VOLUME XXVI NUMBER 1

Kyoto University Economic Review

MEMOIRS OF THE FACULTY OF ECONOMICS
IN THE KYOTO UNIVERSITY

APRIL • 1956

PUBLISHED BY THE FACULTY OF ECONOMICS
KYOTO UNIVERSITY • KYOTO, JAPAN

FATAL ERRORS NEWLY UNCOVERED IN KEYNESIAN THEORY

By Kei SHIBATA*

CONTENTS:

INTRODUCTION

It was just before the publication of Keynes's *General Theory* that I was first informed of its general context. I felt at that time that there was something fundamentally erroneous in it. The thought that this something must be uncovered has thence haunted me and continued to grow with the spread of the domination of Keynesian theory. I tried several times to reveal it. But I was always convinced afterwards that there was still left something uncovered. Now at last I came to the conviction that I succeeded exactly to perceive it.

I found out that it was not hidden, as I had hitherto been suspecting, only in the so-called Keynesian revolution, but that it was also concealed more deeply in his concept of classical economics, which distorted the latter by depriving it of one of the indispensable prerequisites of economics and differed fundamentally from the genuine one. The distortion not only remained unrevised but was aggravated by the so-called Keynesian revolution. Neither he nor his followers nor his criticizers have been aware of it.

He mistook a distorted classical economics for a genuine one, because he failed to realize that he should have improved the theoretical tool before

* The author is professor of Economics at Yamaguchi University

proceeding to analyze the problem with it. It was this that destined him evermore to go astray.

Classical Economics as conceived by Keynes

Keynes did not exactly define what he understood under Classical Economics. But from the scattered statements in his *General Theory* we are able to form an idea of it as follows:

Savings is an exchange of a present wealth for a future one. Present wealth is sacrificed for the future one only in so far as more of the latter is given in exchange for the former, because the former is usually appreciated higher than the latter. Now, what is saved of the present wealth can be made to grow in accordance with the lapse of time only when it is invested. Savings must therefore be invested. Savings must, therefore, equal real investment and constitute an increasing function of the rate of interest on real investment, thus:

$$I=F(i), \qquad \text{(I, 1)}$$

where I and i denote respectively real investment and rate of real interest.

Now, an increase in real investment entails a decrease in its marginal productivity, which equals the rate of real interest, i. Consequently, i is a decreasing function of real investment; or, inversely, real investment is a decreasing function of the rate of real interest, thus:

$$I=f(i) \qquad \text{(I, 2)}$$

Now, real investment and real consumption make up real income:

$$I+C=Y, \qquad \text{(I, 3)}$$

where C and Y signify respectively real consumption and real income.

The amount of capital being assumed provisionally as constant, real income increases in accordance with an increase in the volume of labor, thus:

$$Y=\phi(A), \qquad \text{(I, 4)}$$

where A denotes the volume of labor.

The marginal productivity of labor equals real wage, thus:

$$\frac{dY}{dA}=L, \qquad \text{(I, 5)}$$

where L means real wage.

Now, the supply of labor increases as real wage rises, thus:

$$A=\Psi(L) \qquad \text{(I, 6)}$$

These six equations involve precisely six unknowns; I, i, Y, C, A, and L, and constitute Equation System (I). This is a mathematical expression of what Keynes conceived as Classical Economics.

Fatal Errors underlying Keynes's Concept of Classical Economics

Equation system (I), which Keynes took as representing classical economics, differs fundamentally from the genuine one in that it lacks one of the indispensable prerequisites of economics, which is never omitted in the genuine classical economics. Let us first make this point clear.

Equation (I, 4) expresses the technical relation between an increase in real income and an increase in the volume of labor which is combined with a given amount of capital. This relation may possibly be illustrated by the following example, which is assumed to hold as far as $\frac{dY}{dA}$ is positive:

$$Y=(a-bA)A, \qquad (A,\ 1)$$

where a and b are asumed as constant. The differentiation of this equation will give us:

$$\frac{dY}{dA}=a-2bA, \qquad (A,\ 2)$$

but will never give us equation (I. 5), no matter how much the so-called principle of profit-maximization[1] is taken into consideration in differentiating it.

In order to arrive at equation (I, 5), we must, besides taking that

1) More accurately this should be "the principle of profit maximization as it works under free competition among entrepreneurs".

Whether it is used in this narrow sense or in a broader sense in which it covers even the case of monopoly, the principle of profit maximization concerns the behaviors of businessmen, which do not in themselves constitute the object of study of economics, but of *Betriebslehre.* Economics takes them into consideration only in so far as they partake in the constitution and movement of social economy. Now, every businessman, under free competition, equally endeavors to maximize the rate of profit accepting the wages and prices as they are determined in the market, namely, taking them as constant. Consequently, an example of a businessman's behavior can be taken as representing those af all others under free competition. It is why the theory, built under the assumption of free competition, admits such an equation as (I, 5).

The propriety of the assumption of free competition may be questioned at the present stage of monopolistic capitalism. But, I overlook this problem in this paper, for the following reasons. First, the theory of monopoly, duopoly and so forth is still unfit for replacing the economic theory of free competition, because it still remains that of *Betriebslehre,* in so far as it rests on *given* curves of cost and revenue and lacks the analysis of the reaction of the determination of monopolistic price on these curves. Secondly, incessantly proceeding innovations are always undermining monopolistic positions despite their recurrent re-establishment. And this seems to have the effect of making the economy retain the competitive nature to a great extent. Thirdly, it requires too much space to develop my own economic theory of monopolistic capitalism to be allowed here. Refer to my *A Dynamic Theory of World Capitalism* (1954, Sanwa Shobo, Kyoto) for its rudimentary study.

principle into consideration, be equipped with another condition which prescribes that real income is divided between laborers and capitalists respectively as real wages and real interest in such a way as:

$$Y = AL + Ki, \qquad \text{(I, 4a)}$$

where K stands for the amount of capital, which must be taken as constant in so far as the differentiation process goes, because the production function, which is expressed by equation (I, 4) and is taken into consideration in differentiating equation (I, 4a), is constructed by assuming K as constant.

The amount of capital, K, may be taken either as constant or as unknown in so far as the whole equation system concerns. In the latter case, however, another equation is added to the equation system so as to describe the condition of supply of capital. Accordingly, it does not matter whether K is assumed as constant or as unknown. Only for the sake of simplicity, we will provisionally assume K as constant.

If the conditions, expressed by both equations (I, 4) and (I, 4a), are given, the condition of the so-called principle of profit-maximization will then come to be expressed by equation (I, 5). This means that equation (I, 4a) is indispensable in constructing such an equation system as gives expression to a coherent whole of economic theories.

However, if we add this equation to the Keynesian equation system (I), the latter will come to contain one more equation than the number of unknowns it contains. This must be a serious problem.

One may possibly argue that equation (I, 5) is included in the Keynesian equation system only as one which defines the real wage under free competition, and does not require to be derived by means of equation (I, 4a), because the condition expressed by this equation is involved in that definition of real wage itself, and hence that there is no need for the equation system (I) to involve equation (I, 4a).

Really equation (I, 5) may be taken as giving a definition to real wage. Let us therefore provisionally accept such an interpretation, and see what will result from it. Three equations (I, 4), (I, 5) and (I, 6) contain precisely three unknowns, A, L and Y. Therefore, the values of these three unknowns are determined by these equations alone. Equation (I, 4a), however, holds true no matter whether it is included in the equation system or not, because it cannot be denied that real income is divided between laborers and capitalists. Now this equation contains such unknowns as Y, A, L *and* i. Therefore, this equation will determine the value of i provided that the values of A, L and Y are given. The value of i will thus be determined by the visible equations (I, 4), (I, 5) (1, 6) and the invisible equation (I, 4a), quite independently of the other equations. On the other hand, equations (I, 1) and (I, 2) contain only two unknowns, I and i. Therefore, these two equations suffice to determine the values of these two unknowns. In this case, however, the value of i is determined by equations (I, 1) and (I, 2) quite independently of the other equations. Thus we see that the value of i is determined by two mutually independent courses. It must, however, be unwarranted that the value of i determined by one course will equal that determined by another. In order to assume that these two values of i will necessarily be equal, one must hypothesize not only that K is unknown but also that the value of K must be determined without the participation of the condition of its supply. This hypothesis, however, amounts to giving up the idea of constructing any economic theory. Mere change in the interpretation of the meaning of equation (I, 5) can be of no use in this respect.

