ON THE GENERAL PROFIT RATE

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I. INTRODUCTION

The margin between the proceeds from the sale of products and the sum of the expenditure for labour and for producers' goods consumed in production is called "gross profit", which is composed of 1) capitalists' profit, viz., interest on capital invested in production, 2) ground rent to be paid for the use of land required for production and 3) entrepreneurs' profit. Now ground rent and entrepreneurs' profit which arises from the ability to take advantage of a favourable situation before others can do so can be ignored in the earlier stages of theoretical study. Therefore we propose here to confine ourselves to the study of capitalists' profit (which we will hereafter simply call profit).

The factors which participate in the determination of profit rate can be classified under two groups, viz. changes in prices, and foctors other than such changes. We propose here to confine ourselves to "comparative statics", i.e. to the study of the latter group, postponing the study of those elements of profit that arise from changes in prices for a future study[1].

1) But in doing so, it is vitally necessary to call attention to the difference between 1) such a variation in profit rate as results from a change in price of products when the change in question occurs without accompanying coincident change in the price of cost-goods involved (viz., of the producers' and the labourers'-consumers' goods involved) and 2) a variation in profit rate resulting from a change in the price of products when the change in question is assumed to occur accompanying precisely coincident changes in the price of cost-goods involved. The effect on profit of a change in price of products in the latter case is smaller than in the former precisely by the amount of the change in prices of cost-goods involved. When we propose to postpone the study of such portion of profit as arise from a change in prices,

The problem as defined above will be studied by dividing it into two stages, first in the case where the methods of production are assumed to be independent both of the amount of goods demanded and of the relative prices of the elements of production and secondly in the case where they are dependent on them. We propose to devote ourselves mainly to the study of the foregoing aspect of the problem.

But before proceeding to the main problem, special attention must be paid to the problem of value. I undertake to do this advisedly, notwithstanding the fact that I have already considered this problem elsewhere,[2] because it is sometimes argued that a difference in opinion on this point affects the subsequent conclusions.

II. PROBLEM OF VALUE[3]

The Marxian assertion that, in so far as different products are socially treated as equal through being considered as interchangeable, the individual concrete labours which produced these products must be held to have been reduced to some common entity, viz., social abstract labour, cannot be denied. But it is questionable 1) whether we are not allowed with equal justification to assert that in so far as different products are socially treated as equal the individual concrete utilities of different goods must be considred as being reduced to some common entity, viz., a social abstract utility and 2) whether it is necessary to refer to the social abstract entity or value, at all, whether considered as a

what we precisely mean is to overlook that portion of change in profit which arises from such a change in price of product as occurs, without accompanying coincident change in the price of cost-goods involved; we do not mean to overlook any change in prices which occurs, accompanying coincident change in the price of cost-goods involved.

2) Shibata: The meaning of the theory of value in theoretical economics, Kyoto University Economic Review, Dec., 1933.

3) Readers, not interested in the controversy upon value problem, are advised to skip over this section entirely except that part which begins with the last paragraph on page 43 and ends with the third paragraph on page 45.

social abstract labour or as a social abstract utility. The solution of the first problem, i.e. the problem concerning the controversy between the labour theory of value and the utility theory of value in the above sense depends entirely upon the interests which the investigators happen to be defending. But this is not the problem which I propose to deal with here. It is the second problem, viz. the problem concerning the controversy between the value theory of exchange value and the non-value theory of exchange value, that demands our attention.

It is sometimes argued that the Marxian economics is enabled by its value theory of exchange value to deal with the normal exchange value (or normal price) to which the ephemeral market exchange value (or market price) tends to adhere, and by which it is ultimately governed; while bourgeois economics is deprived of this advantage by its non-value theory of exchange value. It must indeed be acknowledged that bourgeois economics pays more attention than Marxian economics to the supply-and-demand situation. But this difference does not have any necessary logical relation to the difference between the value theory and the non-value theory of exchange value.

First of all, we cannot assert that the explanation of exchange value is completed by merely pointing out that the exchange value of a certain product in terms of some other product is determined by the amount of value contained in the former divided by that contained in the latter, as is sometimes asserted by the Marxists, because value is not an objectively identifiable quantity, but an entity, the determination of which is to be explained, so far as possible, by referring to some other objectively identifiable entity or entities, such as, for instance, a certain amount of specific individual concrete labour. Marx defined the reduction of individual concrete labour to social abstract labour as a process realised through taking the average. But this definition is not complete, because it does not define the specific method of taking the average by which the reduction is actually realized.

The supply-and-demand theory of exchange value, or its developed form, the subjective value theory, actually, though not intentionally, aims at explaining the relation between individual-concrete labour on the one hand and social-abstract labour on the other. Therefore, it is unjust to assert that the supply-and-demand theory can only deal with ephemeral and irregular market values, which fluctuate around the normal exchange value, while the Marxian theory is in a position to explain the latter. If there are two kinds of exchange values, market and normal, the difference in elucidation must not lie in the difference between supply-and-demand conditions and value *per se*, but in the difference between transient supply-and-demand conditions and normal supply-and-demand conditions.

It is, however, possible to study normal prices without referring to demand-and-supply conditions, if we are allowed to assume 1) that the technical co-efficients of production are given independently of variation in the amount of product produced or of the relative prices of the elements of production, 2) that there exists a state of complete competition and 3) that the level of real wages is given, viz., the elasticity of the supply of labour is infinite. The last assumption is introduced here merely for the sake of simplification and has no fundamental relevance to the main reasoning. Should it be required, some other assumption concerning the supply of labour will be introduced in place of it.

In order to facilitate comprehension of the above as well as to prepare for the following discussion, let us assume that there are only five kinds of products; i.e., money, two categories of comsumers' goods[4] and two categories of producers'

4) Of which the one, i.e. consumers' goods 1, is assumed to be consumed only by labourers, while the other, i.e. consumers' goods 2, is assumed to be consumed only by capitalists. Now, consumers' goods are considered to belong to "labourers' consumers' goods" when and in so far as they are consumed by labourers, and to "capitalists' consumers' goods" when and in so far as they are consumed by capitalists. Hence, consumers' goods 1 is meant to represent "labourers' consumers' goods", while consumers' goods 2 "capitalists' consumers' goods".

goods[5], and that their technical co-efficients of production and investment periods of capital (i.e., periods during which capital is considered to be laid down in the form of the respective technical co-efficients of production) and real wages of labourers are as follows :

Technical co-efficient of production	of money	concerning labour	a_0
		concerning producers' goods 2	c_0
	of producers' goods 1	concerning labour	a_{11}
		concerning producers' goods 1	c_{11}
	of consumers' goods 1	concerning labour	a_{21}
		concerning producers' goods 1	c_{21}
	of producers' goods 2	concerning labour	a_{12}
		concerning producers' goods 2	c_{12}
	of consumers' goods 2	concerning labour	a_{22}
		concerning producers' goods 2	c_{22}

	Investment period of capital assumed to be uniformly applicable to all cost items	real wages in terms of consumers' goods 1
Money	t_0 year	W_0
Producers' goods 1	t_{11} year	W_{11}
Consumers' goods 1	t_{21} year	W_{21}
Producers' goods 2	t_{12} year	W_{12}
Consumers' goods 2	t_{22} year	W_{22}

5) Of which the one, i.e. producers' goods 1, is assumed to be used only in its self re-production and in producing " labourers' consumers' goods ", while the other, i.e. producers' goods 2, is assumed to be used only in its self-reproduction and in producing " capitalists' consumers' goods " and money. Now, producers' goods are considered to belong to " labourers' producers' goods " when and in so far as they are used directly or indirectly in producing " labourers' consumers' goods ", and to " capitalists' producers' goods " when and in so far as they are used directly or indirectly in producing " capitalists' consumers' goods ". Hence, producers' goods 1 is meant to represent " labourers' producers' goods ", while producers' goods 2 " capitalists' producers' goods ".

