ON THE THEORIES OF CAPITAL AND DISTRIBUTION

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CONTENTS

On the Theories of Capital and Distribution  Itsumi HISHIYAMA  1

Georg Lukács and the Hungarian Revolution of 1918-19  Toshiko HIRAI  24

State Monopoly Capitalism and Organisation of Struggle for Survival  Jun IEGAMI  46

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In order to understand the nature of the contemporary theory of distribution it is necessary to examine their relationships with the law of diminishing returns of land as so called by the classical economists.

To begin with, we shall turn our attention to the fact that it is not to an individual enterprise or industry but to the macro-scaled agriculture contrasted with the manufacturing industry in general that this law is applicable. Even when the classical economists discussed a matter of wheat or grains and referred to an investment to a particular farm as an illustration, what they wanted to clarify then was for one thing the law of diminishing returns of land, by which among varieties of industries utilizing land=natural resources in general, the agriculture as a typical industry of such kind was affected, and the logic to determine a rent for the other, with which the said law was closely related.

Consequently, it is hardly possible to consider that the classical theory was concerned with the case of diminishing marginal productivity which was analyzed in connection with the so-called

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allocation problem by the neo-classical economists from a micro-viewpoint — from an aspect of an individual enterprise or an industry (which produced commodities in the restricted sense of the word) — and which was applicable not merely to land but also to every factor of production.

Now, generally speaking, in order to examine the behavior of returns in the factors of production, it becomes necessary to make distinction between (1) the changes in the proportion of the factors used up in the productive process and (2) the absolute amount of all invested factors, i.e. the increase in the scale. Now, it is maintained that the diminishing returns are given rise when one of the factors is fixed, while other variable factor is successively being put into the productive process. Under such circumstance, however, the cause of diminishing returns is ascribable not to the increase in the scale but to the changes of relative proportion between the two factors. Hence, in order to freeze any possible disturbing influences caused by the increase in the scale of input, we shall adopt hereafter in this discussion an assumption of constant returns to scale. There is then no need to say that if each of the factors is multiplied by \( \lambda \), the product will be in the exactly same multiple proportion \( \lambda \) under the given technical knowledge.

The factor of production which was considered to be constant from the aspect of the classical law of diminishing returns is the land conceived from a viewpoint of the national economy as a whole. If viewed, however, from an aspect of an individual enterprise or industry, even the land can’t be a constant factor. The reason is because it must be considered that an individual enterprise or even an industry can increase the quantity of land as desired at the sacrifice of other enterprise or industry.

If viewed from such viewpoint of the economy as a whole, it must be considered that the maximum quantity of land is constant under the existing technical knowledge. However, the fact that the maximum quantity is constant means that it is impossible to increase its quantity beyond the maximum level, but that the land can be used in any splitted quantity within the extent of its maximum size. Now then, suppose that the maximum quantity of land which is regarded as constant is expressed by \( x' \), on the assumption that the factors of production other than the land do not exist more than one in kind, let us express its quantity by \( x_1 \). When we assume the possibility of substitution between the two different factors, as the quantity of the variable factor, \( x_2 \), which is engaged in the constant quantity of the land \( x_1 \), undergo a change, its product \( y \) (the quantity of grains) will undoubtedly undergo a change. Yet, the input of the variable factor \( x_2 \) which produces its maximum output (strictly speaking, the average product per unit of the variable factor, i.e. maximum average productivity) is obtainable. Now, supposing that this quantity is expressed tentatively by \( x_2' \), it may safely be said that the \( x_2' \) is the

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2) Ibid., pp. 8-9.
optimum quantity of input in the sense that this input produces the maximum average output. Now, let us hereafter call the ratio between the maximum quantity of the land which is a constant factor \( x_1' \) and the optimum input of the variable factor \( x_2' \) as an optimum ratio and the method of production which is made possible by means of such input ratio between two different factors as an optimum method of production. Then, if the variable factor is used in excess of this ratio, because there is no possibility to use any method of production other than that of inferior efficiency, the returns (strictly speaking, the average productivity) will have to be diminished. When the average productivity diminishes, the marginal productivity of the factor concerned will also have to be inevitably diminished from the definition given to the marginal productivity.\(^3\)

Consequently, the problem is rather concerned with a case where the input of the variable factor does not reach to such optimum ratio than with a case where the variable factor is engaged in excess of the optimum ratio between the two factors. There is no need to say that even under this circumstance the returns of the variable factor, i.e. the average productivity does not reach to the maximum. Yet if viewed from our present standpoint of the economy as a whole we should not forget that, although the maximum quantity of the land is constant, the land after all can be made available in any splitted quantity as desired upon occasion within the said maximum quantity.

Then, if viewed from a standpoint of a rational enterprise, we can say that even in a case where the input of a variable factor does not reach to the optimum quantity \( x_2' \), it will become possible to accrue the maximum returns at all times (in relation to any quantity of variable factor), providing that the quantity of the available land can be so adjusted as to maintain its optimum ratio in relation to the occasional amount of the variable factor not by using the maximum quantity of land \( x_1' \) but by using the smaller land \( x_1 \). In this way it will become possible that the land can maintain the maximum level of returns constantly, in despite of the increased input of the variable factor, until such time when the use of the land reaches

\(^3\) Now, supposing that the variable factor to be used in the productive process is expressed by \( x \) and the output by \( y \), the average product \( \frac{y}{x} \) per unit of the factor concerned, i.e. the average productivity \( AP \) by \( x \): under this circumstance, needless to say, the production function reads as \( y=f(x) \). The marginal productivity of the factor \( x \) can be defined in a usual way as follows:

\[
MP = \frac{dy}{dx} = \frac{d(\pi \cdot x)}{dx} = \frac{d\pi}{dx} + \pi
\]

From this expression the following three cases are deducible by noting that \( \frac{d\pi}{dx} \) of this expression makes the slope of the curve \( \pi \).

