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SAVING AS THE COSTLESS PROCESS (A CRITICISM OF THE WAITING THEORY)

I

My theory of interest regards, on the one hand, the operation of social powers as the most fundamental cause of the continuance of surplus and accordingly of the demand for capital. That is to say, it takes the line that where social powers operate to stabilise wages so that, the imputation of the value of products to productive goods, being necessarily imperfect, the prices of productive goods do not entirely absorb the prices of the products, surplus continues and interest is brought into being. In other words, it seeks to explain the difference between these two prices, creases, so to speak, on the ground of social powers. On the other hand, it also seeks to explain the formation of capital and accordingly its supply on the ground of the operation of social powers. If this explanation is permissible, I think all the fundamentals of interest phenomena can be elucidated by the operation of social powers despite whether viewed from the side of the demand of capital or from the side of its supply. The present article proposes to deal with the latter point.

My thesis according to which the social powers constitute the central basis on which capital is formed, implies many things, as the more important of which the following may be mentioned. According to the abstinence theory, the formation of capital is invariably accompanied by pain or sacrifice of some form or other. It contends that, be it regarded as abstinence or waiting, postponement of the present enjoyment means pain or at least something un-

welcome, and that interest is the reward paid for this unwelcome thing. According to the theory which I propose to set forth here, however, the formation of capital is effected spontaneously, so to speak, and with no pain involved; it is the outcome of the pursuit of maximum satisfaction. The formation of capital embodies nothing more or less than one phase of the operation of the principle of maximum satisfaction. In the consumption of present goods, the marginal utilities of these are equalised between them in so fas as the principle of maximum satisfaction operates. To put certain goods from one use to another in order to bring about such a result does not involve any pain, though it may disappoint the desire which is satisfied in the former use. Similarly, the formation of capital implies neither pain nor "anything unwelcome," it being the process of transferring some parts of income into the future use from the present use, in which they would bring less utilities than in the former.

II

The starting point of my argument is the principle of maximum satisfaction which rules on the side of the subjects of the formation of capital, that is to say, those who save their incomes. This principle, on the one hand, operates, as already stated, as the laws of equi-marginal utilities in the choice of the uses to which incomes consumed at the present time are to be put. Again, it must, on the other hand, inevitably operate in regard to the distribution, in terms of time, of the uses to which incomes are put. Then, the uses of incomes will be regulated for adequate proper distribution, in terms of time, so that the satisfaction of the maximum of desire can be derived from the total income each part of which is used at different period. So long as all other circumstances are the same, this will serve to equalise the marginal utility of the incomes which are employed at different periods. Of course, this point contains many premises. Let me explain these premises, to start with.

1. The principle of maximum satisfaction itself presupposes the subject which operates fully rationally. Only when the subject operates rationally can the equalisation of the marginal utility be looked for in the distribution, in terms of time, of the uses to which incomes are put.

2. That future goods are free from either under-valuation or over-valuation forms another premise. As to cases where this circumstance — the circumstance of the depreciation of future goods especially — is taked into account, I shall consider later on.*

3. The rate of interest, that is to say, the percentage of the automatic future increase of the incomes to be saved, is assumed to be nil. The study of the subject with interest thrown in is a matter which does not claim immediate attention.

4. In order to simplify the circumstances for study, let me confine attention to the present (the current period of revenue, say, this year) and the next period (next period of revenue, say, next year). It is assumed that the capitalistic system as at present rules and that a free choice can be made of the uses to which incomes in the family budget are put.

What will happen, if, in such circumstances, the marginal utility of present income and that of future income are not equal? In regard to the income to be consumed at the present time, it will be so contrived as to make the utilities of the marginal units of income equal in each of the uses to which they are put. That is, the income can be transferred from one object of use to another. There is no

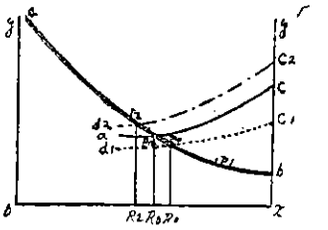
* This premise of the absence of under- and over-valuation about future utilities is of simple methodological nature. I do not deny that there are at least three types of attitude in the valuation of future utilities: that is, the attitude to value future utilities higher or lower than, or equal with, present utilities. Which of these three is most predominant in actual life will be the problem that is important in the theory of interest. Cf. Hayek, Zur Problemstellung der Zinstheorie, Zeitschrift für Sozialwissenschaft u. Sozialpolitik, 1927, Bd. 58, S. 531.

