VOLUME XXIV NUMBER 1

Kyoto University
Economic Review

MEMOIRS *of The* FACULTY *of* ECONOMICS
in
KYOTO UNIVERSITY

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APRIL 1954

PUBLISHED BY
THE FACULTY OF ECONOMICS
KYOTO UNIVERSITY
KYOTO, JAPAN

A TENTATIVE NON-LINEAR THEORY OF ECONOMIC FLUCTUATIONS IN THE PURELY COMPETITIVE ECONOMIC SYSTEM II

By Shin-ichi Ichimura

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8. Analysts of Depression—Regime I.

Under all the assumptions we have made, a system of equations to describe this phase of a cycle is given as follows:

$$((\mathrm{I}))\quad \begin{cases} \mu\dot{p}=C+Ae^{gt}-y & \text{(from (1))} \\ C=ay_h-bp & \text{(from (2.1))} \\ y=a_1p_u-\beta w_u & \text{(from (4.1))} \\ w=\nu p_w. & \text{(from (7))} \end{cases}$$

Note that ν in the last equation of ((I)) may vary depending upon the conditions in labor employment. Eliminating all the variables other than p and taking the first two terms of the Taylor expansion of retarded terms, we get

(8) $B'\dot{p}+Bp=Ae^{gt}$, where

$$B'=[\mu-(1-a)\{a_1u-\beta\nu(u+w)+ah(a_1-\beta\nu)]$$

$$B=\{(1-a)(a_1-\beta\nu)+b\}$$

Within the reasonable range of values of various coefficients involved, B′ and B are both positive. Hence the characteristic equation of (8) will have a negative root. Denoting it by $-\lambda$, we can write the general solution of (8) as

(9) $p(t)=(p_o-A\ )e^{-\lambda t}+A^*e^{gt}\ (A^*=A/B'g+B\,;\ \lambda=B/B')$.

By observing the solution and (8) we can claim the following propositions:

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15) It is plausible to assume that $(a_1-\beta)$ is substantially greater than zero, because when there is some profit to our representative firm and p and w change proportionately, the absolute amount of profit will be increasing; so will be production. This may seem to reject the homogenuity of production function, but whenever there is some marginal profit, it seems more plausible for the economy as a whole that some of marginal firms willl be induced to come into new production. Then, if the change in ν, if any, remain small, we can expect that $(a_1-\beta\nu)>O$. We shall assume this. Furthermore, even if the full capacity point is reached so that a_1 has to be substantiated by a_2 we still assume that $(a_2-\beta\nu)$ can not become negative unless the full demanded real wage is very great, when labor is far above the full employment. Then if $a>O$ but less than unity, both B and B' will be positive.

(i) *Under the assuuotion that* $(\alpha_1-\beta\nu)>O$, *the marginal propensity to consume being less than one is a sufficient condition for stability.*

It the m.p.c. (a) is equal to one, it guarantees the convergence of the price-level to the lower equilibrium path determined *by* A^*d^{gt}. (See Fig. 36.)

(*ii*) *The initial condition, that is, the price-level at the beginning,* p_0 *does not affect the equilibrium path of* $p(t)$.

If the values of parameters such as a, u, or h change, then the time-shape of $p(t)$ and the lower equilibrium path will be changed also. These effects can be looked at from two angles: the effect on the lower equilibrium path, and the effect on the speed of the falling price-level. Combining the two effects we have four possible cases: (A, a) the equilibrium path is raised but the speed of the price fall ($\dot{p}$) is slower; (A. b) the equilibrium path is raised and the speed of the falling price is greater; (B. a) the equilibrium path is lowered and the prie fall is slower; (B. b) the lower equilibrium path is lowered and the price fall become quicker. (See Fig. 8)

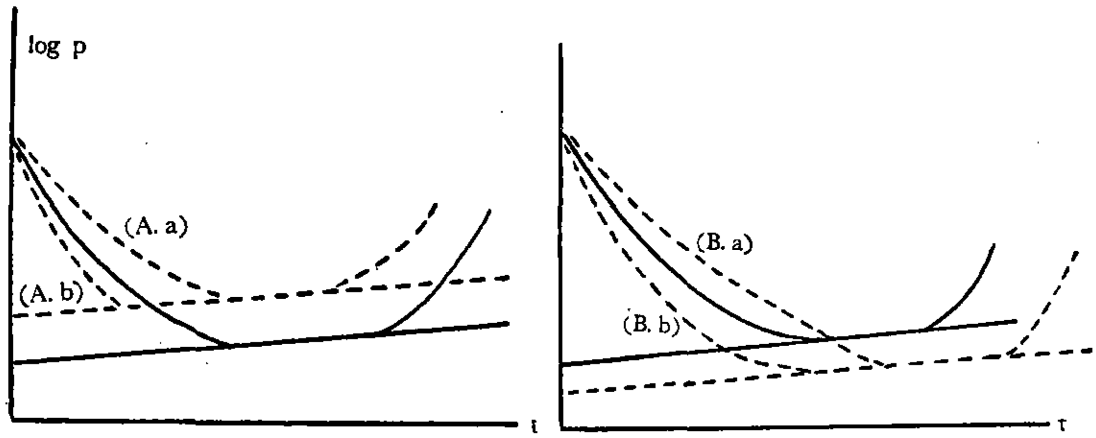


Fig. 8.1 Fig. 8.2

If the equilibrium path is raised(or lowered), depression becomes less deep (or deeper). If we call the time when p becomes zero, the date of Recovery, then (A. b) will certainly speed up the date of recovery; (B. a) postpones it. In the cases of (A. a) and (B. b), the effects are uncertain, but the exact date can be calculated by solving p=0 for it.

According to this classification, the once for all *increase* in parameters have the following effects: a···(A. a), h···(B. a), μ···(B. a), b···(A. a), u···(A. b), ω···(B. a), α_1···(B. b), ν···(B. a), β···(B. a). It should be noted that in the up-swing the converse holds.

In words, we have the following propositions. They are stated for the probable change in parameters in the down-swing.