1. \( \frac{d\pi}{dx} > 0 \rightarrow MP > AP \)
2. \( \frac{d\pi}{dx} = 0 \rightarrow MP = AP \)
3. \( \frac{d\pi}{dx} < 0 \rightarrow MP < AP \)
literally to its maximum to be regarded as a constant factor without being forced to make the uneconomical use of any excessive land in relation to the input of variable factor. It has already been pointed out that, when the maximum quantity of land $x_1$ is reached, if the variable factor is further kept to be engaged, its returns (both the average productivity and the marginal productivity) will diminish. Thus, if the input of the variable factor $x_2$ is shown on the abscissa and the average productivity ($\text{AP}$) and the marginal productivity ($\text{MP}$) on the ordinate, the classical behavior of returns from the macro-viewpoint can be shown as in Figure 1.

![Figure 1](image)

II

It seems that a unit of the variable factor to be put to the land was grasped as a kind of composite unit composed of capital and labor by Ricardo who gave us a classical prototype of the theory of distribution. That is, each unit of the variable factor may be considered to be composed of the so-called labor with spade or

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4) There ought to be a minimum size of the land, too, below which any further splitting cannot be made for the sake of efficiency. Now, the returns from the variable factor used in a land of such minimum unit of size may show an increase at the outset. But, because the macro viewpoint to consider the land of the economy is adopted here, an idea of the minimum size of the land should be ignored in this connection. Besides, since there is another kind of diagraming in the Ricardian system to show a rise of differential rent owing to the moving of the marginal cultivation to the more inferior soil, which is different from what is shown in Figure 1, it rather appears that such kind of diagraming demonstrates the principal analytical apparatus of the Ricardian system. However, at this point in consideration of the comparison with the neo-classical theory, an idea of the rent of the so-called secondary form, i.e. the logic of the rent formation owing to the intensive use of one of the same land should be called into question. See P. Sraffa's aforementioned book: foot-note 1 on page 26.
the labor with tool. Now, disregarding that part \( UP \) of constant returns in Figure 1 for the sake of simplified explanation, let us suppose that the input of the variable factor will start from \( M \). In other words, \( M \) is taken in place of the origin \( O \). Now, if the level of the total demand for grains is supposed to call for the input of the factor \( MN \), then the total amount received by the whole of a composite factor composed of capital and labor will be, if expressed in the quantity of grains, an oblong \( MQRN \), \( RN \) being returns per unit. On the other hand the total amount of the rent will be, if similarly expressed in the quantity of grains, an area \( PQR \) or an oblong \( TQRS \). Under this circumstance, the input in the position of the point \( N \) will show the marginal point: and no rent is given rise at such point: and the net products at that point will be composed of profits and wages which are remunerations for "capital and labor". Each of the inputs in the boundary to the left direction form \( N \) will accrue the rent which can be calculated by a vertical distance of the curve \( MP \) and the segment \( QR \).

In the meantime we have so far seen that the marginal productivity of a composite factor composed of "capital and labor" was graphically formulated in the law of diminishing returns maintained by the classical economists in this way, but the fact that the marginal productivity of \textit{land} in itself was not specifically treated by the classical economists should not be considered as the substantial difference from the graphic formula maintained by the neo-classical school. The reason is because the graphing by the classical school has naturally an implication of the diminution of the marginal productivity of \textit{land}, as it were, a dual logic, on the supposition that the land is a variable factor. The question in a true sense, I should think, rather lies in that the classical school didn't make universal application of the law of diminishing returns in determining the respective remuneration for capital and labor, each of which was grasped separately. Their logics to determine profits and wages are respectively based on different reasonings and these reasonings are also different from their logic to determine the rent. Now, even if it is possible to make graphic formulation of the marginal productivity of "capital and labor" as a composite factor, it would be impossible to analyze such a concept into the marginal productivity of capital and the marginal productivity of labor, each of which should be conceived as an independent factor of production. According to Ricardo, the level of needed subsistence for the laborer and his family is supposed to play a prescriptive role in distributing incomes for capital and labor when the social system and custom are given. In other words, there is no need to say that the subsistence wages, i.e. wages which are given exogenously and the profits as a residue plays a central role in the Ricardian theory of distribution. However, the

\[ \text{5) Today I don't think that this point deserves an additional explanation: for example Blaug calls "composite unit" of Ricardo as "laborer with shovel". See transl. by Kubo, Mazane & Sugihara of M. Blaug, "Economic Theory in Retrospect" (1962), Part I, 1966, p. 101.} \]
so-called marginal theory of distribution (theory of the marginal productivity) was completed by the neo-classical economists by regarding the law of diminishing returns which was originally applicable only to the land from the macro-viewpoint, as being appropriate for every independent factor to be used in the productive process, and particularly by advancing such idea to make it a fundamental formula of distribution of incomes for capital and labor.

III

One focus of the current problems taken up at the Cambridge at present is the criticisms of the neo-classical marginal theory of distribution for capital and labor and an attempt to advance a new theory of distribution to take place of the old one.\(^6\)

\(^6\) There is following book written by Sraffa to back up these problems: “Production of Commodities by Means of Commodities, 1st ed. 1960; transl. by Hishiyama & Yamashita, 1962. It must be noted that this book is subtitled as “Prelude to A Critique of Economic Theory”. What is meant here by “Economic Theory” is the neo-classical marginal theory of value and distribution. One of the sensations created by this work upon the contemporary economics culminated in controversies exchanged by the Cambridge in England and the Cambridge in America on the subject of the so-called recent switchings of technique. About these controversies, see Chapter XIII of the aforementioned book by Sraffa and “Symposium” (participants — L. L. Pasinetti, D. Levhari, P. A. Samuelson, M. Morishima, M. Bruno, E. Burmeister, F. Sheshinski, & P. Garegnani.) in Quarterly Journal of Economics, 1966.

Since the rise of these controversies a flood of the related writings came to be published, but some of principal ones are systematically compiled recently, happy to say, in the following inexpensive edition: “Capital and Growth”, edited by G. C. Harcourt and N. F. Laing, Penguin Modern Economics Readings, 1971.