We shall, if we assume a complete competition among producers under the above conditions, have the following five equations concerning the relation between cost (in price) and price respectively of money, producers' goods 1, consumers' goods 1, producers' goods 2 and consumers' goods 2:

$$\text{A} \quad (c_0 k_2 + a_0 W_0 p_1)(1+i)^{t_0} = 1 \qquad (1)$$

$$(c_{11} k_1 + a_{11} W_{11} p_1)(1+i)^{t_{11}} = k_1 \qquad (2)$$

$$(c_{21} k_1 + a_{21} W_{21} p_1)(1+i)^{t_{21}} = p_1 \qquad (3)$$

$$(c_{12} k_2 + a_{12} W_{12} p_1)(1+i)^{t_{12}} = k_2 \qquad (4)$$

$$(c_{22} k_2 + a_{22} W_{22} p_1)(1+i)^{t_{22}} = p_2 \qquad (5)$$

where k_1 denotes the price of producers' goods 1, k_2 the price of producers' goods 2, p_1 the price of consumers' goods 1, p_2 the price of consumers' goods 2 and i the general profit rate, i.e., the rate of profit to which any producer producing under competition without any privileges ought to have access.

If we assume, in the above case, that the technical co-efficients of production, investment periods of capital and real wages are all given, these five equations contain precisely five unknowns, viz., p_1, p_2, k_1, k_2 and i. Thus we see that the prices and general profit rate are determined without referring to value or to the supply-and-demand conditions of goods.

We have in the above arrived at prices and general profit rate without referring to value but with special reference to objectively identifiable entities, such as technical co-efficients of production, viz., amounts of concrete producers' goods and of labour needed for the production of a unit of the respective products, and real wage levels.

The methods adopted in the foregoing statement may, however, also be applied in determining values and the rate of surplus value. According to the value theory, the capitalist production is, in its essence, carried on with the aim of exploiting as much surplus value as possible out of the labour-power bought with that part of capital which is paid out for acquiring labour-power, viz., from the so-called variable capital. Therefore if this essential feature of capitalist production reveals itself precisely as it is, the rate of surplus value (viz., the surplus value divided by the amount of variable capital paid for the labour-power out of which surplus value is exploited) of any product will become equal to that of any other product. Therefore, if we assume precisely the same conditions in other respects as we have assumed in the above, we shall have the following five equations concerning the relation between cost (in value-price) and value-price (viz., value divided by money' value) respectively of money, producers' goods 1, consumers' goods 1, producers' goods 2 and consumers' goods 2:

$$\text{B} \quad c_0k'_2+a_0W_0p'_1(1+m')=1 \qquad \ldots\ldots (1)'$$
$$c_{11}k'_1+a_{11}W_{11}p'_1(1+m')=k'_1 \qquad \ldots\ldots (2)'$$
$$c_{21}k'_1+a_{21}W_{21}p'_1(1+m')=p'_1 \qquad \ldots\ldots (3)'$$
$$c_{12}k'_2+a_{12}W_{12}p'_1(1+m')=k'_2 \qquad \ldots\ldots (4)'$$
$$c_{22}k'_2+a_{22}W_{22}p'_1(1+m')=p'_2 \qquad \ldots\ldots (5)'$$

where k'_1 denotes the value-price of producers' goods 1, p'_1 that of consumers' goods 1, k'_2 that of producers' goods 2, p'_2 that of consumers' goods 2 and m' the rate of surplus value. Thus we can arrive at value-prices and rate of surplus value without referring to supply-and-demand conditions, without however thereby obscuring, as was the case with the Marxian theory, the relation between value and concrete labour.

The above equation system gives only the value-price, viz. the value expressed in its relation to the value of money. If we wish to arrive at value instead of value-price, we shall have to replace the first equation contained in the above system by the following:

$$W_0p'_1(1+m') = 1$$

because, $W_0p'_1$ denotes the amount of labour needed for the creation of one unit of labour, and consequently $W_0p'_1m'$ denotes the amount of surplus labour exploited from one unit of labour power. Then p'_1, p'_2, k'_1 and k'_2 will, of course, be rendered to denote values instead of value-prices.

Now it may be questioned whether we can arrive at the same prices and general profit rate by reasoning without reference to value, as we do when we reason with special reference to value. In order to examine this problem, we must take into account the theoretical postulate that capitalist producers, striving to make as much profit as possible, naturally force the rates of profit in all branches of production to equilibrate, in so far as there is no disturbance to competition among producers, and that this will make the prices deviate from value-prices when the maintenance of prices at value-prices conflicts with the unification of profit rates. Therefore, if we denote the ratio of the price of producers' goods 1 to their value-price by d_{11}, that of price of producers' goods 2 to their value-price by d_{12}, that of price of consumers' goods 1 to their value-price by d_{21} and that of price of consumers' goods 2 to their value-price by d_{22}, the equation system B will be supplied with the following additional equations:

$$(c_0 k'_2 d_{12} + a_0 p'_1 d_{21})(1+i)^{t_0} = 1 \quad \text{......} (6)'$$
$$(c_{11} k'_1 d_{11} + a_{11} W_{11} p'_1 d_{21})(1+i)^{t_{11}} = k'_1 d_{11} \quad \text{......} (7)'$$
$$(c_{21} k'_1 d_{11} + a_{21} W_{21} p'_1 d_{21})(1+i)^{t_{21}} = p'_1 d_{21} \quad \text{......} (8)'$$
$$(c_{12} k'_2 d_{12} + a_{12} W_{12} p'_1 d_{21})(1+i)^{t_{12}} = k'_2 d_{12} \quad \text{......} (9)'$$
$$(c_{22} k'_2 d_{12} + a_{22} W_{22} p'_1 d_{21})(1+i)^{t_{22}} = p'_2 d_{22} \quad \text{......} (10)'$$

together with the following four equations, denoting the definition of d_{11}, d_{21}, d_{12} and d_{22} respectively:

$$k_1 = k'_1 d_{11}, \quad p_1 = p'_1 d_{21}, \quad k_2 = k'_2 d_{12}, \quad p_2 = p'_2 d_{22}.$$

Equations (6)′–(10)′ can be reduced by means of the above mentioned four equations to the equation system A. This by itself proves that the results concerning prices and the general profit rate which are obtained by the equation system B are precisely the same as those arrived at by means of the equation system A. And this means again that reference to value does not affect the results concerning prices and general profit rate.

III. FACTORS DIRECTLY DETERMINING THE GENERAL PROFIT RATE

It will be clear from the foregoing that an investigation of general profit rate can be undertaken by means of equation system A. Now we obtain from the equations (2) and (3) therein the following equation (6)[6]:

$$1 \quad c_{11}(1+i)^{t_{11}}+a_{21}W_{21}(1+i)^{t_{21}} \\ +(c_{21}a_{11}W_{11}-c_{11}a_{21}W_{21})(1+i)^{t_{11}+t_{21}}$$

This equation implies that the general profit rate depends directly on the technical co-efficients of production and investment periods of capital, of "labourers' goods",[7] and on the real wage levels of labourers employed in producing such goods. In other words the geneıal profit rate depends directly neither on the technical co-efficients of production, nor on the investment periods of capital, of "capitalists' goods",[8] nor on the real wage levels of labourers employed in producing such goods.

The proposition that the general profit rate directly depends neither on the technical co-efficients of production of "capitalists' goods", nor on the investment period of capital of such goods, nor on the real wage levels of labourers employed in producing such goods, does not imply that it is entirely independent of such factors, but that they cannot affect the general profit rate except indirectly, through modifying either technical co-efficients of production, or investment periods of capital of "labourers' goods", or real wage levels of labourers employed in producing such goods. In so far as any of the technical co-efficients of production of "labourers' goods", and the investment periods of capital of

6) We arrive at precisely the same equation by taking, instead of only two equations (2) and (3), all the five equations (1)–(5) into consideration. This is proved as follows.

