumable goods of the first kind be $u_{21}, u_{02} \ldots \ldots \ldots u_{29}$; the amount of labour-power of the first, the second, ......... and the $e$ th kinds be $\mathrm{a}_{21}, \mathrm{a}_{2 g}, \ldots \ldots \ldots \mathrm{a}_{\mathrm{e}}$; the amount of capital goods of the first, the second, ......... and the sth kinds necessary for the production of one unit of consumable goods of the last $n$ - 1 th kind be $\mu_{n 1}, \alpha_{n 2}, \ldots \ldots \ldots \mu_{n 3}$; the amount of labour-power of the first, the second, ......... and the eth kinds be $a_{n 1}, a_{n 2}, \ldots \ldots \ldots a_{n e}$; the amount of capital goods of the first, the second, ......... and the sth kinds needed for
producing one unit of kapital goods of the first kind be $\beta_{11}$, $\beta_{12}, \ldots \ldots . . \beta_{18}$; the amount of labour-power of the first, the second, ...... .. and the eth kinds be $b_{11}, b_{12}, \ldots \ldots . . b_{19} \ldots \ldots .$. ; the amount of capital goods of the first, the second, ......... and the sth kinds required for producing one unit of capital goods of the last sth kind be $\beta_{\mathrm{s} 1}, \beta_{\mathrm{s}}, \ldots \ldots . . \beta_{\mathrm{ss}}$; and the amount of labour-power of the first, the second, and the eth kinds be $b_{s i}, b_{s 2}, \ldots . \ldots b_{s e}$. These are assumed to be known quantities and unalterable by the amount of production. It is further assumed that all of these embody floating capital which is used up in one round of production, that no fixed capital is required, and that in all branches of production the period of the rotation of capital is equal). Then, the fifth set of equations containing $n$ equations can be obtained in regard to money and the consumable goods of each of $n-1$ kinds :

Also, we obtain the sixth set of equations containing $s$ equations in regard to the capital goods of $s$ kinds:

The fifth set of equations contains $s$ new unknown quantities in respect of the prices of each of the capital goods of $s$ kinds, while the sixth set of equations contains no new unknown quantities.

The total amount of the various consumable goods and money in social demand is nothing but the total amount of money and various consumable goods held by all persons
after exchange. Consequently, if the total amount of money held by all persons after exchange is to be denoted by $N_{1}$, and that of the consumable goods of the first, the second, $\ldots \ldots .$. and the last $n$ - 7 th kinds by $\mathrm{N}_{2}, \mathrm{~N}_{3}, \ldots \ldots \ldots \mathrm{~N}_{1}$ respectively, the seventh set of equations containing $n$ equations can be obtained:

This set of equations contains $n$ new unknown quantities in respect of the total amount of money and of each of the consumable goods of $n-1$ kinds held after exchange. If the assumption is accepted that all production is carried on capitalistically and that competition is carried on to the extent of the equilibrium being attained, all the consumable goods held after exchange ought to have been produced capitalistically and all the consumable goods produced ought to be sold out without any portion being left unsold on the hands of capitalist producers. So the quantity of consumable goods of $n-1$ kinds held after exchange, viz. $\mathrm{N}_{2}, \mathrm{~N}_{3}, \ldots .$. $\ldots \mathrm{N}_{\mathrm{n}}$, embodies at the same time the quantity produced. With regard to the money held after exchange, however, it, by its own peculiar nature and also under the present assumption, includes that which was held before exchange. Therefore, the amount of money to be produced is equal to the amount of money held after exchange minus the amount of money held before exchange. Thus, supposing $\mathrm{N}_{1}{ }^{\prime \prime}$ to denote the amount of money to be produced, we obtain the eighth equation:

$$
\begin{equation*}
\mathrm{N}_{1}^{\prime \prime}=\mathrm{N}_{1}-\left(\mathrm{G}_{1}+\mathrm{G}_{2}+\ldots \ldots \ldots+\mathrm{G}_{\mathrm{m}}\right) \tag{VIII}
\end{equation*}
$$

This equation contains one new unknown quantity regarding the amount of money to be produced.

The total amount of labour-power of each kind in social
demand is the sum total of the labour-power of that kind required for the production of money and consumable goods of all kinds and capital goods of all kinds. Consequently (since the quantity of each kind of labour-power necessary for the production of one unit of money and consumable goods and capital goods of all kinds is technically given and the total amount of the money and the consumable goods of all kinds to be produced are denoted, as previously provided, by $\mathrm{N}_{1}{ }^{\prime \prime} \mathrm{N}_{2}, \mathrm{~N}_{3}, \ldots \ldots \ldots \mathrm{~N}_{\mathrm{u}}$ ), if $\mathrm{S}_{1}, \mathrm{~S}_{2}, \ldots \ldots \ldots . \mathrm{S}_{\mathrm{s}}$ be taken to denote the total amount of each of capital goods of the first, the second, ......... the sth kinds to be produced, the ninth set of equations containing $e$ equations can be obtained:

This set of equations contains $e+s$ new unknown quantities, viz. $e$ regarding the total amount of each of labourpower of $e$ kinds to be supplied and $s$ respecting the total amount of each of capital goods of $s$ kinds to be produced.

The total amount of capital goods of each kind in social demand is the total amount of capital goods of that kind required for the production of money and consumable goods of all kinds and capital goods of all kinds. Under the assumption that all production is carried on capitalistically and that competition is carried on to the extent of the equilibrium being attained, all kinds of capital goods in demand ought to have been produced capitalistically. Moreover, as any portion of the producers' goods that have been produced is not to be left unsold on the hands of the capitalist producers concerned after meeting the demand of that branch of production concerned, the total amount of the capital goods of the first, the second, ......... and the sth kinds in demand ought to be equal to that of those produced-which are, as previously provided, denoted by $\mathrm{S}_{3}$,
$\mathrm{S}_{3}, \ldots \ldots \ldots$ and $\mathrm{S}_{3}$. Since, on the other hand, the amount of each of the capital goods of all kinds required for producing one unit of money and of each of the consumable goods and capital goods of all kinds is given, we get the following tenth set of equotions containing $s$ equations :
(X)

This set of equations contains no new unknown quantities.

Unlike capital goods, labour-power is not produced capitalistically. The total amount of labour-power of $e$ kinds supplied socially-which is to be equal to what is denoted by $\mathrm{E}_{1}, \mathrm{E}_{3}, \ldots \ldots \ldots$ and $\mathrm{E}_{\mathrm{e}}$, as previously provided,being the sum total of the amount of labour-power of each of the first, the second, $\qquad$ and the eth kinds supplied by $\theta$ labourers, we obtain the eleventh set of equations containing $e$ equations:

$$
\left\{\begin{array}{l}
\mathrm{E}_{\mathrm{l}}=\mathrm{E}_{11}+\mathrm{E}_{21}+\mathrm{E}_{81}+\ldots \ldots \ldots+\mathrm{E}_{91}  \tag{XI}\\
\mathrm{E}_{\mathrm{E}}=\mathrm{E}_{19}+\mathrm{E}_{2 \mathrm{~g}}+\mathrm{E}_{82}+\ldots \ldots \ldots+\mathrm{E}_{91} \\
\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \\
\mathrm{E}_{\mathrm{e}}=\mathrm{E}_{1 \mathrm{e}}+\mathrm{E}_{2 \mathrm{e}}+\mathrm{E}_{8 \mathrm{e}}+\ldots \ldots \ldots+\mathrm{E}_{\text {ge }}
\end{array}\right.
$$

This set of equations contains no new unknown quantities.