reason why the same thing should not also take place when the marginal utility of income is different in the two periods. If the marginal utility of income for the present period is smaller than that for the future period, part of present income will be saved (so long as the income is represented by the amount of currency, one is free to reserve it for future uses) for future uses. So long as future goods are valued neither higher nor lower, that is, if they are given the same value as present goods, the marginal utility of the present income exclusive of the portion to be saved can be made equal to the future marginal utility of the future income plus this saved portion. In other words, the income is put from one use to another so that the maximum satisfaction of desire can be secured through both periods.

When this much is made clear, it is possible to proceed to consider the matter under the two circumstances, that of the under-valuation of future goods and that of the definite rate of interest. In the case of the under-valuation of future goods, which means, in the present instance, that the satisfaction of desire to be derived from the goods to be purchased in future with the income is under-estimated, the future utility will be discounted to the extent of the rate of this under-valuation. In regard to the utility estimated at discount, the marginal utility of the present and future uses of income will be made equal. I shall here assume that the rate of under-valuation, that is, the discount rate, is uniform in respect of the utility of all units of income.

Even if the factor of interest rate may be let into the subject, there is no occasion for altering the above-mentioned view, though when the rate of interest is given, the money price of the future income at the present time becomes different from that which rules when it is not given. Whereas in the one case, it is represented by 1, in the other case, it will be 1 as divided by the denominator of capital and interest combined (which refer to rates), that is $\frac{1}{1 \text{ (expressing capital)} + \text{rate of interest}}$. The utility of the

money which is used for the purchase of certain goods can be computed easily according to the utility of the latter. In the same way, the utility of the present money can be worked out on the basis of the utility of the income for future uses, and on the basis of this utility, the equality of utility in the present and future uses of the present income can be obtained.*



Now, let ox in this diagram be the total amount of the present income, ab the utility curve (value curve) which forms where all is put to present uses; and cd the curve of future utility for the future uses of the present income. This, however, shows the everted form, and it indicates the utility posterior to the marginal utility of the future income ($x\,x_0$). The future income does not appear in this diagram, but it is $x\,x_0$, that is, from the point x_0 lying to the right of x up to x. Assuming that there is neither interest nor the depreciation of future goods, the portion to be put to present uses will be up to R_0 where the perpendicular from P_0 — the point of intersection between these utility curves — crosses ox, while R_0x, or r, only will be saved for future uses. But, if future goods are under-valued and if the rate of under-valuation is the same in any part of the income, the utility curve, when expressed at the present valuation, will be c_1d_1. That is to say, c_1d_1 embodies the utility curve at the present valuation of the utility devoted to future uses. Next, supposing that interest is non-existent and the fact of the depreciation of future goods only exists, the portion oR_1 will be put to present uses and R_1x will be saved. But if the interest rate operate here, it becomes possible to gain one unit of the future income plus interest rate with one unit of the present income. Such being the

* Ricci, Die Kurve des Geldnutzens und die Theorie des Sparens, Zeitschrift für Nationalökonomie, Band III, Heft 3, S. 327 ff.

case, if the curve at the present valuation of the future uses of the present income is to be sought, on the basis of the curve indicating the present valuation of the utility of future uses, and also on the assumption that from the present income accrues the interest on it, it will be c_2d_2. In this case, R_2x will be saved and oR_2 be put to present uses.

