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CAPITAL AND THE SUBSISTENCE-FUND

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I INTRODUCTION

I once undertook to criticise the Böhm-Bawerkian theory of employment. I then endeavoured to prove that the theory contained certain special assumptions concerning the technical co-efficients of production and that the assumptions in question were responsible for the Böhm-Bawerkian conclusion that the elasticity of the demand for labour can never be smaller than I. Now my conclusions may possibly have been obscured by overlooking the fundamental difference between Capital taken in the usual sense and Subsistence-fund on the one hand and the precise relation between the Böhm-Bawerkian structure of production and the Walrasian on the other. I should therefore like here to restate my criticism in a more convincing manner by taking these points into consideration.

II THE BÖHM-BAWERKIAN THEORY

As is well known, Böhm-Bawerk developed his theory of interest-rate with the aid of the following tables thus:

"Let it be assumed that the demand for and supply of goods meet in the one market,......and that productive power is equal in all sections of production, the increase in productive power resulting from an extention of the period of production being equal also......, and that the total amount of property to be invested is 15 billion gulden, the number of laboureres being 10 million,......while the productive power increases......from 350 gulden to 700 gulden according to the periods of production to be adopted (as is shown in column

II of the following tables)." (Kapital und Kapitalzins, II. Abt. 4. Aufl., p. 444—445)

" Now, (assuming that the capital of 15 billion gulden is divided amount 1.5 million enterprises in equal proportions,) let wages be 300 gulden......(Then) the three years' production is most profitable to an enterprise .....In such circumstances, all enterprises will, of course, attempt to choose this most profitable period of production, but..... as the number of labourers employed......by an entrepreneur with a *capital* of 10 thousand is 22.22......, 33,333,333 labourers will have to be employed in order to operate the total *capital* of 15 billion gulden......As a matter of fact, however, the number of labourers available is only 10 million and 4.5 billion gulden would suffice to employ those 10 million labourers. Then the rest of the *capital* would have to remain idle. But entrepreneurs could not, of course, afford to leave their *capital* idle, nor would they care to do so. (Accordingly) they would vie with one another in an attempt to induce labourers into their respective enterprises by paying them higher wages than their rivals. This competition necessarily forces up wages. Let us, then suppose that wages ......rise to 600 gulden. (Then)......an eight years' production becomes most profitable. But, if the eight years' production is chosen......a *capital* of 10 thousand gulden enables the entrepreneur to employ 4.16 labourers only. Consequently, even with the total national *capital* of 15 billion gulden, the number of labourers employed will not exceed 6.25 million. The other 3.75 million labourers will then be condemned to unemployment......(Thus the equilibrium is attained when the wage is 500 gulden.) In this case 6.66 labourers can be employed with a *capital* of 10 thousand gulden. With the total national *capital* of 15 billion gulden, therefore, exactly 10 million labourers can be employed." (ibid. p. 450—453)

Period of Production	Annual Product per Laborer	TABLE I (Wage 300)			TABLE II (Wage 600)			TABLE III (Wage 500)		
		Annual Profit per Labor.	Number of Employees	Annual Profit per 10,000	Annual Profit per Labor.	Number of Employees	Annual Profit per 10,000	Annual Profit per Labor.	Number of Employees	Annual Profit per 10,000
1	350	50	66.66	3,333	−250	33.33	Loss	−150	40.00	Loss
2	450	150	33.33	5,000	−150	16.66	Loss	−50	20.00	Loss
3	530	230	22.22	5,111	−70	11.11	Loss	30	13.33	400
4	580	280	16.66	4,666	−20	8.33	Loss	80	10.00	800
5	620	320	13.33	4,266	20	6.66	133	120	8.00	960
6	650	350	11.11	3,888	50	5.55	277	150	6.66	1,000
7	670	370	9.52	3,522	70	4.76	333	170	5.71	970
8	685	385	8.33	3,208	85	4.16	345	185	5.00	925
9	695	395	7.40	2,925	95	3.70	351	195	4.44	866
10	700	400	6.66	2,666	100	3.33	333	200	4.00	800

III CRITICISM

According to the Böhm-Bawerkian tables, the changes in the number of labourers employed are in exact inverse ratio to the changes in wages, so long as the period of production remains the same. This means that elasticity of demand for labour is precisely 1 so long as the period of production remains the same, and therefore, that the elasticity is greater than 1 whenever the period of production is extended owing to a rise in wages, or shortened owing to a corresponding fall. This means again that the assumptions concerning the productive power function, on which depends the extent to which the period of production is lengthened owing to a rise in wages, affect only the degree in which the elasticity of demand for labour is *in excess of* 1, but can never make it smaller than 1, because even under the most extreme assumption concerning productive power function, i.e., even under the assumption of an unchanged period of production, the elasticity of demand for labour can never fall below 1.

