ISSN 0023-6055

THE
KYOTO UNIVERSITY
ECONOMIC REVIEW

MEMOIRS OF THE FACULTY OF ECONOMICS
KYOTO UNIVERSITY

56

VOL. LVI, NO. 1 APRIL 1986 Whole No. 120

CONTENTS

Quesnay's *Tableau Economique* and Interindustry Model
Izumi HISHIYAMA 1

Sales System of the Steel Industry
Fumio KONDO 23

'Ownership in Capital' in Marxian Economics
Kiichiro YAGI 50

PUBLISHED BY
THE FACULTY OF ECONOMICS, KYOTO UNIVERSITY
SAKYO-KU, KYOTO, JAPAN

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KYOTO UNIVERSITY

VOLUME LVI NUMBER 1 (APRIL *1986*) WHOLE NUMBER 120

QUESNAY'S *TABLEAU ECONOMIQUE* AND INTERINDUSTRY MODEL[1]

—NOTE ON NEMCHINOV'S ARTICLE ABOUT REPRODUCTION SCHEME—

By Izumi HISHIYAMA*

I Motif and Subject

After a couple of years since I had published an article[2] about The *Tableau Economique* of Quesnay, I knew that it was discussed in an article by Mr. V.S. Nemchinov in the Soviet Union.

The above-mentioned article of Nemchinov is of this title "Some quantitative dependence of reproduction scheme" which was published in *Voprosi Ekonomiki*, a

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1) This article is English translation of that described below which was published in Japanese in 1965. In the article in Japanese Section IV was titled "Expanded reproduction scheme" which is changed to "Price determination in reproductions system" in this English translation as shown below. However, the discussion on the problem of the latter title was not developed in the article in Japanese, although it suggested the problem in its note. This was partly because that Sraffa seemed to have solved the problem quite sufficiently in his epoch-making book in 1960. My article in Japanese was published under the same title of that in English in *Keizai Ronso* (*Economic Review*), Vol. 96, no. 5, November, 1965, Kyoto University Economic Society.

2) I. Hishiyama, "The *Tableau Economique* of Quesnay —Its Analysis, Reconstruction, and Application", *Kyoto University Economic Review*, Vol. 30, No. 1, April, 1960. For the sake of reference in the following, this article will be denoted, as I.

bulletin of the economic department of Soviet Science Academy, No. 2, 1962.[3] The article is a product of practical interests in the reality of Soviet's economy for the purpose to construct models applicable to the planned economy in the Soviet in modern times.

As I concluded the article with the description "The *Tableau Economique* which was conceived just two hundred years ago had the makings suitable to accept that simple but bold ideas hidden within it, and the people who are ambitious to develop it in a strict manner and to apply it to today's economy can still derive a lesson from it" (I. p. 45), I was interested in the practical meaning of the *Tableau Economique*. It is, however, a surprise to me and, on the other hand, arouses my interest that my thought on the *Tableau Economique* attracts notice even in the Soviet's planned economy which has rather a different structure from that of our economy.

It seems to me that the analysis by Nemchinov has two viewpoints which can never be overlooked. According to the language used by Nemchinov himself, one of them is the problem of "the optimal sector-composition of final social products" or "the fundamental optimal national-economic proportions", and the other is the problem of price determination in the *Tableau Economique*, that is, reproduction system. Thus, these two problems constitute the subjects of Sections III and IV of this article respectively. In foregoing Section II, I will discuss somewhat in detail the relation between "the fundamental equations of the *Tableau Economique*" of mine and the fundamental equations that validate Nemchinov's "physical production models". The aim of this discussion is to elucidate to what extent the most fundamental ideas of the *Tableau Economique* (those attributed to the *Tableau* by me) are incorporated in Nemchinov's models.

II Quesnay's *Tableau Economique* and Nemchinov's "Physical Production Models"

Quesnay's *Tableau Economique*. It is needless to say again that the *Tableau Economique* is the first scheme of circular process. It is well-known that when people proceeded to think the problem of circular process fundamentally the *Tableau* always appeared in the front as a phoenix and became, so to speak, a source of cultivation of such thought. In my article I have defined one of its theoretical implications as "a theory to determine the equilibrium values of the sales of respective industries by giving the volume of expenditure and the expenditure coefficient of the non-industry sector" (I. p. 5).

The *Tableau Economique* divides the industries of reproduction into two sectors.

3) В. Немчинов, "Некоторые Кодичественные Зависимости Схемы Воспроизводства" *Вопросы Экономики*, Февраль (2), 1962, ctp. 100. For the sake of reference in the following this article will be denoted as II. Main part of this article is included in the following book by Nemchinov: B.C. Немчинов, *Экономико-Математические Методы и Модели*, 1965. Prof. H. Ishizu has translated Nemchinov's article into Japanese and published in *Kagawa University Economic Review*, Vol. 35, No. 4, October 1962. It was very helpful for making this article.

In other words, it employs the structure of two sectors. Simply speaking, these two sectors can be said to correspond to the division of the agricultural and industrial sectors. However, one of the essential viewpoints of the *Tableau* is as follows: that is, the model of the *Tableau Economique* considers "products of the land" (productions de la terre) or especially agricultural products as a typical commodity among them to be only one basic products. Speaking in a little more detail, agricultural products are used not only as living necessaries of the people who are engaged in the all sectors but also as the input (capital goods or means of production) used in the process of production in these sectors. The *Tableau* considers that the agricultural sector inputs its products into production in the form of seed, feed for livestock and the like (in the form of capital goods), on the other hand the industrial sector also used and processes agricultural products as raw materials. Sraffa considers in his famous book the division between basic products and non-basic products to be a fundamental concept in circular process and finds the criterion of this division in "whether a commodity enters (no matter whether directly or indirectly) into the production of *all* commodities"[4].

