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DICHOTOMY IN THE CLASSICAL MONETARY THEORY

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Introduction

In two articles which appeared in *Econometrica*, Patinkin examined classical monetary theory. The main conclusion of this was that the classical attempt to dichotomize the economic processes of a monetary economy into a real sector, dependent upon and determining relative prices, and a money sector, dependent upon and determining absolute prices, cannot possibly succeed.

These propcsitions were attacked by W. B. Hickman, W. Leontief, C. G. Phipps in criticisms, and Patinkin answerd of them. Here I consider these controversies.

1. Patinkin's first theorem

First of all, Patinkin states the following theorem, the case where cash balance enters into utility function (System B) and the case, where it does not enters into utility function (System A) are mutually exclusive. In system B, both homogeneity postulate and Say's Law hold, but in system A both do not hold. Therefore, if homogeneity postulate hold, the system is necessarily B, and in this case, Say's Law holds. And if Say's Law hold, the system is necessarily B and in this case homogeneity postulate holds. Thus homogeneity postulate is equivalent to Say's Law, in the sense, either the both hold together or does not hold together. I call this proposition, Patinkin's first theorem.

2. Phipps' Criticism

Mr. C. G. Phipps wrote a criticism and Patinkin answered him. In the present section I want to see this controversy. Phipps' criticism is as follows. We get a following theorem from the two assumptions, perfeet competition and maximization of utility.

If a good has no (marginal) utility for any trader, its price relative to a good which has (marginal) utility is zero. In section 2 of his first article, [1] Patinkin introduces money into the problem. This he takes to be paper or token money concerning which he makes the assumption (page 140) " ... people derive no " direct " utility from paper money and therefor [the stock of money] does not enter the utility function". On the following pages, he makes the contradictory assumption that the price of money relative useful good is $1/p_i$. Using these contradictory assumptions, Patinkin finds it quite easy to show the system under discussion to be inconsistent.

Patinkin says, the fallacy of Phipps' argument can be demonstrated as follows. We have for the equilibrium condition for the excess demand for money

$$Z_n(p_1, p_2, \dots, p_n) - \bar{Z}_n = 0$$

where $Z_n(p_1, p_2, \dots p_n)$ is the demand for money and $\bar{Z}_n$ is the stock of money in existence. Under the assumption that money does not enter the utility function, it was shown that $Z_n(p_1, p_2, \dots, p_n) \equiv 0$ identically in the $p_i(i=1, 2, \dots, n)$, Phipps now says that, under this assumption, a necessary condition that the above relation is satisfied is that $p_n = 0$. But this argument is clearly incorrect since this relation can be satisfied if $\bar{Z}_n = 0$. It might be said that this is a trivial case. But it is only in this trivial case that it was even claimed that system B could be consistent.

Now we consider a money which offers service only as an unit of account. In this case, equilibrium condition for money in B system holds, and p_n is determined, more over, as we are assuming money serves as an unit of account $p_n = 1$. Therefore, two assumptions, money, which serves only as an unit of account, does not enter utility function, and the price of which is equal to *1* do not contradict.

I can not agree this Patinkin's answer. He thinks that if equilibrium condition holds for momentary market, price of money p_n is determined. In the economic system in which n commodities (including money) are exchanged, we can determine at most $n-1$ prices, and we can not determine any more. Therefore, in this system, we can determine the prices of all commodities (excluding money) in terms of money and p_n can not be determined.

To avoid this difficulty, if we think that p_n is the price of money in terms of arbitrary chosen commodity, then we can determine the price of money p_n in the market. But in this case the price of money can not be always equal to *1*.

Moreover, Say's Law holds in system B, so in this case, money market can not be said to be in equilibrium, but in this case demand is identically equal to supply. We can not determine any price using this apperent equation. Patinkin's answer to Phipps is not correct.

But I can not agree with Phipps. As well known, when two assumptions, perfect competition and maximization of utility hold, marginal utility of a good is proportional to its price, therefore at the subjective equilibrium point the marginal utility is equivalent to zero when the price of it is zero. But Phipps' propositon, can not be proved thus.

