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A TENTATIVE NON-LINEAR THEORY OF ECONOMIC FLUCTUATIONS IN THE PURELY COMPETITIVE ECONOMIC SYSTEM

By Shin-ichi Ichimura*

1. Basic Assumptions

This paper submits a theory of the so-called Jugler cycle. The questions to be discussed are the behaviors of the price-level, real output, and capital accumulation throughout the cycle. The theory is 'macro-dynamic' because it deals with such aggregative concepts as the price-level and output as a whole, and the inter-temporal relationships of various variables come into the analysis in an essential way. The theory is non-linear, because changes in the functional shape of investment is taken into consideration. Compared with other non-linear macro-dynamic theories such as the Hicks-Goodwin type or the Kalecki-Kaldor type of theories, the following theory has two distinct characteristics.

In the first place, it tries to some extent to dig into the theory of firms and incorporate it into the theory of economic fluctuations. Most of macro-dynamic theories left out the theories of entrepreneurial behavior and began their analyses with the variables in the market. Many criticisms on the recent works by Mr. Harrod and Professor Hicks seem to center around this shortcoming of their theories. It is humbly hoped that the following discussion will cast some light on these controvertial problems. It may be of some interest to note that the following theory seems to have some theoretical similarities to rather old theories of Professors Robertson and Mitchell.

In the second place, we shall proceed throughout the assumption of pure competition; that is to say, we shall almost always neglect the influence on supply which may arise from calculations made by sellers about the influence on prices of sales they make themselves. (Similarly for demand.) It is under this assumption that the price-level rather than demands plays a great role in our analysis.

In the following theory, we take as data (i) the growth of population,

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(ii) the natural resources in the economic system, (iii) technology, (iv) the economic organization of production and distribution, (v) expectations. The effects of the changes in (i), (ii), (iii), and (v) will be discussed to some extent in the following analysis. To simplify the following discussion, we assume that all the prices except the wage rate \(w\) and the rate of interest \(r\) change proportionally so that we can discuss in terms of the general price-level \(p\).

2. The Fundamental Equation of The Price Variation

We assume that the price-level changes proportionally to the excess demand for the aggregated commodities. The Demand consists of Consumption \(C\) and Investment \(I\), and the Supply is assumed to be the current real output \(y\). These assumptions lead to the following Fundamental Equation of the Price Variation, which is the very basis of the following analysis; namely,

\[ \mu p' = C + I - y, \quad (\mu > 0) \]

where \(C\) and \(I\) are real consumption and investment ex-ante, and \(\mu\) is the reciprocal of the so-called price-flexibility.\(^1\)

If there is Autonomous Demand in consumption and investment, it is theoretically more convenient to separate them from the rest of consumption and investment. Let us assume that there is Autonomous Consumption but no Autonomous Investment. The latter will be brought into our discussion in later sections. For the exponential growth of population, we can assume that Autonomous Consumption increases in proportion to population. Letting \(C\) stand for the rest of Consumption, Eq. (1) becomes:

\[ \mu p' = C + I + Ae_{\mu} - y. \]

Here two things are to be noted: (i) the adjustment of inventories to the price-level or output is entirely ignored, (ii) Investment here is Gross Investment in Fixed Capital only so that Investment in Working Capital and Liquid Capital will be left to the later discussions. Then \(y\) is more like real G.N.P.

3. Consumption and The "Income Period"

The real consumption function is assumed to be a linear function of Real Income and the price-level; that is,

\[ C = ay - bp, \]

where \(a\) is the marginal propensity to consume and \(b\) is a coefficient indi-

\(^1\) cf. P. A. Samuelson, *Foundations of Economic Analysis*, 1943

O. Lange, *Price Flexibility and Employment*, 1945


cating the price effect on consumption. It may be assumed that $0.5 < a < 1$ and $0 < b$. In order to incorporate this consumption function into the following analysis, some remarks are necessary as for the realization of disposable income and the behavior of consumers with respect to the price level.