Accordingly, in reference to such division of commodities, Quesnay's *Tableau Economique* might be said to be a model that regards agricultural products (or corn) to be *only one* basic products. Further, it employs "[a method] of singling out corn as the one product which is required both for its own production and for the production of every other commodity" (Sraffa, *ibid.*, p. 93). Even if the agricultural sector purchases the products of the industrial sector from the latter, they are, irrespective of their appearance, essentially nothing but transformed agricultural products (which have been transferred to industry as raw materials). This is because that the essentially same kind of products as those inputted directly in the production of the agricultural sector can be said to have been inputted in the production of the relevant sector (agriculture) in an indirect manner or through, so to speak, a round about way. Such a model that regards agricultural products as only one basic product as above equals essentially the *single-product model* (однопродуктовой модели) defined by Nemchinov.

Another point attracting notice in the *Tableau Economique* is that the non-industry sector is understood as an independent sector separated from the industry sector and the former receives the net product of the latter as revenue (income) which is spent for the products of the both sectors. Of course, this point corresponds to the distinction between physical cost and net product (produit net) in terms of cost (or value) composition. Moreover, the *Tableau* considers that if the amount of net product (revenue) is given, the manner of its spending does exercise a crucial effect on the determination of sales and accordingly that of the scale of output in each production sector. In my view, it can be said that this thesis is indeed not only alpha but also omega for a trial to elucidate the circular scheme expressed in the *Tableau Economique*.

However, the object of the *Tableau Economique* is in fact French economy two

4) P. Sraffa, *Production of Commodities by Means of Commodities: Prelude to a Critique of Economic Theory*, 1960, p. 8.

hundred years ago under the *ancien régime*. Therefore, Quesnay's theoretic ground and his practical interests are decisively different from those of us and of Nemchinov. This point is shown typically in the manner of conceptional construction of his circular scheme, especially seems to be shown intensively in the manner of understanding net product (produit net). More concretely speaking, it is expressed in the manner how to see the material composition and cost (value) composition of net product. However, the principal logic which supports Quesnay's conception has an aspect to be succeeded and developed by us by jumping over an abyss that lies between Quesnay and us —perhaps a classic would always have such a characteristic. To what I pay attention here is just that essential vision which only a classic possesess and which leaps over the extension of time and space developed before us to give light to the elucidation of fundamental problems.

I have tried to represent the implication of the core in the *Tableau Economique* by a simple expression that I have called "the fundamental equations (equation fondamentale) of the *Tableau Economique*" (I. p. 5). As Nemchinov declares (II, p. 107), my equations and Nemchinov's equations (3) can be attributed to the same, which can be said to be, so to speak, the point of contact between his thought and mine. If so, it may be a short cut to the subject to discuss this point. Before developing the discussion on this point, however, I would like to make some preliminary explanations.

Intersector Model. An economic system is understood as structure of transactions of products and production factors which flow between sectors comprising the system. In order to show this simply, the interindustry table in the case where the economy has two industry sectors and one non-industry sector (which corresponds to the sector composition in the *Tableau Economique*) will be shown as in Table 1 (numbers are fictitious).

Table 1

Sale / Purchase	Industry I	Industry II	Non-industry sector	Total products
Industry I	20	30	50	100
Industry II	40	30	30	100
Non-industry sector	40	40	(80)	—
Total products	100	100	—	200

In constructing his "intersector model of physical production" (модель межотраслевых связей материального производства), Nemchinov assumes the transaction structure which has principally the same characteristics as those of Table 1, except the numbers used there. In his view, the row (the third row) of the non-industry sector in Table 1 represents the structure of net products or revenues by sectors (that is, sectors I and II), on the other hand the column (the third column) of the non-industry sector shows the structure of final products by sectors. In Table 1, net products or revenues by sectors are 40 and 40 for sectors I and II respectively, and final products by sectors are 50 and 30 for sectors I and II respectively. These net products (revenues) and final products by sectors correspond to value composition and material composition of national income (by sectors) respectively. The total of net products of each sector represents value composition of national income. The elements of the third column comprise final products 50 and 30 in sections I and II respectively, which total 80 and which represent material composition by sectors of national income.

The block closed by thick lines shows the transaction structure of capital goods (means of production) between sectors, that is, the structure of physical cost. Each element in this block represents the values of means of production spent in the reproduction during this period, at the same time, however, these elements are shown from the both sides of material composition and value (cost) composition.

In short, to say in reference to the language of Nemchinov hismelf, in this table "national income is examined from the three viewpoints, that is, the viewpoints of material, sectors and value, and they represent principally an important essence of the composition of social production scheme" (II, p. 101).

Now, let us pay attentibn to another point. If we read horizontally the row of industry I in Table 1, the figure 20 in the first column shows the amount of internal transaction or intra-sector procurement in this sector, but the following elements 30 and 50 are, from the standpoint of this sector, the sales to other sectors (that is, sector II and the non-industry sector). In other words, these figures represent the purchase (procurement) of the product of sector I by other sectors. Of course, other sectors purchase the product of sector I for different uses: sector II for the use of intermediate products including raw materials etc. and the non-industry sector for that of final products, but as a whole (30+50) is the sales of sector I to other sectors. Similar comment can be made with respect to sector II. I have called these amounts "the sales of industry" and denoted the amount in sector I (30+50) in Table 1 as x and that in sector II (40+30) in Table 1 as y respectively (I, p. 5), and Nemchinov calls the amounts "the extra-sector commodity-procurement amount" for each product and denotes them as Z_1 and Z_2 respectively (II, p. 104).