(*iii*) *If the marginal propensity to consume* (*a*) *becomes larger, it slows down the* *price fall and speeds up recovery.*

(iv) The lower real wage demanded by workers (ν) *will make* $\dot{p}$ *slower and raise the equilibrium path.*

(v) The increased productivity of labor, regarded as an increase of β, *slows down the price fall and raises the equilibrium path.*

(vi) Other things beings equal, the lower marginal cost will make $\dot{p}$ *faster and slump deeper.*

(vii) If the price flexibility becomes smaller (or larger), the slump will be deeper (less deep) and the price fall slower (or faster).

(viii) If the price effect on consumption becomes greater, slump will be less deep and the price fall slowed down. (The Pigou Effect.)

(ix) The shorter output lag will make $\dot{p}$ *slower but deepen slump.*

(x) The longer wage lag makes $\dot{p}$ *slower and slump deeper.*

9. Analysis of Recovery—Regime I continued.

In our narrative in section 7, we have explained the date of recovery as the time when K^* exceeds K_e. This needs further analysis. When price level starts goimg up, it is moving upwards almost in parallel to the lower equilibrium path, and current production is also going up. The general conditions of business is getting better and better. In this circumstance, however, it is more natural to expect the entrepreneurs will start replacement to keep the existing capital intact and prepare themselves for the coming good days. Let us assume that this replacement succeeds in maintaining the existing capital actually intact, so that capital in existence is kept constant after the date of recovery, say, time t_1. Denoting the capital in existence at time t_1 by K_1, we have to substitute the first two equations of ((I)) by

$$((\mathrm{I}'))\quad \begin{aligned} &p=C+I+A_1e^{gt}-y \\ &I=\delta K_1. \end{aligned}$$

where it is to be noted that time is measured anew from t_1 and A_1 is the autonomous consumption of time t_1. Then, instead of (8), we have

$$(10)\quad B'\dot{p}+Bp=A_1e^{gt}+\delta K_1.$$

The general solution of (10) is given by

$$(11)\quad p(t)=(p_1-A_1^*-K_1^*)e^{-\lambda t}+A_1^*e^{gt}+K_1^*,\ \text{where}$$
$$A_1^*=A_1/B'g+B,\ \textit{and}\ K_1^*=K_1/B.$$

Then what happens is that the lower equilibrium path itself is raised. (See Figure 9.1) Then the price-level starts converging not to the old path but to the new equilibrium path. In Fig. 9.1, the dotted curve indicates the path which p would have followed if there had not been any replacement investment started after the date of recovery. The smooth curve is the path of p in the new situation. Here the conditions of capital are changed. (See Fig. 9.2) After $t_1 K_e$ is constant, which may postpone the date of shift to Regime II or Prosperity. But there is another force working in the opposite direction. Because of the faster rising price-level, K is rising more rapidly than before. This effect will more than cancel the first effect. Thus we seem to have the following propositions:

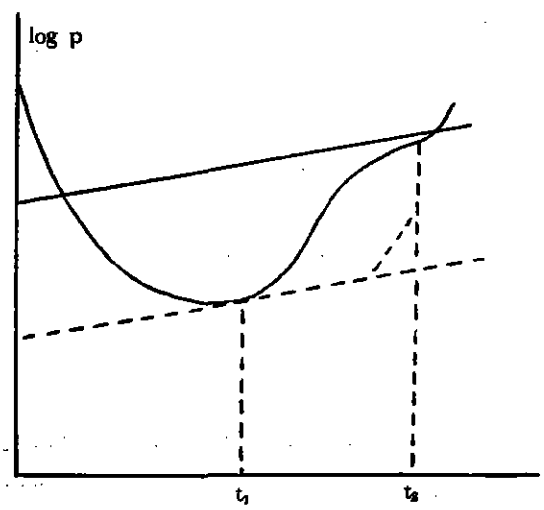


Fig. 9.1

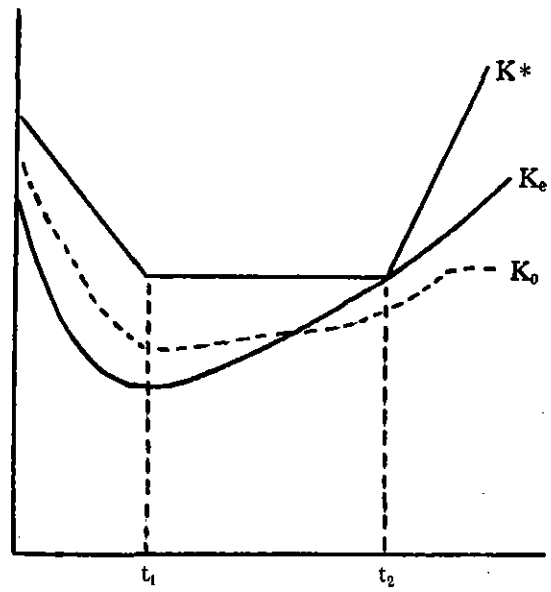


Fig. 9.2

(*xi*) *If replacement investment starts at the end of the down-swing, full recovery to prosperity will come earlier than otherwise. In general the earlier date of recovery will make prosperity come along earlier also.* Needless to say, if replacement investment begins before the date of recovery, the date of recovery will come earlier; so will the date of shift to prosperity.

It is obvious that the greater g, δ, A, will make the date of recovery earlier. We may state this, however, as one of our propositions:

(*xii*) *The greater the growth rate of population, the depreciation rate* (δ), *or autonomous consumption, the earlier will be the date of recovery.*

10. Analysis of Prosperity—Regime II.

With our induced net investment function at work, Prosperity begins. Regime II is characterized by the following system of equations:

$$((\mathrm{II}))\quad \begin{cases} \mu\dot{p}=C+I+A_2e^{gt}-y & \text{(from (1.1))} \\ C=ay_h-bp & \text{(from (2.1))} \\ I=\alpha(p_v+\varepsilon\dot{p}_v)-\dfrac{K}{\theta} & \text{(from (5.1) and (6.1))} \\ y=\alpha_1 p_n-\beta w_n \text{ or } y=\alpha_2 p_n-\beta w_n & \text{(from 4.1, 2))} \\ w=\nu p_w & \text{(from (7))}, \end{cases}$$

where $a=\frac{1}{\theta}\rho a^*$, and (4.1) holds before the full capacity point of y; (4.2) after the point, and A_2 is Autonomous consumption at t_2.

