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A CRITICAL NOTE ON D. H. ROBERTSON'S THEORY OF SAVINGS AND INVESTMENT (I)

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1. INTRODUCTION

The prices which are formed day after day in the market represent the resultant outcome of the decisions of various sellers and buyers — decisions, the contents of which are in turn dependent upon their estimate of what may happen in the future. Thus anticipations of future events can be regarded as one of the factors determining prices equally with available quantities of the elements of production, conditions of technological knowledge and the state of wants for goods on the part of the public. This consideration is of peculiar importance in the study of the process of the formation of prices under changing conditions, because the working of the *factor of anticipation* is particularly important at such time.

How; then, should the moment of anticipation be introduced into the analysis of economic changes? In the studies of those who seek to construct theoretically an economic dynamics from this point of view, it seems to me that we may distinguish two principles of approach — the first characterized by what may be called "*Period Analysis*" and the other "*Momentary Equilibrium Analysis*".¹⁾ Of these two types of

1) The sense in which the expressions "period analysis" and "momentary equilibrium analysis" are here used will be explained later on. At present it is sufficient to say that the term "period analysis" is here used in practically the same sense as it is used by B. Ohlin. Ohlin's use of this term can be deduced from the following extract: "He (i. e. Myrdal) there (i. e. in Ch. V. of "Der Gleichgewichtsbegriff als Hilfsmittel in der geldtheoretischen Analyse," in *Beiträge zur Geldtheorie*, published by F. A. Hayek, 1933) works out in some detail the vitally important distinction between "looking forward" and "looking backward," and show its significance more clearly

analysis, I shall, in this study, attempt to clarify the character of the first and to define its area of validity.

With this end in view I propose to consider D. H. Robertson's method of analysing the fluctuation of prices. In Part 1, I shall restate in terms of period analysis the analysis which he makes of the process of capital formation in his *Banking Policy and the Price Level* (1st ed. 1926, 3rd ed. 1932), thereby offering an elucidation of the essential characteristics of the so-called "period analysis." In Part 2, I shall explain the theory of savings and investment which

than he had done before in Swedish writings and discussions. This analysis of income and capital values with the aid of *ex-post* and *ex-ante* concepts is independent of the timeless equilibrium construction which is expounded in the paper and which is similar to that used in the book of 1927. In fact, it seems most useful in a period analysis of the type which Lindahl and myself are using, while Myrdal views it with some scepticism." (B. Ohlin, "Some Notes on the Stockholm Theory of Savings and Investment," *Economic Journal*, March, 1937, p. 55.) He also says: "With the exception of Myrdal all (economists of the Stockholm School) use a period method of analysis. In this respect, the procedure is similar to D. H. Robertson's." (B. Ohlin, *ibid.* p. 58.) What is here called period analysis is also identical with E. Lundberg's "sequence analysis." (Erik Lundberg, *Studies in the Theory of Economic Expansion*, 1937.)

Now, Myrdal, who repudiates this method of period analysis, contends that students of economic dynamics have to analyse first of all the "tendency of variation which comes into being in the state of momentary price formation, and which is conditioned by anticipations dominant in this state" (Myrdal, *Der Gleichgewichtsbegriff u. s. w. ibid.*, SS. 430-431, 389-390). It is certainly opposed to the point of view of period analysis, although I do not think that this fundamental point of view is consistently carried through in his theory of monetary equilibrium. It seems clear that if he maintains this point of view rigidly, he will eventually have to amend his theory in favour of R. Frisch's view, as outlined in his "Statikk og dynamikk i den økonomiske teori," in the *National-økonomisk Tidsskrift*, 1928, p. 321 ff., discarding the distinction between "*ex-ante*" and "*ex-post*" calculations. This latter point of view is here called momentary equilibrium analysis; since, according to this viewpoint, the market situation is incessantly in a state of dynamic equilibrium in the sense that sales and purchases are always equal in individual commodity markets and that, moreover, there exists a certain relationship of interdependence between individual commodity markets; the actual process of economic development is regarded as a *continuous* series of these momentary dynamic equilibrium situations.

he set forth in his "Saving and Hoarding", in the ECONOMIC JOURNAL, Sept. 1933, and in his "Industrial Fluctuation and the Natural Rate of Interest," in the ECONOMIC JOURNAL, Dec. 1934, regarding this set of studies as an illustration of the application of the method of period analysis, and I shall hope to demonstrate that it is invalid for the analysis of the changing process of price formation, when we come to consider an economic system which involves the transactions of securities (or the "*financial circulation*" of J. M. Keynes). In the course of this exposition and also at the end of Part 1, I shall refer to the controversy between E. Lindahl and G. Myrdal on the problem of monetary equilibrium.

PART 1. D. H. ROBERTSON'S PERIOD ANALYSIS AS
SET FORTH IN HIS "BANKING POLICY AND
THE PRICE LEVEL"

2.