The Fundamental Equations of the *Tableau Economique*. The decisively important parameter in Quesnay's *Tableau Economique* is the coefficient which distributes the expenditure of net product (product net) among sectors. The non-industry sector spends its net revenue obtained by the production during the previous period to sector I at the rate of λ and to sector II at the rate of $(1-\lambda)$. I have called this coefficient "the expenditure coefficient" and denoted it as r. On the other hand, Nemchinov considers it as the coefficient which represents "the weight of sectors in material composition" of final product (national income) Y, simply speaking, "sector composition of final product".

Now, I will denote the sales of sector I to other sectors (that is, "the extra-sector procurement amount" of sector I by Nemchinov) as Z_1, that of sector II as Z_2, the expenditure coefficient (the coefficient representing material composition by Nemchinov) as λ, the total of sales of each industry to the non-industry sector, that is, final product as Y, and total sales (or total purchase amount) by sectors or total products by sectors as X_1 and X_2 respectively. Then, the reproduction construction of the *Tableau Economique* shown in Table 2 can be represented as in Table 3[5].

In these tables, it is supposed that "each industry conforms to this propensity

Table 2

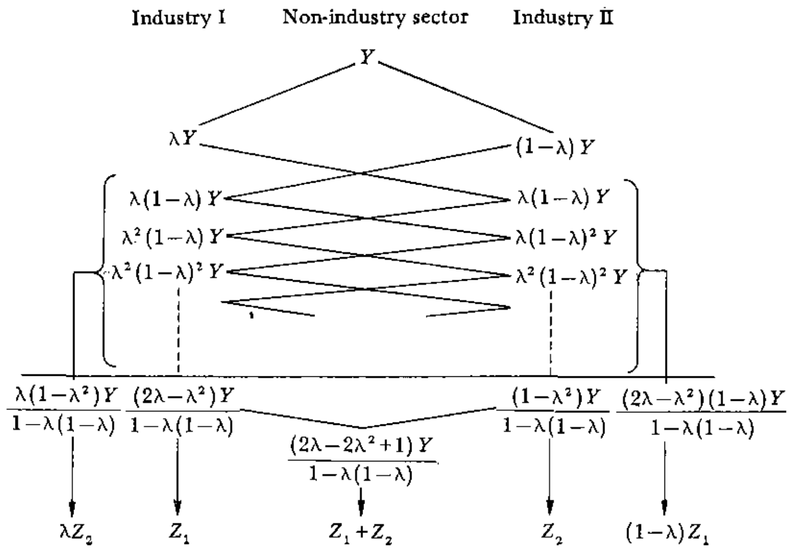

Table 3

	Industry I	Industry II	Non-industry sector (final products)	Total products
Industry I	X_1-Z_1	λZ_2	λY	X_1
Industry II	$(1-\lambda)Z_1$	X_2-Z_2	$(1-\lambda)Y$	X_2
Non-industry sector (net products)	λZ_1	$(1-\lambda)Z_2$	Y	—
Total products	X_1	X_2	—	X

5) I have given following consideration to Table 3 in order to make easy comparison with Nemchinov's model. First, the same symbols as those in Nemchinov have been used to represent economic quantities. Thus, x and y (of mine) representing the sales of industries I and II have been changed to Z_1 and Z_2 respectively, and "the expenditure coefficient" r to λ. Therefore, as far as this point concerned, Table (2.2) in my previous article differs only formally from this Table 3. However, the elements of columns and rows which were blank in Table (2.2) have been entered into Table 3, and the row of "the non-industry sector" has also been added there. This may imply to some extent a substantial difference from the original *Tableau Economique*. But, it is in a sense inevitable in the context of this paper which discusses exclusively the modern meaning of the *Tableau Economique*.

to expend λ in its internal transactions as well as transactions with other industry" (I, p. 3). Further, the expenditure coefficient λ is regarded to represent "the propensity to expend *in any and every* transaction of this system", irrespective of transactions of intermediate products between sectors or those of final products with the non-industry sector (I, p. 3).

For example, if λ is 1/2, the all sectors including the non-industry sector and sectors I and II expend 1/2 of their receipts respectively to the products of sector I, on the other hand expend $(1-\lambda)=1/2$ to the products of sector II. Generally speaking, the non-industry sector expends, as shown in Table 2, the rate of λ of its revenue (Y) to the products of sector I. In other words, sector I sells its products of λY to the non-industry sector. This process is shown in Table 3 with the element λY in the third column of the first row. On the other hand, the non-industry sector spends the amount of $(1-\lambda)Y$ to the products of sector II. That is to say, sector II sells the amount of $(1-\lambda)Y$ to the non-industry sector. This process is shown in Table 3 with the element $(1-\lambda)Y$ in the third column of the second row. As shown in the zigzag processes in Table 2, each sector expends λ to the products of sector I and $(1-\lambda)$ to those of sector II from the amount of its sales, and these expenditure processes are supposed to be repeated to the point where the amount of sales of each sector is reduced to the negligible one.