In the same way as we did about ((I)), we get the following differential equation:

(12) $B_1\ddot{p}+B_2\dot{p}+B_3p=A_2e^{gt}-K_1/\theta$,

when $B_1=a\varepsilon v$,

$B_2=B'-(\varepsilon-v)a$,

$B_3=(B-a)$.

In order to analyze the time-shape of $p(t)$, it is convenient to make the following transformation of p to $q(t)$;

(13) $q(t)=p(t)-A_2^*e_{gt}-K^*$, where

$A_2^*=A_2/B_1g^2+B_2g+B_3$; $K_2^*=K/B_3$.

In terms of this q, (12) can be written as $B_1q+B_2q+B_3q=0$ and the characteristic equation is given by $B_1\lambda^2+B_2\lambda+B_3\lambda=0$. By setting

(14) $2k=B_2/B_1$; $k_1^2=B_3/B_1$,

we have

(15) $\ddot{q}+2k\dot{q}+k_1^2q=0$.

By setting further

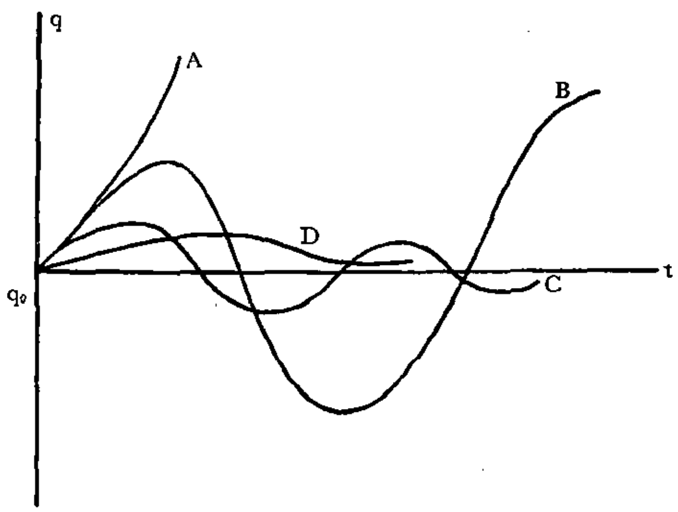
(16) $k_2^2=k_1^2-k^2$,

the general solution of (15) can be written as

(17) $q(t)=e^{-kt}(C_1\cos k_2t+C_2\sin k_2t)C_3e^{-kt}(\cos(k_2t+C_4))$, where if at $t=0$, $q(t)=q_0$ and $\dot{q}(t)=\dot{q}_0$,

$C_1=q_0$; $C_2=q_0+(kq_0/k_2)$; $C_3^2=C_1^2+C_2^2$; $\tan C_4=-C_2/C_1$.

Assuming $q_0>O$ and $q_0>O$, the possible time-shaps of $q(t)$ will be the following four: (See Fig. 10.1)



A: if k^2-$k_1^2>o$
B: if 〃 $<$o and $k<o$
C: if 〃 $<$o and $k>o$
D: if 〃 $>$o and $k>o$

Fig. 10.1

Now considering the signs of B_i's in (12), the signs of B's in the numbered order, we can make the following table:

	B_1	B_2	B_3	the time-shape
1) $if a>B, (\varepsilon-v)a \quad B'$	+	−	−	......A
2) $if a>B, (\varepsilon-v)a<B'$,	+	+	−	......A
3) $if a<B, (\varepsilon-v)a>B'$,	+	−	+	......A or B
4) $if a<B, (\varepsilon-v)a<B'$,	+	+	+	......C or D.

Considering that the value of A_2^* is likely to be negative, the time shpae of $p(t)$ will look like the following: (See Fig. 10.2)

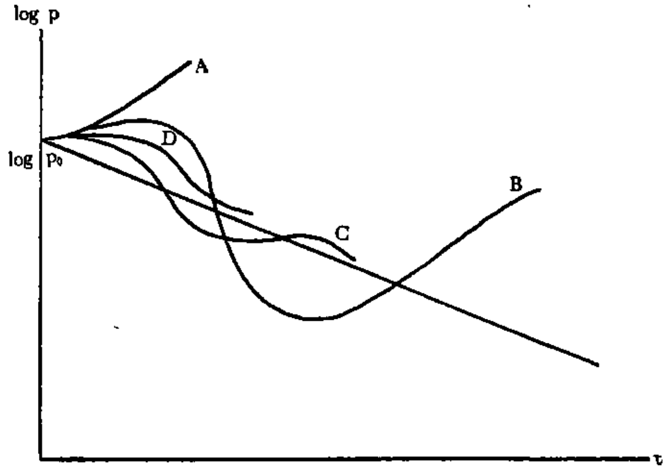


Fig. 10.2

Here we are concerned with the time-shape of $p(t)$ from o to θ only. So that the price-level may not go down at all even during the gestation. Of course it may do so. Leaving the analysis of Case 3) and 4) to later sections, we assume the coefficients have the values such that p has an explosive solution. Then, denoting two real charcteristic roots by λ_1 and λ_2, the general solution for $p(t)$ can be written as

(18) $p(t)=C'_1e^{\lambda_1 t}+C'_2e^{\lambda_2 t}+A_2^*$, where

$C'_1=(p_0-A_2^*+K_2^*)\lambda_2-(A_2^*g\text{-}p_0)/\lambda_2-\lambda_1$

$C'_2=(A_2^*g\text{-}p_0)-\lambda_1(p_0-A_2^*+K_2^*)/\lambda_2-\lambda_1.$

The price-level will keep on rising under the assumption. But it is likely that the rising y reaches the full capacity point in the process of prosperity. (See Figure 11.) The a_1 in the above equations must be chan-