D. H. Robertson's analysis begins with the discussion of a highly hypothetical social economy. In this society, not only are population, productivity and state of wants constant but there exist neither durable capital goods nor any stock of goods, what exists being the stream of goods only. In such a social economy, the quantity theory of money proposed by the Cambridge School, based on the concepts of real balances, may easily be applied in the following form. If M represents the total amount of cash balances which people hold in their purses, safes, etc., and P the price-level of consumers' goods, then the "*real balances*", or "*real hoarding*" viz. the total quantity of goods which people wish to hold in the form of money, H , may be defined thus:

$$(1) \quad H = \frac{M}{P}$$

Now, Robertson entitles the duration of time which elapses, on the average, from the departure of each piece of money from income to its return into income (i. e. the reciprocal

of the income velocity of money) the "*Period of Circulation of Money*" and denotes it by K . If the total amount of production of our community per this period of circulation is T , then :

$$(2) \quad H = T,$$

in so far as our social economy is in a stationary state. In the following study, it is assumed, for convenience' sake, that the length of the period of the circulation of money K is constant.

Besides, the analysis is built on the following assumptions :

(1) Economic development can be divided into a series of economic changes during the finite and indivisible unit periods, and each unit period being regarded as of short duration is called "*Day*."¹⁾

(2) All people make their plans concerning their economic activities of "today" and after at the beginning of each period, assuming that the prices which ruled on the market of yesterday rule also in the market of today and after, and during this period they spend their money according to these plans, even when their expectations are disappointed by the actual pricing in the market of today.²⁾

(3) Even if prices change and today's prices tend to differ from yesterday's, this change does not affect the producer's supply of products. That is to say, the supply is perfectly inelastic, whereas assumption (2) may be taken to mean that the elasticity of demand is equal to 1.³⁾

(4) Corresponding to the above-mentioned assumption that there are no durable capital goods in the economic system as considered here, it is further assumed that there are no "securities" as representing title to claims against

1) Robertson, *Banking Policy and the Price Level*, p. 59. ditto, "Saving and Hoarding," *Economic Journal*, Sept. 1933, p. 399.

2) Although this assumption is not clearly stated, its indispensability is evident, when one observes carefully his whole argument. Herein lies an obstacle to the comprehension of his theory. While E. Lindahl takes the same view-point as Robertson, he states this assumption explicitly. Cf. his *Money and Capital*, 1939, p. 94.

3) Robertson, "Saving and Hoarding", *ibid*, p. 401.

the future yield. Consequently, wealth cannot be held in the form of goods or securities. It can be held only in the form of money. Therefore, when the public is using its income, it has the choice between spending and saving, but, when it has accumulated wealth by saving, it has no option as to the form in which wealth should be held. In parallel with this, it is assumed that no new investments are made.

(5) When there occur any changes in the external conditions of the economic system, the real balances of the public tend to depart from their equilibrium value. But the public's wants for real balances do not change and it endeavours to maintain them at a normal level. Under the above-mentioned assumption (3), this effort of the public to restore balances to their old normal level constitutes the main reaction designed to stimulate the tendency of the economic system towards equilibrium. The way in which this comes about will be precisely defined later.

(6) (a) It is assumed that people suppose that they can produce today the same amount of output as they produced yesterday, having no anticipation of the occurrence of technological changes. Accordingly, the income which they anticipate at the beginning of today to earn today, is, under assumption (2), equal to the product of yesterday's price-level by their production. It is therefore equal to the income received yesterday. Thus the income which is disposable to-day is assumed to be equal to the income received yesterday. (b) The income received yesterday is assumed to be today's disposable income. No matter which of these two alternative hypotheses, (a) or (b), is taken, the result will be the same, but their economic meanings are not necessarily identical. D. H. Robertson seems to favour the second hypothesis (b),¹⁾ but in the present article I shall employ the first hypothesis (a) as the basis of argument, in order to secure simplicity of exposition.

1) Robertson, "Saving and Hoarding," *ibid*, p. 399.

(7) The initial position of our economic system is assumed to be stationary. In other words, the economic situation at the starting point of the economic development considered here is the stationary state. Here, equation (2) is valid. Again, if F is the "daily" *stream of money* poured into market to buy commodities (i. e. the total amount of purchasing power which is spent in the market to buy commodities), and O is the "daily" *stream of output* which is flowing from production and poured into market to meet the flow of money, then obviously :

$$(3) \quad F = M/K; \quad O = T/K.$$

And, by the aid of (1) and (2), we obtain,

$$(4) \quad F = P \cdot O.$$

Since, in this equation, the expression F on the left side is the product of the quantity of money by the velocity of circulation of money, (see equation (3), in which $1/K$ represents the income velocity of money), this equation (4) is simply a variant of the quantity equation of the Fisher type. If E is the *total money income* per "day" which producers receive by the sale of their outputs (or as returns for their work), then :

$$(5) \quad E = F,$$

since there are no savings in the stationary state. As this state is considered as the "base year" when we trace the process of economic development, we shall hereafter regard the values of the various variables in this state as given. In the following, all capital letters such as M, H, P, T, E , etc. denote constants in this sense.

Taking such a state of equilibrium to be the state of the "zero" day, so to speak, D. H. Robertson traces, step by step, the course of changes which will occur when the Government makes an injection of money into the economic system day after day by the amount of X/K , from the first day onward, under the assumptions enumerated above. In other words, these assumptions have framed the problem of credit expansion to measure precisely the effects of inflation

on the prices of commodities as well as on the formation of real capital. Now the fundamental problem of how the price-level is determined on any given day, can be solved as follows. On the one hand, the stream of money on that day is equal to the product of the price-level in question by the amount of daily output, which is assumed to be constant. (See also equation (12) which appears later). On the other hand, according to the above assumptions (2), (4), (5) and (6), this volume of the stream of money for the day is determined by the conditions of the preceding day and the day's inflation, both of which are the knowns of the problem considered here. Thus we can determine the daily stream of money, and consequently the price-level in question can be found, by dividing it by the constant daily output.