He states that the price of a good is equal to zero, when the marginal utility of it is equivalent to zero. In order to see this we must treat the problem in the market equilibrium analysis. Phipps did not notice this point and failed to treat the theorem "the market price of a good of which the marginal utility is equivalent to zero" in the market equilibrium analysis and treated it in the subjective equilibrium analysis are not in contradiction.

Next we prove the theorem "When a good has no marginal utility, the price of it is equivalent to zero" in the market equilibrium analysis. That is, even if we assume that the price of a good, which has no marginal utility, is not equal to zero, it necessarily becomes zero in the market equilibrium analysis. And when this is proved, the assumption that money does not enter utility function and the assumption that the price of money is equivalent to *1* in the subjective equilibrium analysis do not contradict each other.

First, if we put the price of money equal to *1* and assume that it has no (marginal) utility, the amount of money which the consumer plans to hold Z_n^a equal to zero, as Patinkin proves. Therefore, as long as $\bar{Z}_n^a > 0$ there necessarily exists excess demand and the price of it rises. After the rise of that price, there also exists excess supply of money, so there is excess demand for some good, and the prices rise further. Thus, if we assume that the price of money is equal to *1* and that money has no (marginal) utility, we get by market equilibrium analysis the results that in order equilibrium to hold the price must rise. And even if the price rise any further, general equilibrium can not be established. In other words, the price rises infinitely, *i.e.* the price of money in terms of other good falls to zero.

When the price of money in terms of other good falls to zero, the value of the excess supply of money become negligible, so it is just the same as in the case where Say's Law holds. In this case, the ratio of exchange of money in terms of any good is evidently equal to zero. Thus, when we extend Patinkin's method of analysis to the market equilibrium analysis, the market price of money in terms of some good becomes zero. Moreover, in subjective equilibrium analysis, we need not use the market equilibrium prices, *i.e.* we can use arbitrary prices. Thus we can say, that the two assumptions in Patinkins theorem do not contradict each other.

3. Criticism to the first theorem.

Now it is evident that Phipps' criticism to Patinkin is not correct. But I can not agree to Patinkin's theorem. Here is my criticism to it.

Patinkin thinks that possible cases are system A and B. But, I think, these two are not the all possible utility theories. When we classify utility function whether cash balance enters into utility function or not, possible cases are two, one is the utility function which include cash balance as an element and the other is one which exclude it.

Patinkin himself points out the following three kinds of utility function which include cash balance.

$$W^a(Z_1^a,\ Z_2^a, \dots,\ Z_n^a,\ p_1,\ p_2, \dots,\ p_{n-2},\ p,\ r)$$
$$V^a(Z_1^a,\ Z_2^a, \dots,\ Z_{n-2}^a,\ Z_{n-1}^a/rp,\ Z_n^a/p)$$
$$u^a(Z_1^a,\ Z_2^a, \dots,\ Z_{n-1}^a,\ p_n Z_n^a/p).$$

Now, as an example of the case in which cash balance enters utility function we think following system. For convenience, I call this system C. In system C, cash balance and prices enter into utility function. In this case, homogeneity postulate and Say's Law is not equivalent.

Writing individual's utility function.

$$u(Z_1^a,\ Z_2^a, \dots,\ Z_n^a,\ p_1,\ p_2, \dots,\ p_{n-1})$$

and assuming that when all prices and demand for money changes proportionately utiliy does not change, then u^a is homogeneous function of zero degree with respect to $Z_n^a,\ p_1,\ p_2, \dots,\ p_{n-1}$.

In this system C, the necessary and sufficient condition for the existence of homogeneity postulate is that there is no stock of money $\bar{Z}_n = 0$ at the planning date, in this case, the demand for money Z_n^a is homogenoous function of first degree, with respect to all prices. Therefore, in system C, the necessary and sufficient condition for the existence of homogeneity postulate is not the existence of Say's Law.

Thus we can say that Patinkin's first theorem that homogeneity postulate is equivalent to Say's Law is not correct.