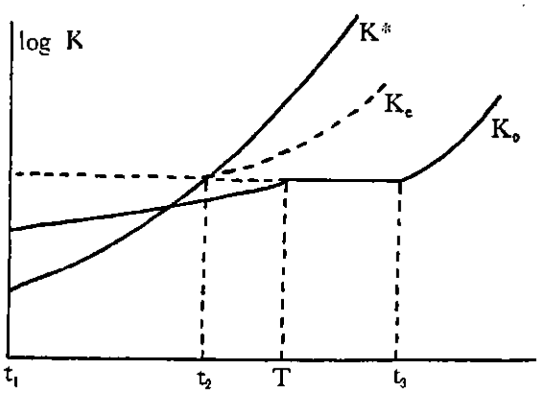


Fig. 11.1

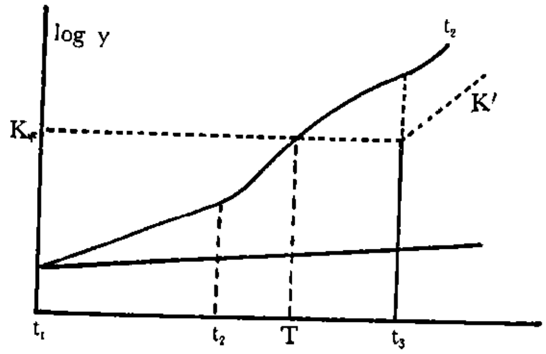


Fig. 11.2

ged to a_2. This change will make very little difference to B' but B much smaller. So that at the beginning of the phase of a cycle, a set of coefficients may be such that Case 3) is the case and the solution has an oscillatory time-shape, but after this point it may become explosive.

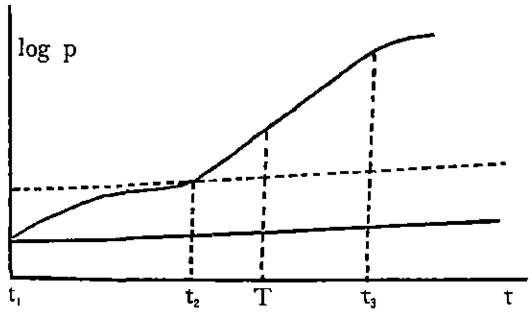


Fig. 11.3

Notice particularly that at time T when current production reaches the full capacity point, capital in operation cannot increase any more. And the explosive process becomes more seriously divergent. On the other hand, current production will be increasing much more slowly. All these are rather obvious by the ob serva tion of the equations and the solution given above. From these observations we can infer the following propositions:

(*xiii*) *If the marginal cost rises, the speed of the rising price-level will become faster, and that of current production slower.*

(xiv) *The greater capital coefficient* (α) *and elasticity of expectation* (ε) *makes the explosive process more serious.* When α is very small and yet an explosive time-shape of the price-level is to occur, degree of optimism must be extremely great.

(xv) *The factors which make* α *smaller will make prosperity less explosioe. They are, for example, more adaptability of fixed capital, rising interest-rate, greater productivity of capital*(γ), *longer gestation period, more substitutability of different capital goods, and so on.*

Finally it should be noted that all the propositions in section 8 still holds in Regime II.

11. *Analysis of Upper-Turnintg Point—Regime III.*

A system of equations characterizing Regime III is nothing but ((I)) except that our investment function must be changed to (6.3); that is, $I=\alpha(p_v+\varepsilon p_v-p_\theta-\varepsilon p_\theta)$. Then we get

(19) $B_1\ddot{p}+B_2\dot{p}+B_3p=A_2e^{gt}-\alpha p_\theta-\alpha\varepsilon\dot{p}_\theta$.

Notice that the time-shape of $\dot{p}$ and p from the beginning (t_2) and the end of Regime II ($t_2+\theta$) is given by (18). Hence we can solve the equation above for the time from the beginning of Regime III ($t_3=t_2+\theta$) till ($t_3+\theta$). Since the homogenoous part of (19) is the same as that of (12), the characteristic roots are the same λ_1 and λ_2. The general solution of (19) is:

$p(t)=(D_1+E_1)e^{\lambda_1 t}+(D_2+E_2)e^{\lambda_2 t}+A_3^*e^{gt}+K_3^*$, where

time is still measured from time t_2, and

$$A_3^* = (A_2 - aA_2^{*-g\theta} - a\varepsilon A_2^* g e^{-g\theta})/B_1 g^2 + B_2 g + B_3$$
$$K_3^* = aK_2^*/B_3$$
$$E_1 = -aC'_1(1+\varepsilon\lambda_1)e^{-\theta g}/B_1\lambda_1^2 + B_2\lambda_1 + B_3,$$
$$E_2 = -aC'_2(1+\varepsilon\lambda_2)e^{-\theta g}/B_1\lambda_2^2 + B_2\lambda + B_3,$$
$$D_1 = (p_3 - A_3^* - E_1 - E_2 - K_3^*)\lambda_2 - (p-_3 - A_3^* g - E_1\lambda_1 - E_2\lambda_2)/\lambda_2 - \lambda_1,$$
$$D_2 = (\dot{p}_3 - A_3^* g - E_1\lambda_1 - E_2\lambda_2) - \lambda_1(p_3 - A_3 - E_1 - E_2 - K_3^*)/\lambda_2 - \lambda_1.$$

In D_1 and D_2, $\dot{p}_3$ and p_3 are $\dot{p}(t)$ and $p(t)$ at $t=t_3$.

Suppose that $\lambda_1 > 0$ and $\lambda_2 < 0$. Then it is very probable that D's are positive but E's are negative. Depending on the magnitudes of these coefficients, the time-shape of $p(t)$ after t_3 would be one of those indicated in Fig. 12.

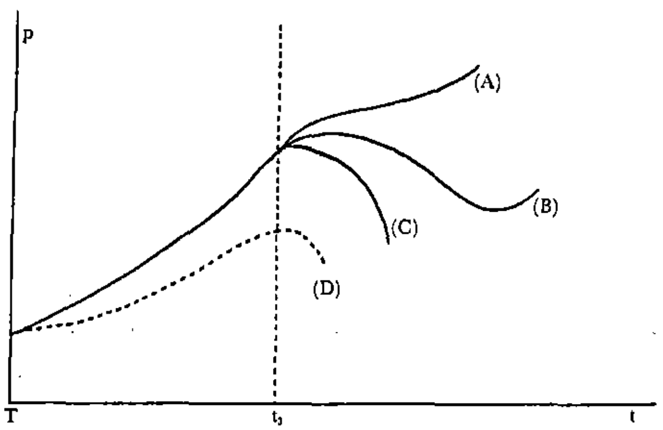


Fig. 12

(A): If both of (D_1+E_1) and (D_2+E_2) are positive, or one of them negative but small in modulus, then the time-shape of $p(t)$ will be (A).

(B): If both have different signs but a negative one is very large in modulus, p will behave as (B).