Let me explain this in other words. Let the present income be x and let it be assumed that of this r is saved. The utility curve of income for present uses shall be indicated by $\varphi(x)$ and that for future uses by $\varphi_0(r)$. The size of r is shown by the following equation:

$$\varphi(x-r) = \varphi_0(r) \quad \cdots\cdots\cdots (1)$$

Let the rate of under- or over-valuation of the utility in future be n, and if the future utility is to be under-valued, then n is positive value which is smaller than 1. If it be assumed that the interest rate is non-existent, the value of r will be shown by the following formula:

$$\varphi(x-r) = n \cdot \varphi_0(r) \quad \cdots\cdots\cdots (2)$$

Again, let it be assumed that the interest rate is taken into consideration and let this interest rate be indicated by i. The future income, 1, can be bought with the present income, $\frac{1}{1+i}=p$. That is, the price of the future income is p. The utility function of any good, $f(y)$, can be rewritten into the utility function of money by taking its price into consideration. If z be taken to indicate the marginal utility of money, and y to indicate the amount bought of that goods, it can be shown by the following formula:

$$z = \frac{1}{p} f\left(\frac{y}{p}\right)^*$$

If this way of thinking is applied to the present case, φ_1, namely, the marginal utility of the present income (money) intended for the purchase of the income for future uses, can be shown as follows. Let it be assumed, first of all, that

* Ricci. a. a. O., S. 308.

future goods are free from under-valuation, then,

$$\varphi_1 = \frac{1}{p}\,\varphi_0\left(\frac{r}{p}\right)$$

Next, let it be assumed that future goods are under-valued, and then,

$$\varphi_1 = \frac{1}{p}\,n\varphi_0\left(\frac{r}{p}\right)$$

If $1+i$ be substituted for $\frac{1}{p}$, the size of

$$\varphi_1 = (1+i)\,n\varphi_0(\overline{1+i}\,r)$$

can be determined by the following equation:

$$\varphi_1(x-r) = \frac{1}{p}\,n\varphi_0\left(\frac{r}{p}\right) \quad \ldots\ldots (3)$$

III

From what I have stated, it will be seen that it is merely in pursuit of maximum satisfaction that a part of present income is saved. It means one automatic process; it hardly implies any painful effort. The view has been predominant for a long time that this saving means sacrifice or pain. But to put present income to future uses entails no more sacrifice than that which is involved when, for example, one gives up the idea of buying sweets and purchases apples instead, in the choice of present uses. To give up the purchase of sweets, if considered by itself, may appear to involve sacrifice, but as it is for the sake of a larger utility derivable from apples, no sacrifice is involved as a whole.

In the same way, although the abandonment of the employment of income for present purposes may appear to imply sacrifice in that it means the postponement of enjoyment, yet in point of the sum total of the satisfaction of desire at present and in the future, there is no sacrifice. Saving, that is, the formation of capital, is effected simply because a larger measure of satisfaction is sought. From this point of view, it seems that the abstinence theory or

the waiting theory stands on doubtful premises. According to the abstinence theory, the supply of capital, in itself, means sacrifice or pain, and the interest is the reward given for this pain. This reward, it contends, serves to stimulate saving, or the supply of capital, at the cost of pain, and there must be saving up to the point where this reward and pain balance. But since saving is, by its own nature, made for the satisfaction of desire, the part assigned to interest cannot be to make up for the pain attending saving. It is true that interest serves to influence the amount of saving, but this is merely because it determines the price at which future utility can be bought with present income. It is not because it is the reward for pain, but because it affects the estimate of the utility of present income for future uses.

Saving being one phase of the distribution, in terms of time, of the uses of income, it is possible to consider what social circumstances are necessary for it to take place.

Let me assume, to begin with, that the relations of social powers are non-existent and that wealth (saving, viz. the income saved) does not mean any power. In such circumstances, there will be no saving of income in quest of powers. Then, income will be spent solely to meet the needs of living sooner or later (at present or in the future). The existence of the maniac and the miser may suggest itself to some minds, but the latter may be put out of the question as his existence is due to the fact that wealth constitutes a social power. As regards the former, his existence may well be considered, independently of the present problem, as a sort of abnormality.

I shall proceed with my study on such assumptions. Let it be assumed that the same amount of income as at present can be obtained in the future. Indeed, such is really the case with most people, though approximately. In such a case, and provided that future goods are not undervalued, the marginal utility of the present income when it is all put to present uses is equal to the marginal utility of future income as it is all put to future uses. Therefore,

there cannot be any saving of present income. The condition, $\varphi(x-r) = P_0(r)$, is fulfilled only when r is zero. It is, however, believed that future utility is generally underestimated more or less. If so, the discounted marginal utility of future income will be smaller than the marginal utility of present income, and the demand for a part of future income being put, if possible, to present uses, that is, the demand for capital, that is for the saving of others will arise. Not that the operation of interest is ignored here. But my present purpose is to make clear what makes the saving of income possible. As interest is the result of accumulation due to this saving, the consideration of interest may well be detached from the present study, for the present study is concerned with saving itself, which brings interest into being.