Then, the total $\left[\frac{Y(2\lambda-\lambda^2)}{1-\lambda(1-\lambda)}\right]$ of the series ranged vertically under the column of sector I in Table 2 will become the sales of sector I (the extra-sector commodity procurement amount for sector I by Nemchinov), Z_1. On the other hand, the total $\left[\frac{Y[1-\lambda^2)}{1-\lambda(1-\lambda)}\right]$ of the series ranged vertically under the column of sector II will represent the sales of sector II, Z_2. Next, in the sales Z_1 of sector I, the part to sector II is the total λZ_2 of the series parenthesized in the left of Table 2. The reason why it equals λZ_2 is that the sales of sector I to sector II is, reversely speaking, nothing but the purchase by sector II from sector I. As sector II is supposed to spend λ of its sales to the products of sector I, the amount thus spent should become λZ_2. This process is shown with the element (λZ_2) in the second column of the first row in Table 3. In the same way, the sales of sector II to sector I will become $(1-\lambda)Z_1$, which is shown in Table 3 with the element in the first column of the second row. As above, all the elements entered into the interindustry table of Table 3 by Gothic types have been explained.

As the total products of industry I is X_1 and its sales to other sectors is Z_1, it would be obvious that the internal transactions of industry I (the element in the first column of the first row in Table 3) will become (X_1-Z_1) according to the definition of (internal transactions) = (value of total products) − (sales to other sectors). In the same way, the internal transactions of industry II will become (X_2-Z_2) as the element shown in the second column of the second row in Table 3. The amount of income payment of industry I to the non-industry sector, namely the net products value of the former, as it is the remaining part of the total products value of the former from which its physical cost is deducted, will become λZ_1 (the element in the first column of the third row in Table 3) according to the equation, $X_1-[(X_1-Z_1)+(1-\lambda)Z_1]=\lambda Z_1$. In the quite same way, the amount of income payment of industry II can be obtained as the element in the second column of the third row, $(1-\lambda)Z_2$. In the non-industry sector, both the elements composing its row and those composing its column, namely the both revenue and expenditure sides of its income will be equal to Y (the element in the third column of the third row) in total respectively. The symbol X entered into the fourth column of the fourth row in Table 3 represents the grand total of the total products by sectors X_1 and X_2.

It is obvious that the intersector model shown in Table 3 is underlain by the following system of equations:

$$\left.\begin{array}{l}\text{[from the row of industry I] } Z_1=\lambda(Z_2+Y)\\ \text{[from the row of industry II] } Z_2=Z_1(1-\lambda)+Y(1-\lambda)\end{array}\right\}\qquad (1)^{6)}$$

The solution of these equations will be:

$$Z_1=\frac{Y(2\lambda-\lambda^2)}{1-\lambda(1-\lambda)},\quad Z_2=\frac{Y(1-\lambda^2)}{1-\lambda(1-\lambda)}$$

From this the following equation can be obtained:

$$Z_1+Z_2=\frac{Y(2\lambda-2\lambda^2+1)}{1-\lambda(1-\lambda)}$$

The system of equations (1) represents my "*équation fondamentale* of the *Tableau Economique*" (I, p. 5) and equals Nemchinov's system of equations (3). (The model underlain by this system of equations will be called "model I" hereinafter.) With this system of equations I have regarded what the *Tableau Economique* implicates theoretically as "a theory to determine the equilibrium sales of industries (i.e. Z_1 and Z_2) by the volume of expenditures Y and the expenditure coefficient λ" (I, p. 5) and put these equations as the basis to reconstruct the *Tableau Economique* as "a first systematic attempt at a *dynamic* theory of development from a macroscopic viewpoint" (I, p. 2). On the other hand, Nemchinov has regarded this system (in case the extra-sector commodity procurement amounts is Z_s in general) as the one which has an implication to "determine quantitatively the interrelation between the sector composiion of social final products ($\lambda_s Y$) and the extra-sector commodity procurement amounts (Z_s)", and tried to put it as the basis of planning of "optimal sector-composition of social final products" (II, pp. 104–5) or "fundamental optimal national-

6) Nemchinov adds to the equations (1) one more equation, $Y=\lambda Z_1+(1-\lambda)Z_2$. But this equation can be derived by the addition of the two equations in the text, and never shows an independent condition. Therefore, it is obvious that the equations representing the fundamental conditions of this model can never be three but only two, as described in the text (and in my previous article).

economic proportions" (II, p. 108). These subjects in Nemchinov will be examined in Section III below.

Nemchinov's Fundamental Equations. Nemchinov tries to construct "an inter-sector model of physical production" (II, p. 103) that is more concrete, or rather useful for modern Soviet economy. Therefore, I will here construct his so-called "two-sector model of physical production" (II, p. 105), which is (considered by him) more general in comparison with the *Tableau Economique*, and the system of equations underlying the model in relation to my analysis of the *Tableau Economique*.

In Section 6 ("The *Tableau Economique* and Its Application) of my article, a trial of "an application of the *Tableau Economique* to an economy which is far advanced than the one originally intended for" (I, p. 2) has been added. The idea behind this trial is as follows: it is supposed in the *Tableau Economique* that the expenditure coefficient λ to the final products of the non-industry sector represents the preference rate of products in *all* the transactions including those of intermediate products between industry sectors; however, "In reality, the expenditure of the non-industry sector and that of industry sectors depend upon different factors of their own, ... Therefore, the expenditure coefficient of income of the non-industry sector λ does not necessarily coincide with the rate of preference of products in the transactions of each industry" (I, p. 38).