I. Market Equilibrium Analysis

Above I considered the problem of Say's Law and Homgeneity postulate from the view point of subjective equilibrium analysis, below I will consider these problems from the view point of market equilibrium analysis. In other words, when the quantities of demands and supplies are determined as functions of prices any way, and when prices of all goods are to be determined by the equilibrium of these demands and suppliers, what will happen if Says Law and/or homogeneity postulate hold to these demand and supply functions.

On this point Dr. Lange says; in the case in which Say's Law holds, a proportional change of the prices of all commodities can not affect the demand and supply of commodities relative to the demand and supply of money. But a proportional change of all prices does not induce a substitution between different commodities either, therefore, the demand and supply functions of commodities are, when Say's Law holds, homogeneous of zero degree *i.e.* a proportional change of all pries does not affect the quantities demanded or offered.

Let us consider a closed system in which n commodities are exchanged, one of them—say the nth commodities—functioning as medium of exchange, *i.e.* as money. By Walras' Law total demand and total supply are identically equal. Therefore, the number of equilibrium conditions which can be used in determining equilibrium prices, is $n-1$. In the general case the $n-1$ equilibrium prices are determined by the $n-1$ equations, which express, for each commodity, the equality of demand and supply.

When Say's Law holds, the number of independent equation is only $n-2$, while the number of equilibrium prices to be determined is $n-1$. Therefore, $(n-2)$ equations determine, in this case, $(n-2)$ prices as functions of the price of the $n-1th$, commodity (which is chosen arbitraly) i.e. $p_i = p_i(p_{n-1})$, $(i = 1, 2, \dots, n-2)$ in this case, the demand and supply depend merely on the relative prices, *i.e.* on the ratio of the prices, $p_i/p_{n-1}(i = 1, 2, \dots, n-2)$.

Thus, Lange states, when Say's Law holds, the equilibrium values of $n-2$ relative prices are determinate, but absolute prices are indeterminate.

Contrally to the Lange's statement that Say's Law is the sufficient condition for the existance of homogeneity postulate, Patinkin does not think so. As above mentioned he states that the one is equivalent to the other, when the demand and supply functions are derived from utility function. But when we consider market equilibrium, demand and supply functions need not be derived from utility function therefore, in the general theory of market equilibrium, one is not equivalent to the other. I suppose Patinkin thought thus, he treated Say's Law and homogeneity postulate as independent conditions in the market equilibrium analysis.

He considered first the economic system, where only homogeneity postulate holds, and Say's Law does not hold. In this case, each excess demand function becomes the function of relative prices, so the number of variables is $n-2$. By Walras' Law the number of equilibrium conditions is $n-1$ and they are all independent as Say's Law does not hold. As the number of independent equations is larger than the number of unknowns, the system is overdetermined.

If say's Law holds in this system, the number of independent equations equals to the number of unknowns, so the relative prices are determined uniquely, but in this case absolute prices are not determined. For, absolute prices are determined from neither commodity side nore money side.

Patinkin states first, when Say's Law does not hold, and only homogeneity postulate holds, the system is overdetermined, against this statement, Professor Hickman and Leontief state that overdeterminacy does not hold and relative prices are determined uniquely in such system.

Lange states that if Say's Law holds, homogeneity postulate hold, so absolute prices are indetermined. So in order to determine the absolute prices, we must abundan Say's Law. Against this, as Patinkin thinks that Say's Law is necessary to save the overdeterminacy of homogeneous system, so if Say's Law does not hold and homogeneity postulate holds, the system becomes overdetermined. Therefore, to determine the absolute prices, we must abundan both Say's Law and homogeneity postulate.

Both Lange and Patinkin state that the relative prices are determined if both Say's Law and homogeneity postulate hold. But I think in this system the relative prices may be overdetermined or underdetermined.

Last, Patinkin thinks that the absolute prices are indetermined by dichotomy, but I do not think so.