The total amount of social capital must be large enough to meet the total cost of production required for the production of social products. Nor does it exceed it in equilibration. Therefore, we get the following twelfth equation ;

$$
\begin{aligned}
\text { (XII) } \mathrm{K}_{1}+ & \mathrm{K}_{2}+\ldots \ldots+\mathrm{K}_{\mathrm{m}}=\mathrm{E}_{1} \mathrm{q}_{1}+\mathrm{E}_{2} \mathrm{q}_{3}+\ldots \ldots+\mathrm{E}_{\mathrm{e}} \mathrm{q}_{\mathrm{e}}+\mathrm{S}_{1} \mathrm{k}_{1}+\mathrm{S}_{2} \mathrm{~K}_{2} \\
& +\ldots \ldots+\mathrm{S}_{\mathrm{s}} \mathrm{k}_{\mathrm{s}}
\end{aligned}
$$

This may be converted into:

$$
\begin{aligned}
& (\mathrm{XIII}) \mathrm{K}_{1}+\mathrm{K}_{2} \ldots \ldots+\mathrm{K}_{\mathrm{m}}=\mathrm{N}_{1}^{\prime \prime}\left(\rho_{11} \mathrm{k}_{1}+\mathrm{a}_{12} \mathrm{k}_{2}+\ldots \ldots+{\sigma_{18}} \mathrm{k}_{\mathrm{s}}+\mathrm{a}_{14} \mathrm{q}_{1}+\right. \\
& \left.\mathrm{a}_{12} \mathrm{q}_{2}+\ldots \ldots+\mathrm{a}_{1 \mathrm{e}} \mathrm{q}_{\mathrm{e}}\right)+\mathrm{N}_{2}\left(\alpha_{21} k_{1}+\alpha_{22} \mathrm{k}_{2}+\ldots \ldots+\alpha_{29} \mathrm{k}_{9}+\mathrm{a}_{21} \mathrm{q}_{1}+\right. \\
& \left.\mathrm{a}_{22} \mathrm{q}_{2}+\ldots \ldots+\mathrm{a}_{2 \mathrm{e}} \mathrm{q}_{\mathrm{e}}\right)+\ldots \ldots+\mathrm{N}_{\mathrm{n}}\left(\alpha_{\mathrm{n} 1} \mathrm{k}_{\mathrm{I}}+\alpha_{n 2} \mathrm{k}_{2}+\ldots \ldots+a_{n \mathrm{~g}} \mathrm{k}_{\mathrm{g}}+\right. \\
& \left.a_{n 1} q_{1}+a_{n 2} q_{2}+\ldots \ldots a_{n e} q_{e}\right)+S_{1}\left(\beta_{11} k_{1}+\beta_{12} k_{2}+\ldots . .+\beta_{1 s} k_{1}+\right. \\
& \left.b_{11} q_{1}+b_{10} q_{2}+b_{1 e} q_{e}\right)+\ldots \ldots+S_{8}\left(\beta_{\mathrm{si}} k_{1}+\beta_{\mathrm{s} 2} k_{2}+\ldots \ldots+\beta_{89} k_{8}+\right. \\
& \left.\mathrm{b}_{81} \mathrm{q}_{1}+\mathrm{b}_{82} \mathrm{q}_{2}+\ldots \ldots+\mathrm{b}_{\mathrm{se}} \mathrm{q}_{\mathrm{e}}\right)
\end{aligned}
$$

Neither the twelfth nor the thirteenth equation contains any new unknown quantities.

What I have so far described will suffice to explain in its abstractest form the organisation of capitalistic production where labour-power alone constitutes the fundamental productive goods, all capital goods represent floating capital, the productive coefficients are given, the production is, without exception, carried on capitalistically and under perfect free competition, no friction exists in the process of circulation, and the time and period of the rotation of capital is equal. To sum up, we see:

| Sets of equations. | Wherein are contained |  |
| :---: | :---: | :---: |
|  | Unknown quantities. | Equations. |
| I | $m \mathrm{n}+\mathrm{m}+\mathrm{n}$ | mn |
| II |  | m |
| III | $(n+e) \theta+e$ | $(\mathrm{n}+\mathrm{e}-1)^{\theta}$ |
| IV |  | $\theta$ |
| V | s | $n$ |
| H |  | s |
| YII | n | n |
| VIII | 1 | 1 |
| IX | $e+s$ | e |
| X |  | 5 |
| XI |  | e |
| XII |  | 1 |
| Total | $\begin{gathered} m n+m+2 n+n^{\theta}+e^{\theta}+ \\ 2 e+2 s+1 \end{gathered}$ | $\begin{gathered} m n+m+2 n+n^{\theta}+e^{\theta}+ \\ 2 e+2 s+2 \end{gathered}$ |

From the above, it will be seen that the number of the equations contained is one in excess of that of the unknown quantities therein contained. But one of the equations contained in the second, fourth and fifth to the twelfth sets of equations is so circumstanced that it can be deduced from the other equations.

In oder to explain it, let us first add together separately the left and the right terms of all the equations contained in the second and the fourth sets of equations and transform them into one equation. Then, we obtain:
(a) $\left(\mathrm{N}_{11}{ }^{\prime}+\mathrm{N}_{21}{ }^{\prime}+\ldots \ldots+\mathrm{N}_{\mathrm{m} 1}{ }^{\prime}+\mathrm{N}_{11}+\mathrm{N}_{21}+\ldots \ldots+\mathrm{N}_{91}\right)+\mathrm{P}_{2}\left(\mathrm{~N}_{12}{ }^{\prime}+\right.$

$$
\left.\mathrm{N}_{22}^{\prime}+\ldots \ldots+\mathrm{N}_{\mathrm{m} 2}^{\prime}+\mathrm{N}_{12}+\mathrm{N}_{22}+\ldots \ldots+\mathrm{N}_{02}\right)+\mathrm{P}_{5}\left(\mathrm{~N}_{\mathrm{n} 3}{ }^{\prime}+\ldots \ldots+\right.
$$

$$
\left.\mathrm{N}_{\mathrm{m}}{ }^{\prime}+\mathrm{N}_{13}+\mathrm{N}_{23}+\ldots \ldots+\mathrm{N}_{83}\right)+\ldots \ldots+\mathrm{P}_{\mathrm{n}}\left(\mathrm{~N}_{1 \mathrm{n}}{ }^{\prime}+\mathrm{N}_{\mathrm{on}^{\prime}}{ }^{\prime}+\ldots \ldots+\right.
$$

$$
\left.\mathrm{N}_{\mathrm{ma}}^{\prime}+\mathrm{N}_{\mathrm{tn}}+\mathrm{N}_{\mathrm{en}_{\mathrm{n}}}+\ldots \ldots+\mathrm{N}_{\mathrm{en}}\right)=\mathrm{p}^{\prime}\left(\mathrm{K}_{\mathrm{l}}+\mathrm{K}_{2}+\ldots \ldots+\mathrm{K}_{\mathrm{m}}\right)+
$$

$$
\mathrm{G}_{1}+\mathrm{G}_{2}+\ldots \ldots+\mathrm{G}_{\mathrm{m}}+\mathrm{q}_{1}\left(\mathrm{E}_{11}+\mathrm{E}_{21}+\ldots \ldots+\mathrm{E}_{81}\right)+\mathrm{q}_{2}\left(\mathrm{E}_{12}+\mathrm{E}_{22}+\right.
$$

$$
\left.\ldots \ldots \mathrm{E}_{\mathrm{q} 2}\right)+\ldots \ldots+\mathrm{q}_{\mathrm{e}}\left(\mathrm{E}_{\mathrm{e}}+\mathrm{E}_{\mathrm{E}_{\mathrm{e}}}+\ldots \ldots+\mathrm{E}_{\mathrm{\theta} \mathrm{e}}\right)
$$

