THE
KYOTO UNIVERSITY
ECONOMIC REVIEW

MEMOIRS OF THE FACULTY OF ECONOMICS
IN THE KYOTO UNIVERSITY

VOL. LI, NO. 1-2	APRIL-OCTOBER 1981 Whole No. 110-111

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PUBLISHED BY
THE FACULTY OF ECONOMICS, KYOTO UNIVERSITY
SAKOYO-KU, KYOTO, JAPAN
Theories which deal with the question of how the real wage rate changes during a "typical" trade cycle usually do not come up with unique answers. Roughly the same may be said about empirical investigations although we would like to argue that a certain "normal" pattern can be distinguished as opposed to some "exceptions". At least in the post-war periods we observe a cyclical pattern which has the following characteristics: if we take deviations from trend the maximum of the real wage rate occurs during the recession phase, whereas the real wage rate reaches its minimum during the expansion phase. Obviously, we can produce this pattern by introducing a lag into the relationship of changes in output (or employment) and changes in the real wage rate. The question, which we want to answer in this paper is the following: What kind of wage-price-adjustment mechanism is likely to produce a cyclical pattern as described above?

There are some models put forward in the literature which would explain the observable cyclical pattern.

Solow and Stiglitz (1968) have constructed a model where aggregate demand depends on the income distribution between wages and profits. Output depends on the real wage rate and aggregate demand for goods. Capital stock is taken as constant, i.e. only short run dynamics are investigated. Price and wage changes are governed by excess demand on the goods and the labour market. In principle the model is compatible with various cyclical patterns of the real wage rate.

A somewhat different model was constructed by Desai (1973) based on a model proposed by Goodwin (1967). Here, investment depends on the distribution of income between wages and profits, and wages and prices also respond to excess demands (although excess demands are measured differently). At least in the simpler version of the model there is a clockwise movement of employment and the real wage rate.

Our model is in some respects a simpler version of the one developed by Solow and Stiglitz (1968). It is simpler because we assume no relationship between aggregate demand and the distribution between wages and profits. The cyclical change of aggregate demand is taken exogenously, i.e. the trade cycle may be due to monetary phenomena, changes in

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* Professor Dr., Universität Mannheim; Guest Scholar, Kyoto University (July–October 1981). An earlier version of this paper was presented at the European Meeting of the Econometric Society in Helsinki, September, 1976. We should like to acknowledge gratefully that one of the authors was supported partly by a grant from the Leverhulme Trust. We thank Professor P. C. B. Phillips, University of Birmingham, for comments on an earlier and Yale draft. Of course, the authors claim responsibility for any shortcomings in the paper.
export and import demand and/or a cyclical variation in investment demand. The model is made simple in this respect as we want to analyse whether an adjustment hypothesis of the tâtonnement variety or a disequilibrium approach with quantity adjustments is more in line with observable cyclical patterns of the real wage rate.

We shall proceed as follows:

- The trade cycle is taken to be caused by a cyclical behaviour of aggregate demand (no repercussions from the wage rate on aggregate demand).
- Changes of the money wage rate and of the price level are assumed to be functions of excess demand for labour and for commodities respectively.
- The structure of the simple macroeconomic model is taken in its standard form.
- Price adjustments as well as quantity adjustments will be considered.
- Some aspects of the wage bargaining process will be accommodated in the model.

Symbols:
- $w$: money wage rate
- $\hat{p}$: price level
- $\frac{w}{\hat{p}}$: real wage rate
- $A$: labour supply
- $N$: demand for labour services
- $L$: actual employment
- $Y$: actual output
- $Y':$ planned output by firms
- $Y^d$: effective demand
- $D$: autonomous demand
- $\bar{K}$: capital stock
- $\dot{w} = \frac{dw}{dt}, \quad \dot{w} = \frac{w}{w}$

**Model A**: adjustment à la tâtonnement. The tâtonnement process has the following characteristics: Demand and supply are determined by prices, and price changes are functions of excess demands. However, we introduce two deviations from the "pure tâtonnement": consumption demand depends on realised income, and actual employment is the smaller amount of labour supply and labour demand. The model is then constructed as follows:

<table>
<thead>
<tr>
<th>labour supply:</th>
<th>$A=A(u); \quad A, \geq 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>labour demand:</td>
<td>$N=N(u); \quad N, &lt; 0; \quad \text{from } \frac{\partial Y}{\partial N} = u$</td>
</tr>
<tr>
<td>planned output:</td>
<td>$Y' = Y(\bar{K}, N(u)) = Y^*(u)$</td>
</tr>
<tr>
<td>aggregate demand:</td>
<td>$Y^d = D + b(Y); \quad 0 &lt; b &lt; 1$</td>
</tr>
<tr>
<td>actual employment:</td>
<td>$L = \min{N(u), A(u)}$</td>
</tr>
<tr>
<td>actual output:</td>
<td>$Y = Y(\bar{K}, L)$</td>
</tr>
<tr>
<td>adjustment hypotheses:</td>
<td>$\dot{w} = h_1(N(u) - A(u))$</td>
</tr>
<tr>
<td></td>
<td>$\dot{\hat{p}} = h_q(Y^d - Y)$</td>
</tr>
</tbody>
</table>

Working of the model:
Full employment equilibrium is achieved if $D = D^*$, $u = u^*$ where $u^*$ clears the labour market and $D^*$ is determined by
\[ Y'(u^*) = D^* + b(Y^t(u^*)) \]

We distinguish three cases:

1. \( D = D^* \) (full employment equilibrium).
   \( u^* \) is stable if \( \delta < 0 \) for \( u > u^* \) and \( \delta > 0 \) for \( u < u^* \). The first condition is easily established as with \( u > u^* \) we have \( h_1(...) < 0 \) and \( h_2(...) > 0 \). The second condition requires an additional assumption as for \( u < u^* \) we have \( h_1(...) > 0 \) but also \( h_2(...) > 0 \) as long as \( A_* > 0 \). For stability of \( u^* \) we have to assume that \( \omega > \bar{\beta} \) for \( u < u^* \). This is justified as firms try to increase employment. They can sell the extra output \( (Y^t - Y) \), and this is profitable as \( \partial Y / \partial N > 0 \). Hence firms have an incentive to raise \( w \) sufficiently to increase \( u \) and thus increase employment and output.

2. \( D < D^* \) (underemployment).
   It can be shown that there exists a stable \( u^* > u^* \) for any given \( D < D^* \). Therefore, we have \( u^* = f(D) \) with \( f' > 0 \). The proof is as follows. Take
   \[ \delta = \omega - \bar{\beta} = h_1(N(u) - A(u)) - h_2(Y^t(D, Y) - Y) \]
   where \( Y = Y(K, A) \) for \( u < u^* \)
   and \( Y = Y(K, N) \) for \( u > u^* \)
   Let \( \psi = 0 \) and differentiate with respect to \( D \):
   \[ 0 = h_1'(N_0 - A_0) \frac{du}{dD} - h_2'(Y_r + (bY_r - Y_r) \frac{du}{dD}) \]
   \[ \frac{du}{dD} = \frac{h_1'(N_0 - A_0) h_2'(Y_r + (bY_r - Y_r) \frac{du}{dD})}{h_2'(N_0 - A_0) + h_2'(1-b) Y_r} < 0 \]
   as long as \( h_1'(N_0 - A_0) < h_2'(1-b) Y_r \). This holds if \( Y_r = Y_0 N_0 < 0 \) but may not be fulfilled if \( Y_r = Y_0 A_* > 0 \). However, as \( Y_r = Y_0 A_* \) can only hold for \( u < u^* \) and \( u^* > u^* \) this case is ruled out.

3. \( D > D^* \) ("inflationary gap").
   It can be shown for any given \( D > D^* \) that there exists a stable \( u' < u^* \) such that \( u' = g(D) \) with \( g' < 0 \). The proof is the same as above.

The relationships \( f(D) \) and \( G(D) \) are represented in fig. 1.

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**Fig. 1**
If we let $D$ move cyclically between $D^*$ and $D'$, we obtain the following cycle of $D$ and $v$ (fig. 2):

![Diagram](image)

Of course, more complicated patterns can be obtained if we start from arbitrary initial conditions, but a closed cycle can only have the indicated direction, i.e. a counter clockwise movement. The real wage rate $v$ reaches its maximum early in the recovery phase ($D'<D<D^*$), and reaches its minimum in the downswing of the trade cycle ($D'>D>D^*$).