II. Homogeneity Postulate and Independency.

1. Patinkin's Second theorem

Let us denote the excess demand function for each commodity as

$$X_i(p_1, p_2, \dots, p_{n-1}) \qquad (i=1, 2, \dots, n)$$

As Walras' Law $\qquad -X_n \equiv \sum_{i=1}^{n-1} p_i X_i$

holds, the number of independent equilibrium conditions is $n-1$. Cancelling money (nth) equation, equilibrium conditions become

$$X_i(p_1, p_2, \dots, p_{n-1}) = 0 \qquad (i=1. 2, \dots, n-1)$$

If we assume that when the number of independent equations is equal to that of unknowns (prices), positive real prices are uniquely determined, we can determine $n-1$ prices. $p_i(i=1, 2, \dots, n-1)$ from $n-1$ equations.

I assume here the homogeneity postulate, the characteristic of classical system, *i.e.* $n-1$ excess demand functions for real goods are homogeneous function of zero degree with respect to all prices, in other words we assume

$$X(p_1, p_2, \dots, p_{n-1}) \equiv X_i(\theta p_1, \theta p_2, \dots, \theta p_{n-1}) (i=1, 2, \dots, n-1), \ \theta \neq 0.$$

Putting $\theta = 1/p_{n-1}$ we get

$$X_i(p_1, \dots, p_2, p_{n-1}) \equiv X_i(\pi_1, \pi_2, \dots \pi_{n-2}) (i=1, 2, \dots, n-1), \ \pi_i = p/p_{n-1}.$$

i.e. variables become $n-2$ relative prices $\pi_i(i=1, 2, \dots, n-2)$. If $n-1$ equations are independent, the number of independent equations is larger than the number of independent variables by one, therefore the system is overdetermined. Thus we get the following theorem,

In the classical economic system, when homogeneity postulate holds, real excess demand functions $X_i(i=1, 2, \dots, n-1)$ are homogneous of degree zero, and if they are independent the system is over determined and the relative prices, satisfying general equilibrium conditions, are indeterminate.

I call this proposition the Patinkin's Second Theorem.

2. Criticisms to the second theorem.

Prof.s Leontief, Hickman criticise this theorem. Leontief says, homogeneity postulate is equivalent to Say's Law by Patinkin's first theorem, so Say's Law holds in the homogeneous system. By Walras' Law the excess demand function for money is dependent on real excess demand functions. Cancelling the monetary function, there is $n-1$ real excess demand functions, among which one is dependent on the others by Say's Law, so the assumptions in the second theorem contradicts to the first theorem. So if the first theorem is correct, the second one is not correct.

We can not agree with Leontief. We can not say that the first theorem contradict to the second one even if we think, as Leontief, the first theorem is correct. The first theorem holds in the economic system which are derived from utility function. But the second theorem is free from this restriction. Therefore the second theorem stands on more wider economic system than the first. In such economic system Patinkin does not thinks that the first theorem holds. Therefore, if we treat the first theorem and the second theorem in the economic system which are derived from utility function, we can say as Leontief. But these two theorems stands on different assumptions, therefore Leontief's statements is not correct.

Hickman states as follows. When homogeneity postulate holds, Jacobian

$$J=\frac{\partial(X_1,\ X_2,\dots,\ X_{n-1})}{\partial(p_1,\ p_2,\dots,\ p_{n-1})}$$

is identically equal to zero.

The necessary and sufficient condition for this is the exsistence of a functional relationship between $X_i(i=1,\ 2,\dots,\ n-1)$.

$$F(X_1,\ X_2,\dots,\ X_{n-1})\equiv 0\ .$$

This means $X_i(i=1,\ 2,\dots,\ n-1)$ are dependent, therefore, if $X_i(i=1,\ 2,\dots,\ n-1)$ satisfies homogeneity postulate, they can not be independent. That is two assumptions in Patinkin's theorem contradict each other. Thus Hickman criticises Patinkin.

3. Classification of functional relationship.

In the following section 4 I criticise Hickman's above statement, here I classify functional relationship.

As well known, the necessary and sufficient condition for the condition that Jacobian is identically equall to zero is the existence of a functional relationship

$$F(X_1,\ X_2,\dots X_{n-1})\equiv 0\ .$$

However, we must notice the fact that there is no relation between the existence of above functional relationship and the equilibrinm conditions of the market $X_i=0,\ (i=1,\ 2,\dots n-1)$. In order to see this point more clearly I classify the above functional relationship from the view point of the equilibrium of the market.