Though the equation directly derived from all the five equations (1)—(5) is: $0=\{c_{12}(1+i)^{t_{12}}-1\}\ \{c_{11}(1+i)^{t_{11}}+a_{21}W_{21}(1+i)^{t_{21}}+(c_{21}a_{11}W_{11}-c_{11}a_{21}W_{21})\ (1+i)^{t_{11}+t_{21}}-1\}$, this can be reduced to equation (6), because $\{c_{12}(1+i)^{t_{12}}-1\}$ can never be zero, since it is theoretically postulated that $a_{12}\geqq 0$, $L_0a_{12}-c_{12}\geqq 0$ for sufficiently large L_0, $W_{12}>0$, $k_2\geqq 0$ and $p_1>0$, while we are given equation (4).

7) Goods are considered to belong to "labourers' goods" when and in so far as they are "labourers' consumers' goods" or "labourers' producers' goods".

8) Goods are considered to belong to "capitalists goods" when and in so far as they are "capitalists' consumers' goods" or "capitalists' producers' goods".

"labourers' goods" and the real wage levels of labourers employed in producing such goods, depends directly or indirectly upon any of the technical co-efficients of production of "capitalists' goods", or on investment periods of capital of "capitalists' goods", or on real wage levels of labourers employed in producing "capitalists' goods", the general profit rate must be considered indirectly to depend upon the latter also.

From equation (6) it is apparent that the technical co-efficients of production of "labourers' goods" and the real wage levels of labourers employed in producing such goods being given, the general profit rate will be higher the shorter the invest-periods of capital of such goods. This is clear at a glance so long as the following condition is fulfilled:

$$c_{11}a_{21}W_{21} \leqq c_{21}a_{11}W_{11}.$$

But even if this condition is not fulfilled, the above mentioned proposition holds true (see Appendix, I).

Equation (6) permits us also to deduce that, the technical co-efficients of production of "labourers' goods" and investment periods of capital of such goods being given, the lower the level of real wage paid to the labourers employed in producing "labourers' goods", the higher the general profit rate will be. This is evident in so far as it regards W_{11}, i.e. the real wage of labourers employed in producing "labourers' producers' goods". But the proposition holds also with regard to W_{21}, i.e. the real wage of labourers employed in producing "labourers' consumers' goods", (see Appendix, II).

We can further deduce from equation (6) that, the investment periods of capital of "labourers' goods" and the real wage levels of the labourers employed in producing such goods being given, the general profit rate will be higher in proportion as the technical co-efficients of production of "labourers' goods" are smaller. This is obvious in so long as c_{21}, viz. the technical co-efficient of production of "labourers' consumers' goods" concerning producers' goods and a_{11} viz. the technical co-efficient of production of "labourers' producers' goods" concerning labour are in question. But the proposition holds also with regard to c_{11}, viz. the technical co-efficient of production of and concerning "labourers' producers' goods", and a_{21}, viz. the technical co-efficient of production of "labourers' consumers' goods" concerning labour (see Appendix, III).

Now, the technical co-efficients concerning labour vary in inverse ratio to the length of the working day or to the intensity of labour, provided wage is calculated per day. Equation (6) leads us to the well known proposition that the longer the working day or the more intensive the labour the higher the general profit rate will be, provided that the alteration in the working day or in the intensity of labour takes place with respect to labourers employed in producing "labourers' goods".

We have in the above ascertained that general profit rate will be higher the smaller the investment period of capital and the technical co-efficients of production, whether concerning labour or concerning producers' goods, of "labourers' goods", and the lower the level of real wages of labourers employed in producing "labourers' goods". Let us now proceed to investigate the effect upon the general profit rate of the elevation of the organic composition of capital, i.e. the change in the method of production prevailing under capitalism whereby reduction in the technical co-efficients of production concerning labour occurs side by side with an enlargement in the technical co-efficients of production concerning producers' goods. The key for the solution of this problem is to be found not in equation (6) as before but in the theoretical postulate that such a variation in the method of production must aim at reducing the cost of production of the goods concerned, and therefore must entail a fall in their purchasing power when it is generally adopted,[9] and therefore must, so long as the

9) If the methods of production are, contrary to the assumption in the text, dependent on the relative prices of the elements of production, the initial innovatory change in the method of production of a certain goods would entail a secondary adaptive change in that of other goods, and the purchasing power of the former goods in terms of the latter goods may or may not

goods concerned are commodities, lower their price unless some counteracting change occurs regarding the production condition of money. Now such an alteration in the method of production of " labourers' goods " as would, provided that the method of production of money remains unchanged, lower their price, whether due to an elevation of their organic composition of capital or not, cannot, in some way or other, but lower the price of the goods constituting the cost elements of money. A fall in the price of the " labourers' goods " constituting the cost elements of money cannot but involve a rise in the general profit rate, as the composition of equation (1) testifies, provided that the fall in price in question occurs without any change in the method of production of money. Consequently, any such change in the method of production of " labourers' goods " as would make their price become lower than it otherwise would be, i.e., as would lower their price unless some counteracting change takes place regarding the productive condition of money, will necessarily involve a rise in the general profit rate.[10)]

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fall so much as it would have done in case that secondary change were absent. But the proposition in the text holds in such a case also, because it is impossible for the secondary adaptive change in the method of production to be so much effective as to counteract the effect of the initial innovatory change.

10) The assumption of unchanged method of production of money is introduced in the above reasoning merely for the sake of ascertaining whether the change in the method of production of " labourers' goods " in question is such as would make their price become lower than it otherwise would be, and has nothing to do with the essential part of our argument. Indeed, if we assume that the method of production of money undergoes precisely the same cost-curtailing variation as that in the method of production of " labourers' goods ", the price of the latter goods would, the price-lowering effect of the change in question in their production-method being precisely offset by the price-raising effect of the change in question in the production-method of money, remain unchanged, and there may be no fall in price of the goods constituting the cost elements of money (and hence no such fall in the cost of money as results therefrom), and the main ground of our argument may appear to be taken away. However, such a change in the production method of money as is assumed here must necessarily consist of a sufficient reduction in the technical co-efficients of production of money — a fact which, with unchanged price of the goods constituting the cost elements of money, must necessarily imply a reduction in the cost of production of money and hence a rise in the general profit rate, as the composition of equation (1) again testifies.

Let us illustrate the above reasoning by inserting into our equation system A the following set of assumptions.

$$
\begin{aligned}
A' \quad & c_{11} = c_{12} = c_{21} = c_{22} && (=c) \\
& a_{11} = a_{12} = a_{21} = a_{22} && (=a) \\
& W_{11} = W_{12} = W_{21} = W_{22} && (=W) \\
& t_{11} = t_{12} = t_{21} = t_{22} && (=1)
\end{aligned}
$$

We shall then obtain from equations (2), (3), (4) and (5),

$$k_1 = p_1 = k_2 = p_2 \qquad (=p)$$

Let us next denote $1/(1+i)$ by λ and assume that $t_0=1$, then equation (1) will be converted to:

$$p(c_0 + a_0 W_0) = \lambda$$

which will mean, if we assume that c_0, a_0 and W_0 are not affected by the change in the method of production of commedities, that:

$$\frac{p}{\lambda} = \text{const.}$$

viz., that the change in λ will be proportional to that in p, and *vice versa*; in other words, that any change in the method of production of "labourers' goods", whose generalisation may cause prices of the goods concerned become lower than it otherwise would be will tend to raise the general profit rate, even if this change is of such a nature as to effect an elevation of the organic composition of capital.

Now capitalist producers normally elevate the organic composition of capital only when such elevation lowers the cost of production, i.e., only when it is of such a character that its generalisation would make the purchasing power of the goods concerned become lower than it otherwise would be. Accordingly, the elevation of the organic composition of capital by capitalists far from causing a decline in the general profit rate actually tends to raise it.