Robertson analyses price fluctuations day by day by this method, and in the following pages I shall attempt to restate his analysis in the form of period analysis.¹⁾

For such a purpose it is necessary to use certain notations. Let us suppose that :

$m(t)$ = the *Total Amount of Stock of Money* which the public holds on "t" day,

$h(t)$ = the *Total Amount of Real Hoarding* of the public on "t" day,

$p(t)$ = the *Price-level of Commodities* in the market on "t" day,

$f(t)$ = the *Total Stream of Money* which is poured into the market by the public as well as by the Government to purchase commodities during "t" day,

$e(t)$ = the *Total Amount of Money Income* which the public receives by the sale of its outputs during "t" day,

$c(t)$ = the *Total Consumption Expenditure* which the public makes during "t" day,

1) See Robertson, *Banking Policy and the Price Level*, Appendix to Chapter V. § 1.

$l'(t)$ = the *Total Amount of Automatic Lacking* of the public during "t" day, (defined later),

$l''(t)$ = the *Total Amount of Induced Lacking* of the public during "t" day, (defined later).

Further, these variables may denote either something that has actually happened (i. e. *retrospective magnitudes*), or the estimates on the part of the public of what may happen in future (i. e. *prospective magnitudes*). In order to distinguish these two cases, I will use R. Frish's notation; in the former case, that is to say, when variables take retrospective values, the symbol \sim will be put above the letters denoting such variables, and letters with no such symbol above them will be supposed to represent variables which can take only prospective values.¹⁾ For example, $p(t)$ is the price which is expected by the public to rule the market on "t" day, $\tilde{p}(t)$ denotes the price-level which actually rules "t" day.

Now, under the assumptions already announced, we are enabled first of all to determine the public's estimate of the prices for the first day, made at the beginning of the day. People have no doubt that the price-level P , which ruled on the previous zero day, will rule on the first day and afterwards as well. They accordingly believe that there will be no change in their money income. On the basis of this belief, they plan the same economic activity as on the previous day, and attempt to purchase products by spending money to the amount of planned consumption expenditure $c(1)$, which is equal to their former income E . But, on the first day, there appears unexpectedly the additional flow of money X/K from Government sources, which competes with the public's expenditure in the market. Thus, the total flow of money poured into the market on the first day $\tilde{f}(1)$ is the sum of these two flows emanating from two different sources. Thus,

1) Ragnar Frisch, *Et Generelt Monetært Begreb og Symbolsystem*, (Mimeographed Lectures), Oslo 1935.

$$\tilde{f}(1) = \frac{M}{K} + \frac{X}{K} = \frac{(M+X)^1}{K}$$

As production is unchanged, despite this increase in purchasing power, prices must rise. According to assumption (3), this rise in prices does not affect the supply of goods. Consequently, as $\tilde{f}(1) = \tilde{p}(1) \cdot O$, the first day's price-level $\tilde{p}(1)$ will find the following level:

$$\tilde{p}(1) = \tilde{f}(1)/O = (M+X) : T.$$

This actual price-level evidently disappoints public expectations. At the end of the first day, when people make out their accounts for the day, they will find that the quantity of good which they have actually secured by spending money to the amount of E , falls short of the quantity which they intended to obtain by this expenditure. Robertson characterises as "*Automatic Stinting*" the decrement of real value in consumption caused by a rise in prices in consequence of the expansion of the flow of money; the cause of which may consist in either the additional issue of currency by the Government or banks or the diminished desire of individuals to hold real balances. To the negative automatic stinting arising from the opposite course he gives the name "*Automatic Splashing*". Accordingly, on the first day,

$$\text{Automatic Stinting} = \frac{E}{P} - \frac{E}{\tilde{p}(1)} = \frac{1}{K} \frac{X}{M+X}$$

Since, the first day's planned consumption of the public is, in terms of money as well as in terms of goods, equal to its first day's expected income, as shown above; and the real value of this planned consumption is consequently equal to current output, which is, in turn, equal to the real value of the actualised income; it is obvious that the real value

1) In this case, the increase in the day's flow of money is relatively greater compared with the increase in the quantity of money. This may be considered to imply the increase of the velocity of the circulation of money. Consequently, the period of the circulation of money K , which we have assumed to be constant, must be taken to be the normal one.

of the first day's actual consumption of the public comes short of the current output. Since this result is not spontaneously intended by the public, it is easy to see that the phenomenon of so-called "forced saving" („*erzwungenes Sparen*“) appears here. When, in automatic stinting, the real value of consumption falls short of the current output, Robertson calls this deficiency "Automatic Lacking." If the automatic lacking during the first day is denoted by $\bar{l}(1)$,

$$\bar{l}(1) = O - \frac{E}{\bar{p}(1)} = \frac{T}{K} \frac{X}{M+X}.$$

In this settlement of accounts, people will also find that, on the one side, the stock of money $\bar{m}(1)$ has increased, compared with the stock of money of the previous day, to

$$\bar{m}(1) = M + (X/K);$$

while, on the other side, the real hoarding $\bar{h}(1)$ has decreased, in consequence of the rise in prices, to

$$\bar{h}(1) = \frac{\bar{m}(1)}{\bar{p}(1)} = \frac{M + (X/K)}{\frac{M+X}{T}} = T - \frac{X}{K} \frac{1}{\bar{p}(1)} (K-1).$$