1°. *if* $X_i = 0 (i = 1, 2, \dots, n-2)$ hold, then $X_{n-1} \equiv 0$
2°. *if* $X_i = 0 (i = 1, 2, \dots, n-2)$ hold, then $X_{n-1} \equiv X^*_{n-1} \neq 0$
(X^*_{n-1} *const*)
3°, *even* $X_i = 0 (i = 1, 2, \dots n-2)$ do not hold.

I call the last case, in which even $X_i = 0 (i = 1, 2, \dots n-2)$ do not hold, strong independency, the second case, in which if $X_i = 0$ $(i = 1, 2, \dots, n-2)$ hold, then $X_{n-1} \equiv X^*_{n-1} \neq 0$ semi-independency semi-dependency, the first case, in which if $X_i = 0$ $(i = 1, 2, \dots, n-1)$ hold, then $X^*_{n-1} \equiv 0$, strong dependency, and the former two (2°, 3°), the independence in the wide sense, and the later two (1°, 2°), the dependence in the narrow sense.

For example, consider the economic system, in which three goods are exchanged by the medium of money (the nth goods) and the excess demand functions are expressed as follows.

$$
(\mathrm{i})\begin{cases} X_1 \equiv \frac{p_1}{p_3}+\frac{p_2}{p_3}+1 \\ X_2 \equiv \frac{p_1}{p_3}+\frac{p_2}{p_2}+2 \\ X_3 \equiv 2\frac{p_1}{p_3}+2\frac{p_2}{p_3}+6 \\ -X_n \equiv p_1X_1+p_2X_2+p_3X_2 \end{cases}
\qquad
(\mathrm{ii})\begin{cases} X_1 \equiv \frac{p_1}{p_3}+\frac{p_2}{p_3}+1 \\ X_3 \equiv \frac{p_1}{p_3}+\frac{p_2}{p_3}+2 \\ X_3 \equiv \frac{p_1}{p_3}+\frac{p_2}{p_3}+2 \\ -X_n \equiv p_1X_1+p_2X_2+p_3X \end{cases}
$$

$$
(\mathrm{iii})\begin{cases} X_1 \equiv -2\frac{p_1}{p_3}+3\frac{p_2}{p_3}+5 \\ X_2 \equiv 2\frac{p_1}{p_3}-5\frac{p_2}{p_3}+3 \\ X_3 \equiv 2\frac{p_1}{p_3}-13\frac{p_2}{p_3}+1 \\ -X_n \equiv p_1X_1+p_2X_2+p_3X_2 \end{cases}
$$

Clearly homogeneity postulate holds in each case and Jacobian $J = \frac{\partial(X_1, X_2, X_3)}{\partial(p_1, p_2, p_3)}$ is identically equal to zero, and in this case, there are following functional relations

(i) $X_1+X_2-X_3+3 \equiv 0$ (ii) $X_1-X_2+X_3+1 \equiv 0$ (iii) $X_1+2X_2-X_3 \equiv 0$.

in case (i), when the first market is in equilibrium, the second and third market can not be in equilibrium, (*i.e.* strong independency), in case (ii) even if both the first and the second markets are in equilibrium, the third market can not be in equilibrium, (*i. e.* semi-independency semi-dependency), in case (iii) if both the first and the second markets are in equilibrium, the third market is necessarily in equilibrium (*i.e.* strong independency).

4. Criticism to Hickman

As is shown in the preceeding section, there are three kinds of functional relationships from the view point of the equilibrium of the market, *i.e.* there are semi-independency semi-dependency and strong independency (independency in the wider sense), as well as strong dependency. Therefore, it is inadequate to call to be dependent when there is functional relation $F(X_1, X_2, X_3) \equiv 0$.

Hickman calls dependency when there is strong dependency. Therefore it is not correct to call to be strongly dependent when there is functional relation $F(X, X_2, X_{n-3}) \equiv 0$. In this case we must tell whether it is strongly dependent or semi-independent semi-dependent or strongly independent.