The rate of preference of products as input between each industry of course depends upon the technical structure of production in each industry. Therefore, let us suppose that the rate of preference of products in each industry depends upon the technical structure θ_1 and θ_2 respectively (in the assumption of the two sector model). Moreover, let us suppose the technical structure of production in industries I and II to belong to the type in which $\theta_1=\theta_2$ holds good, and denote the equal preference rate of capital goods as θ.[7] Then, the composition of this model can be shown with Table 4.

As shown in Table 4, for input into production, namely for productive consumption each industry expends its sales amount at the rate of θ to the products of I and at the rate of $(1-\theta)$ to the products of II. The model shown in Table 4 is underlain by the following system of equations.

7) The preference rate of products, namely, capital goods (supposed to be the same) in each sector represents also the coefficient of distribution between physical cost (of the payment to the products purchased from the other sector) and added value in each sector. Therefore, sector I purchases with its sales revenue (Z_1) the products of sector II at the rate of $(1-\theta)$ and pays to added value, namely, income at the rate of θ. On the other hand, sector II expends the rate of θ of its sales (Z_2) to the products of sector I and expends $(1-\theta)$ to added value. Such a type of technical structure of production, in which an assumption of symmetry of the preference rate of products or value composition (in each sector) is satisfied, could hardly be found in reality. However, as the (similarly fictitious) assumption which supposes the organic composition of capital in each sector to be equal is useful for elucidating some principles, this kind of assumption may also be considered to clarify the principles implied in the *Tableau Economique* and, as discussed below, be useful as a mediator which leads to a model closer to reality.

Table 4

	Industry I	Industry II	Non-industry sector (final products)	Total products
Industry I	X_1-Z_2	θZ_2	λY	X_1
Industry II	$(1-\theta)Z_1$	X_2-Z_2	$(1-\lambda)Y$	X_2
Non-Industry sector (net products)	θZ_1	$(1-\theta)Z_2$	Y	—
Total products	X_1	X_2	—	X

$$\left.\begin{aligned} Z_1 &= \theta Z_2+\lambda Y \\ Z_2 &= (1-\theta)Z_1+(1-\lambda)Y \end{aligned}\right\} \tag{2}$$

This system of equations (2) is the quite same one[8] as my equation (18.1) (I, p. 39). (The model underlain by this system of equations will hereinafter be called "model II".)

Nemchinov's fundamental equations are the one by which my equations (2) have been advanced a step forward, for it represents a physical production model under the condition of $\theta_1 \neq \theta_2$. In other words, the technical structure in industries I and II of Nemchinov's model corresponds to a type more realistic than in my assumption.[9] Under the above condition, industry I expends $(1-\theta_1)$ of its sales amount (the extra-sector commodity procurement amount in sector I by Nemchinov) to the products of industry II for the purpose of input into production, while industry II expends the rate of θ_2 of its sales amount (the extra-sector commodity procurement amount in sector II) to the products of industry I as input. Thus, "the two-sector model of physical production" by Nemchinov will become as Table 5 (II, p. 105).

Table 5

	Sector I	Sector II	Final products	Total products
Sector I	X_1-Z_1	$\theta_2 Z_2$	λY	X_1
Sector II	$(1-\theta_1)Z_1$	X_2-Z_2	$(1-\lambda)Y$	X_2
Net products	$\theta_1 Z_1$	$(1-\theta_2)Z_2$	Y	—
Total products	X_1	X_2	—	X

8) If the symbols in the system of equations (2) are replaced as follows: Z_1 and Z_2 by x and y respectively, θ by r, and Y by a, these equations will become equation (18.1) in my article. However, in this system "net product" (produit net) is regarded to be national income, while it is regarded exclusively as "profit "in my article.

9) The condition in model II in which value composition (θ) in each sector is supposed to show particular symmetry and therefore the technical structure of each sector is supposed to have an interdependent relation to satisfy this symmetry can of course hardly exist in reality (see note 7)). Actually, the technical structure of production in each sector is not symmetric with each other, and accordingly respective value composition (the rate of preference of products in my article) is unique in each sector. Thus, there can never exist the "symmetry" in model II. In this sense, it can be said that model III (that of Nemchinov) is closer to reality than my model II.

The equations which underlie the model shown in this table are as follows.

$$\left.\begin{aligned} Z_1 &= \theta_2 Z_2 + \lambda Y \\ Z_2 &= (1-\theta_1)Z_1 + (1-\lambda)Y \end{aligned}\right\} \qquad (3)^{10)}$$

The system of equations (3) is the fundamental equations of Nemchinov. (The model represented by them will hereinafter be called "model III".)