As this result is not in line with most observable cyclical patterns of the real wage rate, the model has to be changed.

**Model B:** We introduce two changes in turn:
1. We assume that in cases where effective demand is smaller than supply ($Y'<Y'(u)$), there is a rather rapid quantity adjustment of output to effective demand $y_d$. As $Y'$ is determined by $Y'=D+b(Y')$, we can write $Y'=Y'(D)$.

   If $Y'<Y'(u)$ and $Y=Y'$ we should expect that employment tends to be smaller than $N(u)$. Therefore, we introduce
   \begin{equation}
   N=\min(N(u), F(Y'))
   \end{equation}
   where $F(Y')$ indicates the amount of labour services necessary to produce $Y'$.

2. It is desirable to incorporate into the model some aspects of the wage bargaining process. Then $w$ cannot solely be a function of excess demand but must also depend on something else. We try the following simple hypothesis: Assume that unions and employers have certain targets of the real wage rate. Let $\delta$ be the target real wage rate of employers. As unions have a direct influence only on $w$ while firms have a more direct influence on $p$, we may write the following adjustment equations

   \begin{align}
   \tilde{w} &= h_1(N-A) + \phi_1(\delta-v) \\
   \tilde{p} &= h_2(Y'-Y) + \phi_2(u-\delta)
   \end{align}

   where $\phi_i(0)=0$ and $\phi_i>0$.

Introducing assumptions (2) into the model A does not change the cyclical pattern significantly. Depending on $\phi_1$, $\phi_2$, $\delta$, and $\delta$ the line $g(D)$ and $f(D)$ will change its
position and slope, but the slope remains negative, and therefore, there is no significant change in the cyclical pattern of $u$.

However, if we introduce assumption (1) then $f(D)$ becomes positively sloped. This can be shown as follows. For simplicity, let $\phi_1 = \phi_2 = \phi = \text{constant}$. Then we have

$$\dot{\theta} = h_1(N-A) - h_2(Y^d - Y) + \phi(\theta + \delta - 2\psi)$$

Again, we take the three cases ($D = D^*$, $D < D^*$ and $D > D^*$) in turn.

1. $D = D^*$. If $u = u^*$, $\dot{\theta} = \phi(\theta + \delta - 2u^*) \leq 0$ depending on whether $\theta + \delta \geq 2u^*$. There is a stable $\psi^* > \psi^*$ if $\theta + \delta > 2u^*$ and $\psi^* < \psi^*$ if $\theta + \delta < 2u^*$. Note, however, that the "stable" solution $\psi^* > \psi^*$ implies inflation and unemployment.

2. $D < D^*$. If $N(v) \geq F(Y^d)$ we have

$$\dot{\theta} = h_1(F(Y^d) - A(v)) + \phi(\theta + \delta - 2\psi)$$

As $Y^d = Y^d(D)$ and $Y^d > 0$ it is easily established that $\psi = f(D)$ has a positive slope:

$$f' = \frac{h_1 F(Y^d) - 2\phi}{2\phi + h_1 A} > 0, \text{ and also } \psi < \psi^*.$$  

3. $D > D^*$. Now $Y^d(D, Y) - Y > 0$ as $Y$ is constrained by $\min(A(v), N(v))$. Hence:

$$\dot{\theta} = h_1(N(v) - A(v)) - h_2(Y^d(D, Y) - Y^d(v)) + \phi(\theta + \delta - 2\psi)$$

The slope of $\psi = g(D)$ is determined by:

$$\frac{d\psi}{dD} = g' = \frac{h_1 Y^d}{h_1 (N - A_v) + (1-b) h_2 Y L - \phi}$$

If $L_v = N_v$ we have $\frac{d\psi}{dD} = g' < 0$. Only if $L = A$ and $(1-b) h_2 Y L - \phi > 0$ we may have $g' > 0$ for sufficiently small values of $v$. However, this case is only relevant if $\theta + \delta$ is unrealistically small.

Depending on the relative slopes of $f(D)$ and $g(D)$ we can construct three possible cyclical patterns of $u$ and $D$. These are illustrated in fig. 3.