Classical economics, by which Keynes meant to cover all economics prior to his *General Theory*, has never been so unscrupulous as to assume that equation (I, 4a) was dispensable, namely, that the value of K must be determined without the participation of the condition of its supply. Let us refer, for instance, to Boehm-Bawerkian theory, as mathematically expressed by Wicksell. It was composed of the following equations:

Let S' denote the amount of subsistence-fund per laborer, thus;

$$S' = \frac{LT}{2}, \tag{II, 1}$$

where L and T signify respectively real wage and round-about period of production. Real income per laborer, which is denoted by P, is a function of S', thus;

$$P = f(S') \tag{II, 2}$$

Now, P is divided between the laborer and the capitalists, thus;

$$P = L + \frac{S'Z}{2}, \tag{II, 3}$$

where Z denotes the rate of interest on subsistence-fund. The principle of profit-maximization, on the other hand, enables us to derive from the preceding two equations the following condition:

$$\frac{dP}{dS'} = Z \tag{II, s}$$

Now, the total social subsistence-fund equals the total social employment times S', thus:

$$S = AS' = \frac{ALT}{2}, \tag{II, 5}$$

where S and A denote respectively total social subsistence-fund and total social employment. S and A being assumed as given, these five equations contain precisely five unknowns, S', L, T, P and Z.[2]

Let us call this the Wicksellian equation system (II). Equations (II, 2) and (II, 4), involved in this equation system, correspond respectively to equations (I, 4) and (I, 5) involved in the Keynesian equation system (I). It must be emphatically noticed that equation (II, 4) is derived here not directly from equation (II, 2), but via equation (II, 3) which corresponds to equation (I, 4a). Of course, these correspondences appear to be metaphoric in this case, because the Wicksellian equation system is constructed on the basis of a production function which expresses such a real income per laborer as corresponds to *subsistence-fund* per laborer, while the Keynesian is constructed on the basis of a production function which expresses such a real income as corresponds to total *employment*. Let us, for the sake of making the comparison easier, transform the production function, which underlies the Wicksellian equation system (II), as follows:

Let us denote by A' the number of laborers employed per wage-unit subsistence-fund, thus:

$$1=\frac{TA'}{2} \quad \therefore \quad A'=\frac{2}{T} \qquad \text{(II, 1)}'$$

Real income per wage-unit subsistence-fund is a function of A', thus:

$$P'=f_a(A'), \qquad \text{(II, 2)}'$$

where P' stands for real income per wage-unit subsistence-fund. P' is divided between laborers and capitalists, thus:

$$P'=A'L+LZ, \qquad \text{(II, 3)}'$$

where Z denotes the rate of real interest on subsistence-fund. The principle of profit-maximization enables us to derive from the preceding two equations the following condition:

$$\frac{dP'}{dA'}=L \qquad \text{(II, 4)}'$$

Let us denote total social subsistence-fund and total social employment respectively by S and A. S equals L times the ratio of A to A', thus:

$$S=L\frac{A}{A'}=\frac{ALT}{2} \qquad \text{(II, 5)}'$$

S and A being assumed, as in the case of Wicksell, as given, these five

2) To speak the truth, Wicksell avoided the notation S'. Therefore, his equation system was free from equation (II, 1). Moreover, he resorted to the concept of "subsistence-fund in terms of wage-unit" per laborer, i. e., S'/L, instead of to that of "subsistence-fund in real terms" per laborer, i. e., S'. Therefore, he experessed equations (I, 2)~(I, 4) respectively as follows:

$$P=f(T), \quad P=L+\frac{LTZ}{2}, \quad \textit{and} \quad \frac{dP}{dA}=\frac{LZ}{2}$$

equations contain precisely five unknowns, A, T, P', L and Z.

Let us call this the transformed Wicksellian equation system (II)'. It will now be clearer that the equation system, which gives expression to the genuine classical economics, involves in itself equation (II, 3)' which corresponds to equation (I, 4a). Thus we see that equation (I, 4a) constitutes one of the indispensable equations which compose the genuine classical economics, even though it is not included in Keynesian equation system (I), in which Keynes envisaged a representation of classical economics. Keynesian equation system (I), which thus lacks an indispensable eqation (I, 4a), therefore, should suffer from a shortage in equation. In its appearance, however, it seems to suffer from nothing of that sort. It has attained this appearance, because it has this shortage in the number of equations supplemented by the intrusion of equation (I, 2). But the equation (I, 2), which intruded in the equation system, not only lacks the capacity of substituting for equation (I, 4a) but is incompatible with equation (I, 1).

That equation (I, 2) lacks the capacity of substituting for equation (I, 4a) is proved by the fact that it is impossible to derive from equations (I, 2) and (I, 4) such a condition of profit maximization principle as is represented by equation (I, 5).

That equation (I, 2) is incompatible with equation (I, 1) is demonstrated as follows: Equation (I, 1) represents the demand for real investment. It gives expression to the fact that some portion of income is not consumed but saved for investment. Equation (I, 2), too, expresses the demand for real investment. It shows that real investment, which is made for the purpose of exacting real interest, has the rate of that interest lowered as it increases. Thus we see that equations (I, 1) and (I, 2) equally give expression to the demand for real investment. Really they refer to its different aspects. But these different aspects cannot be represented simultaneously in such a manner as are done by means of these two equations, because each of these equations defines the demand for real investment in such a wise that leaves no room for another demand condition to partake in the determination of real investment. Really, if equation (I, 2) plays the role of representing in a somewhat varied manner what is represented by equation (II, 4) involved in the Wicksellian equation system, it may be compatible with equation (I, 1). But it will then come in conflict with equations (II, 4) and (I, 5), because equation (II, 4) and equation (II, 4)', which corresponds to equation (I, 2) in the above defined new sense, represent the condition of the same principle of profit-maximization under the same productive technological condition, which is differently represented only by the use of different basic terms. Moreover, equation (I, 2) in the above defined new sense cannot stand alone, but must accompany to itself what corresponds to equations (II, 2) and (II, 3), which will come in conflict respectively with equations (I, 4) and (I, 4a). Therefore, it is impossible to regard equation (I, 2) as one of the above defined new sense. This means that equation (I, 2) cannot play any other role than that in which it is incompatible with equation (I, 1). To speak the truth, this was precisely the reason why genuine classical economics, which involved equation (I, 1), neglected equation (I, 2).[3]

Keynes, however, was too anxious to include equation (I, 2) in his equation system (I) to pay due attention to what was discussed above. He was convinced that such a condition as was represented by equation (I, 2) was at work in actual economy, and was resolved to take it into consideration in constructing economics. He, however, did not think of the need, under which he was placed when he intended to take it into consideration, of so reconstructing the theoretical tool in advance as to make it fit for the new task. He only put new wine into old wine-skins, heedless of the great danger that that would make the skins burst.[4]

Essential Feature of Keynesian Revolution

It was in the above examined distorted classical economics that Keynes brought about his revolution. He accomplished this by changing some of the equations contained in it.

He first replaces equation (I, 1) with:

$$I=sY, \qquad \text{(I, 1)'}$$

where s stands for the ratio of savings to real income, which he called savings-coefficient.

He asserted that s is an increasing function of real income. And this characterization of s constitutes one of the pillars upon which his revolutionary structure is built. But the grave problem involved in his theory, which we are now going to examine, remains as it is even if we remove this peculiar characterization of s. Moreover, recent statistical investigations have proved more and more the untenability of Keynesian characterization of s. Let us therefore assume s as constant in so far as this paper goes.