(C): If both of them are negative, (C) will be the pattern of $p(t)$.

(D): If $p(t)$ has a solution in Regime II such that the time-shape is B, C, or D, then (D) will be the behavior of $p(t)$ unless it has reached the turning point before t_3.

It is hard to say anything definite about the possibility of which type of behaviors is likely to occur in Regime III. If (A) happens, we can say nothing here until we come to the further inspection of ceilings in later

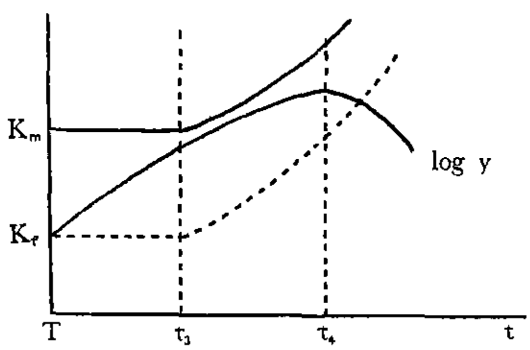


Fig. 13

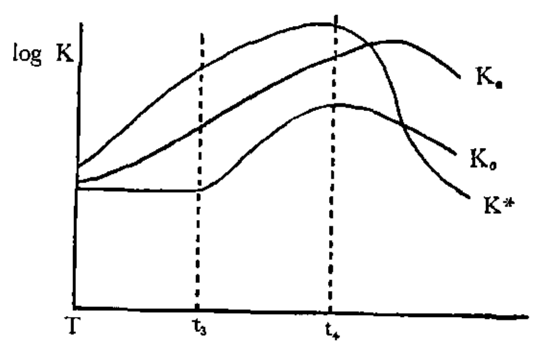


Fig. 13.1

sections. Let us waive to analyze this case at this stage of our theoretical investigation. But considering our proposition (*xv*), we seem to be able to conjecture a few propositions:

(xvi) *If the gestation period is very long, or capital productivity is great, it is very likely that the price level starts falling before output of current production hits any ceilings.* (See Fig. 13.)

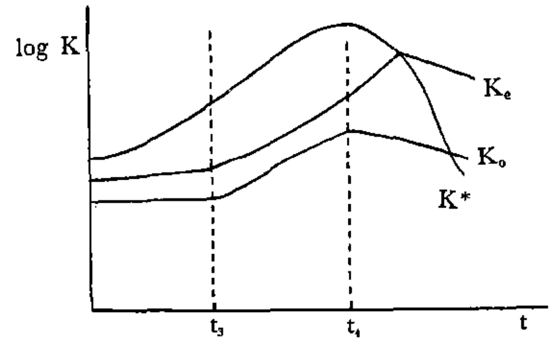


Fig. 13.2

When the price-level goes down, K^* starts falling down very rapidly. For a while, investment may still go on because there is some construction of fixed capital almost completed. But sooner or later, probably very soon, K^* will beceme less than not only K_e but even K_o, capital in operation needed for current production. There is no reason to continue investment any longer than this time. One should say, it is even more likely that investment drops off before that point, (Fig. 14 will give both cases.) The dotted curve shows the case that investment stops when K^* is equal to K_o; the smooth curve indicates the case in which investment becomes zero when $K^*=K_e$. Moreover, there still is a possibility that investment in fixed capital falls to nothing very soon after the turning point of the price-level. There seems no plausible assumption better than the other. Only for the convenience for later discussions, let us assume that investment is stopped when $K^*=K_e$. From the proposition we have mentioned above and the considerations in this section we can state the following propositions:

(*xvi*) *The longer the investment is kept going after the upper turning point of the price-level, the more severe will be "Crisis" and the longer will be the depression following.*

(*xvii*) *The more explosive the prosperity is, the longer will be the following depression.* For if prosperity is very explosive, it means that the investment planned in later phases of prosperity is very great and that great part is left uncompleted, and the uncompleted capital is not useful for production. Hence, we have to reconsider the fore-going analysis of ours and think that capital relevant for determining the lower turning point is not just the relation between K and existing capital but rather between K^* and *existing capital available for production.* Denoting this capital by K'_e, the conditions of capital throughout the whole cycle will be something like Figure 14. This explains the above proposition. Needless to say, our analysis should be adjusted to this new interpretation. One alteration seems to be very important.

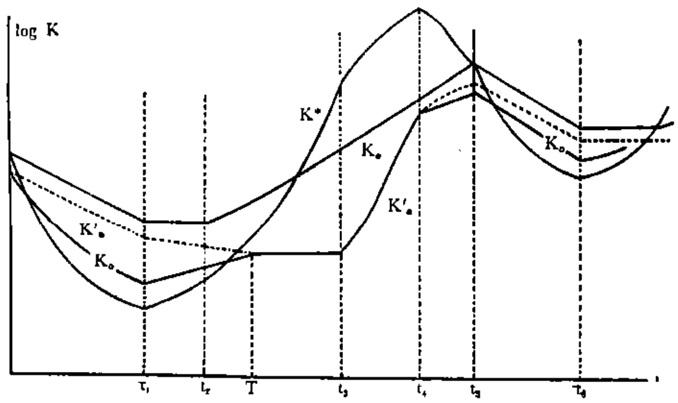


Fig. 14

That is, if there is some uncompleted fixed capital, then investment in the early phase of recovery will immediately increase capital available for production. Hence, it is rather likely that in the upswing output will not hit the full capacity point at all. (See Fig. 15) Then, we are likely to have a cycle which does not hit even the full capacity point.

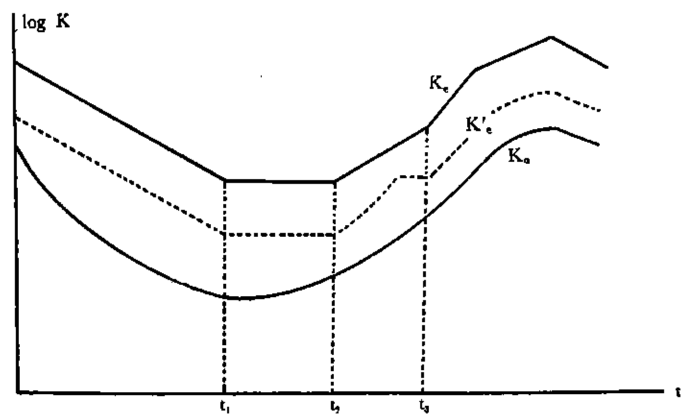


Fig. 15

This seems to be a typical case with smaller capital coefficient. What Hicks called a 'weak cycle' may be corresponding to this case. At any rate, in our theory, what affects time-shape is predominantly the changes in fixed capital. In other words, the movement of various ceilings has very close connections with the time-shape of y^*.