The *Formule* of the *Tableau Economique*. It is quite interesting to see that the system of equations (3) which is called by me Nemchinov's fundamental equations represents also the fundamental equations of the *Formule* of the *Tableau Economique*. The solution of the equations (3) will become as follows.

$$Z_1 = \frac{Y[\lambda+\theta_2(1-\lambda)]}{1-\theta_2(1-\theta_1)}, \quad Z_2 = Y\left\{(1-\lambda) + \frac{(1-\theta_1)\,[\lambda+\theta_2(1-\lambda)]}{1-\theta_2(1-\theta_1)}\right\}$$

The *Formule* represents a particular case of (3), where it is assumed that the expenditure Y of "*revenue*" is 2 billions, λ is 1/2, and θ_1 and θ_2 are 2/3 and 1 respectively. That is, industry I ("*the productive class*") expends $(1-\theta_1)$ namely 1/3 of its sales amount Z_1 to purchase the products of industry II ("*the sterile class*") as input. On the other hand, industry II, as $\theta_2=1$ in it, expends the amount corresponding to the total of its sales amount Z_2 to purchase the products of industry I. As above, industry I spends $(1-\theta_1)$ of its sales amount Z_1 to purchase the products of others and pays the remaining part θ_1 namely 2/3 to net products. As for industry II, as described above, since θ_2 is 1, its sales amount Z_2 is wholly inputted to the products of others and no part is remained to pay for net products (produit net). Moreover, as in the *Formule* there exists no internal transaction of products of industry II, it can be said that $X_2=Z_2$ is supposed there.

In this way, the *Formule* of the *Tableau Economique* does never modify mathematical features of the *Tableau Economique* itself, namely "Tableau fondamental". In short, the *Formule* represents the theory to determine the sales of each sector Z_1 and Z_2, in case the expenditure Y of the non-industry sector and its expenditure coefficient λ are given for the period in which the input coefficients $(1-\theta_1)$ and θ_2 for the productive consumption in each sector are not changed.

Summary. As above, starting from my "fundamental equations of the *Tableau Economique*", I could arrive at the fundamental equations of "The two-sector model of physical production" by Nemchinov. To see from this destination point (model III) retrospectively my fundamental equations (model I), the latter corresponds, as indicated by Nemchinov, to the case of $\lambda=\theta_1=\theta_2$ in his own equations (model III), and my model II underlain by equations (2) corresponds to the case of $\lambda \neq \theta_1=\theta_2$

10) Nemchinov shows the second equation in (3) as $Z_1=\theta_1 Z_1+(1-\lambda)Y$. But, this is clearly a mistake and the equation is corrected as in the text. Moreover, he adds the equation $Y=\theta_1 Z_1+(1-\theta_2)Z_2$ to this system of equations. I think it is proper to delete this equation from (3) for the same reason as that explained in note 6).

in his fundamental equations (model III) (although Nemchinov does not suggest this). Accordingly, it must be recognized that Nemchinov's model, namely model III is closer to reality than either of the models (that is, models I and II) developed by me in that it considers the technical structure in each sector not to be symmetric with each other.[11)]

However, whichever of these models (models I, II and III) can be said to have substantially the same characteristics in the principle underlying its construction. This principle or conception belongs originally to Quesnay's *Tableau Economique* for which I have shown my own view. According to the language by Nemchinov, this principle may be expressed as follows: "the extra-sector commodity procurement amounts [or the sales of industry sector to other sectors] Z_1 and Z_2 are functions of final products [namely, net products Y] and parameters, that is, material composition (λ) and value composition (θ) which characterize final products" (II, p. 106, but parentheses [ ] are added by Hishiyama). Such a view of Nemchinov can be said to be substantially similar to my definition for the case of $\lambda=\theta$ in Quesnay's *Tableau Economique* (model I) as "a theory to determine the sales of each industry [i.e. Z_1 and Z_2] by the expenditure Y and the expenditure coefficient λ" (I, p. 4.).

III Optimum Industrial Construction

Problem of optimum industrial construction. One of the subjects raised (as Nemchinov thinks so) by the model (model III) is the problem of "the optimal sector-composition" of national income. It may be noticed that the model is concerned with such allocations of productive resources among sectors with which the sales, namely the output of commodities must be maximised. This problem is the one familiar to us under the theme of "optimum allocations of resources". However, as explained below, it may be said that the principle of problem forming by Nemchinov is underlain by a different basis from that known by ordinary text-book. At any rate, this problem has been suggested[12)] in my article in the analysis of what I have called "the fundamental figure" (diagrammatically formed fundamental equations) (I, p. 8, diagram (4.1)), but the suggestion has not developed sufficiently.

What coefficient λ means. In considering the problem of "optimum allocations", parameter λ which represents the expenditure coefficient of net products plays a critically important role for either of my models (model I and II) or the model by Nemchinov (model III). Therefore, at first as a preparation of discussion, I will examine the problem from the viewpoint of national income Y. As shown clearly in inter-industry tables of Tables 3–5, national income is grasped from two viewpoints. One

11) For the reasons in detail of this point, see notes 7) and 9).

12) Because "the fundamental figure" shows the series of Z_1, Z_2 or (Z_1+Z_2) respectively determined corresponding to all the variable values which the expenditure coefficient can have between 0 and 1 in my model (model I) of the *Tableau Economique*, and for each series it is clearly shown there visually which element is the maximum.

of them is the viewpoint of material composition (in the third column of each table). From this viewpoint, national income will be grasped in the material form of final products by sectors. This form corresponds to that of the composition by commodities of our so-called real income or real output, namely, to that of "final products" by sectors in Nemchinov. The other viewpoint is that of value (or cost) (in the third row of each table). From this viewpoint, national income is understood as cost or value paid by each industry. That is to say, national income comes up also as added value distributed to each sector, and in Nemchinov in the form of "net product" or "net revenue" (paid by industry sectors). In short, national income Y will be grasped from the both sides of the material or physical viewpoint and that of value or cost by sectors.