The replacement of equation (I, 1) with (I, 1)′ deprives it, on the one

---

3) Of course, it becomes to require a more detailed discussion if we replace such a production function as is represented by Keynes' equation (I, 4) with such as:

$$Y=f(K, A)$$

The essential point, however, remains unaffected even if we change our assumption in this respect.

4) Refer to Appendix.

hand, of its capacity of determining the values of i and I in combination with equation (I, 2), and empowers it, on the other hand, to determine the value of Y in combination with equation (I, 2) independently of both quantities of employment and of capital, provided that the value of i is determined elsewhere, as will be revealed later. By replacing equation (I, 1) with (I, 1)$'$, he thus proceeds further on the road of negligence of the factors which partake in the determination of real income. It must be noticed that such a negligence was precisely that, for which he had paved the way by removing equation (I, 4a) from his equation system (I).

He next replaces equation (I, 6) with:

$$L' = \lambda L, \qquad \text{(I, 6)}'$$

where L' and λ respectively denote nominal wage and price level, of which the former is taken by him as one which will be constant until the volume of employment attains a certain level.

This replacement of equation (I, 6) with (I, 6)$'$ has the result of inserting in the equation system (I) one more unknown, λ: and hence of making the equation system fall short of one equation. He unwittingly makes up for this deficit in the number of equations by the following consideration regarding money.

Financial institutions are, according to him, in a position to increase the amount of money, which will be denoted by M, to any extent. M is thus determined outside the scope of the object of theoretical analysis, and must be regarded as a given factor from the point of view of theoretical analysis.

Now, M is divided into two parts: one part is held for speculative motive, namely, in wait for a more profitable chance of investment, and the other part is held for transaction motive, namely for the purpose of facilitating the transaction of goods, thus:

$$M = M_s + M_t, \qquad \text{(I, 7a)}$$

where M_s and M_t stand respectively for the amount of money held for speculative motive and that held for transaction motive.

M_s is held because its holders prefer holding their wealth in liquid form—namely in a form in which it can be more easily mobilized—to laying it down fixed at the prevailing low rate of interest. Hence, M_s is a decreasing function of the rate of interest, thus:

$$M_s = \varphi_s(i) \qquad \text{(I, 7b)}$$

From these two equations, we obtain:

$$M_t = M - \varphi_s(i) \qquad \text{(I, 7b}'\text{)}$$

This equation shows that the supply of M_t is an increasing function of i, because M is constant and $\varphi_s(i)$ is a decreasing function of i, and can be taken as representing the supply curve of M_t.

As against this supply curve of M_t, there is a demand curve of M_t, which gives expression to the fact that the amount of money held for the purpose of facilitating the transaction of goods increases according as i lowers: because the lowering in i tends both to increase the volume, and to raise the price, of the goods transacted, thus:

$$M_t = \varphi_t(i) \qquad \text{(I, 7c)}$$

By inserting the value of M_t as defined by this equation into equation (I, 7b′), we obtain:

$$M - \varphi_s(i) = \varphi_t(i) \qquad \text{(I, 7)}$$

This is the equation which Keynes inserted in his equation system (I) by taking the monetary condition into consideration.

By the insertion of this equation (I, 7), the Keynesian equation system is remedied of the deficit in the number of equations, which is caused by the replacement of equation (I, 6) with equation (I, 6)′. It comes now to be composed of seven equations, (I, 1)′, (I, 2), (I, 3), (I, 4), (I, 5), (I, 6)′ and (I, 7), which contain precisely seven unknowns, I, Y, i, C, A, L, and λ. Let us refer to this revised equation system as the Keynesian equation system (I)′.[5)]

Now, equation (I, 7) contains only one unknown, i. And hence, this equation suffices to determine the value of i. Once the value of i is thus determined, the value of I can be determined by equation (I, 2). Once the value of I is thus determined, the value of Y can be determined by equation (I, 1)′. Once the value of Y is thus determined, the value of A can be determined by equation (I, 4). Once the value of A is thus determined, the value of L can be detormined by equation (I, 5). Once the value of L is thus determined, the value of λ can be determined by equation (I, 6)′.

The volume of employment, A, is thus determined at a certain level, provided that the amount of money, M, is given. Now, M can be given at will by the bank. And an increase in M will have the effect of lowering i, because it will help the M_t supply curve, defined by equation (I, 7b′), shift rightward. The lowering in i increases I, because I in equation (I, 2) is a decreasing function of i. The increase in I entails an increase in Y, because s in equation (I, 1)′ is constant. The increase in Y entails an increase in A, because Y in equation (I, 4) is an increasing function of A.

5) Keynes announced that he conceived Y, I and C in terms of *wage-unit*. I neglect this announcement in this paper, and take these as conceived in *real terms*. I do so, because we would have to take dY/dA always as unity and conflict Keynes's own argument, if we try to be true to his announcement in this respect and conceive Y in terms of wage-unit.

Thus the financial institutions have it under their power to increase the volume of employment by increasing their supply of money.

The financial institutions, however, cannot wield this power limitlessly, according to Keynes, because an increase in A entails a lowering in L, according to the conditions set by equation (I, 5), and this lowering in L will cause the laborers to stop increasing the supply of labor once it attains the point at which it touches the supply function of labor, which is assumed by Keynes to be at work underneath the surface of the market, and to be ineffective only so long as the real wage obtained under hereditary nominal wage stands at a higher level than is set by itself. Once this point is reached, further supply of money is deprived of its power of increasing A, and begins to consume itself in the form of a rise in price level, λ. He called the volume of employment which corresponds to this point the Full Employment.

Now the banks do not usually supply money enough to make the actual volume of employment grow so much as to attain the level of Full Employment. Hence there arises what may be called Involuntary Unemployment, which comprises those, who are willing to get employment even at the sacrifice of a lowering in real wage (provided that nominal wage remains unaffected) and yet are precluded from getting employment. Involuntary unemployment equals the excess of full employment over actual employment. The banks, therefore, have it under their power to remove the involuntary unemployment merely by becoming more generous in their supply of money. Classical economics lacked an insight into this power of financial institutions because it was ignorant of the mechanism according to which this power worked. The revelation of this mechanism constitutes the very quintessence of the Keynesian revolution.

Fatal Error underlying Keynesian Theory (1)

According to Keynesian equation system (I)′, the value of i is determined by equation (I, 7). The value of i being thus determined, the value of I can be determined by equation (I, 2). Here we see that real investment is now determined independently of equation (I, 1)′, which is a revised form of equation (I, 1).

By way of critically studying the Keynesian concept of classical economics, we noticed previously that the genuine classical economics omitted equation (I, 2), and demonstrated that this was because each of the equations (I, 1) and (I, 2) defined the demand condition of real investment in such a wise as left no room for another demand condition to partake in the determination of real investment. We demonstrated there also that the inclusion of equation (I, 2) besides equation (I, 1) in Keynesian equation system (I)—which meant to let double demand conditions partake in the determination of real investment— destined Keynesian equation system (I) to become overfilled with equations in case it was supplied with the missing equation (I, 4a).

It may seem that Keynes solved this problem in his own way by means of equation (I, 1)′ in his new equation system (I)′, because equation (I, 1)′ which replaces equation (I, 1) is now exempted from the office of conflicting equation (I, 2). Is equation (I, 1)′ then endowed with such a character as enables it to play the role of substituting for equation (I, 4a)? No, equation (I, 1)' does not have the capacity of making it possible for the principle of profit-maximization to attain, in combination with equation (I, 4), such a condition of its realization as is represented by equation (I, 5). Any equation, which is devoid of such a capacity, is unqualified to play the role of a substitute for equation (I, 4a).

Equation (I, 4a) must, therefore, be deemed as indispensable also to the Keynesian new equation system (I)′. However, if this equation is added to it, it becomes to suffer from an excess in the number of equations over that of unknowns involved therein. Moreover, it will begin to work in the following manner:

Equation (I, 7) determines the value of i. The value of i being thus determined, equations (I, 4), (I, 4a) and (I, 5) will determine the values of A, L and Y; while equation (I, 2) will determine the value of I so long as the value of i is thus determined. The values of Y and I being thus determined, equation (I, 3) will determine the value of C. On the other hand, the value of L being thus determined, the value of λ will be determined by equation (I, 6)′. Thus the values of all unknowns contained in the equation system (I)′ will be determined without the participation of equation (I, 1)′. We see that equation (I, 1)′ is put in Keynesian equation system (I)′ as a trouble maker.