Now this conclusion is due either to a confusion by Böhm-Bawerk of capital taken in its usual sense with subsistence-fund or to his use of the term Capital in an unusual connotation. Böhm-Bawerk develops his theory with the promised assumption of a fixed amount of *capital*. The calculation of the above tables, however, is made with the aid of the formula: $S = \frac{ALN}{2}$ (where S indicates the amount of subsistence-fund, A the number of labourers employed, L the wage, and N the period of production), with the assumption that S is a fixed amount, i.e. the calculation is made assuming a fixed amount of subsistence-fund.

Now capital taken in its usual sense (a qualification which we shall avoid repeating hereafter) can never be equal to subsistence-fund so long as the price of capital goods contains the profit of their producer; for the capital goods, whose price is paid out of capital by their purchasers, are regarded in capitalist society as composing capital by the full amount of their price. Therefore, capital exceeds subsistence-fund by the amount of profit which is contained in the price of the capital goods. (See Appendix) Therefore, the number of labourers employed by a certain fixed amount of capital must be smaller than that employed by an equal amount of subsistence-fund, and the larger the profit contained in the price of capital goods the greater this difference must be. Now the rise in wages, causing diminution in the rate of profit necessarily diminishes the amount of profit contained in the price of producers' goods. Therefore, the amount of subsistence-fund contained in a fixed amount of capital increases with the rise in wages. Thus the rate of decrease, due to rise in wages, in the number of labourers employed with a fixed amount of capital, must be smaller than the decrease in the number of labourers employed with a subsistence-fund of such fixed amount as was contained in the capital before the change in wages occurred. Therefore, if under the assumption of a fixed amount of subsistence-fund the elasticity of demand for labour is precisely 1, as is shown above, so long as the period of production remains the same, it must equally be smaller than 1 under the assumption of a fixed amount of capital, so long as the period of production remains the same. But once it is thus proved that the elasticity of demand for labour becomes smaller than 1 under the most extreme assumption concerning productive power function, that is, under the assumption of an unchanged period of production, it becomes a matter of assumptions concerning the productive power function whether elasticity of demand for labour shall appear to be larger than, equal to, or smaller than 1.

This argument can be illustrated by the following tables, which are derived from the above-quoted Böhm-Bawerkian. Column D represents the total sum of profit and subsistence-fund—a sum which is not only equal to the total price of social product including that of capital goods but, in this case, to the amount of capital also, because the supplementary investments are here assumed to take place at infinitesimal intervals. Columns A′, B′, C′ and D′ are derived by multiplying respectively the columns A, B, C and 10,000 by the ratio between the newly assumed amount of capital (=11,000) and such amounts of capital as were presupposed when the amount of subsistence-fund was assumed to be 10,000. The newly assumed amount of capital is fixed at 11,000, because the amount of capital actually presupposed at the point of equilibrium by Böhm-Bawerk through his assumption of the amount of subsistence-fund at 10,000 is actually 11,000.

From the tables offered below we confirm the following facts:— (1) The number of labourers employed unber the assumption of a certain fixed amount of capital is less than that under the assumption of an equal amount of subsistence-fund, so long as the productive method and the wage remain constant, and the production requires capital goods. (2) The rate of decrease in the number of labourers employed caused by the rise in wages under the assumption of a fixed amount of capital is less than that under the assumption of an equal amount of subsistence-fund, the changes in the number of labourers being less than exactly