Now, a key point to set up such problem is related with the concept of the marginal productivity of capital as a fundamental factor to determine the rate of interest or profits. The weakness of such concept is partially rooted in the difficulties to measure capital. Now, then, the law of diminishing returns is, needless to say, based on a presupposition of the production function to indicate the relationships between the input and the output under a given technical knowledge.

Now, let us suppose that the factors of production engaged to produce a specific or composite commodity are composed of two factors such as labor and capital. Of course the capital as one factor of the production function on the input side must be measured by using a physical and technical unit. But, being quite different in nature from the land, since the capital as a produced means of production is, if seen materially, a compound of various heterogeneous commodities, some kind of a physical measuring unit which has definitely something to do with interest or profit to be determined must be established first of all. Yet, the current situation makes it hardly possible to find a measuring unit suitable to the structure of the marginal theory of value and distribution. Again, on the other hand, when the capital is to be expressed in terms of value, a circular reasoning will have to be given rise because the rate of interest to be determined must be presupposed beforehand. Consequently, it is impossible to form a concept of the "quantity of capital" as an independent factor to determine the rate of interest. This is one of the criticisms.

Another criticism is concerned with the problem of appropriateness of the well behaved aggregate production function which is used to make the background of the latest controversies on the subject of switching of techniques between the Cambridge of England and the Cambridge of U.S.A. and on which the neo-classical theory of growth was based.

Although the aggregate production function serves the purpose to indicate the technical relationships between the input and the output in relation to the whole.

7) Aforementioned book written by Sraffa (Part I) and by Garegnani may well be pointed out as recent noteworthy works about these problems. In each of these works in addition to mere critical comments, even a positive 'measure of capital' is demonstrated in the way of commodity approach by one and in the way of labor approach by the other. It was by the Cambridge economists that the importance of such points of the issue in question were repeatedly emphasised, particularly John Robinson being a most outstanding representative of them. For an instance, see her following writings. J. Robinson, "The Production Function and the Theory of Capital" in Review of Economic Studies, Vol. 21 1953-4. (in Capital and Growth, Penguin Modern Economics Readings, pp. 47-64): J. Robinson, "The Measure of Capital: the End of the Controversies", Economic Journal, Vol. 81, 1971. pp. 597-602.

8) Now, referring to the production function $y=f(x)$, if the conditions of $f'(x)>0, f''(x)<0$ and $f'(0)=\infty, f''(0)=0$ are met, such production function is regarded as well-behaved. F. H. Hahn and R. C. O. Matthews, "The Theory of Economic Growth: a Survey" in Surveys of Economic Theory prepared for The American Economic Association and the Royal Economic Society, Vol. 2. 1968. p. 10.
economy, it involves an extremely simplified assumption. In the first place the input and the output are presupposed to be something physically homogeneous in nature, so to speak, a singular commodity, in the same way as used to be thought by Ricardo in his early days. Accordingly, for an example, let us assume that the input and the output are made up of one and the same commodity, i.e. grains. Yet the input factor in question is made up of the homogeneous labor in addition to the capital made up of grains. Hence, it is assumable that in such production system, there is a possibility of infinite numbers of combinations of capital and labor, i.e. limitless numbers of method of production, engaged in such productive process to produce grains under a given technical knowledge.

The production function is characterised with an assumption of constant returns to scale as previously presupposed. In this way, if the two factors used in production are respectively multiplied by $\lambda$, the output in the same multiple proportion $(\lambda)$ is obtainable.

Nextly, if we realise that the capital and labor in such production system, being different from the classical model, should be considered as two different and independent factors, each of these factors is respectively subject to the law of diminishing returns, with emphasis on the point that each of the capital and labor is regarded as independent and separate factor. In other words, if either one of two factors is fixed and the other factor is kept increasing, it is assumable that its marginal productivity is kept diminishing.

Now, if each of the input and the output is respectively represented by the capital per unit of labor (grains in other word) $k$ and the output per unit of labor (grains in other word) $y$, such production function may be shown as in Figure 2. The behavior of returns shown in this diagram has nothing different for the form's sake from the case shown in Figure 1 with an exception of a portion of constant return as shown by $UP$ of Figure 1. However, there is a necessity to take note of

9) According to his 'Introduction' given to Vol. 1 of the "The Works and Correspondence of David Ricardo" edited by Sarfia, it is assumable that the so-called 'corn-ratio theory' was conceived in 'an essay' on the rate of profits and in his letter written in 1814 and in early 1815: "The Works and Correspondence of David Ricardo", Vol. 1, 1951, p. XXXI. However, about these views held by Sraffa, see the Critical essay written by Takuya Hatori, "Early Ricardian Theory of Value and Distribution" in Shogaku Ronso, Vol. 34, No. 3, pp. 91-151.

10) Take reference to page 2 of this essay. However, the adoption of the assumption in the case of the neo-classical school is based on different reason. As is widely known, the theory of marginal productivity is applicable in the strict sense of the words only when the hypothesis of constant returns to scale is presupposed because each factor is supposed to obtain more (or less) remuneration than the marginal productivity of the factor when such presupposition is not applicable. J. E. Meade, "A Neo-Classical Theory of Economic Growth", 2nd ed., 1961, pp. 13-14.
the fact that the capital per unit of labor is indicated on the horizontal axis of Figure 2 and the output per unit of labor on the vertical axis in a different manner from Figure 1.

Now, let us focus our attention upon an optional point $P$ on the curve shown in Figure 2. Needless to say, $P$ shows that the output will be $OQ$ when the capital per unit of labor is $OM$, but at the same time it speaks for the method of production adopted at such time, too. The Merkmal which characterises such method of production can be expressed either (1) by the capital per unit of labor, i.e. the intensity of capital $OM$, or (2) by the output coefficients (reciprocals of capital coefficients) which are expressed by the slope of $OP$. The inclination of tangent coming touch this curve at the point $P$, needless to say, shows the marginal productivity of capital. It will be self-explanatory that, whenever the curve takes the shape shown in Figure 2, as the capital per unit of labor is increased in quantity, or as the method of production is carried on by more and more intensive capital, then the marginal productivity of capital will tend to be diminished. Moreover, the coefficients of capital will tend to become greater at the same time (corresponding to the diminution of coefficients of the output). In short, in proportion to the increase of capital per unit of labor, i.e. the increased intensity of capital, the marginal productivity of capital will tend to be kept diminishing.