Even in this case, however, saving with the following objects or in the following sense is possible. Even if an income of the same amount as at present is assured for the future, there are emergencies to be provided against, such as sickness, misfortunes and unemployment. Due provision will be made against such needs or exigencies that may arise in the future. The necessity of such provident or insurance-like saving causes part of the present income to be put by, with the consequent alteration of the utility curve of income for future use. This provident saving is, however, possible only for the class of people who can afford it, and so long as the amount of income, and accordingly the amount of the cost of living, is given, such saving cannot exceed a certain limit. And this necessary limit is even of an estimable size (a few thousand yen in Japan to-day). Saving of this kind cannot, however, be regarded as the main form of saving in the present capitalistic economy either in its size or in its nature. First, as to its nature. An observation of each subject which figures as the supplier of capital to-day shows that his inclination or effort to save does not slacken when his savings are big. It is rather intensified at such a time. From this fact it may be inferred

that the fundamental motive actuating them to save money is something other than the provision against emergency needs. For, saving for such purposes ought to be rendered unnecessary when the savings attain a certain size. Next, as to its size. The large proportion of the capital in present-day society is supplied by a very wealthy class. This fact is clearly revealed since an inquiry made by help of statistical materials shows that a very large proportion of the nation's capital is held by a limited number of people. If so, the major part of the capital cannot be made up of the provident saving such as has already been explained. It, therefore, follows that such a process of saving can be left out of consideration for the moment in the present study, as it plays only a minor and subservient part in the supply of capital.

If this much can be allowed, I can pass on to the other points without fear of contradiction. I have stated that if present and future incomes are equal (that there is no change in the state of desire is assumed), saving cannot take place. But, as a matter of fact, the income of each subject changes more or less. It increases as he grows older. Nor does his desire remain unchanged. As the size of his family increases, his desire becomes stronger. Then, as regards the utility of income (that is, in regard to the ratio of income and desire), it will show little change, or it may safely be said that its marginal utility is rather smaller in the future (when he has grown older) because of his comparatively high income. This may not be the case with all individual subjects, but it may still be accepted as a general rule. If so, even if the assumption that present and future incomes are equal may be discarded and the conditions such as actually rule be substituted for it, it would seem impossible for present income to be saved. Accordingly, it is difficult to explain the supply of capital on the basis of the given conditions. Then, how can the saving of income or the formation of capital be accounted for? The clue to the solution of this point is furnished by the results

of the study hitherto made of the marginal utility of money.

IV

In my opinion, the utility curve of the present uses of income and that of its future uses (let them be called the present utility curve and the future utility curve respectively for brevity's sake) are entirely different from each other in form and in nature. If otherwise, that is, if both were of the same nature and form, it would be impossible to explain why, as already stated, saving invariably takes place and capital is necessarily formed in consequence. Then, in what respects do they differ in nature and in form?

The form which the utility curve of income and accordingly money takes may be viewed in many ways. It is held that the utility curve of money is elastic or unelastic according as the product of the marginal degree of the utility of money and the amount of money (the amount of money to be employed) progressively increases or decreases with the increase of the latter. Let the amount of money be x, the marginal degree of marginal utility be $y(x)$, and the product of these two be $R(x)$. And let the first differential quotient of $R(x)$ be $R'(x)$. According as $R'(x)>0$, $R'(x)=0$, $R'(x)<0$ (according as $R'(x)\geq 0$, $R'(x)=0$, $R'(x)\leq 0$), the utility curve of money is elastic, anelastic and unelastic.* The intensity or degree of elasticity may vary at every point on the utility curve. If so, various forms of the utility curve are conceivable, from the point of view of elasticity. Ricci gives five different forms in this connection, but here I shall take up only two of them. The first type represents a utility curve which is elastic at the beginning and unelastic later. It is, of course, conceivable that it has an anelastic portion at the point of conversion. The second type is one which is unelastic at the beginning and elastic later. In this case also, there exists an anelastic part at the

* Ricci, a. a. O., S. 313.

point of conversion. These types correspond to Ricci's third and fourth types respectively. When I say "at the beginning," I mean on the part of the curve where the amount of money is small, and by "later" I mean on the part of the curve when it has grown considerably. What is important here is the state after there has occurred the conversion in the degree of elasticity that is the form of the curve after the point at which the elasticity becomes one. On one type of the curve this portion is unelastic and on the other type it is elastic. To which side, then, does the utility curve of money in reality belong? On this point, opinion is divided. According to Ricci, it is unelastic, while Frisch holds that it is elastic.