But Keynes is not satisfied by merely letting the touble maker in the equation system. He wants far more than that. He wants to make more room for the mischivous play of the trouble maker, equation (I, 1)′. So he expells the guardian of any sound economics, equation (I, 4a), out of his equation system, and that, misleadingly enough, in the name of truth to classical economics. Now a play of ghosts begin as follows:

An increase in M will lower i, as will be demonstrated by equation (I, 7). This lowering in i will increase I, as will be demonstrated by equation (I, 2). This increase in I will entail an increase in Y, as will be demonstrated by equation (I, 1)′. This increase in Y will entail an increase in A, as will be demonstrated by equation (I, 4). This increase in A will entail a lowering in L, as will be demonstrated by equation (I, 5). Keynes asserts that this fall in L will not proceed beyond the point at which Full Employment is reached. But what will occur, if more money is supplied? He replies that an inflation will occur. But, how does an inflation occur?

The additional money, too, must lower i. This additional fall in i, too, must increase I. This additional increase in I, too, must entail an increase in Y. Of course, this addtional increase in Y presupposes an additional increase in A. According to Keynes, however, this additional increase in A is not forthcoming. Let us hence assume that this additional increase in A is not forthcoming. The additional increase in Y must then be impracticable. If this additional increase in Y is thus impracticable, additional increase in I must then be impracticable. If this additional increase in I is thus impracticable, additional fall in i must then be impracticable. If this additional fall in i is thus impracticable, additional increase in M must then be impracticable. There can then be no inflation at all. There is no gear in this mechanism which enables the additional money to call forth inflation.

But, inflation must be demonstrated as possible, because it actually occurs sometimes. Additional increase in M must be demonstrated as practicable, because it is actually practicable. Additional increase in M must be demonstrated as causing inflation, because it actually causes it to occur sometimes. If additional increase in M should thus be practicable, additional fall in i must be practicable. If additional fall in i is thus practicable, additional increase in I must be practicable. If additional increase in I is thus practicable, additional increase in Y must be practicable. If additional increase in Y is thus practicable, additional increase in A must be practicable. If additional increase in A is thus practicable, additional fall in L must be practicable. If additional fall in L is thus practicable, additional rise in λ must necessarily occur. We have an inflation here. But how is it made possible? It is made possible evidently through an additional increase in A, which however was assumed by Keynes previously as " not forthcoming ". Thus we see that an additional increase in A, whics is assumed on the hand as "not forthcoming," must be presupposed, on the other hand, as forthcoming in order to cause an inflations to occur! Here we see a ghost.

Accordsng to Keynesian theory, as we have just observed, an increase M entails a fall in i, namely, in the marginal productivity of capital. This fall in i, in its turn, entails an increase in I. This increase in I, in its turn, entails an increase in Y. This increase in Y, in its turn, entails an in increase in A, at least until full employment is attained. This increase in A, in its turn, entails a fall in L, namely, in the marginal productivity of labor. However, to assume that the marginal productivities of all productive elements—in our present case, labor and capital—can fall (or rise) simultaneously, is flatly to contradict the very elementary theory of marginal productivity. How, then, can such an absurdity occur?

Look at what is going on behind the screen, namely, on the surface of equation (I, 4a), where, although all factors are moved according to the movement on the screen, also the dependent movement of that factor, which is not seen on the screen, is visible. The movement of those factors is shown by the diffrentiation of equation (I, 4a) as follows:

$$dY = LdA + AdL + Kdi + idK \qquad (\theta)$$

This equation, combined with equation (I, 5), will be reduced to:

$$O = AdL + Kdi + idK \qquad (\theta)'$$

Now, according to what is going on on the screen, both dL and di are negative. Therefore, dK in equation $(\theta)'$ must be positive. This implies that the movement on the screen presupposes that capital increases according as money increases. Can, however, such an increment in capital as is conjured up at will by a mere increase in money be any thing other than a ghost? Moreover, equation (I, 4a), from which equation $(\theta)'$ is derived, is a collateral of equation (I, 4), which, in its turn, is constructed by assumnig the amount of capital as constant. Therefore, K must be treated as constant in differentiating equation (I, 4). Therefore, dK in equation $(\theta)'$ must necessarily be zero. Therefore, the increment in capital, dK, which is conjured up by an increase in the supply of money, must be a ghost also in this respect.

All these ghosts arise from equation (I, 1)′, which presupposes both that real investment can produce real income by multiplier—inverted saviugs-coefficient—times itself, and that current real investment can augment current real capital.

It is certainly true of real investment that some of the income, created by it, appears in the market as a purchasing power and helps from a demand side the income to increase. It must, however, be also true of real investment that it will result in an increase in real capital when it is added to the latter, and will, as such, help produce more real income. The former effect of real investment is regulated both by savings coefficient and by the time-lag, which is involved in the disposition of income and owes itself to the fact that income is not necessarily discharged instantaneously when it is received. The latter effect of real investment is regulated both by technical coefficient[5] and by the time-lag, which is involved in the technical course of production and owes itself to the fact that real investment (i. e., an increment in the volume of means of production) cannot instantaneously mature in a product, in which additional income is involved, but only after the lapse of the period of production.

Equation (I, 1)′ overlooks the difference between the effect of real investment as a demand creator and that as an increment in the means of production. Consequently, it overlooks both the difference between the magnitude of the effect of real investment as a demand creator and the magnitude of the effect of it as an increment in the means of production, on the one hand, and the difference between the length of the time-lag in the effect of real investment as a demand creator and the length of the time-lag in the effect of it as an increment in the means of production. Moreover, it overlooks entirely the time-lag involved in the effect of real investment as an increment in the means of production.

The magnitude of the effect of real investment as a demand creator is conditioned by savings coefficient[6] in such a wise as:

$$I=\{1+(1-s)+(1-s)^2+\cdots\}=Y \quad \therefore \quad I\frac{1}{s}=Y, \qquad \text{(I, 1)}'$$

while the magnitude of the effect of it as an increment in the means of production is conditioned by technical coefficient[6] in such a manner as:

$$I\frac{1}{c}=\Delta Y, \qquad \text{(X)}$$

where c denotes the technical coefficient.[6] Therefore, even if we overlook the time-lag, it is entirely mistaken to overlook the difference between the conditiones expressd by these two equations. It amounts to an entire overlook of productive technological aspect of economy to regard equation (I, 1)′ as capable of representing equation (X). Moreover, even if we overlook this problem as well as the time-lag, the effect of real investment as an increment in the means of production must be taken into consideration not by means of equation (I, 1)' but by means of an equation representing the supply of real capital. It is therefore entirely mistaken to make equation (I, 1)′, as Keynes does, represent also the effect of real investment as an mcrement in the means of production.

In addition to what is discussed above, there are still more fatal defects involved in equation (I, 1)′. To begin with, it is mistaken to assume, as equation (I, 1)′ presupposes, that the length of time-lag caused by the usage regarding the expenditure of income, necessarily equals the length of time-lag, required by the period of production. The implication of this irrational assumption is far more profound than it may appear to a careless mind.

---

6) Refer to footnote 3) of Appendix.