In the meantime, as an assumption is made here, as far as the production function of constant returns to scale is concerned, if the maximum condition is always satisfactorily met (a determining condition to select the method of production such as the maximum rate of profits under a given wage rate or the maximum wage rate under a given rate of profits), the marginal productivity of capital must always
be equal with the rate of profits. Now then, under such circumstance the major issue, which holds that as the capital per unit of labor, i.e. the intensity of capital tends to increase, the marginal productivity of capital is kept diminishing, does carry an implication that the rate of profits will tend to be diminished as the capital tends to increase. There will be nothing wrong if this is expressed by saying that, if the rate of profits declines, the method of production by using a greater capital comes to be adopted in the whole economy. There will be no difficulty to understand what this means if a circumstance, under which the equilibrium of the production system happened to be at the point \( P \) moves to the point \( P' \), is taken up in our mind. Now, under such circumstance the real wage rate is raised, though no need to say, from \( OR \) to \( OR' \). Consequently, there will be nothing wrong in making a substituted expression to the effect that under such circumstance the more labor-saving method tends to be selected because of the increased wage rate. In this case the equilibrium output per unit of labor, needless to say, will be increased from \( OQ \) to \( OQ' \).

Now, the fact that the rate of profits will be diminished in proportion to the increase in the quantity of capital per unit of labor, or that the quantity of capital

11) Now, supposing that the national income is expressed by \( Y \), capital stock by \( K \), labor by \( L \), rate of profits and wage rate by \( r \) and \( w \) respectively, if all of the national income is to be distributed for profits and wages,

\[ Y = K r + L w \]

By converting this into one unit of labor

\[ (1) \quad y = k r + w \left( \therefore \frac{y}{L} = \frac{y}{L}, \quad k = \frac{K}{L} \right) \]

From (1) the following equation of the rate of profits is obtainable:

\[ (2) \quad r = \frac{y - w}{k} \]

Now, supposing that the following is linear and homogeneous production function (constant returns to scale):

\[ (3) \quad y = f(k) \]

Taking such production function into consideration and the wage rate \( w \) as a datum, the following maximum condition of the rate of profits is obtainable from (2),

\[ \frac{dr}{dk} = \frac{1}{k^2} \left[ k f' - (f - w) \right] = 0 \]

Therefore,

\[ (4) \quad w = f - f' k \]

Substituting (4) to (2),

\[ (5) \quad r = f'(k) \]

That is to say, the rate of profit is equal to the marginal productivity of capital. Such condition is obtainable, needless to say, in the same way by the method to obtain the maximisation of the wage rate \( w \) supposing that the rate of profits \( r \) is to be given.

The reason why the production function of this type is linear and homogeneous is an inevitable result of the presupposition that the basic production function is linear and homogeneous. That is, on the supposition that \( Y = F(K,L) \) is linear and homogeneous function, if the input is multiplied by \( 1/L \), the output should be in the same multiple proportion, \( 1/L \cdot Y = F(1/L \cdot K, 1/L \cdot L) \). Therefore \( y = f(k) \).
will increase in proportion to the decrease in the rate of profits perfunctorily carries exactly the same meaning, for example, as to say that the demand price of apples will decline in proportion to the increase in the quantity of apples in demand, or that the quantity of apples in demand will increase in proportion to the decline in the demand price of apples: the only difference in this circumstance lies in that this main constituent on the part of demand is not the household maintained by the consumers but the enterprises operated by the producers who want to adopt the most suitable method of production on the principle of the profit maximization. Be the matter as it may, since the rate of profits is a kind of price under this circumstance, this rate serves the purpose to play the role of a barometer to measure the scarcity of the quantity of capital: this carries substantially the same meaning as to say that the price of apples is serving the purpose to measure the scarcity of the quantity of apple. Thus, it will be understood without difficulty that the demand curve for capital or investment which indicates a diminishing move in a negative proportion to the rising trend of the rate of interest is explicable on the grounds of the aforementioned logic.

As a conclusion of what has been discussed so far the following points may be summarised here for the time being in connection with other problems to be touched later. In the neo-classical production system as shown in Figure 2, the technique of more and more intensive capital will be chosen as the most favorable (consequently as optimum) method of production in proportion to the decline of the rate of profits (or though the same in its meaning, in proportion to the rise of the wage rate). And such technique will bring forth, needless to say, a greater coefficients of capital (capital output ratio) and a greater output per capita. In other words the neo-classical production function as shown in Figure 2 does indicate a definite order in the way of selecting the most profitable technique. However, the discussion made so far naturally leads to such questions as "To what extent can the said order of selection of technique, i.e. so to speak, an inverse correlation between the rate of profits and the intensity of capital be held to be appropriate?" and "Can the conditions to establish such interrelationships be clarified in a more definite way?" It appears that these questions in themselves constitute one of the key-points of the latest controversies on the subject of reswitching of techniques.

IV

The neo-classical model of production as shown in Figure 2 is based on an extremely simplified assumption as pointed out in the beginning. That is, in this model it is presupposed that both capital on the part of input and the products on the part of output are supposed to be composed physically of one and the same commodity: for an instance, grains. Such assumption is charcterised with an idea that they can easily cope with difficulties involved in measuring capital.
Putting it in the other way, since both of the capital (input) and output are supposed to be made up of the same commodity, i.e. grains, the rate of profits (equivalent to the marginal productivity of capital) is to be expressed by a kind of corn-ratio. Besides, the wage rate is also expressed, needless to say, by the quantity of grains. In short, since there exists no commodity whatsoever except grains, all kinds of economic quantity are to be accurately indicated in this model by a quantity of grains which is a physical unit of measure.