Let this be called $a$ equation.
Next, let us multiply all of the $n$ equations contained in the seventh set of equations by $1, p_{2}, p_{1}, \ldots \ldots \ldots . p_{n}$ successively, and then form one equation by adding together separately the left and the right terms. Then, we obtain:
(b) $\mathrm{N}_{1}+\mathrm{N}_{2} \mathrm{p}_{2}+\ldots \ldots+\mathrm{N}_{\mathrm{n}} \mathrm{p}_{\mathrm{n}}=\left(\mathrm{N}_{\mathrm{n}}{ }^{\prime}+\mathrm{N}_{21}{ }^{\prime}+\ldots \ldots+\mathrm{N}_{\mathrm{m}}{ }^{\prime}+\mathrm{N}_{11}+\right.$

$$
\left.\mathrm{N}_{21}+\ldots \ldots+\mathrm{N}_{\mathrm{al}}\right)+\mathrm{p}_{2}\left(\mathrm{~N}_{\mathrm{va}^{\prime}}+\mathrm{N}_{22}^{\prime}+\ldots \ldots+\mathrm{N}_{\mathrm{m}_{2}^{\prime}}+\mathrm{N}_{12}+\mathrm{N}_{22}+\right.
$$

$$
\left.\ldots \ldots+\mathrm{N}_{82}\right) \ldots \ldots+\mathrm{p}_{\mathrm{n}}\left(\mathrm{~N}_{\mathrm{In}^{\prime}}{ }^{\prime}+\mathrm{N}_{21}{ }^{\prime}+\ldots \ldots+\mathrm{N}_{m \mathrm{n}}{ }^{\prime}+\mathrm{N}_{\mathrm{In}^{n}}+\mathrm{N}_{\mathrm{N}_{4}}+\right.
$$

$$
\left.\ldots \ldots+\mathrm{N}_{\mathrm{en}}\right)
$$

Let this be called $b$ equation.
Next, let us multiply all of $e$ equations contained in the eleventh set of equations by $\mathrm{q}_{1}, \mathrm{q}_{2}, \ldots \ldots \ldots \mathrm{q}_{\mathrm{e}}$ successively, and then form one equation by adding together separately the left and the right terms. Then, we obtain :
(c) $\mathrm{E}_{1} \mathrm{q}_{1}+\mathrm{E}_{2} \mathrm{q}_{2}+\ldots \ldots+\mathrm{E}_{\mathrm{e}} \mathrm{q}_{\mathrm{e}}=\mathrm{q}_{1}\left(\mathrm{E}_{21}+\mathrm{E}_{21}+\ldots \ldots+\mathrm{E}_{01}\right)+\mathrm{q}_{2}\left(\mathrm{E}_{12}+\right.$
$\left.\mathrm{E}_{22}+\ldots \ldots+\mathrm{E}_{92}\right)+\ldots \ldots+\mathrm{q}_{6}\left(\mathrm{E}_{1 \mathrm{e}}+\mathrm{E}_{2 \mathrm{e}}+\ldots \ldots+\mathrm{E}_{89}\right)$
Let this be called $c$ equation.

The right term of $b$ equation is equal to the left term of $a$ equation, while the right term of $c$ equation is equal to the right term of $a$ equation with $\mathrm{p}^{\prime}\left(\mathrm{K}_{1}+\mathrm{K}_{2}+\ldots \ldots+\mathrm{K}_{m}\right)+$ $\mathrm{G}_{1}+\mathrm{G}_{2}+$ $\qquad$ $+\mathrm{G}_{\mathrm{m}}$ struck out. By taking $b$ and $c$ equations into due consideration, $a$ equation may be turned into:
(d) $\mathrm{N}_{1}+\mathrm{N}_{2} \mathrm{p}_{2}+\ldots \ldots+\mathrm{N}_{\mathrm{u}} \mathrm{p}_{\mathrm{u}}=\mathrm{p}^{\prime}\left(\mathrm{K}_{1}+\mathrm{K}_{2}+\ldots \ldots+\mathrm{K}_{\mathrm{m}}\right)+\mathrm{G}_{2}+\mathrm{G}_{3}+$
$\ldots \ldots+\mathrm{G}_{\mathrm{m}}+\mathrm{E}_{1} \mathrm{q}_{\mathrm{l}}+\mathrm{F}_{2} \mathrm{q}_{2}+\ldots \ldots \mathrm{E}_{\mathrm{e}} \mathrm{q}_{\mathrm{e}}$
Let this be called $d$ equation.
Next, let us multiply all of $n+s$ equations contained in the fifth and the sixth sets of equations by $\mathrm{N}_{1}{ }^{\prime \prime}, \mathrm{N}_{2}$, $N_{1}, S_{1}, S_{2}, \ldots \ldots \ldots S_{s}$ respectively, and then form one equation by adding together separately the right and the left terms. Then, we obtain:
(e) $\left\{\mathrm{k}_{1}\left(\alpha_{11} \mathrm{~N}_{1}^{\prime \prime}+u_{21} \mathrm{~N}_{2}+\ldots \ldots+\mu_{n 1} \mathrm{~N}_{n 1}+\beta_{11} \mathrm{~S}_{1}+\beta_{21} \mathrm{~S}_{2}+\ldots \ldots+\beta_{s 1} \mathrm{~S}_{\mathrm{s}}\right)\right.$
$+\mathrm{K}_{2}\left(\alpha_{12} \mathrm{~N}_{1}^{\prime \prime}+\mu_{n ي 2} \mathrm{~N}_{2}+\ldots \ldots+\alpha_{n 2} \mathrm{~N}_{11}+\beta_{12} \mathrm{~S}_{1}+\beta_{22} \mathrm{~S}_{2}+\ldots \ldots+\beta_{s 2} \mathrm{~S}_{\mathrm{s}}\right)$
$+\ldots \ldots+k_{9}\left(\alpha_{19} \mathrm{~N}_{1}{ }^{\prime \prime}+\alpha_{29} \mathrm{~N}_{0}+\ldots \ldots+\mu_{n \mathrm{n}} \mathrm{N}_{\mathrm{n}}+\beta_{19} \mathrm{~S}_{1}+\beta_{29} \mathrm{~S}_{2}+\ldots \ldots\right.$
$\left.+\beta_{\text {se }} \mathrm{S}_{\mathrm{s}}\right)+\mathrm{q}_{1}\left(\mathrm{a}_{11} \mathrm{~N}_{1}^{\prime \prime}+\mathrm{a}_{21} \mathrm{~N}_{2}+\ldots \ldots+\mathrm{a}_{\mathrm{n} 1} \mathrm{~N}_{\mathrm{n}}+\mathrm{b}_{11} \mathrm{~S}_{1}+\mathrm{b}_{21} \mathrm{~S}_{2}+\ldots\right.$
$\left.\ldots+b_{s 1} \mathrm{~S}_{s}\right)+\mathrm{q}_{2}\left(\mathrm{a}_{12} \mathrm{~N}_{1}^{\prime \prime}+\mathrm{a}_{21} \mathrm{~N}_{2}+\ldots \ldots+\mathrm{a}_{12} \mathrm{~N}_{\mathrm{n}}+\mathrm{b}_{12} \mathrm{~S}_{1}+\mathrm{b}_{22} \mathrm{~S}_{2}+\right.$
$\left.\ldots \ldots+b_{s 2} \mathrm{~S}_{\mathrm{g}}\right)+\ldots \ldots+\mathrm{q}_{\mathrm{e}}\left(\mathrm{a}_{\mathrm{e}} \mathrm{N}_{\mathrm{n}}{ }^{\prime \prime}+\ldots \ldots+\mathrm{a}_{10} \mathrm{~N}_{\mathrm{n}}+\mathrm{b}_{1 \mathrm{e}} \mathrm{S}_{1}+\mathrm{b}_{\mathrm{ec}} \mathrm{S}_{2}\right.$
$\left.\left.+\ldots \ldots+\mathrm{b}_{\mathrm{se}} \mathrm{S}_{\mathrm{s}}\right)\right\}\left(1+\mathrm{p}^{\prime}\right)=\mathrm{N}_{1}^{\prime \prime}+\mathrm{N}_{2} \mathrm{p}_{2}+\ldots \ldots+\mathrm{N}_{\mathrm{n}} \mathrm{p}_{\mathrm{n}}+\mathrm{S}_{\mathrm{i}} \mathrm{k}_{1}+$
$\mathrm{S}_{\mathrm{s}} \mathrm{k}_{2}+\ldots \ldots \mathrm{S}_{\mathrm{s}} \mathrm{k}_{\mathrm{g}}$
Let us call this $e$ equation.
Next, if we multiply all of $e+s$ equations contained in ninth and the tenth sets of equations by $\mathrm{q}_{\mathrm{l}}, \mathrm{q}_{3}, \ldots \ldots \ldots . \mathrm{q}_{\mathrm{e}}, \mathrm{k}_{\mathrm{l}}$, $\mathrm{k}_{2}, \ldots \ldots \ldots \mathrm{k}_{\mathrm{s}}$ and then form one equation by adding together separately the left and the right terms, we obtain:

$$
\begin{aligned}
& \text { (f) } \quad \mathrm{E}_{1} \mathrm{q}_{1}+\mathrm{E}_{2} \mathrm{q}_{2}+\ldots \ldots+\mathrm{E}_{\mathrm{e}} \mathrm{q}_{\mathrm{e}}+\mathrm{S}_{1} \mathrm{k}_{1}+\mathrm{S}_{2} \mathrm{k}_{2}+\ldots \ldots+\mathrm{S}_{\mathrm{s}} \mathrm{k}_{\mathrm{s}}=\mathrm{q}_{\mathrm{r}}\left(\mathrm{a}_{11} \mathrm{~N}_{1}^{\prime \prime}\right. \\
& \left.+\mathrm{a}_{21} \mathrm{~N}_{2}+\ldots \ldots+\mathrm{a}_{n 1} \mathrm{~N}_{n}+\mathrm{b}_{11} \mathrm{~S}_{1}+\mathrm{b}_{21} \mathrm{~S}_{2}+\ldots \ldots+\mathrm{b}_{31} \mathrm{~S}_{3}\right)+\mathrm{q}_{2}\left(\mathrm{a}_{12} \mathrm{~N}_{2}^{\prime \prime}\right. \\
& \left.+\mathrm{a}_{22} \mathrm{~N}_{2}+\ldots \ldots+\mathrm{a}_{n 2} \mathrm{~N}_{n}+\mathrm{b}_{12} \mathrm{~S}_{1}+\mathrm{b}_{22} \mathrm{~S}_{2}+\ldots \ldots+\mathrm{b}_{22} \mathrm{~S}_{8}\right)+\ldots \ldots+
\end{aligned}
$$

$$
\begin{aligned}
& +\mathrm{k}_{1}\left(\alpha_{11} \mathrm{~N}_{1}^{\prime \prime}+\alpha_{21} \mathrm{~N}_{2}+\ldots \ldots+\alpha_{n 1} \mathrm{~N}_{\mathrm{n}}+\beta_{11} \mathrm{~S}_{1}+\beta_{22} \mathrm{~S}_{2}+\ldots \ldots+\beta_{\mathrm{s}} \mathrm{~S}_{\mathrm{s}}\right) \\
& +\mathrm{k}_{2}\left(\alpha_{12} \mathrm{~N}_{\mathrm{t}}^{\prime \prime}+\alpha_{29} \mathrm{~N}_{2 n}+\ldots \ldots+u_{\mathrm{n} 2} \mathrm{~N}_{\mathrm{n}}+\beta_{12} \mathrm{~S}_{1}+\beta_{29} \mathrm{~S}_{2}+\ldots \ldots+\beta_{\mathrm{s} 2} \mathrm{~S}_{8}\right) \\
& +\ldots \ldots+k_{9}\left(\omega_{18} \mathrm{~N}_{1}^{\prime \prime}+\omega_{2 \mathrm{~s}} \mathrm{~N}_{2}+\ldots \ldots+u_{\mathrm{nan}} \mathrm{~N}_{\mathrm{n}}+\beta_{18} \mathrm{~S}_{1}+\beta_{29} \mathrm{~S}_{2}+\ldots \ldots\right. \\
& +\mathrm{S}_{\mathrm{sP}} \mathrm{~S}_{\mathrm{s}} \text { ) }
\end{aligned}
$$

Let this be called $f$ equation.
The right term of $f$ equation, multiplied by $1+\mathrm{p}^{\prime}$, is equal to the left term of $e$ equation. Consequently, by taking $f$ equation into consideration, $e$ equation can be turned into:

$$
\begin{aligned}
& \left(\mathrm{E}_{1} \mathrm{q}_{1}+\mathrm{E}_{2} \mathrm{q}_{2}+\ldots \ldots+\mathrm{E}_{e} \mathrm{q}_{\mathrm{e}}+\mathrm{S}_{1} \mathrm{k}_{1}+\mathrm{S}_{2} \mathrm{k}_{2}+\ldots \ldots+\mathrm{S}_{\mathrm{s}} \mathrm{k}_{\mathrm{s}}\right)\left(1+\mathrm{p}^{\prime}\right) \\
& =\mathrm{N}_{\mathrm{s}^{\prime \prime}}+\mathrm{N}_{\mathrm{o}} \mathrm{p}_{2}+\ldots \ldots .+\mathrm{N}_{n} \mathrm{p}_{\mathrm{n}}+\mathrm{S}_{1} \mathrm{k}_{1}+\mathrm{S}_{2} \mathrm{k}_{2}+\ldots \ldots .+\mathrm{S}_{\mathrm{s}} \mathrm{k}_{\mathrm{s}}
\end{aligned}
$$