If it is the length of time-lag, caused by the usage regarding the expenditure of income, that is in question, one may be allowed to assume it as zero without depriving economics of its very essense. But, if it is the length of time-lag, required by the period of production, that is at issue, one cannot assume it as zero without depriving thereby economics of its very essence, because economics which does not deal with economy cannot be true economics; economy which does not involve production in itself cannot be true economy; and production which does not require any time in its productive course cannot be production at all. Therefore, it is depriving economics of its very essence to assume, as equation (I, 1)′ presupposes, that the time, required of the additional means of production for maturing in products, is zero.[7]

We must, therefore, strictly discriminate the effect of real investment as a demand creator from that as an increment in the means of production, and change equation (I, 1)′ so as to make it represent only the former. Let us, in accomplishing this change, assume provisionally, as Keynes did, that the length of time-lag involved in the effect of real investment as a demand creator is nil. Equation (I, 1)′ will then come to express not only, as was the case with Keynes, that the demand for real income is multiplier times real investment but also that the demand for real investment is savings-coefficient times real income. In this new capacity, equation (I, 1)′ becomes one, which is incompatible with equation (I, 2), because it now comes to define the demand condition of real investment, and that in such a mamer as allows no other demand condition to partake in the determination of real investment. If, therefore, equation (I, 1)′ in this new sense is to be included in Keynesian equation system (I)′, equation (I, 2) must be withdrawn from it.

7) In one place, Keynes asseated what amounted *de facto* to replace equation (I, 2) with such as such as defined I as an increasing function of ΔC, namely such as:

$$I=f(\Delta C) \qquad \text{(I, 2)'}$$

His concept of this functional relation was rather tinged with under-consumption theory. It was Samuelson who re-conceived it according to acceleration principle. In either case, equations (I, 1)′, (I, 2)′ and (I, 3) come to suffice to determine the values of C, I and Y, irrespective of the conditions represented by other equations. It has recently become more usual to apply the acceleration principle in such a wise as to regard I as an increasing function of ΔY, thus:

$$I=f(\Delta Y) \qquad \text{(I, 2)''}$$

In this case, equations (I, 1)′ and (I, 2)″ come to suffice to determine the values of I and Y irrespective of the conditions represented by other equations. It is incredible that so many economists canbe so brave as to nefilect the above noticed fundamental error involved in equatipn (I, 1)′ as well as to overlook the importance of other conditions partaking in the determination of Y and$I$, than those expressed by such equations. Moreover, the hypothesis underlying commonly the under-consumption theory and the acceleration principle cannot pass without detailed scrutiny. I must, however, refrain from dwelling on this point any longer because it takes too much space to be allowed hare to do so.

Then the total mechanism will come to work as follows:

Equation (I, 7) determines the value of i. Given the value of i, equations (I, 4), (I, 4a) and (I, 5) will determine the values of Y, A and L. Given the value of Y, equation (I, 1)′ will determine the value of I. Given the values of Y and I, equation (I, 3) will determine the value of C. On the other hand, given the value of L, equation (I, 6)′ will determine the value of λ. And everything will appear to proceed without any serious problem. Let us call this the *Case A*. But it is only in the outer appearance that it does so in this case. The reason is: The values of Y, A and L can be determined in this case by equations (I, 4), (I, 4a) and (I, 5), because the value of i is determined by means of equation (I, 7). This implies that real income is determined without the participation of the supply conditions of labor. It is, however, absurd to assume that the supply conditions of labor cannot participate in the determination of real income. This reveals that there is still another fatal error hidden in the Keynesian equation system.

If, on the contrary, equation (I, 2) is to be included in the Keynesian equation system (I)′, equation (I, 1)′ even in the above revised sense must be withdrawn from it precisely because of its above noticed new character, which makes it incompatible with equation (I, 2). If something akin to equation (I, 1)′ is still to be included side by side with equation (I, 2) in that equation system, therefore, equation (I, 1)′ must in advance be remedied of the above noticed new character which leaves no room for other demand condition to partake in the determination of *real* investment. This can be accomplished only by transforming this equation into one which expresses *nominal* demand either for investment or for consumers′ goods, namely, into such as:

$$\lambda I = sY \text{ or } \lambda C = (1-s)Y, \qquad \text{(I, 1)}''$$

where λ denotes the price level.[8] Let us call this the *Case A′*.

---

8) Notably enough, Keynes, in his Treatise, had *de facto* adopted equation (I, 1)″ by asserting that "what is saved is not necessarily invested". Had he been true to this assertion of his, he should have adopted equation (I, 1)″ instead of equation (I, 1)′. in his *General Theory*.

To stand for an idea, is one thing. To make his theory one representing that idea is another. He is enabled to develop such a theory only when he produces such a theoretical gear as represents exactly that idea and inserts it in a right position in the whole theoretical mechanism. For instance, economists have long since been speaking of real investment, namely, of expansive reproduction. But they failed to develop a real theory of expansive economy, because they continued to make use of the customary theoretical tool, which presupposed a simple reproduction. It is just the same with Keynes. He stressed that what was saved was not necessarily invested. Actually, however, he made use of such a theoretical gear as equation (I, 1)′, which presupposed that what was saved must necessarily be invested.

This equation will be compatible with equation (I, 2). But it will not be so with all the other equations involved in the Keynesian equation system (I)'. This is demonstrated by the fact that this equation system, when it is supplied with equation (I, 4a), still contains one equation in excess of the number of unknowns contained therein. This, too, reveals that there is still another fatal error hidden underneath the Keynesian equation system (I)′. Let us proceed to uncover it.

Fatal Error underlying Keynesian Theory (2)

Keynes did not precisely define equation (I, 7c), from which equation (I, 7) is derived.

According to the hereditary expression, this equation may have the following form:

$$M_t V = \lambda(B+C) \text{ or } = \lambda(I+C),$$

where V and B respectively denote the velocity of circulation of money and the volume of means of production. This sort of monetary equation, however, does not contain i among its component factors, and cannot be deemed to be what is meant by Keynes by equation (I, 7c).

As is widely known, it was Wicksell's *Geldzins und Güterpreise* that paved the way for a really theoretical analysis of the relation between the rate of interest and the price level—a relation, which had been attracting the attention ever since the days of Mercantilism. This contribution by Wicksell has thence been widely accepted and used as a cornerstone in building any sound theory. Keynes's equation (I, 7c) must therefore be taken as having something to do with it.

Now this contribution by Wicksell was composed of two specifications: the specification of the condition which affects price, and the specification of the time of existence of the product, upon whose price that condition gives effects. Namely, he emphasized, on the one hand, that it is neither the rate of *monetary* interest alone nor the rate of *real* interest alone but the *relation* between these two rates of interest that affects the price level; and on the other hand, that this effect appears directly not on the price of the product yet to be produced but on that of the existing product. He, however, overlooked that this effect does not appear directly on the price of all products but on that of the cost goods alone. If we improve his theory in this respect, it will come to be expressed, for instance, by the following monetary equation:

$$M_t V = \frac{1+i}{1+\nu} B_0 + \lambda C_{-1}, \tag{I, 7c$'$}$$

where ν, B_0 and C_{-1} denote respectively the rate of monetary interest, the volume of means of production existing at the beginning of the current period, and the volume of consumers' goods existing at the same time.

We may be allowed to regard V, B_0 and C_{-1} either as constant or as unknown. In the latter case, however, we will be supplied with two additional equations regarding the supply conditions of B_0 and C_{-1} and a further additional equation regarding the conditions determining V. Let us, however, for simplicity's sake, assume them as constant.

Only such a monetary equation as the above cited Wicksellian equation (I, 7c′) can involve in itself factors which represent the rate of interest. Keynes's equation (I, 7c) must, therefore, be taken as implying this sort of monetary equation. Now, this equation (I, 7c′) contains at least two unknowns regarding two rates of interest, i and ν. This demonstrates that it is mistaken to make, as Keynes did, the demand curve of money for transactive motive represented by such an equation as contains only one unknown, i. Keynesian equation (I, 7c) must, therefore, be replaced with such an equation as :[9]

$$M_t = \varphi_t(i, \lambda, \nu) \qquad \text{(I, 7c}''\text{)}$$

Moreover, the rate of interest, which defines the demand curve of money held for speculative motive, must not be assumed as the rate of real interest, i, but as the rate of monetary interest, ν. The Keynesian equation (I, 7b) must, therefore, be replaced with such as :

$$M_s = \varphi_s(\nu) \qquad \text{(I, 7b}'\text{)}$$

Consequently, Keynesian equation (I, 7) must be replaced with such as :

$$M - \varphi_s(\nu) = \varphi_t(i, \lambda, \nu)^{9)} \qquad \text{(I, 7)}'$$

This revision of Keynesian equation (I, 7) has two profound effects. In the first place, it makes Keynesian equation system contain one more unknown, ν. In the second place, it makes both Keynes's own argument and our foregoing argument untenable, both of which started from the assumption that i could be determined by equation (I, 7).