On this point I can not tell anything definite; I must continue to observe the matter itself to attain any convincible insistence. But it seems to me now that the curve shall be unelastic so far as it refers to the utility of money for present uses only, at least when the income is above a certain quantity. Let us suppose a case. If an income to be consumed within a certain fixed period is suddenly increased five times or even ten times without affecting the given social position of the person concerned, it will not make much difference to him. For instance, if a person who has a monthly income of ¥50 and who is living according to his income, has got his monthly income, which must be consumed within that month, increased to ¥500 or ¥5,000, it will be all the same to him, for in whichever case his desire will be satisfied. In this sense, the present utility progressively decreases as the amount of money increases until it reaches zero. Such a thing would be impossible, if the present utility curve were not unelastic beyond a certain point. Even for a class of people, whose social position and whose standard of living are both high, as, for example, Tokyo business men, would find it no easy task to spend over ¥250,000 a year as their cost of living, and to do so would involve much difficulty and ingenious devices. This is another proof that, in so far as present utility is con-

cerned, the marginal utility of money becomes zero at a certain amount of money, and that accordingly its utility curve is unelastic. This conclusion may be reached in the following way also. The present utility of money means its utility in that certain goods can be bought with it. It is also obvious that it consists of the aggregation of the curves of utility which are formed when money is employed for the purchase of various kinds of goods. Now, the uses to which one individual subject can put his income, or, in other words, the kinds of goods which are bought, are limited in number. Because they have their limits and because their number is comparatively small, according to the social position and culture of each individual, the uses can be easily determined. And it is shown that the utility for each of these goods is invariably finite and consequently reaches zero as the quantity increases. So, that of money, which comprises the aggregation of the utility curves of all these goods, reaches zero also. So long as this fact remains, the present utility curve of money is bound to become unelastic when the amount of money exceeds a certain point. Ricci's contention that the utility curve of money belongs to the first type, may, I think, be accepted in so far as it is applied to present utility.

But can his point of view be accepted in regard to the utility of money itself? A regular income can be put to future uses as well as to present uses, and so part of it will be put to present uses and the remainder to future uses, in accordance with the principle of maximum satisfaction. And the utility curve of this income or the amount of money rests on the aggregation of the utilities accruing from all these uses. Now, if future utility is of the same nature as present utility, in other words, if the future utility curve is of the same form as the present utility curve so that the former is, so to speak, a replica of the latter, the utility curve of money itself may well be regarded as belonging to the first type. But is this true?

Let me now consider the nature of the utility curve of

the future uses of money. What does this utility curve represent? It does not differ from the present utility curve in nature in that it embodies the utility of the goods to be consumed in future uses. But in four respects it is different from the present utility curve. First, its form is changed by the interest rate, as has already been explained. Secondly, it embodies the utility arising from the need of providing against various exigencies as well. Thirdly, its form is changed by the over-valuation or under-valuation of future goods. Fourthly, it additionally embodies the utility based on the desire to gain social powers through accumulation. Generally speaking, although the utility of the goods to be consumed in future forms the nucleus of the curve of future utility, the utility curve formed on this basis suffers transformation twice and is qualified by two supplementary utilities newly added. Of these factors, those which can be detached must be set aside for the convenience of theorising. First, interest may be detached on the ground that it can be explained as one item which is rendered possible by saving or accumulation, as already stated. The provident utility, or the utility of provision against future exigencies, may be similarly divorced. Future utility is more liable to be under-valued than to be over-valued. If this general under-valuation is to be assumed, it lowers the curve of future utility to a certain extent. So long as future income is not very different from present income, the impossibility of saving, though it may serve to account for the demand for capital, cannot explain the characteristics of the curve of future utility which make saving inevitable. Attention must, therefore, be concentrated on the utility based on the demand of social powers.