We have so far assumed a complete competition. Let us now discard this assumption. But even if we do so, the general profit rate depends directly neither on the technical co-efficients of production of "capitalists' goods", nor on their investment periods of capital, nor on the real wage levels of labourers employed in producing such goods, nor on the amount of monopoly profit exploited by their producers. This will become apparent if we investigate the problem assuming that the monopoly profit is secured solely by the producers of "capitalists' goods", because the general profit rate would, under such circumstances, be determined directly only by the conditions represented by equation (6), no matter how much monopoly profit is raised by the producers of "capitalists' goods". Hence it follows that the monopoly profit can affect the general profit rate directly only in so far as it is secured by the producers of "labourers' goods".

Let us, therefore, assume that the monopoly profit is obtained only by the producers of "labourers' goods" and that the prices of "labourers' producers' goods" as well as those of "labourers' consumers' goods" are raised respectively to $(1-\alpha_1)$ and $(1-\alpha_2)$ times the imaginary prices arrived at by subtracting monopoly profit from their respective actual prices. Equations (2) and (3) will than be changed to,

$$(c_{11}k_1+a_{11}W_{11}p_1)(1+i)^{t_{11}}(1+\alpha_1)=k_1 \qquad \text{(2)}''$$

$$(c_{21}k_1+a_{21}W_{21}p_1)(1+i)^{t_{21}}(1+\alpha_2)=p_1 \qquad \text{(3)}''$$

These two equations will be reduced to the following variant of equation (6)

$$\begin{aligned}1=\ & c_{11}(1+i)^{t_{11}}(1+\alpha_1)+a_{21}W_{21}(1+i)^{t_{21}}(1+\alpha_2)\\ & +(c_{21}a_{21}W_{11}-c_{11}a_{21}W_{21})(1+i)^{t_{11}+t_{21}}(1+\alpha_1)(1+\alpha_2) \quad \ldots\text{(7)}\end{aligned}$$

We can deduce from this equation that the general profit rate will fall the greater the monopoly profit exploited by the producers of "labourers' goods".[11] This proposition may be proved substantially in the same manner as that in which we testified above the proposition that the general profit rate will rise the shorter the investment period of capital of "labourers' goods" (see Appendix, IV).

It may perhaps be superfluous to note that the above reasoning is not affected by the consideration of fixed capital. Fixed capital first invested can be considered to form so to speak a present value of annuity rents payable at the beginn-

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11) It is easily inferred from this that excise and import duties levied on "laboures' goods" *ceteris paribus* lowers the general profit rate.

ing of each of the investment periods contained in the whole length of time during which the fixed capital is repeatedly used. Such an annuity rent divided by the price of the fixed capital goods corresponds to a technical co-efficient of production concerning producers' goods. Any change in a fixed capital, therefore, can be treated as the change in technical co-efficients of production concerning producers' goods, into which it can be reduced in the manner defined above. Hence no introduction of fixed capital into the scope of our investigation can affect our reasoning above.

IV. ON THE LAW OF DECLINE IN THE GENERAL PROFIT RATE

The tendency of a fall in the general profit rate, which has long since attracted the sincere attention of so many eminent economists, seems to have become more apparent recently. Let us try to give some brief explanation of this fact.

The tendency of a fall in the general profit rate is usually attributed to a rise in the real wage levels and growth of technical co-efficients of production concerning labour resulting either from the shortening of working day or from the lessening of the intensity of labour. It is clear from our investigation above that these factors would certainly lower the general profit rate in so far as they occur regarding the "labourers' goods". It is also clear that these factors have long been constituting the main causes of the phenomenon in question. But as there is no evidence that these factors greatly increased themselves recently, we must look for another additional cause.

As is widely known, Ricardo demonstrated that the recourse to increasingly inferior productive service of land, necessitated by an increase in population, was the main factor responsible for the fall in the general profit rate. It is clear from what has been discussed in section III, that the general profit rate is bound to decline if and in so far as the increase in population makes it necessary for entrepreneurs to depend on any increasingly inferior productive land, because it means ever swelling technical co-efficients of production of "labourers' goods". This factor, however, does not seem to have had quite the importance attributed to it by Ricardo, first because the advance in methods of transportation and communication has tended to solve the problem in question, and secondly because modern times which witness such a striking fall in the general profit rate are at the same time characterised by the fall in the prices of argicultural products in relation to the general price levels.

It may be argued that the increase in the cost of advertisment necessitated by the differentiation of products[12] is the very factor to which the fall in the general profit rate should be imputed. This factor, however, does not appear to have much importance, because it is only with regard to somewhat limited categories of products that advertisment is of any remarkable relevance as a cost factor. Speculative holding in stock of commodities[13] may again be considered as a cause lowering the general profit rate, but this factor also can hardly claim much importance, because it is only within certain narrow limit that merchants can hold goods in stock for speculative purpose.

It seems to me that one of the main factors causing the general profit rate to fall especially in modern times is the exploitation of monopoly-profit by producers enjoying the privileged position as monopolists. The monopolisation extents itself also to goods that are of the nature of "labourers' goods". Now, it is clear from the observations offered in section III that this will lower the general profit rate.

The increase in technical co-efficients of production resulting from interruption of international transactions and

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12) G. D. H. Cole: Practical economics, p. 16 ff.
13) Cole: Principles of economic planning, p. 196 ff.

the rise in import duties, with both which monopoly capitalism abounds, constitute, so far as they take place regarding " labourers' goods ", another factor depressing the general profit rate.

The labour disputes which tend to increase in severity under monopoly capitalism as well as the retardation in the progress of productive power caused by the restricting policies of monopoly capitalism constitute also another, though passive, by no means negligible factor which may, other general-profit-rate lowering factors still operating as hitherto, cause the general profit rate to fall.

As is widely known, Marx has asserted that such a change in the method of production as would increase the technical co-efficients of production concerning producers' goods at the expense of those concerning labour, viz. such a change as would elevate the so-called organic composition of capital, necessarily results in a fall in the general profit rate, notwithstanding that it is introduced for the sake of increasing entrepreneurs' profit by those who first initiate it.

But this peculiar theory of Marx is untenable as is clear from our analysis in section III.

We have ascertained in the course of the foregoing discussion that the technical co-efficients of production which directly participate in the determination of the genaral profit rate are limited to those of " labourers' goods ", or in other words, that the technical co-efficients of production of " capitalists' goods " do not directly participate in the determination of the general profit rate. This means that no change in the technical co-efficients of production of " capitalists' goods " can effect a change in the general profit rate unless the technical co-efficients of production of " labourers' goods " are simultaneously altered. The change in the technical co-efficients of production of " capitalists' goods " can, however, none the less effect a change in the organic composition of capital even if there should be no change in the technical co-efficients of production of " labourers' goods ". It is therefore quite easy to compose a case where the general profit rate remains untouched or even rises irrespective of the rise in the organic composition of total social capital, or *vice versa*. Assume for instance that the organic composition of capital of "capitalists' goods" is raised, say by a change in the methods of production, while no change takes place in the methods of production of "labourers' goods". The organic composition of total social capital would in this case certainly rise, but there would be no change in the general profit rate. Assume again that there is on the one hand a rise in the general profit rate, say, due to such a change in the method of production of "labourers' goods" as would lower the organic composition of capital of such goods, while on the other hand there occurrs a change in the method of production of "capitalists' goods" calculated to elevate the organic composition of capital of the goods concerned to such an extent that it would more than compensate for the fall in the organic composition of total social capital caused by the alteration in the method of production of "labourers' goods". It is further possible even to conceive a case where, other conditions remaining as just stated, the lowering of the organic composition of capital of "labourers' goods" might effect a fall in the rate of surplus value; a case where the general profit rate would rise congruently with an elevation in the organic composition of the total social capital while the rate of surplus value would be lowered. These observations may be taken as disproving the presumption of simple relation between a fall in the general profit rate and an elevation of the organic composition of total social capital. This method of criticising the Marxian theory is, as I pointed out previously[14], precisely the line taken by Bortkiewicz[15]. But it is only fortuitously that the organic composition of capital of "capitalists' goods" undergoes such special

14) Shibata: On the law of decline in the rate of profit, Kyoto University economic review, July, 1934.