This may be converted into:
(g) $\mathrm{E}_{1} \mathrm{q}_{1}+\mathrm{E}_{2} \mathrm{q}_{2}+\ldots \ldots+\mathrm{E}_{\mathrm{e}} \mathrm{q}_{\mathrm{e}}+\left(\mathrm{E}_{1} \mathrm{q}_{1}+\mathrm{E}_{2} \mathrm{q}_{2}+\ldots \ldots+\mathrm{E}_{\mathrm{e}} \mathrm{q}_{\mathrm{e}}+\mathrm{S}_{1} \mathrm{k}_{1}+\right.$ $\left.\mathrm{S}_{2} \mathrm{k}_{2}+\ldots . .+\mathrm{S}_{\mathrm{s}} \mathrm{k}_{\mathrm{s}}\right) \mathrm{p}^{\prime}=\mathrm{N}_{\mathrm{l}}{ }^{\prime \prime}+\mathrm{N}_{\mathrm{o}} \mathrm{p}_{\mathrm{g}}+\ldots \ldots+\mathrm{N}_{\mathrm{n}} \mathrm{p}_{\mathrm{n}}$
This we call $g$ equation.
$\mathrm{N}_{1}{ }^{\prime \prime}$ in $g$ equation is shown in the eighth equation. Therefore, $g$ equation becomes:
(h) $\quad \mathrm{E}_{1} \mathrm{q}_{1}+\mathrm{E}_{2} \mathrm{q}_{\mathrm{e}}+\ldots \ldots+\mathrm{E}_{\mathrm{e}} \mathrm{q}_{\mathrm{e}}+\left(\mathrm{E}_{1} \mathrm{q}_{1}+\mathrm{E}_{2} \mathrm{q}_{2}+\ldots \ldots+\mathrm{E}_{\mathrm{e}} \mathrm{q}_{\mathrm{e}}+\mathrm{S}_{1} \mathrm{k}_{\mathrm{t}}+\right.$
$\left.\mathrm{S}_{\mathrm{s}} \mathrm{q}_{2}+\ldots \ldots \mathrm{S}_{\mathrm{s}} \mathrm{k}_{9}\right) \mathrm{p}^{\prime}+\mathrm{G}_{2}+\mathrm{G}_{2}+\ldots \ldots+\mathrm{G}_{\mathrm{m}}=\mathrm{N}_{1}+\mathrm{N}_{2} \mathrm{p}_{2}+\ldots \ldots+$
$\mathrm{N}_{\mathrm{n}} \mathrm{p}_{\mathrm{n}}$
Let this be called $h$ equation.
As the right term of $h$ equation is equal to the left term of $d$ equation, $d$ equation may be turned into:

$$
\begin{aligned}
& \mathrm{E}_{1} \mathrm{q}_{1}+\mathrm{E}_{2} \mathrm{q}_{2}+\ldots \ldots+\mathrm{E}_{\mathrm{e}} \mathrm{q}_{\mathrm{e}}+\left(\mathrm{E}_{1} \mathrm{q}_{1}+\mathrm{E}_{2} \mathrm{q}_{2}+\ldots \ldots+\mathrm{E}_{\mathrm{e}} \mathrm{q}_{\mathrm{o}}+\mathrm{S}_{1} \mathrm{k}_{1}+\right. \\
& \mathrm{S}_{2} \mathrm{k}_{2}+\ldots \ldots+\mathrm{S}_{\mathrm{s}} \mathrm{k}_{3} \mathrm{p}^{\prime}+\mathrm{G}_{1}+\mathrm{G}_{\mathrm{v}}+\ldots \ldots .+\mathrm{G}_{\mathrm{m}}=\mathrm{p}^{\prime}\left(\mathrm{K}_{1}+\mathrm{K}_{2}+\ldots \ldots \ldots .\right. \\
& \left.+\mathrm{K}_{\mathrm{m}}\right)+\mathrm{G}_{1}+\mathrm{G}_{2}+\ldots \ldots .+\mathrm{G}_{\mathrm{m}}+\mathrm{E}_{1} \mathrm{q}_{1}+\mathrm{E}_{2} \mathrm{q}_{2}+\ldots \ldots .+\mathrm{E}_{\mathrm{e}} \mathrm{q}_{\mathrm{e}}+\ldots
\end{aligned}
$$

This may ultimately be converted into:

$$
\begin{aligned}
& \mathrm{E}_{1} \mathrm{q}_{1}+\mathrm{E}_{2} \mathrm{q}_{2}+\ldots \ldots+\mathrm{E}_{\mathrm{e}} \mathrm{q}_{\mathrm{e}}+\mathrm{S}_{\mathrm{l}} \mathrm{k}_{1}+\mathrm{S}_{2} \mathrm{~K}_{2}+\ldots \ldots+\mathrm{S}_{\mathrm{s}} \mathrm{k}_{\mathrm{s}}=\mathrm{K}_{1}+\mathrm{K}_{2} \\
& +\ldots \ldots .+\mathrm{K}_{\mathrm{m}}
\end{aligned}
$$

This equation is nothing more or less than the twelfth equation.

Thus, it will be seen that one of the equations contained in the second, fourth, and fifth to twelfth sets of equations can be deduced from the other equations. Accordingly, where it appears at first sight that the number of the equa-
tions is one more than that of the unknown functions, the truth is that one of the equations contained is invalid. In reality, the number of the equations and that of the unknown functions are in accord. So long, therefore, as it assumed that the goods can be differentiated and the subject value functions are continuous, a system of equilibrium can be formed. The prices of all commodities and the rate of profit are determined where this equilibrinm is attained.

## 2. POINTS AT ISSUE.

The general equilibrium theory stands on a number of premises which I have laid down in the previous chapter. In most cases, critics of this theory contend that such premises are not permissible. In my opinion, however, they are quite permissible in the process of theoretical study. Certainly the general equilibrium theory so far enunciated is not sufficiently perfect. For instance, such method of representation as has been employed in the preceding chapter is based, in fact, on the atomic view of society, and is naturally open to question on that account. This point is, however, amenable to easy correction. If I accept the somewhat defective representation as it stands, it is because correction tends to and to the complexity of the whole problem, and also because such correction has no direct relation to what this article is concerned with.

As will be clear from what I have already explained, the organisation of capitalistic production, as shown in the general equilibrium theory, is very complex. Although expressed in figures, these figures baffle calculation, in actual practice. If calculation is impossible, it must be hopeless to analyse the organisation of capitalistic production by means of these figures. If they are of no practical help in analysing the organisation of capitalistic production, they can be of no use in the way of grasping the laws of development as seen in their necessary relationship to the laws of organisation. No matter how elaborate the figures,
actual analysis follows the lines different from those sug. gested by these figures; it is of the kind to be effected without the aid of such figures. It is for this very reason that the general equilibrium theory hitherto advanced has proved too ineffective in analysing the organisation of capitalistic production itself or in the grasp of the laws of development based on it, if it is of some help in the analysis of the mental structure of individuals. This is why I maintain that the structure of the general equilibrium theory as it stands is such as to render a logical analysis of the organisation of capitalistic production and accordingly the grasp of the laws of its development impossible.

But the fact that the general equilibrium theory is so complexly constituted that mathematical calcutation is impossible in practice does not by any means imply that the theory is mistaken. The capitalistic society which we see now is far more complex. The general equilibrium theory has been evolved by idealising and simplifying some special phases of capitalistic society of this highly complex nature. The complexity of the theory is a mere illustration of the fact that the complexity of the subject under discussion has not been sufficiently and duely simplified yet. It by no means implies that the theory is mistaken. But if a complex reality is to be dealt with as it presents itself in all its complexity, it will be hopeless to grasp it theoretically. In order to grasp it theoretically, we must single out simple phases which are amenable to our power of reasoning. By starting from the analysis of these phases, we must gradually proceed to add more complex rules so far as calculation is possible until we reach that point thence we must face the reality armed with the theories so long attained. If we want to apply the power of mathematical reasoning, we must first simplify the problem so that it can be properly exercised. This process of simplification is wanting in the general equilibrium theory hitherto advanced. How, then, can such simplification as admits of the exercise of our power of mathematical calculation be possible? This is the
problem which the present article proposes to discuss.