Present income can be saved so that it may be put to future uses. By merely holding possession of it also, instead of consuming it, desire for social powers can be satisfied. Let me call utility accuring from the satisfaction of this desire power utility. The portion of income saved has power utility. The most striking feature of this utility is its

positive elasticity. The desire for certain consumable goods is very limited, and the degree of the progressive decrease of this utility due to the increase of the quantity is remarkable. Because there is a wide choice of the kinds of goods, however, the degree of the progressive decrease of the utility of the present uses of the money with which any of these goods can be purchased is small. It nevertheless becomes zero after all, as already stated. As to the desire for social powers, it cannot be satisfied even by the possession of property of several thousand million yen, which is the highest property of individuals in the history of humanity. This desire is, so to speak, insatiable, and there is an endless pursuit of accumulation. As the saved portion of income is used towards the satisfaction of this desire, its elasticity is extremely large. In so far as it contains the element of this power utility, the curve of future utility is entirely different in form from the curve of present utility. The former is of an extremely elastic nature because of this power utility. Thus, the marginal utility of present income for present uses is far smaller than its marginal utility (inclusive of power utility) for future uses, provided it is assumed that there is no substantial difference between present and future incomes and that there is no change in the state of desire. This necessarily leads to saving. In short, the curve of future utility has a far stronger elasticity than the curve of present utility. The former is, in other words, very elastic. It is this difference in elasticity that makes saving certain. And it is the desire for social powers that brings about this difference in elasticity. This desire is the outcome of the relations of social powers, upon which it rests. Thus, it is solely because of these relations of social powers that saving is effected and capital is formed.

I have already explained that the curve of future utility is different in nature from the curve of present utility. The utility curve of present income is, however, a combination of these two things. In other words, it consists of the curve of present utility of that portion of present income which is

put to present uses and the curve of future utility of that portion of it which is saved for future uses. Let me consider how Ricci's contention, already referred to, stands in this respect. His contention that the utility curve of money belongs to the first type may be accepted, in so far as it concerns the curve of present utility of present income, but it cannot be accepted in regard to the utility curve of present income itself. Because the progressive decrease of power utility is very slow, it seems fair to regard the latter as belonging to the second type rather than to the first type in so far as the utility curve of the present income must be the synthesis of the curve of its present utility and that of its future utility. At least, the utility curve of money has to become unelastic when its quantity exceeds a certain point.

V

No one has yet tried to investigate the form of utility curve of money including the income which is to be saved. The knowledge of the utility curve of income only for present consumption can not give, after all, any conclusive answer to our problem, because the utility of money to be saved is now very important to know. Only by way of the investigation I shall look into the hitherto attained result about the present utility curve.

Of course I shall not here dwell on the methods, recently invented and tried by Frisch, of measuring the utility of money. Owing to the limited materials available, his study is not sufficiently extensive in scope, but it is the sole attempt that has been made to find the marginal utility of money from actual facts. It is, therefore, necessary to refer to the results of his study.

In his research of 1926, Frisch made his calculation by the materials furnished by a certain cooperative union in Paris, materials which cover the period of 1920—1922, while in his study of 1932, his calculation is based on the statis-

tical figures compiled by the Labour Statistical Bureau the United States of America regarding the cost of living in 92 cities in the period of from 1918 to 1919. Since the materials used and the methods of calculation adopted in both cases are different, it is obviously necessary, strictly speaking, to allow for many things in making a comparative study of them, and yet I do not think it is absolutely irrelevant to compare them as they are. What Frisch has worked out is the reciprocal of the elasticity of the utility of money, which he calls the flexibility of the marginal utility of money or money flexibility. Let the prices of commodities be P, and the total amount of income ρ, then $\frac{\rho}{P}$ shows the size of the real income, which is signified by r. The marginal utility of money, w, is the function of real income, r, and it is denoted by $w(r)$.