Patinkin points out the fact that there is a functional relation $F(X_1, X_2, X_3) \equiv 0$ under the homogeneity postulate. Therefore, Hickman's criticisrm is not correct. Assumptions in the second theorem do not contradict each other and we get the conclusion of over determinacy.

III. Say's Law and Relative Prices

1. Patinkin's third theorem.

We saw in the above section that Patinkin's second theorem is correct. When real excess dernand functions are all independent, in the Classical system where homogeneity postulate holds, the system is overdetermined, *i.e.* in this system we can not determine relative prices which satisfy general equilibrium conditions $X_i = 0$ $(i = 1, 2, \cdots, n-1)$.

Following this he states. In the classical system we assume Say's Law $\sum_{i=1}^{n-1} p_i X_i \equiv 0$ as well as homogeneity postulate. In this case, real excess demand functions are dependent so we can save overdeterminacy. So we get the following theorem.

When Say's Law holds as well as homogeneity postulate, we can determine relative prices in this system. I call this proposition Patinkin's third theorem.

2. Criticism to this third theorem.

I think this theorem is wrong. As already saw, Jacobian is identically zero if homogeneity postulate holds. In order to determine prices in the market, there must be a strong relation between X_i $(i=1, 2, \cdots, n-1)$. However, a strong relation between X_i and Say's Law is not equivalent. Patinkin failed to see this point. For example, in a system, in which three goods (including money) are exchanged.

When each excess demand function is expressed as

$$X_1 \equiv 9\frac{p_2}{p_1} - 4\frac{p_1}{p_2}$$
$$X_2 \equiv -18\frac{p_2}{p_1} + 8\frac{p_1}{p_2}$$
$$-X_n \equiv p_1X_1 + p_2X_2 \equiv (p_1-p_2)X_1 .$$

If the first market is in equilibrium, other markets are in equilibrium and relative price is determined as $p_1/p_2=1.5$. It is clear that, this homogeneous system is saved from over determinacy not by Say's Law but by strong relation $2X_1+X_2\equiv 0$.

Here I want to point out the fact that even if both homogeneity postulate and Say's Law are fullfiled, the system also remains under determined. For example, if excess demand functions are expressed as

$$X_1 \equiv a(w+c)$$
$$X_2 \equiv \frac{p_1}{p_2} b(w+c)$$
$$X_3 \equiv -\frac{p_1}{p_3}(a+b)(w+c)$$
$$-X_n \equiv p_1(a+b-a-b)(w+c) \equiv 0$$
$$w \equiv \frac{p_1q_1'+p_2q_2'+p_3q_3'}{p_1q_1+p_2q_2+p_3q_3}$$

In this system both homogeneity postulate and Say's Law hold but we can not determine relative prices, *i.e.* we can only determine price level w. In other words such system is underdetermined even if both homogeneity postulate and Say's Law hold.

Thus we can say that in order to determine relative prices in the homogeneous system Say's Law can save neither underdetermination nor overdetermination.

Say's Law reduces the number of independent equations by one, and we can construct a system in which Say's Law hold and we can determine absolute prices. Therefore Say's Law can not be used for the discrimination whether we can determine relative prices or not.

VI. Homogeneity Postulate and Relative Prices

1. Dichotomy

We assume that, in the homogeneous system in which strong dependency holds, $n-2$ relative prices $\pi_i = p_i/p_{n-1}$ are determined by $n-2$ equilibrium conditions, in this case if multiplicative factor p_{n-1} is determined in the remaining market, we can determine all the absolute prices by multiplying all relative prices by this multiplicative factor.

The equilibrium conditions, which can be used for the determination of this factor p_{n-1} is $X_{n-1}=0$ or $X_n=0$. But as they are equivalent to each other by Walras' Law, equilibrium conditions $X_i=0$, $(i=1,2,\cdots,n-2)$ and $p_{n-1} \neq 0$. Therefore, here I use $X_n=0$ to determine p_{n-1}. Our present problem is to inquire, wether this multiplicative factor is truely determined by $X_n=0$ or not.

I call the method of determining absolute prices, as Dichotomy, in which we seperate the excess demand functions into two, one expressing the excess demand for real goods, the other expressing that of money, and determine relative prices in the real excess demand functions, and determine multiplicative factor, therefore absolute prices in the excess demand function for money.