15) L. v. Bortkiewicz: Zur Berichtigung der grundlegenden theoretischen Konstruktion von Marx im dritten Band des Kapital, Jahrb. f. Nationaloekonomie und Statistik, 1907.

variation as was assumed by Bortkiewicz, and therefore, no amount of discussion of merely fortuitous cases can disprove the Marxian theory concerning the general law of the development of capitalism. An examination of the Marxian theory as well as the development of the general theory must be divorced from such special contingencies. If we thus discard the Bortkiewicz method of criticising the Marxian theory, everything may be found to hinge upon whether the elevation willingly undertaken by capitalist producers of the organic composition of capital of "labourers' goods" would necessarily fail to raise the rate of surplus value sufficiently to maintain, let alone augment, the general profit rate. Marx presupposes that there is such necessity, while his critics assert the contrary. It is, however, no solution of the issue for both sides simply to maintain their respective views as being selfevident. The problem must be solved by starting from the premise commonly admitted by both the adherents and the opponents of the Marxian theory of the fall in the general profit rate. That is why we based our study of the problem in question upon the theoretical postulate that the change in the method of production willingly undertaken by the capitalist producers must normally be such as would make the purchasing power of the goods concerned become lower than it otherwise would be.

The foregoing criticism of the Marxian theory by the writer which was published previously in a more detailed form[16] has drawn from Mr. Tsuru the following illuminating criticism.

"Let us......make p and k (in the case simplified by the introduction of A′ set of assumptions) denote values and use in place of the monetary cost equation (1) the following monetary value equation, i.e..

$$\frac{aWp+(ck+aWp)i}{a}=1 \quad \text{(a)}$$

16) Shibata: On the law of decline etc.

then, k being equal to p, we obtain from this equation the following:

$$\lambda=(c+aW)\frac{p}{a+cp}$$

(Now any one of the equations (2), (3), (4) and (5) can, under our present assumptions be reduced to $\lambda=c+aW$). We deduce then from the combination of (these two equations)... that:

$$p=\frac{a}{1-c}$$

It will be clear from this that the condition for the lowering of p is

$$dc<-da\left(\frac{1-c}{a}\right)$$

(while the condition for the lowering of the general profit rate is $dc>-W\cdot da$, — a condition which may be derived from the equation $\lambda=c+aW$). It will be clear even from the following example that the above defined condition (for the lowering of p)... is compatible with the condition (for the lowering of the general profit rate) also defined above... Namely: if we assume that in the case when c is 2/3 and a is 1/30 (and W is 5), and therefore when i is 20% and p 0.1, the technical co-efficients of production are changed to $c=207/300$ and $a=9/300$, i will become 19.0476% and p 0.09677... In so far as a change in the absolute level of value is relevant, we must not neglect the equation (a) unless such changes are invariably represented by corresponding changes in the absolute level of price. Is it not true, then that to get at the root of the law of value is to fully apprehend the effectiveness of the equation (a)?"

Now, as must be clear from our previous discussion, and as Mr. Tsuru also admits, any change in the method of production of commodities which involves a fall in the general profit rate will necessarily be such as would make the price of the commodities concerned become higher than it otherwise would be. Therefore, to advocate "the use of the monetary value equation (a) in place of the monetary cost equation (1)" amounts to presupposing that the capitalist producers will willingly adopt any new method of production provided that it only entails a fall in value of the goods concerned; even if it does follow a rise in the cost (in price), and consequently make the price of the commodities concerned become higher than it otherwise would be. Therefore, if we take for example the case which Mr. Tsuru has suggested as a model to demonstrate the possibitity for an elevation of the organic composition of capital coinciding not only with a fall in profit rate but also with a fall in *value*, we shall see that the elevation which effects a fall in the general profit rate not only raises the price of the commodities concerned from

$$\frac{1}{(2/3+5/30)\times 1.2}=1,$$

to

$$\frac{1}{(2/3+5/30)\times 1.190476}=1.008,$$

but also raises the cost (in price) of production of the commodities concerned from

$$(2/3+2/30)\times 1=0.83333,$$

to

$$(207/300+5.9/300)\times 1.008=0.84672.$$

Can we admit the assertion that "the capitalist producers will willingly adopt any new method of production no matter how much it may raise the cost (in price) of production, and consequently the price, of the commodities concerned, if it only effects a fall in *value* of the commodities concerned?

We must of course admit that capitalist producers do sometimes adopt a method of production that will raise the cost (in price) of production, such, for instance, as was pointed out by Mr. Cole. But evidently such is not what Mr. Tsuru or Marx has in mind.

I hope I have made it clear that the elevation of the organic composition of capital normally achieved by capitalist producers does not cause a reduction in the general profit rate, but that on the contrary it tends to bring about a rise. This, however, by no means implies that a fall in the general profit rate may not coincide with a rise in the organic composition of capital. It simply shows that if such coincidence occurs the fall in the general profit rate must be explained not by the elevation of the organic composition of capital as was done by Marx, but by other factors, such as the simultaneous rise in wages or the simultaneous shortening of the working day and what not. Let us assume for instance that the wage of labourers rises. This rise in wages will encroach upon the general profit rate. The capitalist producers will try by replacing labourers by machinery to evade the depressing effect upon the general profit rate of the rise in wage. This will tend on the one hand to elevate the organic composition of capital which may be great enough to overcome the contrary effect arising from the rise in wages and on the other to counteract to some extent but not enough to offset the falling tendency in the general profit rate due to the rise in wages. We shall then observe the phenomenal coincidence of a fall in the general profit rate and an elevation of the organic composition of capital. The fall in "interest rate" coinciding with the "lengthening" of the period of production as demonstrated by Böhm-Bawerk is an example of such a case, because the fall in "interest rate" in this case results essentially not from the "lengthening" but from the rise in wages which necessitates the "lengthening", and is essentially different from what was advocated by Marx.

Now one may ask if we are not allowed to ascribe the recent fall in general profit rate to the Böhm-Bawerkian lengthening of the period of production. Our answer to this question is already given by our observation that there is no evidence of remarkable rise in real wage levels recently.

APPENDIX

This will be demonstrated by the analysis given below, which I owe, especially in so far as the differentiation procedure concerns, to the kind help of Mr. Midutani, professor in the Kobe Commercial University. The analysis given below is based on the following theoretical postulates:—

$$c_{11}\geqq 0,\ a_{11}\geqq 0,\ W_{11}>0,\ t_{11}>0,\ k_1\geqq 0,\ \alpha_1\geqq 0,\ c_{21}+a_{21}>0,\ i\geqq 0,$$
$$c_{21}\geqq 0,\ a_{21}\geqq 0,\ W_{21}>0,\ t_{21}>0,\ p_1>0,\ \alpha_2\geqq 0,\ L_1a_{11}-c_{21}\geqq 0,\ L_2a_{11}-c_{11}\geqq 0.$$

(for sufficiently large L_1 and L_2).