Nevertheless, in actual realities, speaking of economy as a whole or a certain industry, or even a specific enterprise, the capital, if put in a concrete shape, is something composed of an aggregate of varied kinds of commodities of materially heterogeneous nature which are put to engage in the productive process. Now then, speaking of such aggregate production model, as already pointed out, the problem to measure the capital for the purpose of aggregation by using a physical and technical unit must be solved as an essential prerequisite, but we shall not intend here to go into further consideration of this problem. Be the matter as it may, if it is kept in our mind that the existing production system is in realities composed of a great number of varied kinds of commodities on each part of input or output, no one will make objection against the fact that such single commodity model conceived by the neo-classical economists represents an extremely abstracted idea from realities, of which complex and technical relationships are simplified. However, seeing that any theory is composed more or less of abstractions of realities, the question is, I should rather think, whether the neo-classical single commodity model — of which noteworthy characteristics have a proposition that a technique of more intensive capital will be selected to cope with the declined rate of profits for the sake of one and whole economy — can demonstrate in a generalized manner the fundamental characteristics of the actually existing production system which is composed of a variety of commodities, both as input and output.

12) Such problem of measuring method has a long history since the "invariable standard of value" of the classical economists. Such varied kinds of numéraire as a single particular commodity, period of production, dated labor (direct or indirect labor), etc. But none of them brought forth any satisfactory result. Today, only solution of such problem to be attended in a logical, consistent and elegant manners can be sought in the composition of "standard commodity" described in Chapter 4 in the aforementioned book of Sraffa. Besides, about this point, see the following works: aforementioned book of Garegnani and also "Translator's Postscript" by Hiroshi Yamashita contained in the last part of the said book and also Kajita's aforementioned English essay, pp. 2-8.

"The rate of returns for investment" by Solow, who is one representative of the neo-classical economists, can't be considered to be a concept to avoid the necessity to measure capital as intended by the advocates, because a general rate of profits must be presupposed. Besides, as to the way how the concept of Fisher-Solow's "rate of returns" was formed in the analytical structure of Sraffa-type and its criticism, see the following essay written by a Cambridge champion, L. L. Pasinetti, "Switches of Technique and the Rate of Returns in Capital Theory" in Economic Journal, Vol. 79, 1969 (in 'Capital and Growth'. pp. 261-266).
From the aforementioned viewpoint the present writer hereby would like to make a brief summary of a few points of the latest, so-called Cambridge controversies having some bearings upon the economic theory.

One substantial point in this sense is that one of the main issues implied in the neo-classical production model—the issue that a technique of more intensive capital will be selected in proportion to the diminished rate of profits is not of such nature that can generally throw light on the fundamental nature of actually existing complex system of production technique. Putting it in the other way, it doesn't follow that a simple and abstract representation of the whole existing production system by using the production function as indicated in Figure 2 can make a good demonstration of the fundamental characteristics of the actually existing production system. Generally speaking, when the rate of profits diminishes, on some occasions it may so happen that the more capital using technique may be selected as in the neo-classical cases, but on the contrary on some other occasions the more capital saving technique may be selected. Furthermore, these two cases may take place at the same time by turns one after another. Let the matter be as it may, such restricted sequence of order about the selection of one particular technique as indicated by the neo-classical economists can neither be considered to indicate a general tendency of the existing production system, nor to have any logical inevitability whatsoever.\(^{13}\)

As already discussed, in the said neo-classical model, the marginal theory of distribution to maintain that each marginal productivity for the two factors such as capital and labor will tend to be equal with the rate of profits and the wage rate in equilibrium of production can well be established, but since such marginal theory of distribution can only be held good in concert with the main neo-classical...
proposition previously mentioned in connection with the selection of technique, once the general appropriateness of such proposition comes to be denied, needless to say, it will lead undoubtedly to a conclusion that the said marginal theory of distribution will lose its general applicability.

Why so? Why is it that the neo-classical proposition pertaining to the selection of technique and the marginal theory of distribution can’t have a general applicability? Now, a few important points about such question may be pointed out here without going into further technical details for demonstration.

The first point which may deserve our attention is the fact that either side of arguers is using the model of the cost of production worked out by Ricardo-Sraffa in which the demand functions do not play any definite role or some other type of model which is very much similar to it, instead of using a traditional system of supply and demand equilibrium (worked out by Marshall or Walras) as a reliable model of the actual production system. Therefore, the principal characteristics of the actual production system which provide the common basis to both sides may be summarized as follows.14)

In such production system the commodities are produced (either in a single or in a joint form) by means of commodities (i.e. means of production or capital goods) and labour which were used up in various productive processes. And if the term “technique” is defined as the whole of a collection of methods of production (the degree of capital intensity being their Merkmal) adopted in realities by each industry or productive process, this production system is supposed to produce the net product which can be distributed for wages and profits under a given technique in addition to the mere replacement of means of production or capital goods. In a


About the way of looking at the matter from an angle of the mutual interdependence of markets or the mutual interdependence of productions which was lying in the background of Nell’s idea, see the following book by Mizokawa who by inquiring into the Say’s law had made a distinction between a pattern of ‘exchange theory’ and a pattern of ‘production theory’ in earlier days than Nell: Kiichi Mizokawa, “Classical Political Economy and Theory of Market”, 1966.

similar way as in a case of previously mentioned case, on the one hand the constant returns to scale is presupposed in each industry or process, while on the other the relative price for all commodities and the wage rate are measured in terms of any given commodity or a certain composite commodity.

The substantial characteristic of such production system lies in that it is a system with one degree of freedom. In other words, it is not until either one of the wage rate or the rate of profits which was considered to be uniform for each industry or process is given exogenously that this system can be brought to its self-completion. For an example, supposing that the rate of profits is chosen as an independent variable, when it is determined by some exogenous factors, the wage rate and the relative prices of various commodities come to be determined simultaneously as its results.