In classical system, we usually use the Cambridge monetary equation

$$M - p_{n-1}K\sum_{i=1}^{n-1}\pi_i S_i = 0$$

in determining multiplicative factor p_{n-1}, where (real) quantities of supply S_i, relative prices π_i, are determined by the equilibrium conditions of real goods. Therefore, assuming M is given, we can determine unknowns p_{n-1}.

3. Patinkins Forth Theorem

Patinkin states that absolute prices are indeterminate even if we use the above Cambridge monetary equation.

In classical system we postulate homogeneity, therefore, unknowns are $n-2$ relative prices π_i $(i=1,2,\cdots,n-2)$. If $n-1$ real excess demand functions are independent (in the wider sence) the system is usually over determinant (second theorem) but classical system uses Sey's Law to avoid this overdeterminacy, *i.e.* in classical system, as both homogeneity postulate and Says' Law hold, we can determine relative prices (third theorem). But in this system we can not determine absolute prices.

In such case, if we state that by Cambridge monetary equation we can determine multiplicative factor p_{n-1} and multiplying relative prices π_i $(i=1, 2, \cdots, n-2)$ by this p_{n-1} determine absolute prices, we are comitting logical errors. When we try to determine prices by Dichotomy we can not determine absolute prices, and if we state that we can determine them we are committing error.

4. Criticism to Hickman

Against to this, Hickman states as follows: In Cambridge monetary equation M, K, is given, $\sum_{i=1}^{n-1} \pi_i S_i$ is homogeneous of degree zero with respect to all prices. So

$$M \equiv K \sum \pi_i S_i$$

cannot hold for all prices. Thus Patinkin criticises classical system. But this is not correct. Cambridge monetary equation is not identity but condition, therefore is not contradictory.

For example, in a economic system in which two goods are exchanged, when cxcess demand functions are expressed as

$$X_1 \equiv d \frac{p_2}{p_3} - S$$
$$X_2 \equiv -cX_1$$
$$-X_n \equiv p_1 X_1 + p_2 X_2$$

we can determine relative prices. In this case, if we use Cambridge monetary equation

$$M = p_2 KD(\pi)$$

as a condition, we can determine p_2.

Hickman states thus. But even if Cambridge equation is condition, (not identity) which expresses equilibrium condition for money, it is equivalent to X_n, so we can not determine absolute prices. Therefore, Hickman's criticism to Patinkin is wrong.

5. Criticism to the fourth theorem

Above I pointed out the errorness of Hickman's statement, but I cannot agree with Patinkins fourth theorem.

Now we consider a closed system, where n commodities and securities (0 th commodity) are exchanged, one of them say the n th commodity functioning as a medium of exchange.

We have assumed that homogeneity postulate holds, *i.e.* $n-1$ real excess demand functions are all homogeneous function of degree zero with respect to all prices. In this case, the sum of excess demand functions for money and that of securities is homogeneous zero degree with respect to all prices but any of them need not be so.

Now in our system, $n-2$ relative prices are determined, as $n-2$ excess demand functions for real goods are in equilibrium.

In this case, p_{n-1} and r are still to be determined and remaining excess demand functions which can be used for the determination of p_{n-1} and r are that of money and securities but these two X_0, X_n are mutually equivalent.

Therefore, in our system we can determine one of r and p_{n-1} and can not determine both of them. Now we use monetary equation to determine p_{n-1}. If, in this case, equilibrium condition for money is Cambridge equation, we can determine p_{n-1} with this. Multiplying relative prices with this p_{n-1} we can determine all absolute prices.

Thus we can determine absolute prices by dichotomy, but in this case we can not determine interest rate r.

V. Conclusions

First, homogeneity postulate and Say's Law is not equilivalent. Second, in order to determine relative prices in homogeneous system, there must be strong dependency in this system. Third, Says' Law and/or homogeneity postulate have no relation with the determination of relative prices. Fourth, we can determine absolute prices with dichotomy, but in this case one variable, interest rate, remains indeterminate.

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