Looking back our analysis of the whole cycle, we may state the following propositions:

(xviii) *The period of a single cycle depends very heavily on the gestation period.* The period here is understood as the time necessary for the price level to start with the lower turning-point and end up there after going up and down once.[16)]

(xix) *If all the parameters involved in our system of equations in three regimes do not change, then there will be the only pattern of economic fluctuations which*

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16) Hicks notes on this possibility in his new book but does not go into any further analysis of it. It seems that in a theory of economic fluctuations this factor may not be ignored. See J. R. Hicks, *Trade Cycle*, 1950, pp. 96n, 36-37n, 123.

More important is that our theory seems to see the effects of technologican improvements upon the course of real output, whereas such theories as Hicks do no seem to; at least it is not very easy to see as they are presented, *is regular in period and amplitude.* If, however, output hits some sort of ceiling and then falls down, then this is not a sufficient condition for regularity.

(xx) *Throughout all the phases of cycle, there always exists some surplus capacity in the sense that operating capital exceeds existing* capital.[17]

12. *Steady Growth and Its Instability.*

If $p(t)$ does not die a natural death, then it raises current output to a certain level above which y cannot go up so fast as it is wanted. The further inspection of these ceilings are in order. But before doing it, let us investigat a 'steady growth.' The question here is: Is it possible for output to grow steadily? Under what conditions can real output grow steadily?

In our model, autonomous consumption is steadily growing exponentially at the rate of g. The steady growth in output will, therefore, require that y must grow at the same rate. Then, from (4.1) it is clear that p and w must also be going up. Suppose that p is rising as

(21.1) $p = Pe^{gt}$,

then we get from (4.1) and (7); and (2.1);

(21.2) $w = \nu Pe^{gt}$

(21.3) $y = (ae^{-ug} - \beta\nu e^{-(u+w)g})Pe^{gt}$

(21.4) $c = a(a_1 e^{-(h+u)g} - \beta\nu e^{-(u+h+w)g})Pe^{gt}$

In the steadily growing economy, however, the above y in (21.3) must be moving exactly along the full capacity path ($K'K_f$ in Fig. 35.) For otherwise the fluctuations in investment is bound to occur. Then fixed capital operating for current production must exactly be the amount technically needed. This required amount of operating capital can be calculated from (4.1,2) and (5.0) as

(22.1) $K_0 = a^* e^{-ug} p.$

Our investment function (6.1) is then:

(23.1) $I = \frac{a^*}{\theta}\{(\rho 1 + \varepsilon g) e^{-vg} - e^{-ug}\} p.$

So long as investment function could be maintained in this form, p, w, y, C, I, and autonomous consumption is going up all at the same exponential rate. But there is a factor which is likely to cause instability in investment. Entrepreneurs want to invest at time t by the amount of I given by (23.1), but they can only succeed in doing so at the rate of $(y - C - Ae^{gt}) = (I - \mu p)$. In other words, part of their investment plans is not realized. This is bound to cause dissatisfaction on the part of entrepreneurs. But if current production is large enough, then operating capital may be increased

17) S. Kuznes,t "Relation between capital goods and finished product,"

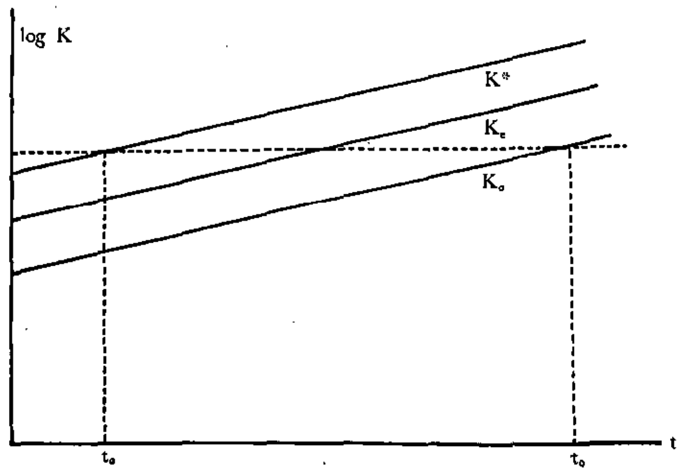

Fig. 16

as much as technically needed, though capital in existence will have to be always below the desired amount of capital. (See Fig. 16.) The ivnestment which technically required is:

(23.2) $I'=h_o=ga^*e^{-ug}p.$

Thus, the condition that investment *ex-post* must not be less than technically required invesment can be expressed as $(y-C-Ae^{gt})\cdots I\geqq 0$, from which we can get:

(24) $\{(1-ae^{-hg})\ (\alpha_1e^{-ug}-\beta\nu e^{-(u+w)g})+b-A\}>ga^*e^{-ug}$, or

$\{(1-ae^{-hg})\ (\alpha_2e^{-ug}-\beta\nu e^{-(u+w)g})+b-A\}>(g-\gamma)a^*e^{-ug}.$

(Condition 1.) Current production must have the margin after consumption that is large enough to supply at least technically required amount of investment.

The real dissatisfaction will come when the desired amount of capital is not completed after the gestation period. Denote by $K^*(t-\theta)$ the amount of capital planned at time $t-\theta--v$ to complete during the period from $t-\theta$ to t, and by K_o the technically needed capital in operation at time t. Then to stisfy entrpreneurs, both must be equal. (See Fig. 16.)