As for the mechanism of distribution and spending, we may think that entrepreneurs distribute their earnings in sales after some time—"The Distribution Lag"—, and receivers of such remunerations, now as consumers, spend their incomes also after some time—"The Expenditure Lag." The sum of these two lags, as we call the "Income Period," will be noted by $h$.\(^1\) Investigations by some economists seem to confirm that $h$ can safely be assumed around three months or even shorter. Taking calendar years as our unit of time, $h$ is, say, $0.15 \sim 0.3$.\(^2\)

As for the reaction of consumers to the price change, we assume that they react simultaneously or spontaneously to the change in the price-level. Thus, Consumption function can more properly be written as

$$C = ay_h - bp,$$

where $y_h$ is real income at time, $t-h$ and disposable at time $t$.

It may be interesting to note a possible interpretation of $-bp$, because of the inclusion of this term, $C$ in (2.1) is not only the schedule of consumption but also may well be regarded as the realized consumption. For consumers plan to spend as much as $ay_h - bp$ without expecting any change in the price-level but can only purchase as much as $ay_h - bp$; hence, the difference $-b(p-p_h)$ is the forced savings if $(p-p_h) > 0$ or the forced dissavings if $(p-p_h) < 0$ so that it is not improper to think that $C$ is the realized consumption as well as the schedule of consumption. We shall take throughout the following analysis this interpretation of our consumption function. Another

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1) This income period should not be confused with the Robertsonian Income Velocity Period which is merely the reciprocal of the income velocity of money and has little to do with the income period. Professor Goodwin's attempt to combine the velocity period with the multiplier does not seem to be justified.

R. M. Goodwin, "The Multiplier," in New Economics, ed. by S. Harris, p. 488. See also
D. H. Robertson, Essays in Monetary Theory, 1940, pp. 117—121.

As Professor Hicks said, it would be rash to conclude that the income lag is generally negligible. Though it may look small, the order of magnitude should be compared with those of other magnitudes of coefficients which may also be small.
thing to be noted is that when \((p - p_0)\) is negative, this term takes care of the so-called Pigou Effect to some extent, because it is the rise in the demand for consumers' goods due to the fall of the price-level.

4. Marginal Cost and Production

The Investment Function and the Production schedule are both determined by entrepreneurial behavior. Take a representative entrepreneur. He has to make two decisions: the Short-Run Planning and the Long-Run Planning. In the former he takes capital stock \((K)\) as given and plans how much output to produce and accordingly employs productive factors, and in the latter he makes his Investment Plan. As the ordinary theory of firms of Marshallian type shows, the profit maximizing behavior of entrepreneurs in the short-run under pure competition determines the level of output so as to equate the price of output to the marginal cost corresponding to that level of output.

Let us assume that the short-run marginal cost curve (SMC) is as is shown in Figure 1. This SMC is drawn for the fixed capital stock \((K_i)\) and the fixed wage rate \((w_i)\), and for the given technology. From A to B, SMC rises very slowly but from B to C, very sharply, and after C, output \((y)\) cannot increase. Then, if the price-level is given, for example, at \(p_1\), output will be planned to be at \(y_1\). B is the point where the existing capital \((K_i)\) becomes much less "adaptable" to increasing quantities of variable productive services. Let us assume that if \(p\) is given above \(p_0\), then the representative entrepreneur desires to increase capital; that is, starts planning to invest.

This critical point, \(B_1\), of adaptability naturally shifts to the right if \(K\) increases. (See Fig. 2) We assume that equal successive increases in \(K\)

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2) Whether we should define the so-called "full capacity" point by B, or C, or some other point is a matter of convenience. Since, however, the purpose of defining the "full capacity" point in trade cycle theories is to consider it as the point where Investment starts, the assumption in the text suggests that B serves for the purpose better. However, in the case of an individual firm whose capital is "indivisible", the full capacity point can more properly be defined by the amount of output of which the total cost with a given amount of capital is exceeded by the total cost of the output producible with that capital plus an additional unit of capital. (to next page)
volve equal successive displacements of those steeper segments to the right. The above two assumptions give us the Long-run Marginal Cost Curve as an extension of the portion of SMC, with a gentle slope.