Annual Wage per Labourer	Period of Production	Annual Profit per Labourer	Subsistence-Fund = 10,000: Number of Labourers Employed A	Total Annual Wage B	Total Annual Profit C	Amount of Capital D	Capital = 11,000: Number of Labourers Employed A′	Total Annual Wage B′	Total Annual Profit C′	Subsistence-Fund D′
300	1	50	66.66	20,000	3,333	13,333	55.00	16,500	2,750	8,250
	2	150	33.33	10,000	5,000	15.000	24.44	7,332	3,667	7,333
	3	230	22.22	6,666	5,111	15,111	16.02	4,806	3,720	7,280
	4	280	16.66	5,000	4,666	14,666	12.50	3,750	3,500	7,500
	5	320	13.33	4,000	4,266	14,266	10.29	3,084	3,290	7,710
	6	350	11.11	3,333	3,888	13,888	8.80	2,640	3,080	7,920
	7	370	9.52	2,857	3,522	13,522	7.52	2,324	2,865	8,135
	8	385	8.33	2,500	3,208	13,208	6.94	2,082	2,672	8,328
	9	395	7.40	2,222	2,925	12,925	6.30	1,891	2,489	8,511
	10	400	6.66	2,000	2,666	12,666	5.79	1,737	2,315	8,685
600	5	20	6.66	4,000	133	10,133	7.24	4,342	145	10,855
	6	50	5.55	3,333	277	10,277	5.94	3,567	297	10,703
	7	70	4.76	2,857	333	10,333	5.07	3,041	355	10,645
	8	85	4.16	2,500	354	10,354	4.42	2,656	376	10,624
	9	95	3.70	2,222	351	10,351	3.93	2,362	374	10,626
	10	100	3.33	2,000	333	10,333	3.54	2,129	355	10,645
500	3	30	13.33	6,666	400	10,400	14.10	7,050	423	10,577
	4	80	10.00	5,000	800	10,800	10.19	5.095	815	10,185
	5	120	8.00	4,000	960	10,960	8.03	4,015	964	10,036
	6	150	6.66	3,333	1,000	11,000	6.66	3.333	1,000	10,000
	7	170	5.71	2,857	970	10,970	5.72	2,865	974	10,026
	8	185	5.00	2,500	925	10,925	5.04	2,520	932	10,068
	9	195	4.44	2,222	866	10,866	4.50	2,250	877	10,123
	10	200	4.00	2,000	800	10,800	4.07	2,035	815	10,185

reciprocal to the changes in wages under the assumption of unchanged productive method.

But, in the above case, the elasticity of demand for labour does not become less than 1 even if we alter the assumption of a fixed amount of *subsistence-fund* to that of a fixed amount of *capital*, strictly discriminating capital from subsistence-fund. However, once it is proved that the elasticity of demand for labour becomes smaller than 1 under the most extreme assumption of unchanged productive method, it becomes a matter of assumption concerning the production function itself whether the elasticity of demand for labour shall appear to be smaller than 1 or not. Let us now illustrate this.

Let it be assumed that the productive power function is such as is indicated below:

Period of Production	5.0	5.1	5.2	5.3	5.4	5.5
Annual Product per Labourer	620.0	623.0	625.5	627.5	629.0	630.0

and that the amount of capital is 11,168.3,—an amount which is presupposed to contain 10,000 of Subsistence-fund at the point of equilibrium under the assumption that the number of labourers available is 8,170.

Let us first develop our reasoning with the assumption that the amount of subsistence-fund is 10,000, and transfer these results to the case which involves a capital outlay of 11,168.3. We shall then obtain the following table.

The point of equilibrium then is attained when the wage is 480 and the period of production is 5.1, as is proved by the first three lines of the table.

Now, if in this case, the wage is maintained at the artificially higher level of 510, the 5.2-year-production period becomes the most profitable to the entrepreneur, as is proved by the next three lines of the table. If the wage is maintained at 540 the 5.3-year-production period, and if it is maintained at 570 the 5.4-year-production period will be the most profitable to the entrepreneur, as is proved respectively by the third and the fourth groups of figures in the table. The number of labourers employed with a capital of 11,168.3,