It is considered that such system is applicable not only to a stationary economy where no net investment exists (consequently all of the net products are consumed) but also to a growing economy where net investment is being made. Besides, this system is applicable to a case where one industry is inclusively producing only a single commodity or where one productive process is producing multiple commodities in a joint manner as previously suggested. It is possible for this system to be expanded without difficulty from a simple case where the means of production is so composed of only circulating capital that all of its means of production will be exhausted in a specific period of time (for an example in a year) to a case where operation is carried on with a fixed capital to some extent, having a more close resemblance to realities.

15) The assumption of constant returns to scale was not made in the Sraffa's work. However, he admitted to adopt such assumption as a temporary working hypothesis. See the "Preface" in his aforementioned book. But the participants in the controversy generally adopted this assumption.

16) This point is, indeed, one of the substantial features which distinguishes the Sraffa's system from Walras'. The Walras' general equilibrium is rather characterised with the system without degree of freedom, correspondingly more like a deterministic system.


18) There are possibly two ways to deal with fixed capital—one is the method to indicate the 'rate of mortality' of fixed capital explicitly in production equations or the other is the method to base on the equations of the joint production: in other words to regard all machines and products for a given year as joint products of machines and labor in the preceding one year. The method held by Samuelson, Garegnani, Spaventa, etc. belongs to the former, but the more general method is the one based on equations of joint production which is developed by Sraffa, von Neumann, etc. The strong point of this method lies in that this method will give "the 'correct' answer in every case, no matter how, complex over the life of a durable instrument of production, may be the pattern of falling productivity or increasing maintenance or repairs". See aforementioned Sraffa's book, p. 110.
"How can the relationships between the wage rate and the rate of profits under a particular technique in such production system composed of the whole aggregate of various existing methods of production in each industry or process be graphically shown?" — this question gives us a clue to shed light on the problem for the time being (i.e. the problem concerned with the appropriateness of the definite order to select a particular technique and the marginal theory of distribution which were conceived by the neo-classical economists).

Now, Figure 3 is a graph showing the relationships between the wage rate and the rate of profits (hereinafter referred to as "wage curve") in coping with three possible kinds of techniques under a given technical knowledge, in which the wage rate \( w \) is represented on the vertical axis and the rate of profits \( r \) on the horizontal axis respectively. The maximum rate of wage under a circumstance where the rate of profits happens to be zero is represented by \( W \), but under a circumstance where a stationary economy is presupposed, the maximum net products per capita can also be measured by it as well. On the other hand the maximum rate of profits under a circumstance where the wage rate happens to be zero is represented by \( R \). The actual wage rate as well as the rate of profits in realities are determined, needless to say, at the lower level than respective maximum level.

What must be noted in this connection is the fact that the interrelation between \( w \) and \( r \) will be expressed in a straight line like \( A \) as shown in Figure 3 under a circumstance where the intensity of capital for each method of production actually adopted by each industry happens to be equal, but that the said interrelation will be expressed in a curve like \( B \) or \( C \) under a circumstance where the intensity of capital of each industry happens to be unequal, i.e. under a situation bearing a closer

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resemblance to realities: and that it is only to the case of a straight line like $A$ among the above-mentioned three kinds of the wage curves — i.e. a specific case of a most unlikely condition — that the marginal theory of distribution (an idea that the rate of remuneration for each factor will be determined by the marginal productivity) should be, strictly speaking, adequately applicable.

Now, let us direct our eyes to Figure 4 which is formulated by extracting the case of $B$. Then, supposing that the net product, i.e. income, is defined to be exclusively composed of profits and wages. Such defined formula is considered, needless to say, to hold good always without any exception irrespective of (1) whatever hypothesis may be adopted in connection with the distribution on income for wages and profits, or (2) whatever technical conditions may be presupposed. Hence, the following equation is obtainable from such definition:

\[
\begin{align*}
(1) \quad \text{Capital per unit of labor} &= \frac{(\text{Output per unit of labor}) - \text{Wage rate}}{\text{Rate of profits}}
\end{align*}
\]

In the meantime, if the point $P$ in Figure 4 is supposed to be an equilibrium

20) This point at issue may be demonstrated here by using 2-commodities model. Supposing that consumption good (grains) $A$ and the capital good (machines) $C$ are produced by the labor and machines respectively, constant returns to scale are presupposed for each industry. Now, the technique $\alpha$ adopted under this economy is expressed by the following four different input coefficients each of which is fixed in nature, i.e. coefficient of labor $l_{A}^{a}\alpha$, coefficient of capital $C_{C}^{a}\alpha$ concerned with production of consumption good and coefficient of labor, $l_{A}^{a}\alpha$, coefficient of capital $C_{C}^{a}\alpha$ concerned with production of capital good.

Now, let us suppose, to make the matter simpler, that this economy is composed only of circulating capital and that the uniform rate of profits $r$ and the uniform wage rate $w$ are established there, and the consumption good (grains) is numéraire, then the following price equations are obtainable on the assumption of the technique $\alpha$:

\[
\begin{align*}
(1) \quad 1 &= l_{A}w + C_{C}P_{C}(r+1) \\
P_{C} &= l_{C}w + C_{C}P_{C}(r+1)
\end{align*}
\]

Under this system, $P_{C}$ is the price of capital good (machines) in terms of grains and $r$, $w$, $P_{C}$ are the unknown quantity and the four input coefficients are the given quantities.

Consequently, the equation of the wage curve under the technique $\alpha$ is obtainable by eliminating $P_{C}$ of the equation (1): then,

\[
\begin{align*}
(2) \quad w &= \frac{1 - C_{C}(r+1)}{l_{C} + (l_{A}C_{A} - l_{C}C_{C})(r+1)}
\end{align*}
\]

Now, what is decisive for the curvature of such wage curve (correspondingly for the kind of technique, too) is the value of the second term of the denominator in the right side, i.e. $(l_{A}C_{A} - l_{C}C_{C})$. Now, if the ratio of the capital intensity as to two commodities, i.e. grains and machines $C_{C}/C_{A}$ is supposed to be $m$ and if the formula (2) is rewritten as $w = f(r)$, we will obtain $f' < 0$ and $f'' < 0$ in answer to $m = 1$.