The relative change in the marginal utility $w(r)$, corresponding to a small relative change in the real income, r, is called the flexibility of the marginal utility of money. This is indicated by $\breve{w}$. If it be assumed that there is no fluctuation in the prices of commodities, what can be said of the utility of real income is also true of the utility of money, so this designation has a definite reason. Now, as money flexibility, as Frisch calls it, represents the relative change in $w(r)$ corresponding to the relative change in r, its reciprocal is the relative change in income corresponding to the relative change in the utility of money, that is, the elasticity of the utility of money. Let the former be $\breve{w}$, and the latter e. Then $\breve{w}$ is shown by the following formula:—*

$$\breve{w} = \breve{w}(r) = \frac{dw(r)}{w(r)} \Big/ \frac{dr}{r} = \frac{d \log w(r)}{d \log r}$$

In Frisch's study in 1926, the flexibility of the marginal utility of money in the sense already described is sought within the limits of comparatively low incomes. Its value ranges from 3.55 to 1.28. The larger the income, the smaller

* Ragner Frisch, New-Methods of Measuring Marginal Utility, Tübingen, 1932.

the flexibility. And in this case, its value is larger than 1. His study in 1932 was made in regard to annual incomes of over $ 1,800, and in this case, the money flexibility, as is shown in the following table, does not exceed 0.617, the lowest being 0.261. It is also observable that it decreases as the income increases.

Real income (r)	Marginal utility of money (w)	Value of money flexibility ($\breve{w}$)
2.40	10.00	.617
2.62	9.50	.559
2.90	9.03	.510
3.17	8.63	.467
3.48	8.30	.428
3.80	7.98	.396
4.16	7.72	.362
4.55	7.48	.333
5.00	7.26	.312
5.40	7.07	.294
5.91	6.89	.278
6.50	6.72	.261

In this case, the money flexibility is far smaller than 1. In this regard, Schultz says that if the study of 1932 had extended to annual incomes of under $ 1,800, there might have been found cases where money flexibility was larger than 1, or it might have been found to be 1 in the case of an income of certain size. When two instances are considered in conjunction, it seems possible to conclude that the utility curve of money has a very little elasticity while income, or the quantity of money, is small (that flexibility is larger than 1 shows that elasticity is smaller than 1 and that the curve of utility is unelastic), that elasticity becomes 1 when the quantity of money increases to a certain point, that a still further increase enhances elasticity considerably, that the utility curve of money belongs to the second type, and that elasticity is small at the beginning and grows considerably later. When we judge from the above result

at which Frisch attained, it seems to us that the utility curve of money for present uses is also elastic, contrary to the insistence which I have already made. But I think it is not improbable that the utility curve abovementioned may be unelastic beyond the certain quantity of income which is greater than that treated by Frisch and that the form of the curve may be unelastic-elastic-unelastic instead of being unelastic-elastic. However it is needless to say, the scope of investigation must be extended so as to cover much larger incomes before we can tell something definite about this point.

In any case, it does not matter for the prersent purpose, whichever form the utility curve of money may take. Important for me is only that the utility curve of income for present uses are different in form from that for future uses.

VI

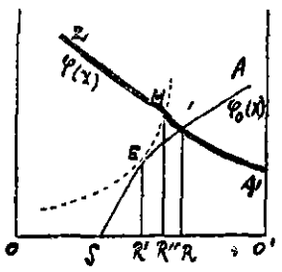
In explaining how saving is possible, I have so far abstracted interest. The reason is that the formation of interest presupposes the supply of capital, and the supply of capital presupposes saving. Such being the case, in explaining how saving is possible, interest ought to be abstracted. Let me now take this abstracted circumstance into consideration and study how the interest rate or its fluctuation affects the degree of saving.

As has so far been done, I shall leave the capital already accumulated out of consideration. To take it into consideration is a matter to be attended to later. If interest is to be paid, the under-valuation of future goods, if it take place, will tend to neutralise its effects, so that the result will be somewhat similar to what may be brought about where there is neither interest nor the under-valuation of future goods. I will, however, refrain from a detailed exposition of this point now, and will proceed on the assumption that there is no under-valuation either in future goods or in future utility.