Annual Wage per Labourer	Period of Production	Annual Profit per Labourer	Subsistence-Fund=10,000				Capital=11,168.3			
			Number of Labourers Employed	Total Annual Wage	Total Annual Profit	Amount of Capital	Number of Labourers Employed	Total Annual Wage	Total Annual Profit	Subsistence-Fund
480	5.0	140.0	8.333	4,000.0	1,166.7	11,166.7	8.335	4,000.5	1,166.8	10,001.0
	5.1	143.0	8.170	3,921.6	1,168.3	11,168.3	8.170	3,921.6	1,168.3	10,000.0
	5.2	145.5	8.013	3,846.1	1,165.9	11,165.9	8.015	3,847.0	1,166.1	10,002.1
510	5.1	113.0	7.689	3,921.6	868.9	10,868.9	7.901	4,029.6	892.8	10,275.5
	5.2	115.5	7.541	3,846.2	871.0	10,871.0	7.748	3,951.3	894.9	10,273.5
	5.3	117.5	7.399	3,773.6	869.4	10,869.4	7.603	3.877.4	893.3	10,275.0
540	5.2	85.5	7.123	3,846.1	609.0	10,609.0	7.498	4,048.9	641.1	10,527.2
	5.3	87.5	6.988	3,773.6	611.5	10,611.5	7.355	3,971.6	643.5	10,524.7
	5.4	89.0	6.859	3,703.7	610.4	10,610.4	7.219	3.898.4	642.5	10,525.8
570	5.3	57.5	6.620	3,773.6	380.7	10,380.7	7.123	4,059.9	409.5	10,758.7
	5.4	59.0	6.498	3,703.7	383.4	10,383.4	6.989	3,983.7	412.3	10,755.9
	5.5	60.0	6.380	3,636.4	382.8	10,382.8	6.862	3,911.5	411.7	10,756.5

which will be 8.170 at the point of equilibrium, will be reduced to 7.748, 7.355 and 6.989 in accordance with the artificial elevation of wages to 510, 540 and 570.

The total annual wage will therefore be raised from its equilibrium level, which is 3,921.6, to 3,591.3, 3,971.6 and 3,983.7 as a result of raising the wage from 480 to 510, 540 and 570.

IV TWO STRUCTURES OF PRODUCTION

When in the previous article referred to above I attributed the Böhm-Bawerkian conclusion that the elasticity of demand for labour can never be smaller than 1 to his special assumptions concerning the technical co-efficients of production, I not only overlooked the above proved difference between Capital and Subsistence-Fund but also disregarded the precise relation between the Böhm-Bawerkian structure of production and the Walrasian. Let us now proceed to study this latter point.

To begin with, it is necessary to define some of the concepts which I propose to employ, viz. *period of production, stringent period of production, time of production and average period of production.* The first is the time required for conversion of the labour first expended in producing the remotest capital goods into consumers' goods, the second is the time required for conversion of any labour expended in producing any goods into consumers' goods, while the third is the time required for conversion of the labour first expended in producing any goods into the respective categories of goods, all under the assumption that labourers are paid precisely at the time when labour is begun. What I mean by average period of production is an arithmetical average of the stringent periods of production of all the labours invested at all the stages in the production of consumers' goods. It is, therefore, exactly comparable with what Böhm-Bawerk implies by the term.

Now that the necessary definitions have been offered I shall proceed to ascertain the relations between the two structures of production.

Beginning with Böhm-Bawerk, I should like to point out that he assumes a special structure of production, that is, a unilateral integration of successive stages of production. That is to say, for instance, that if a given a quantity of labour first gives rise to capital goods, C_1, then another a quantity of labour together with this C_1 is used in making still another category and quantity of capital goods, C_2, then a further a quantity of labour with this C_2 is used in making another category of capital goods, C_3, etc. Thus, if we assume that there are N of these stages, that is, that a given a quantity of labour and capital goods C_{N-1} are used in making one unit of consumers' goods, the total amount of labour is aN. If we assume, in this case, that the time of production of each of these products is uniformly one year, the period of production will be N years, while as the labour expended in the production of C_1 is converted into consumers' goods in N years, and that expended in the production of C_2 in $N-1$ years, and that expended in the production of C_3 in $N-2$ years, etc., the total sum of the stringent periods of production of all the labours expended at all the stages in producing one unit of consumers' goods corresponds to the following progression:

$$a\ (1+2+3+4+\ldots\ldots+N),$$

therefore:

$$\frac{(1+N)Na}{2}.$$

Therefore, the average period of production will be, in this case:

$$\frac{1+N}{2},$$

and the subsistence-fund, S, will be:

$$\frac{(1+N)AL}{2},$$

because, as was stated above, the subsistence-fund must, in the state of simple reproduction, be equal to wages multiplied by both the number of labourers and the average period of production. I will refer here-after to the structure of production assumed in the above as annual supplementary investment of the type of arithmetical progression.