I. Differentiating both sides of equation (6) partially with respect to t_{11}, i being a function of t_{11}, while the others being kept constant, we have:

$$0=c_{11}(1+i)^{t_{11}}\log(1+i)+c_{11}t_{11}(1+i)^{t_{11}-1}\frac{\partial i}{\partial t_{11}}+a_{21}W_{21}t_{21}(1+i)^{t_{21}-1}\frac{\partial i}{\partial t_{11}}$$
$$+(c_{21}a_{11}W_{11}-c_{11}a_{21}W_{21})\left\{(t_{11}+t_{21})(1+i)^{t_{11}+t_{21}-1}\frac{\partial i}{\partial t_{11}}+(1+i)^{t_{11}+t_{21}}\log(1+i)\right\}.$$

Hence, solving this equation for $\frac{\partial i}{\partial t_{11}}$, we have: $\frac{\partial i}{\partial t_{11}}=-\frac{N_{11}+N_{12}}{D}$,

where: $D=D_1+D_2+D_3$, $D_1=M_1c_{11}t_{11}(1+i)^{t_{11}-1}$,
$D_2=M_2a_{21}W_{21}t_{21}(1+i)^{t_{21}-1}$, $D_3=c_{21}a_{11}W_{11}(t_{11}+t_{21})(1+i)^{t_{11}+t_{21}-1}$
$M_1=1-a_{21}W_{21}(1+i)^{t_{21}}$, $M_2=1-c_{11}(1+i)^{t_{11}}$
$N_1=M_1c_{11}(1+i)^{t_{11}}\log(1+i)$, $N_2=c_{21}a_{11}W_{11}(1+i)^{t_{11}+t_{21}}\log(1+i)$.

Now equation (3), can be reduced to: $\left(c_{21}\frac{k_1}{p_1}+a_{21}W_{21}\right)(1+i)^{t_{21}}=1$, where $c_{21}\frac{k_1}{p_1}\geqq 0$, since $c_{21}\geqq 0$, $k_1\geqq 0$ and $p_1>0$; while $i\geqq 0$. Hence $a_{21}W_{21}(1+i)^{t_{21}}\leqq 1$. Hence $M_1\geqq 0$. Hence $D_1\geqq 0$, because $c_{11}\geqq 0$, $t_{11}>0$ and $i\geqq 0$ Next, equation (2) can be transformed into: $\left(c_{11}+a_{11}W_{11}\frac{p_1}{k_1}\right)(1+i)^{t_{11}}=1$, so long as $k_1>0$. Now, $c_{11}(1+i)^{t_{11}}<1$, since $a_{11}\geqq 0$, $W_{11}>0$, $p_1>0$, $i\geqq 0$ and $L_2a_{11}-c_{11}\geqq 0$. (If $k_1=0$, equation (2) leads us to the conclusion that $c_{11}(1+i)^{t_{11}}=0$, because $L_2a_{11}-c_{11}\geqq 0$, $p_1>0$, $W_{11}>0$ and $i\geqq 0$). Hence $M_2>0$. Hence $D_2\geqq 0$, because $a_{21}\geqq 0$, $W_{21}>0$, $t_{21}>0$ and $i\geqq 0$. Lastly, $D_3\geqq 0$, since $c_{21}\geqq 0$, $a_{11}\geqq 0$, $W_{11}>0$, $t_{11}>0$, $t_{21}>0$ and $i\geqq 0$. But it is impossible for both D_2 and D_3 to be zero at the same time, because, whereas $(D_2=0)$ presupposes $(a_{21}=0)$ and $(D_3=0)$ presupposes $(a_{11}c_{21}=0)$, it is impossible for both a_{21} and $a_{11}\cdot c_{21}$ to be zero at the same time, since $L_1a_{11}-c_{21}\geqq 0$ and $c_{21}+a_{21}>0$. Hence $D>0$.

$N_1\geqq 0$ even if $i>0$, since $c_{11}\geqq 0$, and $t_{11}>0$ while $M_1\geqq 0$, as was proved above. $N_2>0$, so long as $c_{21}>0$ and $i>0$, because, whereas $W_{11}>0$, $t_{11}>0$ and $t_{21}>0$, a_{11} must then be positive since $L_1a_{11}-c_{21}\geqq 0$. Hence $N_1+N_2>0$, so long as $c_{21}>0$ and $i>0$.

Hence it follows that $\frac{\partial i}{\partial t_{11}}<0$, so long as $c_{21}>0$ and $i>0$.

× × × × ×

Proceeding just in the same manner as above, the partial differentiation of equation (6) with respect to t_{21} gives:

$$0=c_{11}t_{11}(1+i)^{t_{11}-1}\frac{\partial i}{\partial t_{21}}+a_{21}W_{21}t_{21}(1+i)^{t_{21}-1}\frac{\partial i}{\partial t_{21}}+a_{21}W_{21}(1+i)^{t_{21}}\log(1+i)$$
$$+(c_{21}a_{11}W_{11}-c_{11}a_{21}W_{21})(1+i)^{t_{11}+t_{21}}\log(1+i)$$
$$+(c_{21}a_{11}W_{11}-c_{11}a_{21}W_{21})(t_{11}+t_{21})(1+i)^{t_{11}+t_{21}-1}\frac{\partial i}{\partial t_{21}}.$$

Solving this equation for $\frac{\partial i}{\partial t_{21}}$, we have: $\frac{\partial i}{\partial t_{21}}=-\frac{N_{21}+N_{22}}{D}$,

where: $N_{21}=M_2a_{21}W_{21}(1+i)^{t_{21}}\log(1+i)$, $N_{22}=c_{21}a_{11}W_{11}(1+i)^{t_{11}+t_{21}}\log(1+i)$.

Now, on the one hand $N_{21}\geqq 0$ even if $i>0$, because $a_{21}\geqq 0$, $W_{21}>0$, and $t_{21}>0$, while $M_2>0$ as was proved above, and on the other hand $N_{22}\geqq 0$ even if $i>0$, because $c_{21}\geqq 0$, $a_{11}\geqq 0$, $W_{11}>0$, $t_{11}>0$ and $t_{21}>0$. But , so long as $i>0$, it is impossible for both N_{21} and N_{22} to be zero at the same time, because, whereas $(N_{21}=0)$ then presupposes $(a_{21}=0)$ and $(N_{22}=0)$ $(c_{21}\cdot a_{11}=0)$, it is impossible for both a_{21} and $c_{21}\cdot a_{11}$ to be zero simultaneously, since $L_1a_{11}-c_{21}\geqq 0$ and $c_{21}+a_{21}>0$. Hence, $N_{21}+N_{22}>0$ so long as $i>0$, and consequently $\frac{\partial i}{\partial t_{21}}<0$ so long as, because $D>0$ as was proved above.

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II. Proceeding just in the same manner as above, the partial differentiation of equation (6) with respect to W_{21} gives:

$$0=c_{11}t_{11}(1+i)^{t_{11}-1}\frac{\partial i}{\partial W_{21}}+a_{21}(1+i)^{t_{21}}+a_{21}W_{21}t_{21}(1+i)^{t_{21}-1}\frac{\partial i}{\partial W_{21}}$$
$$-c_{11}a_{21}(1+i)^{t_{11}+t_{21}}+(c_{21}a_{11}W_{11}-c_{11}a_{21}W_{21})(t_{11}+t_{21})(1+i)^{t_{11}+t_{21}-1}\frac{\partial i}{\partial W_{21}}.$$

Solving this equation for $\frac{\partial i}{\partial W_{21}}$, we have: $\frac{\partial i}{\partial W_{21}}=-\frac{N_{30}}{D}$,

where: $N_{30}=M_2a_{21}(1+i)^{t_{21}}$.

Now $N_{30}>0$, so long as $a_{21}>0$, because, whereas $i\geqq 0$ and $t_{21}>0$, $M_2>0$ as was proved above. Hence $\frac{\partial i}{\partial W_{21}}<0$, so long as $a_{21}>0$, because $D>0$ as was proved above.

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III. Proceeding just in the same manner as above, the partial differentiation of equation (6) with respect to c_{11} gives:

$$0=(1+i)^{t_{11}}+c_{11}t_{11}(1+i)^{t_{11}-1}\frac{\partial i}{\partial c_{11}}+a_{21}W_{21}t_{21}(1+i)^{t_{21}-1}\frac{\partial i}{\partial c_{11}}$$
$$-a_{21}W_{21}(1+i)^{t_{11}+t_{21}}+(c_{21}a_{11}W_{11}-c_{11}a_{21}W_{21})(t_{11}+t_{21})(1+i)^{t_{11}+t_{21}-1}\frac{\partial i}{\partial c_{11}}.$$

The solution of this equation for $\frac{\partial i}{\partial c_{11}}$ gives: $\frac{\partial i}{\partial c_{11}}=-\frac{N_{40}}{D}$,

where: $N_{40}=M_1(1+i)^{t_{11}}$.