Here, the public reacts to the given economic change according to assumption (5). When the public plans for the second day, it will strive to improve this situation so that its real hoarding may be restored to the old normal level H . In making plans for the second day, the public expects that the price-level for the second day $\bar{p}(2)$ will be equal to the actual price-level for the first day $\bar{p}(1)$. Thus, $\bar{p}(2) = \bar{p}(1)$. Similarly, the price-level of the third day, of the fourth day and so on, as anticipated on the day, is also estimated to be equal to $\bar{p}(1)$ (according to assumption (2)). Accordingly, people may conclude that in order to make up for the first day's decrease of real hoarding to the extent of $\frac{1}{\bar{p}(1)} \frac{X}{K} (K-1)$, they have only to increase the stock of money by $\frac{X}{K} (K-1)$. But Robertson assumes here, for the sake of simplicity, that for this purpose they would attempt to

increase the stock of money to the desired extent in course of $(K-1)$ days by hoarding daily X/K out of income.¹⁾ Robertson calls the restriction of consumption which takes place in this way "*Induced Lacking*", measuring it in terms of goods. Induced lacking differs from automatic lacking in that it is intended by the public, but it is nevertheless one kind of forced saving, since its cause is traceable to inflation by the Government or banks, and does not consist in an alteration in the public's demand for real balances.

Because of this induced lacking, the planned consumption expenditure for the second day falls, in terms of money, below the anticipated income. In other words, the public plans to save. But the market situation on the second day is formed in exactly the same manner as on the first day, being influenced by the second day's economic plan, which has been made at the beginning of the day. The analysis will be extended day by day in the same way, as shown by the table on the next page.

3.

Let me expound briefly the fundamental of this so-called "step-by-step method of analysis."

As has already been shown, today's economic plan on the part of the public can be determined, when the market situation of the preceding day is given. I shall explain this process by the "closed determinate system of equations," as it is called by R. Frisch,²⁾ that is, by the system of equations which contains just as many equations as unknowns. To

1) Meeting Pigou's criticism, Robertson answers that this hypothesis is simply for the purpose of simplification. cf. ditto, "Saving and Hoarding," *ibid.* p. 409. We do not attach importance to this point either. It is not a matter of principle.

2) R. Frisch: "Propagation Problems and Impulse Problems in Dynamic Economics," in *Economic Essays in Honour of Gustav Cassel*. See also J. Tinbergen: "Suggestions on Quantitative Business Cycle Theory," *Econometrica*, Vol. III, No. 3, July, 1935.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)		
	Price-level <i>p</i>	Public's Outlay <i>c</i>	Money Injection	Flow of Money=Income <i>e & f</i>	Stock of Money <i>m</i>	Real Hoarding <i>h</i>	Lacking		
							Sponta- neous <i>l</i>	Automatic <i>l'</i>	Induced <i>l''</i>
" <i>t</i> " Day									
0 { Prospective Values	} M/T	M/K	—	M/K	M	T	0	0	0
Retrospective Values									
1 { Prospective "	$(M+X)/T$	"	X/K	$(M+X)/K$	$M+(X/K)$	$T - \frac{X}{K} \cdot \frac{1}{M+X/T} (K-1)$	0	$\frac{T}{K} \cdot \frac{X}{M+X}$	—
Retrospective "									
2 { Prospective "	"	"	0	"	$M+2(X/K)$	$T - \frac{X}{K} \cdot \frac{1}{M+X/T} (K-2)$	0	—	$\frac{X}{K} \cdot \frac{1}{(M+X)/T}$
Retrospective "									
.....
.....
K { Prospective "	"	"	0	$(M+X)/K$	$M+X$	T	0	—	$\frac{X}{K} \cdot \frac{1}{(M+X)/T}$
Retrospective "									
$K+1$ { Prospective "	$(M+2X)/T$	"	X/K	"	"	"	0	—	0
Retrospective "									
.....
.....
$K+1$ { Prospective "	$(M+2X)/T$	"	X/K	$(M+2X)/K$	$M+X+(X/K)$	$T - \frac{X}{K} \cdot \frac{1}{(M+2X)/T} (K-1)$	0	$\frac{T}{K} \cdot \frac{X}{M+2X}$	—
Retrospective "									
.....

begin with, assumption (2) can be represented symbolically by the following equation:

$$(6) \quad \tilde{p}(t-1) = p(t) = (\text{price-level anticipated at the beginning of "t" day, for the days subsequent to "t" day}).$$

Again, by assumption (6),

$$(7) \quad e(t) = p(t) \cdot O \quad \text{or} \quad e(t) = \tilde{e}(t-1).$$

Further, induced lacking $l''(t)$ is defined, according to the definition and assumption explained above, by the equation:

$$(8) \quad l''(t) = \frac{1}{K-(t-1)} \left\{ T - \frac{\tilde{m}(t-1)}{p(t)} \right\}^{1)}$$

Since income is equal to the sum of consumption and savings,

$$(9) \quad e(t) = c(t) + p(t) \cdot l''(t).$$

These four equations are sufficient to determine the values of four unknowns, $p(t)$, $e(t)$, $l''(t)$ and $c(t)$. Thus, the public determines its today's plans correspondingly to the market situation of the preceding day. In this connection the rôles of assumptions (2), (5) and (6) are noteworthy. Of these assumption, (5) is obviously based on the idea of the so-called "cash-balance approach" in the theory of money, and only by its aid can the working of money in the process of economic changes be grasped in its fundamental aspect²⁾.