C is the point where the existing capital becomes absolutely "non-adaptable" to any increase of variable productive services. We shall call B's the "full capacity" points and C's the "maximum capacity" points. Disregarding productive factors other than Labor, Capital, and Natural Resources—the productive factors which cannot be produced in the economic system in question; that is, what Classical economists called "Land," suppose that natural resources are abundant and almost perfectly adaptable, and that the full capacity points and the maximum capacity points are given by the definite ratios of Labor and Capital. If, moreover, the production function is homogeneous of the first degree, then the slopes of LMC and the line connecting C's are the same; otherwise, they will not be the same. Thus, it is clear that in some circumstances output \( y \) may not be able to reach the maximum capacity level \( y_c \), or that the SMC, may start rising before the full capacity point.

First of all, if the wage rate rises, the SMC will increase. Let us assume that the wage effect on the SMC is linearly to shift it to the left. (See Fig. 3.) Thus, given the price-level, the wage rate, capital available for operation, and technology, output \( y \) will be linearly dependent on \( p \),

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The equilibrium conditions to determine the level of planned output are given by the following equations:

\( y = a_1 p - \beta w \) if \( p \) is less than \( p_1 \)’s;
\( y = a_2 p - \beta w + \gamma k \) if \( p \) is between \( p_1 \)’s and \( p_2 \)’s;
\( y = \gamma k \) if \( p \) is greater than \( p_2 \)’s,

where \( a_1 \) and \( a_2 \) are the reciprocals of the slopes of AB and BC; \( \beta \) the wage effect on \( y \); \( \gamma \) the effect of the increase of capital on BC’s; and they are positive constants. Clearly, \( a_1 > a_2 \), and as capital in operation changes, the limiting points between (3.1), (3.2), and (3.3) change. (See Fig. 4.)

Secondly, before output reaches \( y_a \), the supply function of Labor may become horizontal and bends back. Then the employment of labor cannot be increased beyond that point—the “Maximum Employment” point, \( (y_a) \). This maximum employment point \( y_a \) may come before or after \( y_c \) but is unlikely to come before \( y_b \) in an economic system.

Needless to say, if the labor force is increasing due to the population growth, the maximum employment point will also be increasing. The accumulation of capital will probably have very little to do with this point.

Thirdly, Natural Resources (or Land) may become much less adaptable before \( y \) reaches \( y_b \), \( y_c \), or \( y_a \), although it is most unlikely that the point of “non-adaptability” of Land comes before any one of them. Let us call this point the “Resource Limit” and denote it by \( y_r \). Then after \( y_r \), the Law of Diminishing Returns works very strongly, so that the marginal cost will start rising. The effect would be to make \( a_1 \), \( a_2 \), and \( \gamma \) smaller after this point. This Resource Limit would not be influenced by the increase of labor force but will probably be enhanced by the accumulation of capital. In what order these ceiling points appear depends on the conditions of the economic system in question and the lapse of time. Figure 5 shows one of the possible situations where population is increasing exponentially but capital.

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1. It should be noted that if technical improvements should lower the marginal cost, the effect would be to increase \( a_1 \), \( \beta \), and \( \gamma \) proportionately.
2. The accumulation of capital will probably shift the supply function of labor to the left so that the maximum employment point is more likely to decline.
is constant. The dotted line indicates the fluctuations in the ceilings due to the sinusoidal change in operating capital.

PP' is the exponential growth of population and so of the labor force. KK_c' is the full capacity point; KK_m' the maximum capacity point; LL' the maximum employment point; and RR' the Resource Limit. The dotted curve CC' is a sinusoidal curve of capital in operation through time. Figure 5 is the picture of the same economic system except that here capital in operation is increasing at the same rate as the labor force. It should particularly be noticed that the effect of capital accumulation on RR' is positive but damping, which would seem to be the case with no technological improvements.