(25) $K^*(t-\theta)=e^{-\gamma\theta}a^*(1+\varepsilon g)e^{-g(\theta+v)}p=a^*e^{-ug}p=K_o.$

$\therefore\ e^{\gamma\theta+g(\theta+v)}=(1+\varepsilon g)e^{ug}$: *hence*, $r=\frac{1}{\theta}\{log\ (1+\varepsilon g)+(u-\theta-v)g\}.$

This condition would permit two interpretations. Firstly, all the parameters being given, it may be considered as determining the *equilibrium of interest.* The interst rate consitent with steady growth is not only constant but must satisfy the last equation in (25).[18] Secondly, it may be said that given the rate of interest, the elasticity of expectation of entrepreneurs must be such that the foresighted conditions of operating capital turn out to be exactly the same as they are realized. This implies more than the perfect foresight in the sense that enterpreneurs anticipate the price level after the gestation

18) (25) may seem to imply that if $g=0$, $r=0$; that is, thore is not interest rate in the stationary state. But (25) does not mean that. What it implies is that in the stationary state there is no force working on the determination of interest rate from the side of long term investment. Whether or not the Shumpetarian conclusion follows therefrom depends on other considerations about the determination of interest rate.

period at the time of decision-making as correctly as it will turn out to be. Assume, for instance, perfect foresiget in this sense, and the conditon (25) of moving equilibrium has to be replaced by

(26) $e^{-r\theta}=e^{-ug}$; hence, $r=ug/\theta$.

If, and only if, the output lag is regarded as the same as the gestation period, and the rate of interest is equal to the growth rate, prefect foresight is a necessary condition for steady growth. This seems to be the hypothesis underlying the Austrian theory of capital. Under our hypotheses, however, we have to rely on the condition (25):

(Condition 2.) Civen the state of expectation, the rate of interest must be determined at the equibrium level given by (25): or given the rate of interest, the long-term expectation must be such that with that interest rate and elasticity of expectation, enterpreneurs can anticipate correctly and technically required capital for production after the gestation period.

Knowing the interest rate given in the market, the technically necessary capital in operation after θ years, and the fact that the price has been changing steadily as Pe^{gt}, the wisdom of enterpreneurship may succeed in accomplishing this nevertheless difficult coordination. Note that if (25) is satisfied, then our investment function can be written as

(33.2) $I=a(1+\varepsilon g)\ (1-e^{-g\theta})e^{-gv}p.$

Thus, it may be possible under the two conditions that despite the dissatisfaction on the part of investment planners, no alteration in investment decisions is made. But here arises one of potential factors which may easily diverge the economy from steady growth. For so long as current production is greater than consumption *plus* technically needed investment, entrepreneurs can always and may cut down their planned investment.

If, however, p, w, y, C, and Investment are changing along the path given by these equations, then the market mechanism we have assumed is such that the price-level changes according to (1.1). Hence,

(27) $\mu gPe^{gt}=\{-(1-ae^{-hg})(a_1e^{-ug}-\beta\nu e^{-(u+w)g})-p(1+\varepsilon g)(1-e^{-g\theta})e^{-gv}\}pe^{gt}+Ae^{gt}$

Since the price-level must be positive, it implies the condition:

(27.1) $\{\mu g+(1-ae^{-hg})\ (a_1e^{-ug}-\beta\nu^{-(u+w)g})-\frac{a}{\theta}(1+\varepsilon g)\ (1-e^{-g\theta})e^{-gv}\}>O.$

This condition necessary for steady growth can be stated roughly as follows:

(Condition 3.) The marginal propensity to save, the price effect on consumtion, the reciprocal of the price flexibility, are large enough to make it possible for the price level to go up steadily while current production is meeting the demand for capital technically required for the increase of production.

In such conditions the price-level and real income are steadily rising,

so that money income is going up more rapidly than either one of them. The transaction domand for money, therefore, must be increasing faster than p or y. Supply of money, then, must increase to meet this demand, while still keeping the rate of interest constant.
(Condition 4.) The monetary system must remain 'passive' and meet the increasing demand for fund and still keep the rate of interest consistent with steady growth constant.

Now output (y) is moving along the full capacity path under these four conditions. There is no reason, however, that this full capacity path is that which keeps the employment of labor along the full employment path. If it is the full employment path, the changing wage rate will make the steady growth impossible. If it is below the full employment path, then the rising price-level will eventually cause the wage rise and keep it as $w=\nu p_{\omega}$. Thus steady growth may be possible, but so long as the actual employment of labar is below the full employment, there always exists workers dissatisfaction. The increasingly greater number of unemployed workers will have to depend on the incomes of employed workers. To kill this potential instability, the full capacity path must coincide with the full employment path. If both coincides at one time-point, then our hypothesis of steadily growing population proves that they coincide forever.
(Condition 5.) The full capacity path must not be above the full employment path. In order that steady growth may not be broken by the rise of demanded real wages, the full capacity path must coincide with the full employment path.

If, however, y is rising, it will sooner or later come to the 'Resource Limit' (RR′ in Figure 6.) Even if K_n is steadily increasing, the resourece cannot be increased at the same rate. For the Principles of diminishing returns to scale is now in action. Then, steady growth is only possible, if the change of technology makes it possible to use or substitute abundant factors for scarce factors. This technological improvement must come in constantly so that the resource limit will be increased at least at the same rate. Moreover, they should not change the various parameters of the original system: for a slight disturbance of the latter will touch off the unstable system.
(Condition 6.) Technological improvements must incessantly be taking place and introduced in such a way that they increase the resource limit at least proportionately to other moving equilibrium paths and yet do not change original parameters of the economic system.

This is the most unrealizable condition of all. Inventions, discoveries, and economic innovations all come in admittedly in sparks. The fact that technological improvements cannot proceed as smoothly as the population growth is, as wo have to rocognize, the most deeply seated reason why economic fluctuations do *exist.* Thus the old opinions of Knut Wicksell we alrcady quoted is now ours: " There is one particular fact in the human economy which by necessity must produce a disturbance in it. It cannot proceed evenly from one year to another as long as there is an increase in population. The increase in population, which goes on all the time, does not only require that the new men get employed like the old, nor is it enough that capital accumulation goes on at the same rate as the increase in population, but it requires in addition—because the large factor nature is unchanged—that there are all the time introduced new methods of production, that is, technical progress goes on. The question then is if this technical progress can proceed according to a curve that increases as smoothly as the curve of the increasing poulation. It is difficult to escape the conclusion that here there must be a certain lack of harmony. The technical progress will either come a little before or a little after the increase in population. In the former case there ought to be an increase and in the latter case temporarily a decrease in our requirements to regularity. It may be, however, that there is something else which is responsible for the periodicity; namely, the structure of the human society itself. The difference between technical progress and human wants causes a jerk in the organism, and this jerk is transformed into a wave proceeding in a ceratain rhythm because of the structure of the human society itself. It takes for instance a certain time before one summons courage after having passed through economic disasters, etc. I have many times used the analogy that if one hits a rocking horse with a hammer, the blows may fall quite irregularly and still the movement of the rocking horse be more or less regular because of its own form."[19]