The structure of production which underlies the previously-quoted calculation of Böhm-Bawerk assumes that supplementary investments are made at each successive period of infinitesimal interval. Now, if in the above case it is supposed that the time of production is $1/n$ year, and that there are nN successive stages of production, and that the quantity of labour invested in each stage of production is equally a/n, the total sum of the stringent periods of production of all the labours expended at all the stages in producing one unit of consumers' goods will correspond to the following progression:

$$\frac{a}{n}(1+2+3+4+\ldots\ldots+nN),$$

therefore;

$$\frac{(1+nN)Na}{2n},$$

while the total amount of labour is still aN; therefore, the average period of production will be:

$$\frac{1+nN}{2n},$$

and the subsistence-fund, S, will be:

$$\frac{(1+nN)AL}{2n}.$$

Therefore, if the supplementary investments are made at each successive period of infinitesimal interval, that is, if n is infinite, these three formulae will resolve into the following:

$$\frac{N^2a}{2}, \quad \frac{N}{2}, \quad \frac{ALN}{2},$$

I will refer, hereafter, to the assumptions of this last case as those of momentary supplementary investment of the type of arithmetical progression. This is precisely what has been assumed by Böhm-Bawerk.

Now let us discard the assumption of Böhm-Bawerk, and assume that there are some categories of capital goods in the production of which these goods themselves are essential.

Taking the simplest case, we will first assume that there is only one kind of capital goods, C, and only one kind of consumers' goods, and that technical co-efficients of production are equal for both products, and that c (which is assumed to be smaller than I) of C and a of labour are necessary for the production of one unit of each of the products: also that the time of production is uniformly one year. Then, since the production of c of C requires ac of labour and c^2 of C, and the production of c^2 of C requires ac^2 of labour and c^3 of C, and so on *ad infinitum*, we can infer that the production of one unit of the consumers' goods requires labour of such an amount as will correspond to the following progression:

$$a(1+c+c^2+c^3+\dots\dots),$$

therefore:

$$\frac{a}{1-c},$$

while as the labour expended in the production of consumers' goods is converted into the latter in one year and that expended in the production of capital goods which are used in producing the consumers' goods in two years, and that expended in the production of the capital goods which are used in the production of the capital goods which are used in producing the consumers' goods in three years and so on *ad infinitum*, the total sum of the stringent period of production of all the labours expended at all the stages in producing one unit of consumers' goods will correspond to the following progression:

$$a(1+2c+3c^2+4c^3+\ldots\ldots),$$

therefore:

$$\frac{a}{(1-c)^2}.$$

Therefore, in this case, the average period of production will be:

$$\frac{1}{1-c},$$

and the subsistence-fund, S, will be:

$$\frac{AL}{1-c},$$

the period of production being infinite. I will refer, hereafter, to this case as that of the assumptions of annual supplementary investment of the type of geometrical progression.

The above formulation confirms the contention that the difference between the Böhm-Bawerkian structure of production, whether the supplementary investments are made annually or at infinitesimal intervals, and the Walrasian structure of production can be reduced to the difference in the form in which the supplementary investments are assumed to take place, viz. the Böhm-Bawerkian structure of production is reduced to supplementary investments of equal sums taking place only for a definite period, while the Walrasian is reduced to supplementary investments beginning with an infinitesimal magnitude from a time infinitely before and ever increasing until it attains a certain level.

Once the above analysis is confirmed, it is quite easy to arrive at a formula defining the numerical relation between such a technical co-efficient of production concerning capital goods (i.e. the quantity of capital goods necessary for producing one unit of each product) under the Walrasian structure of production and such a period of production under the Böhm-Bawerkian as would have a common magnitude of subsistence-fund, i.e.

$$\frac{1+N}{2}=\frac{1}{1-c}$$

where the left-hand number is the formula for the average period of production under the assumption of annual supplementary investments of the type of arithmetical progression and the right hand number is the formula for the average period of production under the assumption of annual supplementary investment of the type of geometrical progression ; or

$$\frac{N}{2}=\frac{1}{1-c}$$

where the left-hand number is the formula for the average period of production under the assumption of momentary supplementary investment of the type of arithmetical progression.

A formula defining such a technical co-efficient concerning labour under Walrasian structure of production which would precisely correspond to the annual product per labourer under the Böhm-Bawerkian structure of production would be :

$$\frac{1}{P}=\frac{a}{1-c}$$

where P represents annual product per labourer as assumed by Böhm-Bawerk. This follows because the right-hand number of the equation is the formula for the total amount of labour required for the production of one unit of consumers' goods under the assumption of annual supplementary investment of the type of geometrical progression; while the left-hand number of the equation also denotes the total amount of labour required for the production of one unit of consumers' goods.