1) If $t > K$, taking t' to be the integral remainder obtained by dividing t by K , $l''(t)$ is expressed by

$$(8') \quad l''(t) = \frac{1}{K-(t'-1)} \left\{ T - \frac{\tilde{m}(t-1)}{p(t)} \right\}$$

2) It is hardly necessary to say that the characteristic of the "cash-balance approach" is that it attempts to explain the demand for money by the propensity of individuals to hold real balances. From this point of view, when the equilibrium is disturbed, such reactions against disturbances as correspond to the given propensity to hold real balances must emerge from the sphere of demand for money. Thus, the essential characteristic of the phenomenon of "induced lacking" lies in this reaction from the monetary side to the disturbance of the equilibrium. Due heed of appreciation must be accorded to Robertson for his effort to elucidate this reaction on the part of money by the help of this concept of "induced lacking".

Assumptions (2) and (6) take into account the effects of the factor of anticipations on the planning of the public. Equations (8) and (9) show clearly, though in a simple form, how the public's plan of consumption and saving is dependent upon the factor of anticipations.

The daily market situation is evidently the resultant of the various decisions of the public based on anticipations, and the external factors given for the day concerned. Resorting to the closed system of equations also, we can describe this process of the formation of the actual market situation in a similar manner. The public spends money according to its plan, regardless of the actual conditions of the market, according to assumption (2). Accordingly,

$$(10) \quad \bar{c}(t) = c(t).$$

This flow of money is combined with the money emanating from the Government source, and the total amount of the purchasing power

$$(11) \quad \tilde{f}(t) = \bar{c}(t) + (X/K),$$

is poured into the market to purchase output O . Consequently, the price-level is determined so that

$$(12) \quad \tilde{f}(t) = \tilde{p}(t) \cdot O.$$

Thus, the public as producers receive income as determined by the equation:

$$(13) \quad \tilde{e}(t) = \tilde{p}(t) \cdot O.$$

Accordingly, the amounts of the stocks of money and of the real hoardings of the public are given respectively by the equation:

$$(14) \quad \tilde{m}(t) = \tilde{m}(t-1) + (\tilde{e}(t) - \bar{c}(t))$$

$$(15) \quad \text{and} \quad \tilde{h}(t) = \frac{\tilde{m}(t)}{\tilde{p}(t)}.$$

Further, automatic lacking $\tilde{l}'(t)$ is defined by the equation:

$$(16) \quad \tilde{l}'(t) = O - \frac{\bar{c}(t)}{\tilde{p}(t)}$$

Here, again, we have as many equations as unknowns. In this case, as in the previous case, equation (10) is justified

by the latter half of assumption (2), and equation (12) by assumption (3). In short, the daily market prices are formed as is shown above, under the assumptions that both the supply and the expenditure are not affected in the least by the changing prices in the market of the day concerned.

To sum up, Robertson's system is one in which it is shown that the anticipations of the public are a determining factor of the market prices. In fact, by the aid of the equations from (7) to (12), the actual price-level $\hat{p}(t)$ can be represented as the function of the anticipated price-level $p(t)$. In this case, of course, the factor of anticipation is introduced in a highly simplified manner, and therefore this point is apt to be lost sight of. It must be reminded, however, that Robertson's analysis evidently takes account of the working of anticipation in the pricing process under changing conditions.

Up to this point I have endeavoured to elucidate the substance of the method of the so-called "period analysis." I shall now attempt to define its general characteristics as follows:

(1) Economic development is divided into a series of economic changes during definite and indivisible "*Periods*" and during each period determinate and unchangeable prices rule. Consequently, if we represent graphically the price-level as a function of time, we get a discontinuous stair-like curve.

(2) At the beginning of each period, individuals plan their economic activities in accordance with their anticipations of future events. This is what is called *ex ante* calculation. The action of individuals during this period is considered as the execution of these plans without revision.

(3) During the period, as the resultant of the planned action of individuals and concurrent external circumstances, a certain definite market situation comes out. What is called *ex post* calculation is performed in regard to this actual market situation. Corresponding with the fact that various economic quantities assume definite values at given

points of time, the mechanism of the formation of prices in this market is described by a system of equations which contains as many independent equations as unknowns.

(4) The process through which individuals plan, on the basis of the knowledge of events that have actually happened, is also described by the closed determinate system of equations.

(5) Period analysis aims at tracing causally the process of economic change by the above-mentioned method, starting from certain initial conditions of the economic system.

4.

Up to this point I have been restating Robertson's analysis as set forth in his *Banking Policy and the Price Level*, in terms of "period analysis." Robertson adopted his method because "the internal mechanics, so to speak, of a process of inflation are almost as hard to visualize as those of the atom, and seem to require the same kind of hypothesis of discontinuous motion."¹ His analysis was found by many to be quite difficult of comprehension, but it is now evident that it is, in its essentials, nothing but a classic construction of what is now called "period analysis". As soon as one regard it as a variant of period analysis, its nature can be grasped accurately and easily.