Putting it in the other way, as far as the wage curve slanting down the righthand direction is concerned, when the capital intensity of two industries is on an equal level (that is, in a case of $m = 1$), the wage curve will be indicated in the form of a straight line, but when on unequal level (that is, in a case of $m = \neq 1$), the wage curve will likely be indicated in a concave form to the origin or in a convex form to the origin. P. Caregnani, loc. cit., p. 408; L. Spaventa, loc. cit., p. 130.
point, the wage rate and the rate of profits will be represented respectively by $\bar{w}$ and $\bar{r}$. Since the maximum wage rate of $W$ is the output per capita as already mentioned, the numerator in the right side of this equation is a segment $\bar{w}W$. The rate of profits designated by the denominator is equal to $O\bar{r}$ and consequently being equal to $\bar{w}P$, too. Therefore, the equation (1) can be re-written in the following way:

\[
(2) \quad \text{Capital per unit of labor} = \frac{W\bar{w}}{P\bar{w}}
\]

In other words, it follows that the quantity of capital per unit of labor is measurable by the slope of $WP$. This idea should be, needless to say, always held good irrespective of whatever hypothesis may be adopted in connection with distribution or whatever technical conditions may be presupposed.

Yet, under a circumstance where a hypothesis based on the marginal productivity of the factors is adopted in connection with the distribution of the product, since the quantity of capital per unit of labor is indicated by the slope of $WP$.

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21) The equation (1) of foot-note (11) is taken up here again.

\[
(1) \quad y = kr + w
\]

Now, the difference between the two outputs under the equilibrium situations is shown by the following equation:

\[
dy = dkr + drk + dw \quad \therefore \frac{dy}{dk} = r + \left( k \frac{dr}{dk} + \frac{dw}{dk} \right)
\]

The condition to which the marginal productivity theory is applicable requires that the term in parentheses in the right side of this equation should become zero.

\[
\left( k \frac{dr}{dk} + \frac{dw}{dk} \right) = 0 \quad \therefore k = - \frac{dw}{dr}
\]

Thus, when the marginal productivity is applicable, it follows that the quantity of capital will be shown by the slope of $MP$ in Figure 4. As mentioned already, since the marginal productivity theory presupposes the linear and homogeneous function and the condition of maximisation,
but by the slope of $MP$, the adoption of a theory of marginal productivity contradicts against the presupposition of the wage curve as shown in Figure 4. There is no need to say that the same holds good of the case of the wage curve like C in Figure 3. Consequently, it is only to a circumstance where the wage curve is formed in a straight line like A in Figure 3 under a condition where the slope of $MP$ becomes identical with the slope of $WP$ that the hypothesis of the so-called marginal theory of distribution (i.e. the theory of marginal productivity) is applicable in the strict sense of the words. It has been already pointed out that such circumstance—the assumption that the capital intensity of each industry is the same in entire economy—is an exceptional or fictitious circumstance which is very unlikely to exist in realities or a logically specific case.

VI

One more problem has been left untouched so far. When a certain neo-classical equilibrium of production is compared with the other equilibrium of production, the particular technique to cope with the lower rate of profits is found to have a greater intensity of capital. In other words, it is the problem concerned with the neo-classical definite order to select production technique on the assumption that if the rate of profits declines, the degree of mechanisation of the economy will be furthered.

Although one specific kind of technique was presupposed in the explanation given so far, let us now proceed to presuppose several kinds of technique which are available for optional selection and to consider the switching over to one of those techniques. In Figure 5 are shown three wage curves in the form of 3 straight lines corresponding to three kinds of techniques named as $\alpha$, $\beta$, $\gamma$ for the sake of simplification. Now, either one of the three, for an instance, the wage curve in the case of technique $\beta$ corresponds to a circumstance where the capital intensity happens to be the same in every industry and the theory of marginal productivity is applicable in the strict sense of the words.

\[ \frac{\text{d}w}{\text{d}k} = -\frac{f}{k} \]
\[ \frac{\text{d}r}{\text{d}k} = \frac{f'}{k} \]

Then the following equation is obtainable from (6) and (7):

\[ k = -\frac{\text{d}w}{\text{d}r} \]

applicable in a strict sense of the words to such circumstance. It will be understood without difficulty that the aforementioned theory holds good with respects to $\alpha$-system as well as the $\gamma$-system only if the wage curve for each of them is indicated in the form of a straight line. Nevertheless, as the technique in question is switched over from $\alpha$ to $\beta$ and from $\beta$ to $\gamma$, the capital intensity itself keeps undergoing changes in the economy as a whole. In Figure 5 only three kinds of techniques are indicated, but in fact an innumerable number of different techniques ought to be taken into consideration. Thus, each point on an envelope curve coming to touch from the outside with an innumerable number of wage curves (to be expressed in the form of a straight line) which comes into existence under each of innumerable kinds of techniques will represent the respective production system characterised with a specific technique.

Such envelope $WR$ represents a formulation of a comprehensive relationships between the wage rate and the rate of profits under a circumstance where a certain technique is successively switched over. Furthermore, since every point of this envelope comes to contact with the wage curve in the form of a straight line under a specific technique, the theory of marginal productivity is always applicable in a strict sense of the words and the rate of remuneration for each factor is found to be equal with the marginal productivity of the respective factors. Also it becomes clear from Figure 5 that as the rate of profits diminishes, the capital intensity of the technique to be selected in the economy increases. That is to say, when the rate of profits is $r_1$, the technique to be selected is $\alpha$-system expressed by the wage curve in the form of a straight line coming touch with the point $P$ and the intensity of its capital (i.e. capital per unit of labor) is measurable by the slope of the segment $PP'$. When the rate of profits is $r_2$, there is no need to say that the technique $\beta$ is adopted and the intensity of its capital is measurable by the slope of $QQ'$. As is understandable from this graph, as the rate of profits diminishes in the way of
the technique of this production system is switched over in the way of $\alpha \rightarrow \beta \rightarrow \gamma$ and the capital intensity in the whole economy keeps ascending.