In order to illustrate the use of these formulae, let us assume that the productive power function under the assumption of *annual* supplementary investment of the type of, geometrical progression is such as is indicated in column II'. Then the technical co-efficients of production arrived at by the application of the above formulae would be such as are indicated in columns III' and IV'.

Period of Production I'	Annual Product per Labourer II'	Technical co-efficients of Production applying to both goods	
		Concerning Producers' Goods III'	Concerning Labour IV'
5.0	620.0	2/3	1/1860
5.1	623.0	41/61	20/38003
5.2	625.5	21/31	20/38781
5.3	627.5	43/63	40/79065
5.4	629.0	11/16	5/10064
5.5	630.0	9/13	2/4095

Let us further assume that the number of labourers available per enterprise is 5.62698703, while either the Subsistence-fund per enterprise or the amount of capital per enterprise remains at 10,000.

We can determine from the above assumptions the tables offered below. The annual profit per labourer can be determined either by subtracting the wage from the annual product of consumers' goods per labourer or by the following formula:

$$\left\{1-\frac{a\mathrm{L}}{1-c}\right\}\div\frac{a}{1-c},$$

because the numerator corresponds to the profit per product, while the common denominator corresponds to the labour necessary for one unit of consumers' goods.

Column A can be determined either from the formula:

$$S=(1+N)\frac{1}{2}AL,$$

or from the formula:

$$S=\frac{AL}{1-c},$$

S representing 10,000.

Column B is derived by multiplying the annual profit per labourer by the number of labourers employed.

Column C is determined by dividing the total cost of unit production by a, that is, by the quantity of labour used in the production of one unit product, and then multiplying the result by the number of labourers employed. Thus we obtain the total capital per entrepreneur, because the total cost of one unit of product divided by a gives the amount of capital necessary to employ one unit of labour. Here, the total cost of one unit of product is calculated as indicated below. Denoting by P' the price of the producers' goods and by i the annual rate of interest, we have two equations concerning the relation between price and cost of producers' and consumers' goods, viz.,

(1) $(cP'+aL)(1+i)=P'$

(2) $(cP'+aL)(1+i)=1.$

From these we obtain the values of P' and i, and then the total cost of one unit of product as $(cP'+aL)$. The equation (2) presupposes that the price of the consumers' goods is 1. This assumption is of great relevance with regard to certain related problems, but it does not constitute an essential part of the problem in hand. It is introduced here only in connection with Böhm-Bawerk's example.

Columns A′, B′ and C′ may be determined in two ways: First, they can be derived by multiplying respectively the columns A, B and 10,000 by the ratio between the newly assumed amount of capital and such amounts of capital as were presupposed when the amount of subsistence-fund was assumed to be 10,000. Secondly, they may also be derived independently of columns A, B and C, thus:—

Column A' may be determined by dividing the total amount of capital per entrepreneur by the quantity of capital necessary in employing one unit of labour, which again is derived by dividing the total cost per unit of product by the quantity of labour used in producing it.

Column B′ may be derived by multiplying the total amount of capital by the rate of profit, which can be obtained from the above two equations.

Column C′ may be derived by dividing the subsistence-fund necessary in producing one unit of consumers' goods by the total quantity of labour involved in its production and by multiplying this result by the number of labourers employed.

Equilibrium is attained when the wage is 500 and the production-period is 5.1, as is proved in table I′.

Now, if in this case the wage is maintained at the artificially higher level of 517, the 5.2 year production-period becomes the most profitable to the entrepreneur, as is proved in table II′. If the wage is maintained at 534 the 5.3 year production-period, and if it is maintained at 550 the 5.4 year production-period will be the most profitable to the entrepreneur, as is proved respectively in table III′ and IV′. The number of labourers employed will be, at the point of equilibrium, 5.627, and will be reduced to 5.463, 5.310, and 5.171 respectively, that is, to 0.971, 0.944 and 0.919 times the equilibrium number respectively, as the wage is raised artificially to 517, 534, and 550. Therefore, the total income of labourers will be increased as a result of raising the wage from 500 to 517, 534 and 550, from

$$500 \times 5.627 = 2813.5$$

to respectively,

$$517 \times 5.463 = 2824.3$$
$$534 \times 5.310 = 2835.7$$
$$550 \times 5.171 = 2844.1$$