## 3. SIMPLIFICATION OF THE GENERAL EQUILIBRIUM EQUATION SYSTEM.

The devices which I propose to apply in order to simplify the general equilibrium equation system so as to make it amenable to calculation is to assume denifite real wages for the suppliers of labour-power of all kinds, on the one hand, and a definite ratio of the demand of capitalists for all kinds of goods, on the other. On what grounds are such assumptions permissible. How can such assumptions be expected to render the calculation of the general equilibrium equations possible? How does the addition of these assumptions affect the general equilibrium equation system? In the present chapter, I propose to deal with these points.

What amount of what kinds of goods do the suppliers of labourpower of various kinds demand? This varies greatly according to individuals, time and place. But it is quite permissible in the process of theoretical study to fix it in some definite form. For orily by conducting our study on the basis of a certain fixed form first and then on the basis of a different fixed form can we obtain clues by means of which we can infer the influences changes in the demand of various kinds of labourers exert. Moreover, as the changes in the demand of various kinds of labourers are not, on a general estimate, very sudden and drastic, it seems not very irrelevant to put, for the sake of a general survey, the respective demand of various kinds of labourers in a certain definite form and infer conclusions therefrom.

Now, let it be supposed that $1_{11}, l_{12}, \ldots \ldots .$. and $1_{1_{n}}$ represent the quantities of money and $n \cdot l$ kinds of consumable goods demanded by the suppliers of labour-power of the first kind, $1_{21}, 1_{2,}, \ldots \ldots .$. and $1_{2_{n}}$ represent those demanded by the suppliers of labour-power of the second kind, and $\mathrm{I}_{\mathrm{e} 1}$, $\mathrm{I}_{\mathrm{e}}$, $\qquad$ and $I_{\text {en }}$ represent those demanded by the suppliers of labour-power of the last eth kind. As each of the prices
of labour-power of $e$ kinds, that is, each wage, ought to be equal normally to the sum total of the products of the quantities and prices of the various goods demanded by the suppliers of labour-power in question, we obtain, on such an assumption, the $I^{\prime}$ set of equations :

A study of this $I^{\prime}$ set of equations and the fifth and sixth sets of equations reveal the fact that they contain $s$ unknown quantities regarding the prices of capital goods of $s$ kinds, $e$ unknown quantities regarding the prices of labourpower of $e$ kinds, 1 unknown quantity regarding the average rate of profit, and $n-1$ unknown quantities regarding the prices of consumable goods of $n-1$ kinds, making a total of $n+e+s$ unknown quantities, and that the fifth set of equations contains $n$ equations, the sixth set of equations $s$ equations, and the $I^{\prime}$ set of equations $e$ equations, making a total of $n+e+s$ equations. That is to say, the number of equations and the number of unknown functions which they contain are the same. This fect means that the unknown functions contained in them-the prices of the various consumable goods, capital goods, and labour-power and the rate of profit-can be calculated without taking the complex demand functions into consideration. Here it is assumed that the quantity of various goods to be produced and various productive goods can be of infinite variety. In actual study, however, it will be possible to limit these to the necessary minimum. If so, it will be easy to calculate the influences exerted by the prices of the various consumable goods, of various capital goods. and of various kinds of labour-power and by the rate of profit upon one another. It appears that herein lies one of the reasons why the system of the so-called labour value theory, and, there-
fore, the Marxian theory especially, has proved very cogent in the systematic grasp of the organisation of capitalist society and the laws of its development. For instance, Marx says: "The value of labour-power, like that of all other commodities, is determined by the hours of labour necessary for the production, and, therefore, the reproduction of this particular goods........ The hours of labour necessary for the production of labour-power means, after all, the hours of labour necessary for the production of the necessaries of life for labourers........ Natural desires for nourishment, clothing, fuel and housing vary according to the climatic and other natural features of the countries in which people live. On the other hand, the scope of and the form of satisfying what is called the necessary desires are in themselves historical products, which largely depend on the degree of civilisation attained by the countries concerned. Above all and essentially, it depends on the conditions under which-and accordingly with what customs and demands of livelihood-the classes of free workers have come into being. Thus, unlike all other commodities, historical and moral factors enter into the determination of the value of labour-power. But in regard to a certain specified country and a certain specified period, the average scope of the necessaries of life is determined." (Italics are those of the writer of the present article). On this score he proceeds with his analytic work by putting the labourers' necessaries of life in a definite form.

However, the problems which can be handled by giving definite form to the requirements (necessaries of life) of labourers of all kinds are limited to those relating to the prices of various capital goods, various kinds of labourpower and various consumable goods, and the rate of profit. In order to study the movement of social capital, it is necessary to consider, furthermore, the total quantities of various capital goods, various kinds of labour-power and various consumable goods, the sum total of capital and profit, the requirements of capitalists, etc. The assumption
which I propose to provide in order to make the general equilibrium equation system amenable to practical calculation in this regard also is to fix the ratios of each capitalist's demands for various goods and the amount of his investments.

The extent of demand on the part of each capitalist for various goods vary greatly according to individuals, time and place. The ratio of his demand for various goods-as, for instance, the ratio of his purchases of coal or silk clothes to the quantity of rice bought-is by no means definite, but what I have already said in regard to the demand of labourers holds true in this case also. That is to say, by putting the ratio of the capitalist's demand in definite form first and carrying on further study by converting it into another form next can we obtain the clues by which to infer the influences exerted by changes in the capitalist's demand. Again, since, on a general view, the ratio of the capitalists' demands are not subject to very sudden and drastic changes, the inference to be drawn in this way does not seem to lead us very far from the general truth. Such being the case, it is quite allowable in the process of theoretical study that the capitalists' demands should be given some definite form.

Let us now suppose that the ratios of demands of capitalists for money and the consumable goods of the first, the second, ........ and the $n-1$ th kinds are given. If, then, the ratios of the quantities of money and all kinds of consumable goods held by the first capitalist after exchange are $\mathrm{l}_{11}{ }^{\prime}: \mathrm{I}_{12}{ }^{\prime}: \ldots \ldots . . \mathrm{l}_{1 \mathrm{I}}{ }^{\prime}$, that of the second capitalist $1_{91}^{\prime}: l_{22}^{\prime}: \ldots \ldots \ldots l_{24}^{\prime} \ldots \ldots \ldots$, and that of the $m$ th capitalist $l_{m 1}{ }^{\prime}$ : $1_{\mathrm{m} 2}^{\prime}: \mathrm{l}_{\mathrm{mn}}{ }^{\prime}$, we can get the $\Pi^{\prime}$ set of equations containing ( $\mathrm{n}-\mathrm{l}) \mathrm{m}$ equations:
(II')

$$
\left\{\begin{array}{l}
\frac{\mathrm{N}_{11}^{\prime}}{\mathrm{I}_{11}^{\prime}}=\frac{\mathrm{N}_{12}^{\prime}}{\mathrm{I}_{12}^{\prime}}=\ldots \ldots \ldots \ldots \ldots=\frac{\mathrm{N}_{1 \mathrm{n}}^{\prime}}{\mathrm{I}_{14}^{\prime}} \\
\frac{\mathrm{N}_{21}^{\prime}}{\mathrm{l}_{21}^{\prime}}=\frac{\mathrm{N}_{22}^{\prime}}{\mathrm{I}_{22}^{\prime}}=\ldots \ldots \ldots \ldots \ldots \ldots=\frac{\mathrm{N}_{21}^{\prime}}{\mathrm{I}_{2 \mathrm{n}}^{\prime}} \\
\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots
\end{array}\right.
$$

$$
\left(\frac{\mathrm{N}_{\mathrm{m} 1}^{\prime}}{1_{\mathrm{m} 1}^{\prime}}=\frac{\mathrm{N}_{\mathrm{m} 2}^{\prime}}{1_{\mathrm{m} 2}^{\prime}}=\ldots \ldots \ldots \ldots \ldots \ldots=\frac{\mathrm{N}_{\mathrm{mn}}^{\prime}}{1_{\mathrm{mu}}^{\prime}}\right.
$$