Now $N_{40}>0$, so long as $M_1>0$, because $i\geqq 0$ and $t_{11}>0$. On the other hand, $M_1=0$ only when $c_{21}=0$ or $k_1=0$, as will be clear from both the theoretical postulate $(p>0)$ and the equation $\left(c_{21}\frac{k_1}{p_1}+a_{21}W_{21}\right)(1+i)^{t_{21}}=1$, which was derived above from equation (3). Hence it follows that $\frac{\partial i}{\partial c_{11}}<0$, so long as $c_{21}>0$, $k_1>0$, because $D>0$, as was proved above.

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Proceeding just in the same manner as above, the partial differentiation of equation (6) with respect to a_{21} gives:

$$0=c_{11}t_{11}(1+i)^{t_{11}-1}\frac{\partial i}{\partial a_{21}}+W_{21}(1+i)^{t_{21}}+a_{21}W_{21}t_{21}(1+i)^{t_{21}-1}\frac{\partial i}{\partial a_{21}}$$
$$-c_{11}W_{21}(1+i)^{t_{11}+t_{21}}+(c_{21}a_{11}W_{11}-c_{11}a_{21}W_{21})(t_{11}+t_{21})(1+i)^{t_{11}+t_{21}-1}\frac{\partial i}{\partial a_{21}}.$$

Hence, solving this equation for $\frac{\partial i}{\partial a_{21}}$, we have: $\frac{\partial i}{\partial a_{21}}=-\frac{N_{50}}{D}$,

where: $N_{50}=M_2W_{21}(1+i)^{t_{21}}$.

Now $N_{50}>0$, because $W_{21}>0$, $i\geqq0$ and $t_{21}>0$, while $M_2>0$, as was proved above. Hence $\frac{\partial i}{\partial a_{21}}<0$, because $D>0$, as was proved above.

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IV. Proceeding precisely in the same way as above, the partial differentiation of equation (7) with respect to α_1 gives:

$$0=c_{11}(1+i)^{t_{11}}+c_{11}t_{11}(1+i)^{t_{11}-1}(1+\alpha_1)\frac{\partial i}{\partial \alpha_1}+a_{21}W_{21}t_{21}(1+i)^{t_{21}-1}(1+\alpha_2)\frac{\partial i}{\partial \alpha_2}$$
$$+(c_{21}a_{11}W_{11}-c_{11}a_{21}W_{21})\Big\{(t_{11}+t_{21})(1+i)^{t_{11}+t_{21}-1}(1+\alpha_1)(1+\alpha_2)\frac{\partial i}{\partial \alpha_1}$$
$$+(1+i)^{t_{11}+t_{21}}(1+\alpha_2)\Big\}.$$

Hence, solving this equation for $\frac{\partial i}{\partial \alpha_1}$, we have: $\frac{\partial i}{\partial \alpha_1}=-\frac{N_{11}'+N_{12}'}{D'}$,

where: $D'=D_1'+D_2'+D_3'$, $D_1'=M_1'c_{11}t_{11}(1+i)^{t_{11}-1}(1+\alpha_1)$,
$D_2'=M_2'a_{21}W_{21}t_{21}(1+i)^{t_{21}-1}(1+\alpha_2)$,
$D_3'=c_{21}a_{11}W_{11}(1+\alpha_1)(1+\alpha_2)(t_{11}+t_{21})(1+i)^{t_{11}+t_{21}-1}$,
$M_1'=1-a_{21}W_{21}(1+i)^{t_{21}}(1+\alpha_2)$, $M_2'=1-c_{11}(1+i)^{t_{11}}(1+\alpha_1)$,
$N_{11}'=M_1'c_{11}(1+i)^{t_{11}}$, $N_{12}'=c_{21}a_{11}W_{11}(1+i)^{t_{11}+t_{21}}(1+\alpha_2)$.

Now equation (3)'' can be reduced to $\left(c_{21}\frac{k_1}{p_1}+a_{21}W_{21}\right)(1+i)^{t_{21}}(1+\alpha_2)=1$, where $c_{21}\cdot\frac{k_1}{p_1}\geqq0$, since $c_{21}\geqq0$, $k_1\geqq0$ and $p_1>0$; while $i\geqq0$. Hence $a_{21}W_{21}(1+i)^{t_{21}}(1+\alpha_2)\geqq1$. Hence, $M_1'\geqq0$. Hence $D_1'\geqq0$, because $c_{11}\geqq0$, $t_{11}>0$, $i\geqq0$ and $\alpha_1\geqq0$. Next, equation (2)'' can be transformed to: $\left(c_{11}+a_{11}W_{11}\frac{p_1}{k_1}\right)(1+i)^{t_{11}}(1+\alpha_1)=1$, so long as $k_1>0$. Now, since $a_{11}\geqq0$, $W_{11}>0$, $p_1>0$, $i\geqq0$ and $L_2a_{11}-c_{11}\geqq0$, we have $c_{11}(1+i)^{t_{11}}(1+\alpha_1)<1$. (If $k=0$, equation (2)'' leads us to the conclusion that $c_{11}(1+i)^{t_{11}}(1+\alpha_1)=0$, because $L_2a_{11}-c_{11}\geqq0$, $p_1>0$, $W_{11}>0$, $i\geqq0$, $\alpha_1\geqq0$ and $t_{11}>0$). Hence $M_2'>0$. Hence $D_2'\geqq0$, because $a_{21}\geqq0$, $W_{21}>0$, $t_{21}>0$, $i\geqq0$ and $\alpha_2\geqq0$. Lastly, $D_3'\geqq0$, since $c_{21}\geqq0$, $a_{11}\geqq0$, $W_{11}>0$, $t_{11}>0$, $t_{21}>0$, $i\geqq0$, $\alpha_1\geqq0$ and $\alpha_2\geqq0$. But it is impossible for both D_2' and D_3' to be zero at the same time, because, whereas ($D_2'=0$) presupposes ($a_{21}=0$) and ($D_3'=0$) presupposes ($c_{21}a_{11}=0$), it is impossible for both a_{21} aad $a_{11}\cdot c_{21}$ to be zero at the same time, since $L_1a_{11}-c_{21}\geqq0$ and $c_{21}+a_{21}>0$. Hence $D'>0$.

$N_{11}'\geqq0$, since $c_{11}\geqq0$, $i\geqq0$ and $t_{11}>0$ while $M_1'\geqq0$ as was proved above. $N_{12}'>0$, so long as $c_{21}>0$, because, whereas $W_{11}>0$, $i\geqq0$, $t_{11}>0$, $t_{21}>0$ and $\alpha_2\geqq0$, a_{11} must then be positive since $L_1a_{11}-c_{21}\geqq0$. Hence $N_{11}'+N_{12}'>0$, so long as $c_{21}>0$.

Hence it follows that $\frac{\partial i}{\partial \alpha_{11}}<0$, so long as $c_{21}>0$.