The fear that the replacement of equation (I, 7) with equation (I, 7)′ would undermine Keynesian total argument has caused the defenders of Keynesian theory to assert that an equation such as :

---

9) The replacement of equation (I, 7) with equation (I, 7)′ makes it more appropriate to alter equation (I, 1)″ into such as :

$$\lambda C_{-1} = (1-s)Y_{-1},$$

because the assumption underlying equation (I, 1)″, that the time-lag involved in the disposition of income is nil, is incompatible with that underlying equation (I, 7)′. It is incompatible with the latter, because it neglects what is emphased by the latter, namly, the specificity of the time of existence of the product, whose price is affected by the inequality of two rates of interest.

$$\dot{i}=\nu, \tag{Z}$$

was involved in the very concept of equilibrium, and hence, also in the Keynesian equation system, because it, too, dealt with an equilibrium.

This sort of defence of Keynesian theory, however, is entirely futile. The reason is: Such a condition, as is described by equation (Z) refers to a case where money is neutral, namely, to a case where sufficient amount of money is actually supplied so as to enable the normal price, which is normally determined at a certain fixed level independently of the actual supply of money, to maintain itself. In the case of Keynesian market equilibrium, however, price is supposed to vary because the actual supply of money does not necessarily conform to such a requirement. This consideration reveals that there is no room left for such a defensive argument. Moreover, such an equilibrium as conforms to the condition set by equation (Z) is incompatible with the existence of involuntary unemployment, which Keynesian equilibrium allows to exist. This fact, too, demonstrates how futile such a defenceve argument is.

Seeing that the above defence was useless, some defenders of Keynesian theory began to resort to its peculiar interpretation which takes him as assuming that the price level will remain constant until full employment is reached. This, however, is an entire misinterpretation. The reason is: ① If this interpretation holds true, λ in equation (I, 6)′ must be taken as constant. Then the number of unknowns contained in Keynesian equation system becomes smaller than that of equations contained therein. ② λ being constant, equation (I, 6)′ will enable us to detemine the value of L, because L', too, is constant. The value of L being thus given, equations (I, 5) and (I, 4) will enable us to determine the values of A and Y independently of the supply of money. This, however, radically conflicts Keynes's assertion. ③ Equations (I, 7)′ and (Z) determines the value of i. The value of i being thus determined, equation (I, 2) will enable us to determine the value of I. The value of I being thus determined, equation (I, 1)′ will enable us to determine the value of Y. There can, however, be no warrant that the value of Y, thus determined independently of equations (I, 4), (I, 5) and (I, 6)′ will equal such a value of Y, as is determined by the latter three equations independently of the other equations according to the manner discussed under ②. ④ Even if we overlook the possibility of determination of the values of L, A and Y which takes place along the course discussed under ②, the value of Y, determined along the course discussed under of ③, will enable us to determine the value of A by means of equation (I, 4). The value of A, determined in such a manner, will be increased by increasing the supply of money. But this increase in the volume of employment will not necessarily be stopped at the point of full employment. The reason is: The point of full employment will have to be defined, under such an interpretation of Keynesian theory, as the point at which the supply curve of labor—which is defined in terms of real wage—ends to be horizontal and begins to rise along rightwardly rising slope. This, however, flatly conflicts Keynesian definition of Full employment. ⑤ According to such an interpretation of Keynesian theory, labor supply curve, which is defined in terms of real wage, will always have to partake in the determination of real wage and employment. This, however, conflicts the very part of Keynesian assertion, where he boasted of his own novelty, namely that part where he emphasized that disutility of labor—i. e., labor supply curve, defined in terms of real wage—does not partake in the determination of real wage. ⑥ Moreover, this interpretation contradicts Keynesian definition of involuntary unemployment, according to which laborers are regarded as involuntarily unemployed only in so far as they are ready to accept a curtailment in real wage for the sake of getting employment.

Keynesians are perplexed by our demonstration of the theoretical necessity of replacing equation (I, 7) with equation (I, 7)′, and resort to nonsensical arguments in their desperate effort for refuting it. They do so solely because they are not yet aware of the fact that this replacement, far from impairing Keynesian theory, rescues it from the difficulty from which it still suffers—as was proved in the last part of the preceding section—even after it is corrected of the error involved in equation (I, 1)′. It is precisely this replacement that helps remove the difficulty inherent in the mechanism under the *Case A′*, because it supplies the equation system representing this case with the missing unknown. Of course, it is not so simple in regard to the *Case A*, because it makes the equation system representing this case fall short of one equation, and renders the unknowns contained therein indeterminable. It necessitates, therefore, a further amendment in this case. This amendment may be taken as possibly accomplished by simply rehabilitating equation (I, 2) into the equation system. This, however, will not do, because equations (I, 1)′, (I, 2), (I, 4), (I, 4a) and (I, 5) will then be enabled to determine the values of A, i, I, L and Y independently of the supply conditions both of labor and of money, and these conditions will come to have their power of affecting their values denied. If this absurdity is to be avoided, the above amendment must be accompanied by a simultaneous replacement of equation (I, 1)′ with equation (I, 1)″. This, however, leads us to the same result as we have just arrived at in regard to the *Case A′*.

Fatal Error underlying Keynesian Theory (3)

Keynes assumed that the quantity of money held for speculative motive was a decreasing function of the rate of monetary interest. He referred to it merely as liquidity preference, although it is more appropriate to call it liquidity preference of the market and set it against liquidity preference of the bank.

If we assume that the quantity of money existing in a society is given, the quantity of money held for transaction motive decreases as much as the quantity of money held for speculative motive increases, and *vice versa.* Therefore, the quantity of money existing in a society being given, the quantity of supply of transaction money must be an *increasing* function of the rate of monetary interest so long as the quantity of money held for speculative motive is a *decreasing* function of it. Presumably, he assumed that the quantity of speculative money was a decreasing function of the money rate of interest because the quantity of *supply* of transaction money then becomes an increasing function of the money rate of interest. He wished to have such a supply function of transaction money, presumably because the *demand* for transaction money is a decreasing function of the money rate of interest.

It is, however, mistaken to assume that the volume of speculative money is a decreasing function of money rate of interest. Suppose that the money rate of interest falls. The result thereof will be a rise in prices not only of goods and shares but also of national bonds and debentures. The price of national bonds and debentures rises in this case, because they represent fixed amounts of interest and these fixed amounts of interest divided by the money rate of interest result in their price. Consequently, a lowering in the money rate of interest must not be assumed, as Keynes did, as inducing the quantity of speculative money to *increase*, but, on the contrary, as inducing it to *decrease.* This is because the gradual rise in price of national bonds and debentures gives rise to a rush to them which aims at gaining the margin caused by their price rise. Of course, this rush will exhaust, and even reverse, itself in time when their price is risen too much and one becomes deffident of its further rise and rather fearful of its reactionary fall. Once such a reactionary movement sets in, it would not be stopped by a slight lowering in the money rate of interest, and the quantity of speculative money would become a decreasing function of the money rate of interest as Keynes assumed. But it is nothing but an exceptional phenomenon which arises only on such a particular stage of change, and cannot be taken as the basis of a 'general' theory.

Keynes seems to have been neccessitated to take the quantity of speculative money as a decreasing function of the money rate of interest at the flat contradiction to this fact, solely because he overlooked the liquidity preference of the *bank* and believed that a bank could supply any amount of money it wished. He was ignorant of the fact that the total amount of transactsons in the world has been obliged in the long run to maintain a certain fixed numerical relation to the amount of monetary gold existing at each respective time in the world under *de facto* (i. e., not necessarily legal) gold standard system, a fact, which is established irrefutably by statistical survey and has essential relevance to the actual movement of world capitalism.