Before closing this section, there is only one thing to mention in connection with the production plan in the short-run. That is the time lags between the price-level given in the market and the production actually carried out at time t. Given p, w, and K, entrepreneurs would expect in the short-run that they will be all the same in the very near future—the Short Term Expectation—and accordingly plans how much to produce. This lag may be called the "Production Planning Period." There will, however, be a lapse of some time again until this planned output

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1) It is of course an idealization necessary for the theoretical analysis that we have defined the ceilings as though Capital and Natural Resources consist of one homogeneous good. As Professor Robertson's excellent classic, *A Study of Industrial Fluctuation*, 1915 and Professor Hicks' *Trade Cycle*, op. cit., remind us, the bottle-necks of various productive factors come one after another. These are the things which we have to remember when we come to an historical study of economic fluctuations but we may put aside for the following analysis.

2) J. M. Keynes, *General Theory*, pp. 46-48; 50-51; 148. Keynes says: the "short-term expectation is concerned with the price which a manufacturer can expect to get for his finished output at the time when he commits himself to starting the process which will produce it." (p. 46) "It is sensible for producers to base the assumption that the most recently realized results will continue."
is actually produced. Borrowing a term of Professor Metzler, let us call this lag the "Output lag". If we denote the sum of these two lags by $u$, this $u$ is longer and more significant than the "Income Period". It would probably be safe to assume that $u$ is, say, $0.3 \sim 0.5$. Thus, under the assumption of static expectations in the short-run, (3.1, 2, 3) should be written as:

\begin{align*}
(4.1) & \quad y = a_1p_t - \beta w_u \\
(4.2) & \quad y = a_2p_t - \beta w_u + \gamma K_0 \\
(4.3) & \quad y = \gamma K_u,
\end{align*}

where $K_0$ denotes Fixed Capital in operation and $y$ is output actually produced at time $t$, and subscripts $u$ indicates that those letters with them are the variables at time $t-u$.

5. Investment and Expectation

Now we come to the Long-Run Planning; that is, Investment Decisions. Suppose that the existing capital is $K_1$ and the price is given at $p_t$. (See Fig. 4.) Then, the output that our representative entrepreneur plans to produce in the short-run will be $y_t^*$, but in the long-run he will plan to produce $y_t\gamma$ by increasing his Fixed Capital from $K_1$ to $K_2$. This desired amount of capital $K_2$ can be calculated by (3.1, 2): $y_t^* = a_1p_t - \beta w_2 = a_2p_t - \beta w + \gamma K_2$; hence, $K_2 = p(a_1 - a_2)/\gamma$. Thus, the desired amount of capital ($K^*$) is given by

\begin{equation}
K^* = a^* p, \quad \text{where} \quad a^* = (a_1 - a_2)/\gamma.
\end{equation}

This desired capital ($K^*$) must clearly be distinguished from capital in existence ($K_2$) and capital in operation ($K_0$). $K^*$ will not be realized until the gestation period elapses. In other words, in order to complete their desired capital entrepreneurs must keep on investing under the Investment Plan for $\theta$ years. We assume that they invest at a constant time rate. Then, investment at time $t$ is:

\begin{equation}
I = \frac{1}{\theta} (a^* p - K_t).
\end{equation}

If, however, investment is planned to be completed only after the gestation period, the price relevant to such decisions must be expected price

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1) L. A. Metzler, op. cit.

Since $K_2$ when it is completed, will be used for many years, the relevant expected price is not only the price at time $t$ to $t+\theta$ but also the prices thereafter. Only for simplicity, we are taking as the representative price some sort of the average price of them and considering as though it were the price-level at time $t+\theta$. Needless to say, this is a radical simplification. Since, however, allowances for risk-elements increase as time passes, this simplification will not damage our analysis very much. cf. M. Kalecki, Essays in the Theory of Economic Fluctuations, 1939, pp. 95–406, on the principles of increasing risk.
prevailing after the gestation period. More precisely, it is the discounted price-level of the price expected to be most probably prevailing after $\theta$ years. Furthermore, it is more proper to take as the representative price the most probable price ± an allowance for the uncertainty of the expectation; that is to say, an allowance of risk.