The pattern in fig. 3(c) produces the clockwise movement which we would identify with the "normal" observable pattern. The diagram suggests that a clockwise movement requires a rather flat $f(D)$ - and rather steep $g(D)$ - line, i.e. a rather large $f'$ and a rather small $-g'$. As $f'$ increases with $h_1'$ and $-g'$ decreases with $h_1'$ this parameter is obviously a candidate for determining the required slopes. The size of $h_1'$ is a measure
of the influence of excess demand for labour in the wage bargaining process. The more important the role of excess demand the flatter the line \( f(D) \) in fig. 3 and the more likely the clockwise movement. However, a prerequisite for this is a rather rapid quantity adjustment of output and labour demand to aggregate demand for commodities.

If the targets \( \phi \) and \( \delta \) become more important we have a larger \( \phi \) and hence a smaller \( f' \) and a smaller \(-g'\) which means that both lines \( g(D) \) and \( f(D) \) in fig. 3 become steeper. A sufficiently large \( \phi \) reduces the difference between \( v^{\text{max}} \) and \( v^{\text{min}} \) in the trade cycle, and we should expect to observe a smaller impact of trade cycle phenomena on the real wage rate.

It may be worth noting that (8), (9) or (10) can be interpreted as a version of the celebrated Phillips curve. In our model it is the change of the real wage rate rather than the change of money wages or prices which is determined by excess demands and some "institutional factors".

**Estimation procedure:** After eliminating the trend from real wage rates and output by taking deviations from the linear trend we tried as a first step

\[
v(t) = a + BY(t-\theta), \theta = 1, 2, 3.
\]

Our study uses annual data covering the period 1955 to 1974. The source of our data is various issues of the Annual Abstract of Statistics and Statistisches Jahrbuch der B. R. D. We experimented with several measures of the real wage and real output. The most satisfactory results obtained so far using the RALS programme developed by D. F. Hendry are presented below. For the real wage rate, \( W \), we use average gross earnings deflated by the Index of Retail Prices. We use two measures of real output, \( y^* \) and \( y^b \) defined as the Index of Industrial Production and Gross Domestic Product respectively. The variables \( W, y^* \) and \( y^b \) are all measured in deviations from trend.

### Results

**U. K. 1955–1969**

\[
W = -2.98 - 0.572 y^*_{t-3}
\]

\( R^2 = 0.79 \)

\( d. W. = 1.73 \)

(14, 38) (4, 69)

\[
W = -3.08 - 0.898 y^*_{t-3}
\]

\( R^2 = 0.72 \)

\( d. W. = 1.68 \)

(12, 71) (3, 96)

**West Germany 1955–1969**

\[
W = -0.92 + 0.66 y^*_{t-1}
\]

\( R^2 = 0.75 \)

\( d. W. = 1.5 \)

(1, 86) (4, 61)

\[
W = -0.02 + 1.2 y^b_{t-1}
\]

\( R^2 = 0.71 \)

\( d. W. = 1.4 \)

(0, 03) (4, 16)

**West Germany 1955–1974**

\[
W = -0.089 + 0.541 y^*_{t-1}
\]

\( R^2 = 0.75 \)

\( d. W. = 1.17 \)

\( \dot{u}_t = 0.638 \dot{u}_{t-1} + \epsilon_t \)

(2, 2)

\[
W = 0.074 + 1.21 y^*_{t-3}
\]

\( R^2 = 0.83 \)

\( d. W. = 1.5 \)

(0, 193) (7, 25)
From the results so far we can establish a distinct cyclical pattern of the real wage rate in both countries. This pattern corroborates the hypothesis of rather rapid quantity adjustments when aggregate demand falls short of full employment output. The period 1969-1974 in the U. K. does not fit this pattern. Possible reasons for this are:

1. the length of the cycle changed drastically
2. the underlying behavioural functions changed due to
   a. a modification in the wage bargaining process (e.g. $\phi$ increased and $h_i$ decreased).
   b. the significantly higher rates of inflation tend to overrule the excess demand mechanism.
   c. structural changes in the economy
3. the model is misspecified

Further statistical analysis is necessary in order to discriminate between these possible causes.

The fact that we got better results for the U. K. with a lag of three periods and for Germany with a lag of one period is no evidence against the theory: with a cycle length of approximately 4 periods we would expect a positive coefficient for a one period lag and a negative coefficient for a three period lag.

(Date of receipt of final typescript: October 1981)

References