In such circumstances, if the utility curve of money

(and accordingly, chiefly the curve of future utility) belongs to the first type, to which reference has already been made (let it be called Ricci's type — elastic at the beginning and unelastic later), the portion to be saved will increase until the rate of interest attains a certain height. If, however, the interest rate exceeds this height, it will decrease. Not so with the second type (which I may call Frisch's type — unelastic at the beginning and elastic later). In the case of this type, the higher the interest rate, the larger the portion to be saved. But this applies to incomes, the size of which exceeds a certain point. Where the elasticity of the utility of money does not exceed 1, the portion to be saved will rather become smaller because of the interest rate.

The influence of the interest-rate on saving with the Ricci's type is explained by Ricci himself as follows:—



Let AS be the curve of future utility (everted), and A'Z the curve of present utility.* The former shows $\varphi_0(x)$ and the latter $\varphi(x)$. Let E be the point on AS where the elasticity is 1. Let the curve of xy, that is a rectangular hyperbola, be drawn through E, and the point where it crosses A'Z be M. A perpendicular is then let down to the abscissa from EM and from 1, where the two utility curves cross each other, and let the points where each perpendicular crosses the abscissa be R', R'' and R respectively. If there is no interest, OR is saved, while if there is interest, the maximum saving is OR', and interest R'R''. No matter what form the curve of present utility, A'Z, may take, the maximum saving remains OR'. When the two utility curves cross at a point lower than E, that is to say, when the point of their intersection lies left of E, the size of interest when there is the maximum saving is negative. Such is Ricci's contention in this respect. It is easy to prove all

* Ricci, a. a. O., S. 330.

this, but I will abstain from doing so to save trouble.

The same observation must be made about the second type of the money utility curve. I shall consider this point a little further.

It is conceivable that the curve of future utility takes different forms as the incomes of individuals are different. Of course, it may be that with all individuals, the future utility curve is unelastic at the beginning and elastic later, but what it matters here is only its form elastic or unelastic near the point where it crosses the curve of present utility.

In the case of the class of people whose annual incomes are comparatively small, the elasticity of the future utility curve of money is, in the above sense, also small. In such cases the following observation may be valid. The curve of future utility transformed by the rate of interest $\varphi_1(r)$ referred to already is in the inner side of the original curve of future utility, $\varphi_0(r)$, so long as the arc elasticity of the original curve of future utility is smaller than 1 and the interest rate is positive. This means that saving decreases when the interest rate is positive as compared with when it is zero. Saving will increase when the interest rate is negative. Now, as to the arc in arc elasticity. Let the quantity of money corresponding to a point p optionally chosen on the curve of future utility be x, and the principal in x, which embodies the total of principal and interest, be x': $x=x'(1+i)$. Let the point on the curve of future utility corresponding to this x' be p'. The elasticity of the utility curve in this arc of pp' is what is under discussion. When we talk of low arc elasticity, it means that the elasticity in the arc pp' is low. So, it does not necessarily run parallel with low point elasticity of the utility curve. This must be quite obvious.

This may be explained factually as follows: When the interest rate is high, future goods and accordingly future utility can be bought cheaply with present money, and so the marginal portion for saving, which would be saved when there was no interest, will be employed for the purchase of

what has low marginal utility. And then, it would be found profitable to put it to present uses with comparatively high marginal utility. Thus, what may have been saved in case there was no interest is put to present uses, beginning with the marginal portion for saving, until both uses become balanced. It is conceivable that for the large majority of people, the utility elasticity of their money is smaller than 1. If this is the case, the high interest rate tends to reduce saving, instead of increasing it.

The situation is entirely different in regard to the class of people who have big incomes. The higher the rate of interest, the larger the proportion to be saved. If the elasticity of the curve of the future utility of money (with which the nature of the curve of present utility has nothing to do) is high, the E point referred to cannot exist, and consequently the point is absent which marks the limit, the rise of the interest rate beyond which has the effect of reducing the portion to be saved. The higher arc elasticity and the higher the rate of interest, the higher becomes the transformed curve of future utility, $\varphi_1(x)$, than the original curve of future utility, $\varphi_0(x)$, and their point of intersection must move so as to indicate the increase of the portion to be saved.

Thus, even if property, or accumulation in the past, may be left out of consideration, it is impossible to say that a high rate of interest increases saving. And if property is taken into consideration, the problem becomes more complex. A high rate of interest increases future income, and changes the form of the curve of future utility in the diagram given above. I shall reserve a detailed exposition of this point of view for some future occasion.

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