Let $p^*$ stand for the long-term expected (representative) price-level and $r$ for the long-term expected rate of interest, and the price-level ($p$) in Eqs. (5.0) and (6.0) had to be $e^{-\theta}p^*$. We assume that this $p^*$ is given by $p^* = p_v + p'_v$, where a subscript $v$ indicates that $p$ and $p'$ are those at time $t-v$, and $\varepsilon$ may be regarded as the elasticity of expectation. Here $v$ is a lag due to the "Investment Planning Period." Since there is no time lag involved to produce anything, this $v$ will probably be much shorter than $u$. Thus, $v$ may safely be assumed, say, around $0.15-0.25$. Since $\varepsilon = (p^* - p_v)p'_v$ expresses an elasticity of expectation over $\theta$ years. If $\varepsilon = 0$, $p^* = p_v$. This means that the long-term expectation as well as the short-term expectation is static. We might call it the "Robertsonian assumption on expectation."\(^1\)

If $\varepsilon$ is equal to $\theta$, it may be interpreted as the expectation which corresponds to the case of the elasticity of expectation being one in the ordinary theory of firm. (Similarly for the case where $\varepsilon \leq \theta$.) Because of what Kalecki called the Principles of Increasing Risk, $\varepsilon$ is likely to be greater than 0 but less than $\theta$. At the present stage of our discussion, we assume that the banking system is perfectly "passive" so that the current rate of interest is constant and the long-term expected rate is the same as the current rate. Thus, by setting $\rho = e^{-\theta}$, we have

$$K^* = \rho a^*(p_v + \varepsilon p'_v)$$

(6.1) $I = \frac{1}{\theta}(K^* - K'_v)$

What if $K^*$ is less than $K'_v$? Clearly, there is no reason to make $I$. $K_v$ will naturally be left to depreciate itself. This is the irreversibility of Investment which we have to take into consideration. Let us assume that whether the life-time of any kind of capital goods is long or short, the percentage of its existing amount must be depreciating. Then, if $K^* < K'_v$,

$$I_v = -\delta K_v e^{-\theta t},$$

(6.2) where $I_v$ shows that Investment here is merely due to depreciation (disinvestment), and that it is the Investment function working in the Down-swing. Notice, however, this depreciation investment does not appear in the market so that when we are analyzing the time-shape of the price-level, we have to drop the investment function entirely. If $K$ exceeds $K'_v$, our

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Gross investment may be understood as including replacement investment.

Needless to say, investment in the above equations is meant to be investment _ex-ante_, not investment _ex-post_. If there were no capital goods available other than those of current production, the difference between the planned investment—that is, $I$ in our notation—and the available capital goods at time $t$ ($y - C - Ae^u$) would not be realized. Then, it is clear that in the up-swing the planned investment exceeds the realized investment, and that the converse is true in the down-swing. If the realized investment is understood as 'saving' in the sense that consumers are willing to leave that much to the future satisfaction, whatever it means, then the above statements can be worded in terms of Saving and Investment. Clearly this is not the situation that will actually happen. The current excess-demand will be met at least partially by the liquidation of stocks. If the entire excess-demand is met by the decreased stocks of capital goods, then the price rise may not occur. It is more likely, however, that the excess-demand will be met partly by the disposed stocks and partly by the raised price-level. We keep, however, our analysis of investment or disinvestment in Liquid Capital until we come to the analysis of the inventory cycle, and simply assume that the price-level rises whenever there is excess demand in the market.