				10,000 of Subsistence-Fund			10.000 of Capital		
Table	wage	Period of Production*	Annual Profit per Labourer	Number of Labourers Employed A	Annual Profit B	Amount of Capital C	Number of Labourers Employed A′	Annual Profit B′	Amount of Subsistence-Fund D
		5.0	120.0	6.666667	800.0000	11,600.0	5.747126	689.6552	8,620.7
I′	500	5.1	123.0	6.557377	806.5574	11,653.4	**5.626987**	**692.1194**	8,581.2
		5.2	125.5	6.451613	809.6774	11,700.3	5.514047	692.0129	8,546.8
		5.1	106.0	6.341757	672.2263	11,378.1	5.573670	590.8090	8,788.4
II′	517	5.2	108.5	6.239471	676.9826	11,421.7	**5.462839**	**592.7180**	8,755.3
		5.3	110.5	6.140432	678.5177	11,458.8	5.358698	592.1361	8,726.9
		5.2	91.5	6.040836	552.7365	11,607.5	5.412573	495.2505	8,960.0
III′	534	5.3	93.5	5.944950	555.8528	11,195.1	**5.310322**	**496.5151**	8,932.5
		5.4	95.0	5.852060	555.9457	11,223.1	5.214308	495.3593	8,910.2
		5.3	77.5	5.772058	447.3347	10,961.9	5.265583	408.0827	9,122.5
IV′	550	5.4	79.0	5.681818	448.8636	10,987.5	**5.171166**	**408.5221**	9,101.3
		5.5	80.0	5.594406	447.5525	11,007.0	5.082592	406.6074	9,085.1

* "Period of Production" is in this case used not in its ordinary sense but as an index for the difference in the technical composition of capital.

In referring to the Walrasian structure of production we have so far assumed a case in which there is no difference between the technical composition of capital (viz. technical co-efficient *concerning* capital goods divided by that *concerning* labour) *of* capital goods and that *of* consumers' goods. Let us now briefly show how the formulae are changed when we discard this assumption.

Let us assume that the quantity of capital goods necessary to produce one unit of themselves is equal to d times the quantity necessary to produce one unit of consumers' goods, while the remaining assumptions remain the same as before. Then since c of C, which is required to produce one unit of consumers' goods, requires ac of labour and c^2d of C in its production, and the production of c^2d of C requires ac^2d of labour and c^3d^2 of C, and so on *ad* infinitum, we can infer that one unit of consumers' goods requires for its production the amount of labour which corresponds to the following progression:

$$a(1+c+c^2d+c^3d^2+c^4d^3+\ldots\ldots)$$

$$\therefore\ \frac{a(1+c-cd)}{1-cd},$$

while as the labour expended in the production of consumers' goods is converted into the latter in one year, and that expended in the production of capital goods which are used in producing consumers' goods in two years, and that expended in the production of capital goods which are used in producing the capital goods which are used in the production of consumers' goods in three years, and so on *ad infinitum*, the total sum of the stringent periods of production of all the labours expended at all the stages in producing one unit of consumers' goods can be denoted by the following progression;

$$a(1+2c+3c^2d+4c^3d^2+5c^4d^3+\ldots\ldots)$$

$$\therefore\ \frac{a\{(1+c-cd)(1-cd)+c\}}{(1-cd)^2}.$$

Therefore, in this case, the average period of production will be:

$$\frac{a\{(1+c-cd)(1-cd)+c\}}{(1-cd)^2} - \frac{a(1+c-cd)}{1-cd}$$

$$\therefore\ 1+\frac{c}{(1+c-cd)(1-cd)},$$

and the subsistence-fund, S, will be:

$$\left\{1+\frac{c}{(1+c-cd)(1-cd)}\right\}AL,$$

the period of production being infinite. Let us then denote this case as one assuming an *annual supplementary investment of the type of special geometrical progression.*

The formula defining the numerical relation between such technical co-efficients concerning capital goods under the Walrasian structure of production and such a period of production under the Böhm-Bawerkian as would have a common magnitude of subsistence-fund would in this case be:

$$\frac{1+N}{2}=1+\frac{c}{(1+c-cd)(1-cd)},$$

of which the left-hand number is the formula for the average period of production under the assumption of annual supplementary investment of the type of arithmetical progression, and the right-hand number is formula for the average period of production under the assumption of annual supplementary investment of the type of special geometrical progression.