According to the above discussion, coefficient λ in my model of the *Tableau Economique,* namely model I has two meanings (see Table 3). In one of them, it shows the ratio of material composition of national income (as in the third column of Table 3). In other words, it shows the ratio (or weight) representing the composition of national income by commodities or sectors. In the other meaning, coefficient λ shows value composition or cost composition (by sectors) of national income (as in the third row of Table 3). That is to say, it shows the composition ratio of added value by sectors. In short, in my model I (Table 3), coefficient λ shows as a uniform parameter (λ) the both sides of material composition and value composition of national income simultaneously.

However, material composition and value composition of national income usually do not constitute equal ratio. Because the factors (exogenous to this model) on which each of them depends are different, as indicated in my article (I, p. 38). That is to say, irrespective of the difference in social structure of economy, material composition will depend on the factors dominating the expenditure propensity in the economic society, and value composition on the technical structure of industrial sectors. Therefore, if (the ratio of) material composition of national income is λ, and (the ratio of) value composition is θ, the particular case where $\theta=\lambda$ will be the condition to make my model I valid. While, in case $\lambda\neq\theta$ and the technical structure of each sector has an interdependent relation which satisfies some kind of "symmetry" (namely, the case where $\theta_1=\theta_2$), this model will be my model II (Table 4), and the case where the technical structure of each sector is not symmetric with each other, namely the case where $\theta_1\neq\theta_2$ will be model III, that is, Nemchinov's model (Table 5).

As Nemchinov knows well, the principle underlying optimum construction or optimum allocations appears rather straightly in the simple model (model I) constructed by me. This point will be examined somewhat in detail in the following.

Figure 6 is the one which is called "the fundamental figure" in my article (I, p. 8). At first, let us pay attention to the figure. In the figure, OM equals ON and they represent national income Y supposed to be of given amount. The straight line MN written in a fat solid line is the one which is called the *expenditure line,* and coordinates of each point on the line show the all combinations of λY and $(1-\lambda)Y$. In other

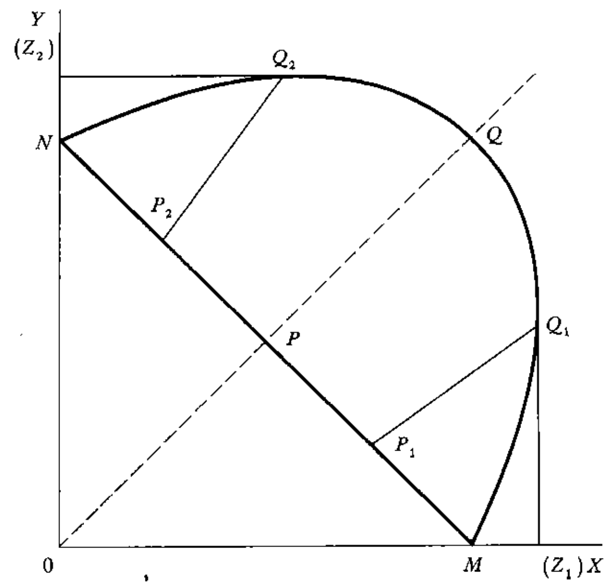

Fig. 6

words, the all points on MN represent the all possible combinations in material composition of national income. Elliptic curve MQN depicted in a fat solid line is called the *sales curve*, and the abscissa of a point on the curve determines the sales Z_1 of industry I, while the ordinate determines the sales Z_2 of industry II. In short, the *expenditure line* MN in this figure represents the all possible conditions of material composition λ of national income, while the *sales curve* MQN shows exhaustively the conditions of the sales in each industry (Z_1 and Z_2) which vary in accordance with the change in material composition λ. Nemchinov shows the quite same matter by a numerical example (II, p. 108).

Optimum construction. The problem of optimum construction raised by this model is the problem to obtain material or sector composition λ of national income which may ensure the global maximum of sales, namely the maximum of (Z_1+Z_2), or the relative maximum of sales, namely the maximum of Z_1 or Z_2.

For example, as shown in Table 7,[13] the maximum of the total sales (Z_1+Z_2) can be achieved when coefficient λ, namely material composition of national income is 0.5, and at this time the total sales will be as much as twice of national income Y. On the other hand, when coefficient λ is 0.7, that is, when material composition of national income (final products) shows the ratio of 0.7 for the products of industry

13) This Table 7 is simplified, but it is the same one with Table 5 of Nemchinov. In his table there were a number of errors in numerals which were corrected by Hishiyama. Needless to say, this numerical example is a numerical example of solution of "the fundamental figure" of mine shown in Fig. 6.

I and that of 0.3 for the products of industry II, the sales of industry I Z_1 will reach the maximum (as much as 1.15 times of national income). Therefore, the optimum value of the material composition λ of national income which ensures the relative maximum of Z_1 will be 0.7. While, coefficient λ that ensures the maximum of sales Z_2 in industry II will obviously be 0.3 (as shown in Table 7).

Table 7 Variation of Z_1 and Z_2 (as the ratio to Y) corresponding to that of λ

λ	Z_1	Z_2	Z_1+Z_2
0.1	0.21	1.09	1.30
0.2	0.43	1.14	1.57
0.3	0.64	**1.15**	1.79
0.4	0.86	1.10	1.96
0.5	1.00	1.00	**2.00**
0.6	1.10	0.86	1.96
0.7	**1.15**	0.64	1.79
0.8	1.14	0.43	1.57
0.9	1.09	0.21	1.30

In "the fundamental figure" of mine (Fig. 6), the three optimal values of coefficient λ described above are indicated by points P, P_1 and P_2 on the *expenditure line* corresponding to points Q, Q_1 and Q_2 on the *sales curve*.