13. *Further Inspection of Ceilings.*

In the foregoing analysis, our theory was limited to the cases in which the price-evel has the natural turning-point before output hits the maximum capacity point or the maximum employment point. Either if y hits the ceiling before p turns down, orif p does not have any turning point in Regime III (Case (A) in section 1), then y will not be able to increase as our systems of equations describe. It is to be noticed, however, that there is some difference between the maximum capacity ceiling and the maximum employment ceiling. The latter is determined by the growth of population

19) Quoted from R. Frisch, "Knut Wicksell," in *The Development of Econmic Thought*, ed. by H. W. Spiegel, 1952, pp. 698-699.

and independent of the investment being made, whereas the formar path is rising as capital in operation is being increasd. Figure 17 shows three possible cases of the tmeshape of y near the ceilings. If y hits one of the

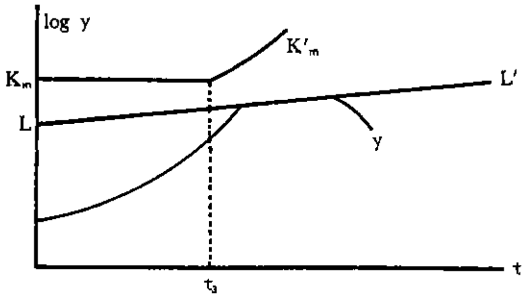

Fig. 17.2

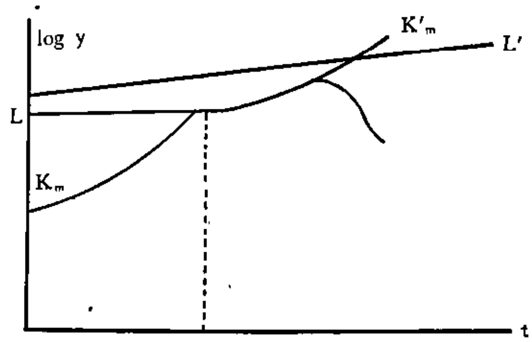

Fig. 17.1

ceilings, the the inflation of the price-level will burst out. But there are some reasons—real and monetary—why this cannot go on and on. First on the real factor which makes the price go down, if y hits a ceiling, then the decrease in α^* will inevitably occur because of the following reason. Suppose that the originally expected price was p_1^* and that as p has risen, the new expected price is p_2^*, and the capital in operation our representative enterpreneurs would have wanted to have might have been K_1 and K_2 respectively if y did not hit the ceiling. Since y hits it, they realize that there is no point of starting a new investment plan; they will stick only to the old investment plan. This means that the capital in operation planned according to the new expected price p_2^* is the same as K_1; that is $K_1 = \alpha^* p_1^* = \bar{\alpha}^* p_2^*$. Hence, $\bar{\alpha}^*$ is smaller than α^*. As the difficulty of obtaining capital goods become incresingly difficult, the capital cefficient will become smaller and smaller. As we analzed in section 10 and 11 α^* will produce a cyclical movement in the upper phase of prosperity. The price level will go down, and so will output.

Next, Monetary factor seems to play a great role in stopping the price-inflation. If the price-level rises very fast without being accompanied by the increase of output, then the newly created purchasing power by the injected new credit is no longer 'productive.' Then it is natural for the monetary system to reconsider the easy monetary policy and rate of interest. There will also come a pressure from consumers. Because of the decreasing real consumption, suffering consumers will complain of such an easy policy of banks. The higher rate of interst makes the discounted expected price lower, which deters investment. When banks impose some strain on credit, the effect may be very strong. For such a restriction of credit would

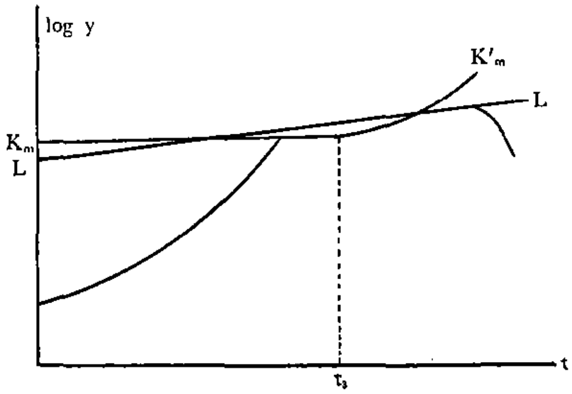

Fig. 17.3

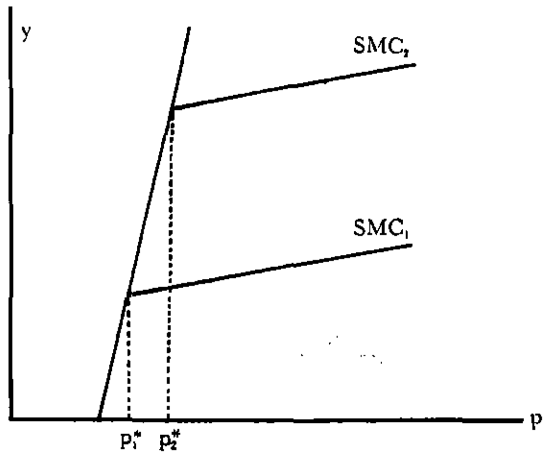

Fig. 17.4

mean to the entrepreneurs who could not borrow money from banks would have to rely on some other source of fund, which usually means to pay an extraordinarily high rate of interest. These real and monetary factors will sooner or later stop the price inflation and the price will eventually go down, unless the investment demand is something special like the demand for ammunitions in war time so that it does not decrease in spite of these factors working. In this case our assumptions of pure competition itself has to be reconsidered.

The cases in which y hits the ceiling are more unlikely to have regularity in the cyclical behaviors. But the way in which banking system react to the rising price-level and money income and entrepreneurs decrease their capital coefficient may have regularity through cycles. Then still regularity will be prevailing in economic fluctuations.