Though only three kinds of techniques are picked up in the explanation given so far, it should be understood, strictly speaking, that the technique will keep undergoing changes in succession as the rate of profits keeps diminishing and the capital intensity in the whole economy will also keep increasing successively: that will become clear if due consideration is given to the fact that any point of the envelope is accommodated by unique system of production technique. Be the matter as it may, this proves that the order of selection of technique drawn from the neo-classical production function shown in Figure 2 has been confirmed by Figure 5. It may not be a mistake to make the following statement for the said reason: it has been confirmed that the single-commodity model of the neo-classical school (or the homogeneous capital model) is nothing more than one extremely specific case among all actually existing multiple-commodity model (or heterogeneous capital model) — i.e. a completely fictitious case where the intensity of the capital of each component commodity (or each industry) is in uniform conditions with respect to any technique to be selected. Contrariwise, if a case where a certain technique is actually adopted is taken for an example, the intensity of the capital of each industry is found in realities to be unequal and moreover it may safely be added to say that the extent of such unevenness undergoes a change as often as the technique in question is switched over to another one.

That is to say, when a certain technique is specified, the wage curve under such technique ($\alpha$) will assumably be formed not in a straight line but in a curved line and furthermore when the type of technique switched over to another type as a result of a change in the rate of profits, it should be considered that the wage curve under such new technique ($\beta$) will normally be shown up in a different form. thus, the envelope coming to contact from the outer side with the said innumerable numbers of wage curves could never be assumed necessarily to be shown up in a unique and definite form (like the convex form against the original point in Figure 5) and furthermore it would be impossible, in a case where the rate of profits diminished, to give a certain pre-determined sequence of order with respect to the capital intensity for the optimum technique to be selected for the whole economy, and under this circumstance the matter of selecting a technique for the economy would tend to become substantially uncertain. Being quite different from the neo-classical production model, it is presupposable in a case where the rate of profits declines that the capital intensity might undergo successive changes or the increase and the decrease might take place reciprocally one after another or the same intensiveness of the capital might appear more than once.22) Furthermore,

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22) For an instance, see the shape of the envelope conceived by Garegnani and Nuti. P. Garegnani, loc. cit., p. 413; D. M. Nuti, loc. cit., in “Capital and Growth”, p. 323 and 327.
since the assumption of the neo-classical production model is by no means anything probable as already been discussed, it would become possible even to contend that the presupposition here adopted has a much higher probability.

VII

In what specific way does the standpoint of such economics carry its bearings upon the standpoint of the so-called neo-classical synthesis? The writer would like to make a generalization on this point. Although the standpoint of neo-classical synthesis appeared to be a kind of popular practice in these days, needless to say it was none other than one of the targets of Samuelson’s economics which happened to be a standard textbook of the contemporary economics. And it is commonly known that it was the Cambridge economists like Kaldor, John Robinson, etc. who stood up as the severest criticisers against the said standpoint of the neo-classical synthesis.

Now, there is no need to say that the standpoint of neo-classical synthesis was so intended as to form a synthesis of Marshallian theory of prices and Keynesian macro theory of income as understandable from the “Systematic Chart of Economics” attached to the Samuelson’s economics. That is, every effort was so made as to eliminate unemployment by achieving the full employment scheme under current mixed economy, basing on the principle of Keynesian effective demand, by making a full use of fiscal and monetary policies. In this way their idea had it that, once their full employment scheme is achieved to its full extent, the neo-classical theory of price can be put in full practice. Putting it in the other way, they were of opinions that the Keynesian macro theory could be applicable to the problem of how to settle the national income and employment in the economy as a whole, and that the traditional Marshallian theory of prices could be applicable to those old problems which were left unsolved such as how the national income under the full employment would be distributed to each factor of production, how the productive resources would be allocated to each industry or use and how the relative prices of commodities would be determined.

The neo-classical synthesis on the one hand is advocating that the combined policies designed to lighten credit situation and to tighten financial condition would be effective toward the aim of economic growth without causing inflation under the present mixed economy, and on the other hand a cheap money policy would facilitate to deepen capital (the increase of quantity of capital per capita, i.e. the rise of capital intensity) and at the same time the tightening of financial condition would generally help to raise the rate of economic growth without causing the so-called

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23) About this, see the following work. “Contemporary Economics” comp. by Aoyama, & Yasui, 1968, pp. 306–309.
"demand-pull inflation." In other words their idea has it that, when it is planned to increase the national income by maintaining the level of full employment, the aforementioned combined policies would have a restrictive effect upon consumption and an encouraging effect upon investment, resulting in the increase of the proportional weight of investment as a whole in relation to the national income and would have a facilitating effect for the economic growth, but it would be possible to keep the aggregate expenditures under complete control so that level of full employment income might be maintained without causing any inflationary trend.

Now, on the background of their idea to see that such mechanism could operate satisfactorily to achieve their desired aims, an assumption is made to the effect that it would be possible to heighten the capital intensity or the capital coefficient for the sake of the economy as a whole by lowering the rate of interest. Putting it in the other way such mechanism is designed on an assumption that the particular technique backed up by the more intensive capital would be selected by the more lowered rate of interest. There is no need, I believe, to make further explanation in addition to the statement that such definite order of selection of technique is an inevitable conclusion drawn from the well-behaved neo-classical aggregate production function.

In this connection what should not be neglected to note is the fact that the use of the production function as shown in Figure 2 is presupposed in that mechanism, in which the said combined policies could operate satisfactorily. That is to say, a possibility of adequateness of such combined policies has an immediate connection with the adequateness of the neo-classical production function. Consequently, if the neo-classical production function couldn't be anything more than fictitious or extremely specific indications of the fundamental characteristics of an actual production system and if the sequential order to select the techniques to be caused by changes in the rate of interest should be in fact something substantially uncertain, then the aforementioned combined policy maintained by the neo-classical synthesis would certainly lose the ground for its adequateness.