Now, in so far as his theory is constructed according to the method of period analysis, it is obvious that of the set of assumptions referred to, some can easily be eliminated, so that a more general theory may be expounded. But I shall here confine attention to the method of period analysis itself, instead of extending the scope of discussion by such generalizations.

In examining the method of period analysis, its strong point must be stressed, first of all. It must be recognized that it serves to prevent, as Robertson says, "the peril of confounding causes with results and processes of change

1) Robertson, *Banking Policy and the Price Level*, p. 49.

with states of abnormality."¹⁾ Secondly, it has brought the analysis of economic changes a step nearer to reality by introducing anticipations as a determining factor in the course of economic change and furthermore by treating them as dependent on objective circumstances.²⁾ Thirdly, it is noteworthy that he has theoretically reproduced the process of economic change by means of the closed determinate system of equations.

As has already been made clear, however, Robertson's period analysis is based on many assumptions, some of which may be eliminated, while others cannot be. Those assumptions which cannot be eliminated must be regarded as inherent in period analysis itself. Among these assumptions we must pay attention particularly to the assumptions which are required to render the quantity equation valid. As has already been shown, in order to make the quantity equation (12), which is indispensable in the analysis, valid, assumptions (2) and (3) are necessary. It is needless to contend that these assumptions are unrealistic; because, when prices change and sellers and buyers are disappointed in their expectations, they are bound to recast, sooner or later, their plans for selling and buying, their supplies and demands being decided according to these new plans, leaving their former plans unexecuted. The equality between the current supply of and the current demand for goods in the daily market, which is an undeniable fact, is brought about only by these adaptations of the people's attitudes to changing prices. Now, is it possible to replace this assumption by a more realistic assumption, namely, that the current prices are determined in a manner to insure the equality between supply and demand? Can the method of period analysis be brought a step nearer to the actuality by introducing such an assumption?

1) Robertson, "Industrial Fluctuation and the Natural Rate of Interest", *Economic Journal*, Dec. 1934, p. 650.

2) Compare E. Lunderg's criticisms of G. Myrdal and J. M. Keynes in his *Studies in the Theory of Economic Expansion*, p. 185.

The answer to this question is negative ; because, when we discard the latter half of assumption (2) and assumption (3) and accordingly rewrite equation (10) into another form, it brings us inevitably to an abandonment of reliance on the method of a "period analysis", such as is defined above. I shall now proceed to explain this point in greater detail. To begin with, period analysis is based on the distinction between and the combination of prospective magnitudes and retrospective magnitudes (i. e. *ex ante* calculations and *ex post* calculations). Furthermore, it is the distinctive characteristic of the method of period analysis that the factor of anticipation is introduced in such a way that anticipations shall influence the process of economic change through the execution of plans based on them without any modification of plans whatsoever. In the system of equations already suggested, equation (10) has a decisive significance, being the one which defines the relationship between prospective magnitudes and retrospective magnitudes. In period analysis the factor of anticipations always plays a part in the determination of actual prices through such a relationship. If, therefore, the latter half of assumption (2) is discarded and equation (10) is abandoned, this connection is lost to the system. More precisely, if the expression $\tilde{c}(t)$ in equation (10) is regarded as the function of the current prices (i. e. prices which are actually ruling and which are changing in order to ensure the equality between supply and demand in the market), $c(t)$, as prospective magnitude, will cease to have anything to do with the actual process of the formation of prices. According to the point of view of what we call "momentary equilibrium analysis", such fixed relationships do not exist any longer. In the momentary equilibrium analysis, sellers' and buyers' plans are regarded as being subject to incessant alteration, even during a single period, owing to changes of prices, which come about to ensure an equality between supply and demand in the market and, in their turn, act upon sellers and buyers to amend their attitudes during the period ; so that expenditure actually performed

during the period can differ from the prospective expenditure for the period as estimated at the outset. Hence we cannot equate the expression $\bar{c}(t)$ with the expression $c(t)$. The manner in which the factor of anticipation is introduced into momentary equilibrium analysis is in this respect different from the way of introducing it into period analysis.

In short, the fundamental nature of period analysis consists in the consideration that the prices of commodities rise or fall in direct proportion to the increase or decrease in the amount of purchasing power, completely disregarding the selfevident fact that the price of a commodity is determined at the point where the supply curve of it and the demand curve for it intersect each other. This being the case, although period analysis may be of much value in the study of changes in the value of money, there are distinct limits to its validity. It must further be admitted that, even in the study of changes in the value of money, the area of application is restricted considerably by these limitations. In Part 2, in order to demonstrate these limitations, I shall consider the case where assumption (4) is discarded, that is, where the public is supposed to have the option to choose the forms in which the accumulated wealth is to be held and I shall criticize Robertson's treatment of this case as based on the method of period analysis, discussing appropriately the problem of the equilibrium between savings and investment, which J. M. Keynes deals with in *A Treatise on Money*.

5.