The formula defining such a technical co-efficient concerning labour under the Walrasian structure of production as would precisely correspond to the annual product per labourer under the Böhm-Bawerkian structure of production would be:

$$\frac{1}{P}=\frac{a(1+c-cd)}{1-cd}$$

where P is the annual product per labourer as assumed by Böhm-Bawerk. This follows because the right-hand number of the equation is the formula for the total amount of labour required for the production of one unit of consumers' goods, under the assumption of annual supplementary investment of the type of geometrical progression, while the left-hand number of the equation also denotes the total amount of labour required for the production of one unit of consumers' goods.

Appendix

Mr. Hideo Aoyama has, on hearing my report, handed me the following paper, giving a mathematical expression to this part of my argument, which I take the liberty of introducing.

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In order to give a mathematical expression to the amount of capital C, it is convenient to use Prof. Hayek's construction of investment function. (See F. A. von Hayek, "On the Relationship between Investment and Output", Economic Journal, June 1934.) Let us suppose that $f(t)$ is the investment function and that it is defined for the interval between $t=0$ and $t=N$, where N measures the period of production. We propose, for convenience' sake, to consider the amount of capital at $t=N$. Then $f(t)$ measures such part of labour invested at t as is not matured until after N, still constituting a part of capital stock at N. The value of that part of capital stock can be represented by $f(t)e^{\rho(N-t)}$, when we suppose that interest is convertible continuously and the force of interest (or the instantaneous rate of interest) is ρ. Since the amount of capital is the total sum of the values of the intermediate products, existing at t=N, produced by labours applied in the time interval between t=0 and t=N, we have

$$C=\int_0^N f(t)e^{\rho(N-t)}dt.$$

In the simpler case, in which labour is invested at the constant rate as Böhm-Bawerk assumes, the investment function is linear and in fact

$$f(t)=\frac{A\cdot L}{N}\cdot t,$$

where A represents the number of labours employed and L the annual wage. Then we have

$$C=\int_0^N \frac{A\cdot L}{N}\cdot t\cdot e^{\rho(N-t)}\,dt=\frac{A\cdot L}{N}e^{\rho N}\int_0^N \frac{t}{e^{\rho t}}dt,$$

$$\int \frac{t}{e^{\rho t}}dt=-\frac{1}{\rho^2}\cdot\frac{\rho t+1}{e^{\rho t}};$$

thus $$C=\frac{A\cdot L}{N}\cdot\frac{e^{\rho N}-\rho N-1}{\rho^2}.$$

Now, remembering that $e^{\rho N}=1+\rho N+\frac{\rho^2N^2}{2!}+\frac{\rho^3N^3}{3!}+\cdots\cdots$, we have, on the other hand, the following relations:

$$\frac{e^{\rho N}-\rho N-1}{\rho^2}=\frac{\frac{\rho^2N^2}{2!}+\frac{\rho^3N^3}{3!}+\cdots\cdots}{\rho^2}$$

hence,

$$\text{if } \rho=0,\quad \frac{e^{\rho N}-\rho N-1}{\rho^2}=\lim_{\rho\to 0}\frac{e^{\rho N}-\rho N-1}{\rho^2}$$

$$=\lim_{\rho\to 0}\left\{\frac{N^2}{2}+\rho\sum_{i=3}^{\infty}\frac{\rho^{(i-3)}N^i}{i\,!}\right\}=\frac{N^2}{2},$$

$$\text{if } \rho>0,\quad \frac{e^{\rho N}-\rho N-1}{\rho^2}=\frac{N^2}{2}+\rho\sum_{i=3}^{\infty}\frac{\rho^{(i-3)}N^i}{i\,!}>\frac{N^2}{2}.$$

Thus

$$C\gtreqless\frac{A\cdot L\cdot N}{2}\qquad\text{according as}\qquad \rho\gtreqless 0.$$

Since $\frac{A\cdot L\cdot N}{2}$ is defined as subsistence-fund by Böhm-Bawerk and Wicksell, the above relation can be rewritten as follows:

$$C\gtreqless S\qquad\text{according as}\qquad \rho\gtreqless 0,$$

where S represents the subsistence-fund.

Let us generalize our representation. S may be defined as follows:

$$S=\int_0^N f(t)\,dt.$$

Since $f(t)>0$ in the interval $0<t\leq N$

and $e^{\rho t}=1$, if $\rho=0$,

$e^{\rho t}>1$, if $\rho>0$, $t>0$,

we have

$$C\gtreqless S\qquad\text{according as}\qquad \rho\gtreqless 0.$$

Hideo Aoyama.