× × × × ×

Proceeding just in the same manner as above, the partial differentiation of equation (7) with respect to α_2 gives

$$0=c_{11}t_{11}(1+i)^{t_{11}-1}(1+\alpha_1)\frac{\partial i}{\partial \alpha_2}+a_{21}W_{21}(1+i)^{t_{21}}+a_{21}W_{21}t_{21}(1+i)^{t_{21}-1}\frac{\partial i}{\partial \alpha_2}$$
$$+(c_{21}a_{11}W_{11}-c_{11}a_{21}W_{21})\Big\{(1+i)^{t_{11}+t_{21}}(1+\alpha_1)+(t_{11}+t_{21})(1+i)^{t_{11}+t_{21}-1}$$
$$(1+\alpha_1)(1+\alpha_2)\frac{\partial i}{\partial \alpha_2}\Big\}.$$

Hence, solving this equation for $\frac{\partial i}{\partial \alpha_2}$ we have: $\frac{\partial i}{\partial \alpha_2}=-\frac{N_{21}'+N_{22}'}{D'}$,
where: $N_{21}'=M_2'a_{21}W_{21}(1+i)^{t_{21}}$, $N_{22}'=c_{21}a_{11}W_{11}(1+i)^{t_{11}+t_{21}}(1+\alpha_1)$.

Now, on the one hand $N_{21}'\geqq$, because $a_{21}\geqq 0$, $W_{21}>0$, $i\geqq 0$ and $t_{21}>0$, while $M_2'>0$ as was proved above, and on the other hand $N_{22}'\geqq 0$, because $c_{21}\geqq 0$, $a_{11}\geqq 0$, $W_{11}>0$, $t_{11}>0$, $t_{21}>0$, $i\geqq 0$ and $\alpha_1\geqq 0$. But it is impossible for both N_{21}' and N_{22}' to be zero at the same time, because, whereas $(N_{21}'=0)$ presupposes $(a_{21}=0)$ and $(N_{22}'=0)$ presupposes $(c_{21}\cdot a_{11}=0)$, it is impossible for both a_{21} and $c_{21}\cdot a_{11}$ to be zero at the same time, since $L_1a_{11}-c_{21}\geqq 0$ and $c_{21}+a_{21}>0$. Hence, $N_{21}'+N_{22}'>0$, and consequently $\frac{\partial i}{\partial \alpha_2}<0$, because $D'>0$ as was proved above.

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It will almost be superfluous to note that the partial differentiation of equation (7) with respect to t_{11}, t_{21}, W_{21}, c_{11} and a_{21} in turn leads us to the same conclusion as that arrived at by the partial differentiation of equation (6) regarding t_{11}, t_{21}, W_{21}, c_{11} and a_{21} respectively. Therefore, only their mathematical treatment will be given below:

1) Partial differentiation of equation (7) with respect to t_{11}:

$$0=c_{11}(1+i)^{t_{11}}(1+\alpha_1)\log(1+i)+c_{11}t_{11}(1+i)^{t_{11}-1}(1+\alpha_1)\frac{\partial i}{\partial t_{11}}$$
$$+a_{21}W_{21}t_{21}(1+i)^{t_{21}-1}(1+\alpha_2)\frac{\partial i}{\partial t_{11}}$$
$$+(c_{21}a_{11}W_{11}-c_{11}a_{21}W_{21})(1+\alpha_1)(1+\alpha_2)\Big\{(t_{11}+t_{21})(1+i)^{t_{11}+t_{21}-1}\frac{\partial i}{\partial t_{11}}$$
$$+(1+i)^{t_{11}+t_{21}}\log(1+i)\Big\}.$$

Hence: $\frac{\partial i}{\partial t_{11}}=-\frac{N_{31}'+N_{32}'}{D'}$,

where: $N_{31}'=M_1'c_{11}(1+i)^{t_{11}}(1+\alpha_1)\log(1+i)$,
$N_{32}'=c_{21}a_{11}W_{11}(1+\alpha_1)(1+\alpha_2)(1+i)^{t_{11}+t_{21}}\log(1+i)$.

2) Partial differentiation of equation (7) respecting to t_{21}:

$$0=c_{11}t_{11}(1+i)^{t_{11}-1}(1+\alpha_1)\frac{\partial i}{\partial t_{21}}+a_{21}W_{21}t_{21}(1+i)^{t_{21}-1}(1+\alpha_2)\frac{\partial i}{\partial t_{21}}$$
$$+a_{21}W_{21}(1+i)^{t_{21}}(1+\alpha_2)\log(1+i)$$
$$+(c_{21}a_{11}W_{11}-c_{11}a_{21}W_{21})(1+\alpha_1)(1+\alpha_2)\Big\{(t_{11}+t_{21})(1+i)^{t_{11}+t_{21}-1}\frac{\partial i}{\partial t_{21}}$$
$$+(1+i)^{t_{11}+t_{21}}\log(1+i)\Big\}.$$

Hence: $\dfrac{\partial i}{\partial t_{21}}=-\dfrac{N_{41}'+N_{42}'}{D'}$,

where: $N_{41}'=M_2'a_{21}W_{21}(1+i)^{t_{21}}(1+\alpha_2)\log(1+i)$,

$N_{42}'=c_{21}a_{11}W_{11}(1+\alpha_1)(1+\alpha_2)(1+i)^{t_{11}+t_{21}}\log(1+i)$.

3) Partial differentiation of equation (7) respecting to W_{21}:

$$0=c_{11}t_{11}(1+i)^{t_{11}-1}(1+\alpha_1)\frac{\partial i}{\partial W_{21}}+a_{21}(1+i)^{t_{21}}(1+\alpha_2)$$
$$+a_{21}W_{21}t_{21}(1+i)^{t_{21}-1}(1+\alpha_2)\frac{\partial i}{\partial W_{21}}-c_{11}a_{21}(1+i)^{t_{11}+t_{21}}(1+\alpha_1)(1+\alpha_2)$$
$$+(c_{21}a_{11}W_{11}-c_{11}a_{21}W_{21})(1+\alpha_1)(1+\alpha_2)(t_{11}+t_{21})(1+i)^{t_{11}+t_{21}-1}\frac{\partial i}{\partial W_{21}}.$$

Hence: $\dfrac{\partial i}{\partial W_{21}}=-\dfrac{N_{50}'}{D'}$,

where: $N_{50}'=M_2'a_{21}(1+i)^{t_{21}}(1+\alpha_2)$.

4) Partial differentiation of equation (7) respecting to c_{11}:

$$0=(1+i)^{t_{11}}(1+\alpha_1)+c_{11}t_{11}(1+i)^{t_{11}-1}(1+\alpha_1)\frac{\partial i}{\partial c_{11}}$$
$$+a_{21}W_{21}t_{21}(1+i)^{t_{21}-1}(1+\alpha_2)\frac{\partial i}{\partial c_{11}}-a_{21}W_{21}(1+i)^{t_{11}+t_{21}}(1+\alpha_1)(1+\alpha_2)$$
$$+(c_{21}a_{11}W_{11}-c_{11}a_{21}W_{21})(t_{11}+t_{21})(1+i)^{t_{11}+t_{21}-1}(1+\alpha_1)(1+\alpha_2).$$

Hence: $\dfrac{\partial i}{\partial c_{11}}=-\dfrac{N_{60}'}{D'}$,

where: $N_{60}'=M_1'(1+i)^{t_{11}}(1+\alpha_1)$.

5) Partial differention of equation (7) respecting to a_{21}:

$$0=c_{11}t_{11}(1+i)^{t_{11}-1}(1+\alpha_1)\frac{\partial i}{\partial a_{21}}+W_{21}(1+i)^{t_{21}}(1+\alpha_2)$$
$$+a_{21}W_{21}t_{21}(1+i)^{t_{21}-1}(1+\alpha_2)\frac{\partial i}{\partial a_{21}}-c_{11}W_{21}(1+i)^{t_{11}+t_{21}}(1+\alpha_1)(1+\alpha_2)$$
$$+(c_{21}a_{11}W_{11}-c_{11}a_{21}W_{21})(1+\alpha_1)(1+\alpha_2)(t_{11}+t_{21})(1+i)^{t_{11}+t_{21}-1}\frac{\partial i}{\partial a_{21}}.$$

Hence: $\dfrac{\partial i}{\partial a_{21}}=-\dfrac{N_{70}'}{D'}$,

where: $N_{70}'=M_2'W_{21}(1+i)^{t_{21}}(1+\alpha_2)$.