I will take the present opportunity to examine, the controversy which took place between E. Lindahl and G. Myrdal in Sweden in connection with the problem of monetary equilibrium in its formal aspects.¹⁾ Lindahl, in

1) E. Lindahl, "Prisbildningsproblemets upplägning från kapitalteoretisk synpunkt," *Ekonomisk Tidskrift*, 1929, No. 2. ditto, *Penningspolitikens medel*, 1930. ditto, *Studies in the Theory of Money and Capital*, 1939.

his *Penningpolitikens medel* (1930)¹⁾, attempted an elaborate reconstruction of the famous Wicksell's theory of the "cumulative process". In doing so, he assumes that changes in external conditions and in prices occur at the moment of transition from one period to another only, and that within any period the economic system is in state of equilibrium as "economic statics" describes. In other words, he regards the process of economic development as a series of momentary and timeless changes occurring at the moments of transition which are placed sporadically and equidistantly between tranquil "periods of equilibrium." This "equilibrium method", as he calls it, is similar to Robertson's method of analysing the process of inflation, the only difference being that it explicitly states the relevant assumptions in making clear its construction. Now, Myrdal comments on such a procedure of Lindahl as follows:

"Lindahl divides the dynamic process into two moments which appear turn and turn about incessantly. Let these be called here *Moment A* and *Moment B*, for convenience' sake. The *Moment A* is timeless; it simply represents transition between two successive *Moments B*. On the contrary, the *Moment B* is not timeless, though it is short in duration. The *Moment A* contains all the changes not expected by individual entrepreneurs. In other words, it contains the total developments, for it is so postulated that perfect equilibrium governs between prices, supply and demand, in the *Moment B*.

"Now,..... timeless *Moment A*, in which appear all price changes caused by changes in the external conditions, arises in the starting situation itself, and this timeless process is, in this case, left unanalysed. The *Moment B*, in which it is

G. Myrdal, *Prisbildningsproblemet och föränderligheten*, 1927. ditto, "Om penningteoretisk jämvikt," *Ekonomisk Tidskrift*, 1931, No. 5-6. ditto, "Der Gleichgewichtsbegriff als Instrument der geldtheoretischen Analyse," in *Beiträge zur Geldtheorie*, edited by F. A. Hayek, 1933. ditto, *Monetary Equilibrium*, 1939.

1) Lindahl, *Money and Capital*, Part II, "The Rate of Interest and Price Level" is an English translation of the main part of the work.

assumed that the equilibrium governs and no unexpected changes occur suddenly, is assumed to appear later. Lindahl attempts to consider the equilibrium in the (capital) market at this moment, but this equilibrium is actually assumed to exist there. But, as the cause of the changes contained in the first Moment A continues to exist, a new Moment A suddenly and unexpectedly succeeds the Moment B, and all changes again occur herein timelessly and exhaustively. This Moment A is then succeeded by another Moment B, and in this way the process is repeated *ad infinitum*.

"This method is fallacious and not valid in any case. Above all, it leaves unexplained what ought to be explained, as has already been pointed out. It may further be asked why the Moment B passes into a new Moment A and why it does not arise at the beginning. Again, how can the Moment B develop out of the Moment A and how is it that the disturbances of the equilibrium do not continuously occur? Are incidents which are relevant to the individuals' income or outlay not unanticipated changes?"

"The only possible way of studying the process in question is evidently to study the tendency in a certain state or the movement from one state to another (that is, during a certain period)."³⁾

Indeed, the process of the formation of prices in the market is one thing, and the process of individuals' decisions as regards selling and buying and of the actual performances of such decisions is another thing; these two processes can be distinguished conceptually. I acknowledge validity of Lindahl's attempt in this respect. On the other hand, however, it is obvious that these two processes actually stand in a complex interrelationship to each other and also that the variation of prices is the result of the interactions of these two processes. Thus it is the fundamental for the study of economic dynamics to make clear these interactions.

3) Myrdal, "Om penningteoretisk jämvikt", *ibid*, pp. 228-229. See also his *Monetary Equilibrium*, pp. 121-122.

In Lindahl's analysis these interactions are left out of consideration. This is the point where difficulty arises, as Myrdal has pertinently remarked. Moreover, in order to connect appropriately the Moment A with the Moment B, these interactions must be elucidated. In short, as Myrdal asserts, Lindahl's scheme is invalid "for it is just the intervening changes — the deviations from the earlier anticipations — which are of interest in monetary analysis, and obviously they cannot be comprehended by this method."¹⁾

Let us study this point a little further. Between the formation of prices and the determination of the attitudes of individuals in relation to supply and demand, there exist interactions capable of being repeated infinitely during any period, however short it may be. This fact is obviously disregarded in Lindahl's assumption of the "period of equilibrium." Now, in order to take this fact into consideration, we must use concepts of supply and demand which differ radically from those of Lindahl's. How should we formulate the concepts required for this purpose? I shall answer this question, in criticizing Lindahl's procedure. As already pointed out, Lindahl assumes the existence of the equilibrium during the period in the sense that the equality between supply and demand is maintained in each period. But the concepts of "supply" and "demand" in this assumption are very ambiguous. Now, the circumstance that this "equilibrium method" is originally intended for the application of the statical apparatus to economic dynamics, implies that the notions of supply and demand must necessarily be those dealt with in the static theory. However, it is very doubtful whether, when the process of economic change comes to be the object of our study, we can achieve success merely by the application of such statical concepts. In fact Lindahl himself is obliged to use these concepts in a manner which differs from their use in the static theory, (e. g. in the case of unemployment). Therefore, this assumption cannot be consistent with