I shall not, however, dwell on this important problem any longer, because I have already discussed it in detail in my *A Dynamic Theory of World Capitalism.*

Postscript

The fatal errors underlying Keynesian theory, newly uncovered in this paper, may be taken as accounting for the helpless bewilderment from which economics has been becoming ever more to suffer and is still suffering even now. It remains, however, yet to be answered why so many economists in the world could become Keynesians and remain as such for so long a time without noticing any of the above revealed fatal errors. Does it reveal that it was I myself that was mislead by an illusion to regard what really were nothing erroneous as fatally erroneous, or that capitalism, at least of the present stage, has something in itself which deprives economists of their devotion to the truth?

Appendix: General Trait of Keynesian Theory

It is not, however, in respect to equation (I, 2) alone that he put new wine into old wine-skins. He was conversed with actual economy, and was deeply interested in its theoretical grasp. Consequenly, he was always endeavoring to make economic theory deal with more actual problems. But he was always careless about the capacity of the theoretical tool, which he had inherited from his predecessors.

For instance, he intended to deal with real investment, namely, with expansive economy. He, however, carelessly accepted the traditional theoretical tool which was constructed by assuming unilateral structure of production. This production structure, however, is bound to give rise to a special problem when it is applied to the case of an expansive reproduction, first because the ratio of real income involved in a product to its value (in real terms) necessarily differs, under such a structure of production, according to the sort of product, and secondly because a change in the rate of expension of economy necessarily entails a change in the relative magnitudes of different products produced. This problem arises no matter whether the economic expansion assumes the form of widening or of deepening. Let us, for simplicity's sake, take case of widening.

Let us assume the period of production of each production stage is equally one year, the round-about period of production is T years, the total number of laborers currently employed is A, the number of laborers employed in each production stage is equally a, and the annual real wage is L.

If simple reproduction takes place under such conditions, the total number of laborers currently employed and the amount of subsistence-fund will respectively be as follows:

$$A=a(1+1+\cdots\cdots)=aT \qquad \therefore \qquad a=\frac{A}{T} \tag{B, 1}$$

$$S=aL\{T+(T-1)+(T-2)+\cdots\cdots\}=\frac{AL(T+1)}{2}, \tag{B, 2}$$

where S denotes the amount of subsistence-fund. And the number of laborers employed per wage-unit subsistence-fund will be:

$$L=\frac{A'L(T+1)}{2} \qquad \therefore \quad A'=\frac{2}{T+1}, \tag{B, 3}$$

where A' denotes the number of laborers employed per wage-unit subsistence-fund It is, therefore, possible to regard in this case the real income per wage-unit subsistence-fund as a function of a sole A', as follows:

$$P'=f(A'), \tag{B, 4}$$

where P' denotes the real income per wage-unit subsistence-fund.

If, on the contrary, production expands at the annual rate of η under such conditions, the total number of laborers currently employed and the amount of subsistence-fund will respectively become as follows:

$$A=a(1+\eta+\eta^2+\eta^3+\cdots\eta^{T-1})=\frac{(\eta^T-1)a}{\eta-1} \quad \therefore \quad a=\frac{(\eta-1)A}{\eta^T-1} \tag{Ba, 1}$$

$$S=aL\{T+(T-1)\eta+(T-2)\eta^2+\cdots\eta^{T-1}\}=\frac{\{\eta(\eta^T-1)-(\eta-1)T\}AL}{(\eta^T-1)(\eta-1)} \tag{Ba, 2}$$

Therefore, the number of laborers employed per wage-unit subsistence-fund will now be:

$$A'=\frac{(\eta^{T}-1)\ (\eta-1)}{\eta(\eta^{T}-1)-(\eta-1)T} \qquad \text{(Ba, 3)}$$

It becomes, therefore, necessary to regard the real income per wage-unit subsistence-fund as a function of both A' and η, as follows:

$$P'=f(A',\ \eta) \qquad \text{(Ba, 3)}$$

Keynes, however, overlooked it when he dealt with an expansive economy, and adhered to the production function which corresponded to equation (B, 4).

To take another example, Keynes developed his theory using the concept of *capital* instead of that of *subsistence-fund.* Now, *capital* differs from *subsistence-fund* in that it involves in itself the profit, which is involved in the price of producers' goods, and which subsistence-fund does not involve in itself. In the above case of simple reproduction, therefore, *capital* will be:

$$K=a\{(1+i)^{T-1}+2(1+i)^{T-2}+\cdots\cdots T\}L$$
$$=\frac{[(1+i)\{(1+i)^{T}-1\}-i\,T]AL}{i^{2}T}, \qquad \text{(B, 2)}'$$

where K and i show respectively the amount of capital and the rate of real interest. Consequently, the number of laborers employed per wage-unit capital will be:

$$A''=\frac{i^{2}T}{(1+i)\{(1+i)^{T}-1\}-iT}, \qquad \text{(B, 3)}'$$

where A'' denotes the number of laborers employed per wage-unit *capital*. It becomes, therefore, necessary to regard real income per wage-unit capital as a function of both A'' and i even in the case of simple reproduction, as follows:

$$P''=f(A'',\ i), \qquad \text{(B, 4)}'$$

where P'' denotes real income per wage-unit capital. Keynes seems to have never thought of such an implication of the use of the concept of *capital* as this.

If it is assumed in this case that production widens at the annual rate of η, capital will come to be:

$$K=a[\{(1+i)^{T-1}+(1+i)^{T-2}+\cdots 1\}+\{(1+i)^{T-2}+(1+i)^{T-3}+\cdots 1\}\eta+\cdots\eta^{T-1}]L$$
$$=\frac{[(1+i)\{(1+i)^{T}-\eta^{T}\}(\eta-1)-(\eta^{T}-1)(1+i-\eta)]AL}{(1+i-\eta)(\eta^{T}-1)i} \qquad \text{(B, 2a)}'$$

Hence, the number of laborers employed per wage-unit capital will come to be;

$$A''=\frac{(1+i-\eta)(\eta^{T}-1)i}{(1+i)\{(1+i)^{T}-\eta^{T}\}(\eta-1)-(\eta^{T}-1)(1+i-\eta)}, \qquad \text{(B, 3a)}'$$

and real income per wage-unit capital will have to be regarded as a function of A'', i and η, thus:

$$P'' = f(A'', \text{i}, \eta) \qquad (\text{B}, 4\text{a})'$$

Keynes, however, neglected it entirely when he dealt with an expansive economy by resorting to the concept of capital.

To speak the truth, the theoretical tool, which Keynes made use of in developing his theory, was a more complicated one. It was such as presupposed that the number of laborers who were employed in the successive production stages decreased in such a manner as made the increment in *capital* required by the successive production stage remain equal,[1] and secondly that the period of production of each production stage was *n*th year, where the *n*th converged on zero.[2] It was perhaps Robertson's *Price Level and Banking Policy* that resorted to such a theoretical tool first. Keynes seems to have adopted it. This theoretical tool makes the composition of K and hence of A'' far more complicated than is represented by (B, 2a)′ or by (B, 3a)′. But the formal expression of production function remains as equation (B, 4a)′ also in this case.

As is demonstrated by these examples, Keynes was almost always careless about the capacity of the theoretical tool in launching upon a study of a new problem with it. Naturally, theoretical tool differs from the theory itself. A theory is what is discovered by analyzing the object of study by means of a theoretical tool. However, a theory cannot be developed but by means of some theoretical tool or other. There is, therefore, always a danger that an inappropriate or even unworkable theoretical tool is used and the result is mistaken for the property of the object of study. In the natural scientific study, therefore, it has long since become a commonplace usage exactly to specify, and examine the propriety and workability of the theoretical tool used, before proceeding to the analysis of the object of study with it. It may not be too much to say that this largely accounts for the steady and speedy progress achieved by natural science. In economics, however, it is quite different. Economics has been accustomed to be rather careless about the theoretical tool. It has been so accustomed presumably because, in economics, the carelessness about the theoretical tool does not always invalidate the study, and even when it does, its destructive power

---

1) Let us assume the periods of production of successive stages of producers' goods as being equally one year, and denote by a_1, a_2, a_3, and i respectively the number of laborers employed in producing the producers' goods of the first stage, that of the second stage, that of the teird stage, and annual rate of interest. a_2 and a_3, assumed by Keynes, will then be as follows:

$$a_2 = 2a_1 - (1+i)a_1$$
$$a_3 = 3a_1 - \{2a_1 - (1+i)a_1\}(1+i)$$

2) It is only under this assumption that I, in the sense defined in the text, becomes equal to ΔK, and savings out of the income of a certain period becomes necessarily equal to ΔK within the same period.

is not so easily discovered as in natural science. But it is only by chance, that it can pass without entailing a destructive effect. This seems to be why in economics, which is too much accustomed to such favorable chances, cases occur when the whole theoretical structure is demolished for a while by the prevalence of a new theory, which mistakes the result arising from an unworkable theoretical tool for the very property of economy itself. This was precisely the case with Keynesian theory.