If investment is being made successfully, the existing capital is increasing to the extent that it is realized. But because of our formulation of investment function and assumptions, we have to be careful in using the investment function. That is to say, though $K_s$ is increasing during the gestation period, we have to consider $K_s$ in Eq. (6.1) as though it were constant. This implies that the replacement part of investment is immediately becoming effective but the rest of investment is kept on being made. After the gestation period is over, the investment planned before $\theta$ years becomes available for operation. Hence, our Investment function should be written then as

$$I = a(p_v + \varepsilon p_v - p_0 - \varepsilon p_v^\prime), \quad (a = pa^\prime / \theta)$$

where $p_0$ stands for the price-level at time $t-\theta-v$.

6. Determination and Change in the Rate of Wages

Throughout the process of economic fluctuations, the rate of wages of

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1) It should be noted that if there is no gestation period nor the investment planning lag and $\varepsilon$ is zero; that is, long-term expectations are static, we get $I = ap$. This is nothing but the familiar Acceleration Principle under the conditions of pure competition.

2) There is another way of looking at the price change. That is, assuming that the excess demand for capital goods is met by stocks, we may consider the price variation as being caused by the accumulation or the decumulation of stocks. Then, $1/\mu$ may be interpreted as the speed of the price adjustment on the part of the sellers. But we do not adopt this interpretation. See the following section.
course changes. According to the familiar Keynesian assumption, the wage rate is fixed if the employment of labor is below the full employment. But even if the employment is less than full employment, the wage rate will rise if the rising price-level brings the real wage-rate below the "minimum real wage" ($v_1$) which workers would insist on maintaining. We assume that if the real wage rate falls down to this level, then the supply side of the labor market determines the wage rate so as to keep the real wage equal to the "minimum real wage" but after the "wage-lag," say, $\omega$ periods. Thus, in this situation we have

$$(7.1) \quad w = v_1 p_{t-\omega},$$

where $p_{t-\omega}$ is the price-level at time $t-\omega$.

Secondly, if the employment of labor reaches the full employment level in the Keynesian sense, workers will demand higher and higher real wages to supply more labor. We assume that after the bargaining between employers and employees, workers succeed in keeping up the real wages. Then we may write

$$(7.2) \quad w = v_2 p_{t-\omega},$$

where $v_2$ is the demanded real wage rate so that it is an increasing function of the price-level.

Thirdly, if the real wage reaches a certain level, say $v_3$, then workers would not supply any more labor but rather decrease the supply of labor. Let us assume, however, that after this point they still insist upon having the same real wage ($v_3$). Then for this stage we can write

$$(7.3) \quad w = v_3 p_{t-\omega}.$$
like the simple multiplier is at work, so that the falling price-level converges to a certain level. The down-swing has a bottom. Due to the autonomous consumption, the price-level keeps on going up. Then there will sooner or later come the time when the desired amount of capital exceeds the existing capital. (See Fig. 7.) For the existing capital is steadily decreasing at the rate of depreciation; whereas the desired amount of capital is rapidly rising once the price-level starts going up. Then investment begins. This first phase of a cycle we call Regime I. If investment starts, demand suddenly increases. But current production does not increase immediately, because there is the output lag. The price rises and will keep on rising because of the investment which demands more and more capital goods. As $\rho$ is going up, output is also rising but with some time lag. At the beginning of this phase of a cycle, the rise of current production may be very sharp. But very likely before the gestation period is over, the full capacity level of output will be reached. For though existing capital is increasing at the rate of realized investment, it is not yet completed so that it is not available yet for operation. After the full capacity point, current production does not increase as rapidly as before. But the price-level rises more and more sharply. The mere efflux of time will, however, bring the gestation period to its close. Regime II is over, and Regime III has come. Then the fixed capital having been constructed suddenly becomes available for production; capital in operation increases; production jumps up. Thus it is natural to expect that the rising price-level will taper off, and that most likely it begins falling. After a while, production itself falls down. Everything is now in the downswing. Regime III is over; we are back in Regime I. (to be continued)