1) Myrdal, *Monetary Equilibrium*, p. 122.

the contents of his theory itself. It is clear that the dynamic concepts of supply and demand functions are indispensable in economic dynamics.¹⁾ Furthermore, owing to the circumstance that, when this assumption is made, the static concepts of supply and demand are mistakenly regarded as applicable also in economic dynamics without modification, the process of the determination of supply and demand is left unanalysed, so that we cannot use these concepts as fundamental tools in economic dynamics. Thus the assumption that the economic system is in a state of static equilibrium during each period must be abandoned and dynamic concepts of supply and demand functions, differing completely from the static concepts, must be formed. From such a point of view, if we define the concepts of supply and demand in conformity with the actualities of a changing economy and seek to render them applicable in economic dynamics, we must take into account, in the first instances, the fact that, in actuality, what is purchased is always identical with what is sold. But such self-evident facts of equality as between the current supply of and the current demand for goods is nothing more than the starting-point of our study. Next, we must inquire into the technical reasons and the psychological motives which lie behind the current supply and the current demand, and determine the shapes and positions of the respective curves. In short, economic dynamics must have for its starting-point the self-evident fact that the price is determined so as to balance supply and demand; and we must begin with the analysis of the process through which the quantity of supply and demand are determined.²⁾

If the assertion which I have so far urged is correct, it is obvious that neither the method employed by Myrdal in developing his theory of monetary equilibrium nor the

1) It is the merit of R. Frisch that he has made clear the nature of of these concepts. cf. R. Frisch, "Statikk og dynamikk i den økonomiske teori," *Nationaløkonomisk Tidsskrift*, 1928, pp. 322-349.

2) It seems to me that this is the point of view which J. R. Hicks has been taking recently. See J. R. Hicks, *Value and Capital*, 1939, p. 131. I find the

“disequilibrium method” which Lindahl advocates in his recent work, *Money and Capital*, Part 1, is tenable. Let us first deal with Myrdal’s method. The point at issue refers to the famous distinction between the *ex ante* calculation and the *ex post* calculation. Now, this distinction is only valid, when every plan which individuals have once determined is carried out without modification during the period concerned. This distinction loses its significance, if errors of expectation are continually rectified and the plans once established are altered from time to time during the period concerned. Now, if Myrdal had carried his criticism of Lindahl’s theory to its logical conclusion, he would have been obliged to acknowledge the continuous rectifications of errors, and therefore to conclude the incompatibility of this distinction of calculations with his point of view and the meaninglessness of this distinction.¹⁾ Although I do not propose to discuss here the relation of this argument to the substance of Myrdal’s theory of monetary equilibrium, it is indisputable that, so long as one repudiates the period analysis methodologically, as Myrdal does, one must also repudiate the distinction between the *ex ante* calculation and *ex post* calculation.

Next, as to Lindahl’s “disequilibrium method.” This method which was recently advocated by him, divides the process of economic changes into “disequilibrium periods” instead of into “equilibrium periods”. Here too, as it is assumed that the economic plans of individuals remain unaltered and the prices are unchangeable during each period, the concept of “period” retains its economic significance. Consequently, it can hardly be said that this method takes interactions between the formation of prices in the

most prominent advocate of this view-point in R. Frisch, and calling it “momentary equilibrium analysis,” contrast it with “period analysis”. Hicks’ “temporary equilibrium analysis” is much nearer to the former than to the latter. In this sense, it seems to me that it is not correct to regard Hicks’ method, as Lindahl does, as a form of the “equilibrium method” in the Lindahl’s sense.

1) See also J. R. Hicks’ interpretation in his *Value and Capital*, p. 183.

market and the economic calculation of individuals into full consideration. This defect of the disequilibrium method becomes the clearer, the more fully its contents are studied. For this purpose, let us now consider, for convenience' sake, the market of an individual commodity, in which, according to Lindahl's hypothesis, only one particular seller among many sellers offers the price of the commodity considered. This hypothesis means that every remaining seller who has not offered the price immediately accepts the price offered by one particular seller, and concurrently supplies a certain quantity of the commodity, up to which his supply curve permits sale at the given price. Even if it were both possible and advantageous for him to offer a cheaper price and to sell more than the given quantity, the seller in question abides by the disadvantage of accepting the offered price until the period of disadvantage is over. This is what this hypothesis claims. Again, as regards buyers, they also demand the commodity at the offered price, considering the relation of such price to their planned expenditures for that commodity; therefore they demand it without regard to the relation of the price to the demand curve for the commodity in question. Thus, in this reconstruction of Lindahl, the demand curve, the traditional apparatus of economic theory, is of no help. The implication of this hypothesis is unrealistic, but even greater absurdity prevails in the case of the disequilibrium which is brought about by this hypothesis during the given period. If, after sellers have supplied the commodity in such quantity as they expect to be bought, it is found that the buyers' demand does not suffice to absorb this market supply, the market will surely fall into a state of disequilibrium through excess-supply. Now we have a right to ask Lindahl, how the quantity of demand, which falls short of the quantity of the supply, is apportioned to the different sellers in such a case. This question, though it is solved in actual practice, cannot be answered from Lindahl's point of view.

(To be continued.)