In order to discriminate those Keynesian carelessnesses about the theoretical tool which are harmless from those which are not, let us resort to the theoretical tool, which I initiated in 1933 and which presupposes a circulatory structure of production.

Let us assume that there is only one sort of producers' goods, only one sort of consumers' goods and only one sort of labor, that the period of production is equally one year, that there is no fixed capital, that c_1 of producers' goods and a_1 of labor are required in producing one unit of producers' goods, and c_2 of producers' goods and a_2 of labor are required in producing one unit of consumers' goods, and that

$$\frac{c_1}{a_1}=\frac{c_2}{a_2}=\alpha \qquad \therefore \quad c_1=\alpha a_1\ , \quad c_2=\alpha a_2 \qquad \text{(C, 1)}$$

Let us further denote by B, I, k, C and A respectively the total price of producers' goods consumed in the course of current production, the excess of the total price of producers' goods currently produced over that consumed in the course of current production, relative price of a unit of producers' goods, the total volume of consumers' goods currently produced, and the total volume of employment. A equals the sum of laborers employed in producing the total current product, which is composed of producers' goods and consumers' goods, thus:

$$A=a_1(B+I)/k+a_2C \qquad \text{(C, 2)}$$

Let us next denote the rate of growth of production by η, and assume that the expansion takes place solely according to the method of widening. The total volume of producers' goods currently produced will then equal η times the total volume of producers' goods currently consumed, thus:

$$B+I=\alpha\{a_1(B+1)+ka_2C\}\eta \qquad \text{(C, 3)}$$

From these two equations we will obtain:

$$k\alpha\eta A=B+I \qquad \text{(C, 3)}'$$

Let us next denote by L and i respectively real annual wage and real interest rate. Price must equal the sum of production-cost and real interest thereon, i, e., γ times production-cost, thus:

$$(1+i)a_1(\alpha k+L)=k \qquad \text{(C, 4)}$$

$$(1+i)a_2(\alpha k+L)=1 \qquad \text{(C, 5)}$$

From these two equations we obtain:

$$\frac{a_1}{a_2}=k \tag{C, 5$'$}$$

Now, the excess of the value of a unit product over the value of producers' goods consumed in the course of its production constitutes the real income, which is involved in a unit product. Hence we have:

$$Y=(1-c_1)(B+1)+(1-c_2k)C, \tag{C, 6}$$

where Y denotes total real income. By combining equations (C, 3), (C, 3)$'$, (C, 5)$'$ and (C, 6), we will obtain:

$$Y=\frac{(1-\alpha a_1)A}{a_2} \tag{C, 6$'$}$$

We thus arrive at an equation, which defines the volume of real income in relation solely to the volume of employment. If we further assume as follows :[3]

$$a_1=a_2=a, \tag{C, 7}$$

equations (C, 4) and (C, 6)$'$ will be further simplified as follows:

$$Y=\frac{(1-\alpha a)A}{a} \tag{C, 7$''$}$$

This equation defines more clearly the volume of real income in relation solely to the volume of employment. We arrived at such an equation in spite of the fact that we used the concept of capital—as is manifested by equations (C, 4) and (C, 5)—and took an expansive economy into consideration (as is manifested by equation C, 3). This shows that we will be enabled, if we only change the theoretical tool we use, to assume, as Keynes did, such a real income as is a function of employment alone, and that the above developed theoretical tool was precisely what Keynes *de facto* presupposed in conceiving his production function. This means that Keynes's carelessness about his theoretical tool is harmless in so far as it concsrned his concept of production function.

Let us now proceed to develop an economic theory by means of our tool defined above, which Keynes *de facto* made use of.

---

3) This assumption reduces equation (C, 5)$'$ into $1=k$. Due to this assumption and what was assumed by equation (III, 1), we obtain $c_1=c_2$. Let c represent c_1, and hence c_2, of this case. We refer to this c merely as "technical coefficient". Of course, technical caefficient usually means the quantity of a certain productive element which is required in producing a unit of a certain product. Hence it is usually defined in relation both to the sort of productive element whose quantity it implies, and to the sort of the product, for whose production it is required. Hence it gives as many technical coefficients as equal the number of the sorts of productive element times the number of the sorts of product. In this paper, however, we mean by technical coefficient merely c in its above defined sense.

Let P' denote the real income per unit product. According to equation (C, 7)$''$, we see that the real income per laborer is $(1-\alpha a)/a$. Now, the volume of labor required in producing a unit product under our assumption is a. Hence we will obtain the following equation:

$$P'''=1-\alpha a \qquad \text{(III, 1)}$$

It is now assumed that there is a relation of substitution between the volume of producers' goods and the volume of labor required for the production of a unit product, namely, between α and a, thus:

$$\alpha=\phi(a) \qquad \text{(III, 2)}$$

These two equations define our production function. Now the amount of capital required per unit product under the above defined assumption is $a(\alpha+L)$. Hence we will obtain:

$$P'''=aL+(\alpha+L)ai \qquad \text{(III, 3)}$$

Taking these two equations into consideration, the condition of the principle of profit maximization will be given by:

$$L=\frac{dP'''}{da}\text{[4)]} \qquad \text{(III, 4)}$$

The total volume of employment equals the sum of labors required in the production of total product, thus:

$$A=a(C+B+I), \qquad \text{(III, 5)}$$

while the total volume of producers' goods consumed in the course of current production equals the sum of producers goods consumed in the production of total product, thus:

$$B=\alpha a(C+B+I) \qquad \text{(III, 6)}$$

Total product times real-income-per-unit-product equals total income, thus:

$$Y=(1-\alpha a)(C+B+I) \qquad \text{(III, 7)}$$

By combining equations (III, 6) and (III, 7), we are enabled, if we wish, to arrive at:

$$Y=C+I$$

Now the supply of labor is a function of real wage, as is assumed in Keynesian equation (I, 6), thus:

$$A=\Psi(L), \qquad \text{(III, 8)}$$

while savings, and hence real investment, will be assumed, as was assumed in Keynesian equation (I, 1), as a function of the rate of real interest, as follows:

4) Because equation (III, 3) can be changed to:

$$P'''=aL+i/(1+i),$$

for equations (III. 1) and (III, 3) can be reduced to:

$$1-\alpha a=aL+a(\alpha+L)i \qquad \therefore\ a(\alpha+L)=1/(1+i)$$

$I=F(i)$ (III, 9)

These nine equations contain precisely nine unknowns, P''', α, a, L, i, I, A, C and Y, provided that we assume B as given. If we assume B as unknown, we will have to add to the above another equation such as represents the supply condition of B or of $(B+AL)$, which means capital. It does not, therefore, matter whether we assume B as given or not. Let us, for simplicity's sake, assume B as given. We have here an equation system, which represents the genuine classical economics, as transformed by the use of a different theoretical tool, which differs from the hereditary one in that it presupposes a circulatory structure of production instead of the traditional unilateral one. We see here again that a genuine economics involves what corresponds to the previously noticed equation (I, 4a), namely, equation (III, 3), and that Keynes's carelessness about the theoretical tool is fatal in so far as it concerns his neglect of equation (I, 4a).