Let us further assume that the amounts of investments made by each capitalist, $\mathrm{K}_{1}, \mathrm{~K}_{2}, \ldots \ldots . . \mathrm{K}_{\mathrm{m}}$, are given. ${ }^{5)}$ This enables us to obtain $m$ equations showing the values of $\mathrm{K}_{\mathrm{l}}$, $\mathrm{K}_{2}, \ldots \ldots \ldots . \mathrm{K}_{\mathrm{m}}$. Let these equations be called the III' set of equations. A study of the $\Pi^{\prime}$ and the $\Pi^{\prime}{ }^{\prime}$ sets of equations, together with the second set of equations, reveals the fact that while they contain $m n$ unknown quontities regarding the amount of money held by each capitalist after exchange, m unknown quantities regarding the amount of investments made by each capitalist, a total of ( $\mathrm{mn}+\mathrm{m}$ ) unknown quantities (because the prices of all consumable goods and the rate of profit are already known by the fifth, the sixth and the $I^{\prime}$ sets of equations, and the amounts of money held before exchange are assumed to be known), the second set of equations contains $m$ equations, and the $\mathrm{II}^{\prime}$ set of equations $(\mathrm{n}-l)_{\mathrm{m}}$ equations, a total of $(\mathrm{mn}+\mathrm{m})$ equations. That is to say, they contain the same number of equations and unknown functions. Consequently, the unknown quantities contained in them-the quantity of each goods held by each capitalist after exchange-can be found by means of these equations without any outside help. It may appear too unreasonable to put the demand ratio of each capitalist in definite form, and, indeed, this would be hardly permissible in the study of the question of individual capitalists. But it is permissible when studying the case of the whole body of capitalists or their groups. As our present object is actually to find the way to apply the general equilibrium theory to the study of the question of capitalists as a whole or in groups, it is permissible here. Moreover, in the actual

[^2]study of the whole body of capitalists or their groups, the number of capitalists need not be made so large as we make it here. As it can be limited to the necessary minimum, calculation will become all the easier.

Inasmuch as the quantity of each of the necessaries of life for the suppliers of labourpower of each kind, or, in other words, the quantity of each kind of goods to be consumed to maintain one unit of each kind of labour-power, viz. the quantity of each of the goods constituting real wage of each labour-power is assumed to be determined the quantity of the goods which the whole of the suppliers of the labour-power concerned demand ought to be equal' to the product of the total quantity of the goods forming real wages multiplied by the sum total of social demand for such labour-power. Thus, we see that the quantity of the consumable goods of the first kind demanded by all the suppliers of labour-power of the first kind is $1_{12} \mathrm{E}_{1}$, that demanded by all the suppliers of labour-power of the second kind is $1_{92} \mathrm{E}_{2}, \ldots \ldots . .$. and that demanded by all the suppliers of labour-power of the $e$ th kind is $\mathrm{I}_{\mathrm{e} 2} \mathrm{E}_{\mathrm{e}}$. The aggregate of these represents the total quantity of the consumable goods of the first kind in demand by all labourers. In the same way, $\left(l_{11} E_{1}+l_{0_{2}} E_{2}+\ldots \ldots \ldots+l_{e_{e}} E_{e}\right)$ represents the amount of the money held by all labourers after exchange, $\left(1_{15} \mathrm{E}_{1}+\mathrm{l}_{23} \mathrm{E}_{2}\right.$ $+\ldots \ldots \ldots+\mathrm{l}_{e 3} \mathrm{E}_{\mathrm{e}}$ represents the quantity of the consumable goods of the second kind held by all labourers after exchange, $\ldots \ldots \ldots$ and ( $1_{1 n} \mathrm{E}_{1}+\mathrm{l}_{2 n} \mathrm{E}_{\mathrm{e}}+\ldots \ldots . .+\mathrm{l}_{\mathrm{en}} \mathrm{E}_{\mathrm{e}}$ ) represents the quantity of consumable goods of the $n-1$ th kind held by all labourers after exchange. Accordingly, the seventh set of equations can be converted into:
(IV')

We shall call these equations the $\mathbb{I}{ }^{\prime}$ set of equations.

We notice that the fourth, tile eighth and the ninth and the tenth sets of equations contain (because the quantity of each of the goods held by capitalists after exchange is already known by the II' $^{\prime}$, the III', and the second sets of equations, and the productiv coefficients, the quantity of all goods constituting real wages, and the amounts of money held before exchange are assumed to be known) $n$ unknown quantities in regard to the total quantity of each of the money and the consumable goods of the $n-1$ th kinds held after exchange, $s$ unknown quantities in regard to the total quantity of each of the capital goods of the $s$ kinds, $e$ unknown quantities regarding the total quantity of each of the labour-power of $e$ kinds, and 1 unknown quantity regarding the amount of money to be produced, making a total of ( $e+n+s+1$ ) unknown quantities, while the $\mathrm{IV}^{\prime}$ set of equations contains $n$ equations, the eighth contain 1 equation, ninth set of equations contains $e$ equations, and the tenth set of equations contains $s$ equations, making a total of ( $e+n+s+1$ ) equations. This means that all of the unknown quantities contained in them-the total amount of money and of each consumable goods, held after exchange the total amount of each capital goods and money, produced, and the total amount of each labour-power, can be found.

Inasmuch, however, as attention has so far been directed exclusively to the consideration of the analysis oi the organisation of capitalistic production, individual sersonal conditions have been left out of consideration except under unavoidsble circumstances (as, for example, the Exing of the demand ratios of individual capitalists). Conserguently, the amounts of labour-power of various kinds surplied by individual labourers, which are handled in the general equilibrium theory, have been ignored. That is to say, the amount of each kind of labour-power supplied by mividual labourers is left out of account. This is an sevitable outcome of the assumption that real wages are g:- It is for this reason that in the system of the labre value theory, this phase of the question is not fully $d=t$ with.

Nor is this improper because there are many other more important problems claiming attention in regard to the organisation of capitalistic production. But, seeing that the general equilibrium theory has been evolved with regard also for individual personal conditions, let me further assume, for the convenience of relating the theories above developed to the general equilibrium theory, (1) that each labourer supplies one kind of labour-power only and (2) that the total social demand for each kind of labourr-power is equally distributed among the labourers who supply the same labour-power-the number of the suppliers of labourpower of the first, the second, ......... and the last eth kinds being assumed to be $r_{1}, r_{2}, \ldots \ldots \ldots$ and $r_{\mathrm{e}}$ respectively. The first assumption is quite proper, as it is, on a general view, justified by actuality, but the second assumption is entirefy arbitrary. This assumption may, nevertheless, be allowed, as it is provided, not because it has any direct relation to the subject of the present article, but simply because it is deemed convenient for observing the relation in which the general equilibrium theory so rewritten as to facilitate the discussion of the subject in the present article stands to the one previously described. Owing to the firstmentioned assumption, we obtain the $\mathrm{V}^{\prime}$ set of equations containing $(\mathrm{e}-\mathrm{l}) \theta$ equations:
( $\mathrm{V}^{\prime}$ )

and the $\mathrm{X}^{\prime}$ set of equations containing $\mathrm{n} \theta$ equations:

$$
\begin{aligned}
& \text { (VI') }
\end{aligned}
$$

The second assumption enables us to have the VIII' set of equations containing $\theta$ equations :

$$
\begin{gathered}
\left(\mathrm{VII}^{\prime}\right) \mathrm{E}_{11}=\mathrm{E}_{21}=\ldots \ldots=\frac{\mathrm{E}_{1}}{\mathrm{r}_{1}}, \mathrm{E}_{\mathrm{r} 1+1,2}=\mathrm{E}_{\mathrm{r}+2,2,2}=\ldots \ldots=\frac{\mathrm{E}_{2}}{\mathrm{r}_{2}}, \ldots \ldots, \\
\ldots \ldots=\mathrm{E}_{\theta-1, \mathrm{a}}=\mathrm{E}_{\theta \mathrm{e}}=\frac{\mathrm{E}_{9}}{\mathrm{r}_{\mathrm{e}}}
\end{gathered}
$$

These three sets of equations contain $\theta e$ unknown quantities regarding the amount of labour-power of the various kinds supplied by $\theta$ labourers, $\theta$ n unknown quantities regarding the amount of money and consumable goods of $n-1$ kinds held by $\theta$ labourers after exchange, making a total of $(\mathrm{e}+\mathrm{n}) \theta$ unknown quantities. On the other hand, the $\nabla^{\prime}$ set of equations contains $(e-1) \theta$ equations, the $\bar{W}^{\prime}$ set of equations $\mathrm{n}^{\prime}$ equations, and the $\mathrm{VIII}^{\prime}$ set of equations $\theta$ equations, making a total of $(e+n) \theta$ equations. Thus we can calculate the amount of labour-power offered by each labourer and the quantities of goods demanded by him.

I have now formed the general equilibrium system by resorting to the second, fifth, sixth, eighth, ninth, tenth, and $I^{\prime}, I^{\prime}, I^{\prime}, \mathbb{I V}^{\prime} \nabla^{\prime} \mathrm{VI}^{\prime}$ and $\mathrm{III}^{\prime}$, sets of equations in such a
way as is amenable to calculation. In this system, the.first, third, fourth, seventh, eleventh and the twelveth sets of equations, which appear in the general equilibrium system already described, are left out, while the $I^{\prime}, \Pi^{\prime}, \Pi^{\prime}, \mathbb{V}^{\prime}$, $\nabla^{\prime} \cdot \bar{Z}^{\prime}$, and $V_{I \prime}^{\prime}$ sets of equations are newly added, though there is no change in the number of unknown quantities. In order to make clear the bearings of the general equilibrium equation system I have now evolved on the one previously described, I must now proceed to explain the connection between the sets of equations left out and those newly added. Let me examine the sets of equations in general, in that order as they are cast off. The fixing of the demand ratios of capitalists (by which the $\Pi^{\prime}$ set of equations was formed) and the fixing of the amount of capital invested by each capitalist (by which the III' set of equations was formed) mean, in effect, the putting in definite form of the demand and supply of capitalists, which are indicated formally in the first set of equations, and consequently the first set of equations becomes invalid. By this process of fixing the above-mentioned factors, $(n-1) \mathrm{m}$ and $m$ equations, a total of $m n$ equations, were added by the $\Pi^{\prime}$ and the $I I l^{\prime}$ sets of equations, while, on the other hand, the first set of equations containing $m n$ equations is invalidatad. Next, the assumption of definite real wages (by which the $I^{\prime}$ set of equations was formed) and the assumption that each labourer can offer labour-power of some one kind only (which enabled the $V^{\prime}$ set of equations to be formed and which, combined with the previous assumptions, brought also the $\mathrm{II}^{\prime}$ set of equations into existence) are to put in definite form the quantity of the various goods demanded and supplied by labourers as indicated in the third set of equations, and consequently the third set of equations is rendered null and void. The fixing of these factors also means the fixing in a definite direction of the equilibrium of the revenue and expenditure of the labourers, which is shown only formally in the fourth set of equations. Therefore, it invalidates not only the third set of equations
but the fourth set of equations also. This process of fixing them resulted in the addition of $e,(e-1) \theta$, and $n^{\prime \prime}$ equations respectively, $(\mathrm{e}+\mathrm{n}-1) \theta+e$ equations in all, in the $I^{\prime}$, the $\nabla^{\prime}$, and the $\mathrm{VI}^{\prime}$ sets of equations, while, on the other hand, the third and the fourth sets of equations which were nullified contained $(\mathrm{n}+\mathrm{e}-1) \theta$ and $\theta$ equations respectively, $(\mathrm{n}+\mathrm{e}) \theta$ equations in all. Thus, it will be seen that in this case the number of the equations newly added is $\theta$-e less. This disparity is accounted for by the circumstance arising from the assumption that the total social demand for labour-power of any one kind is equally distributed among the suppliers of labour-power of that kind concerned (by which the $\mathrm{ZII}^{\prime}$ set of equations was formed). This assumption gives definite form to the substance of the eleventh set of equations, in which the relation between the total amount of social supply of labour-power of all kinds and the amount of supply by individual labourers is. formally indicated, and for this reason the eleventh set of equations is rendered invalid. But whereas this eleventh set of equations containe $e$ equations, the $\mathrm{VII}^{\prime}$ set of equations which supplants it contains $\theta$ equations. This is to say, the newly added set of equations has $\theta$ ee more equations than the old one. The deletion of the seventh set of equations and the addition of the new $\mathrm{IV}^{\prime}$ set of equations mean nothing more or less than the rewriting of the seventh set of equations into the $\mathrm{N}^{\prime}$ set of equations. This conversion was rendered possible by the assumption of definite real wages. I have now explained all the sets of equations newly added and all but one set of equations newly eliminated. The remaining one equation which was eliminated is the twelfth equation. This equation was, however, as I have previonsly explained, was invalid from the very beginning. I think I have now made clear the connection between the simplified general equilibrium system and the one previously described.

## CONCLUSION.

In the previous chapters, I have sought the reason for the impotence of the general equilibrium theory for the grasp of the organisation of capitalistic society and the system of the laws of its development in the fact that too complex and inadaquate rules were adopted in it from the beginning so that practical calculation has been rendered impossible, and stated my views as to how it can be made calculable in practice. How, then, can the general equifbrium theory which has thus been made calculable contribute to the theoretical economic study ? I propose to deal with this phase of the question in another article at some future. date.

Kei Shibata r


[^0]:    1) By the term "possessed......... after exchange" it is not necessarily meant that one possesses things in substantial form. Things may as well have been consumed already. They can be zero too. This definition applies to all cases hereunder mentioned.
[^1]:    2) Some of these may denote zero.
    3) To be exact, the quantity of all goods so combined as to procure the highest whole value differentiated unilaterally in regard to money.
    4) $K\left(l+\mathrm{P}^{\prime}\right)$ is what is purchased by the investment of the K capital. $w\left\{K\left(I+p^{\prime}\right)\right\}$ denotes the function of the present value of the future goods $K\left(I+p^{\prime}\right)$. Now, a marginal value of a commodity, divided by its price, is
[^2]:    5) To fix the total amount of investments by capitalists, as is done in the present article, is to fix the attitude of all capitalists towards investment, if viewed from the standpoint of the general equilibrium theory, and is, therefore, most natural. In actual practice, however, the matter deserves further study. Here, it has been so fixed for convenience' sake, deferring further study to a future occasion.