Introduction of value composition θ. The introduction of parameter θ which characterizes value composition of national income does not mean a substantial modification of the essence of the optimum problem in comparison with the case of my simple model (model I). As obvious in the systems of equations [(2) and (3)] underlying my model II and Nemchinov's model (model III) respectively, sales Z_1 and Z_2 of each sector are the functions of two parameters characterizing national income, namely its material composition λ and value composition θ, and of national income Y. As already described, since θ reflects the technical structure of production in each sector, it can be considered to be given for a comparatively short period of reproduction. Or, in other words, if we select as the basic unit the period during which there is no change of the technical structure in each sector composing the economy and discuss this unit period, we will reach the same conclusion. That is, in such a case, the assumption to consider value composition of each sector to be given will become reasonable.

If value composition θ of national income is given and, as in the above case, national income Y is also given, we will be led again to the same problem. This is the problem of interrelation between material composition λ of national income and sales Z_1 and Z_2 of each sector determined by the value of λ. Thus, we will confront also in this case the same problem as above to obtain the optimal sector-composition of national income (the optimum value of λ) that may ensure the global maximum of sales or the relative maximum of sales. According to the language of Nemchinov, it can be said such "a result of analysis has an important meaning in planning of the fundamental optimal national-economic proportions "(II, p. 108).

On what does coefficient λ depend? As above mentioned, coefficient λ representing material composition of national income is, in other words, the one that characterizes its composition by sectors and by commodities. For example, if we assume the products of industry I to represent means of production (capital goods) such as raw materials, energy and the like, and the products of industry II to represent consumption goods, coefficient λ will relate to the composition ratios of capital-goods output and consumption goods output in real national income (final products). In such a case, on which factors does coefficient λ depend substantially?

[*Theory I*] As known well, concerning the capitalist economy, the theory has acquired a fairly great number of devotees that defines the composition of national income to be determined through the mechanism of simultaneous equilibrium mediated by so-called "parametric functions of prices" in the competitive market. Formalizing the view of this school, D.H. Robertson of Cambridge described as follows: "I shall therefore proceed by examining the forces determining the *composition* of the national income; and I shall do this by building up a *theory of value*, ...".[14] It is obvious that the "theory of value" in the quotation means just the equilibrium theory of demand and supply by Marshall. O. Lange also formalizes this theory finely in his famous article on the theory of socialism.[15] What this theory implies is, in short, nothing but the thesis that if the social organization of the economic system is given the economic equilibrium will be achieved automatically in the competitive market under the condition where all the individuals (both enterpreneurs and households) composing the system attain the maximum of their respective gains through rational behaviours and also the condition where equilibrium prices of all goods are determined by the equality of their demand and supply.

Thus, according to this theory, planners are many independent individuals composing the system and they, who have the target of planning to achieve the maximum of their own respective gains and use the parametric functions of prices in the competitive market as a guide post, by being given incentives and restrictions to their behaviours by these functions, attain *spontaneously* the status of economic equilibrium after repeating trial and error. As known well, such economic equilibrium implies as its part the determination of prices and *output*[16] (namely, the production scale of each

14) D.H. Robertson, *Lectures on Economic Principles*, Vol. 1, 1957, p. 61.

15) O. Lange, "On the Economic Theory of Socialism", *Review of Economic Studies*, Vol. 4, Nos. 1 and 2, Oct. 1936 and Feb. 1937.

16) Since Quesnay's *Tableau Economique*, as indicated in my article (I, p. 35), includes the first theoretical conception which divides the price determination mechanism from the output determination mechanism, it should be considered to intend to be a theory of value different not only from Marshall-Robertson's theory of value but also from Walras-Lange's theory of value. See section IV below.

sector) of commodities.

Therefore, according to this concept, material composition, namely composition λ by sectors and commodities of national income is determined by simultaneous equilibrium in the competitive market, and what underlies λ can be said to be finally in the two fundamental conditions themselves that define competitive equilibrium.

[*Theory II*] Now, let us remove the parametric functions of prices assumed in the above theory and employ the assumption of constant prices. This is just the assumption underlying "the *Tableau Economique*", and one of its implications is, as already indicated in my article (I, p. 35), to "isolate the price determination mechanism and the mechanism that functions to determine the amount of output and that of net products, laying a greater emphasis on the consideration of the latter". According to this theory, material composition λ of national income should be said fundamentally to depend on the expenditure propensity of the economic society.

Now, for the sake of simplification, let us suppose that industry I in the two-sector model displayed by us is the capital goods sector and industry II the consumption goods sector. Then, as described above, coefficient λ will relate to the ratio of flows of capital goods and consumption goods composing national income (final products). More exactly speaking, λ is the ratio of net output of capital goods to national income (final products), namely the investment ratio, and $(1-\lambda)$ is nothing but the ratio of output of consumption goods to national income, namely the consumption ratio.

[*Theory II-a*] In the capitalist economy, to see aggregately, if the consumption propensity of the public is given, investment ratio $\lambda(I/Y$ or $\lambda Y/Y)$ will obviously depend, in principle, on the investment decision of individual enterpreneurs who are independent planners. Thus, also in this theory, similarly in principle as in the case of theory I, it is certain that coefficient λ depends on behaviours independent with each other of individual enterpreneurs who aim at the maximum of profits. In this case, however, it is different that for individual enterpreneurs prices have lost the role of a guidepost (parameter) to lead their individual investment activities and come down to a silent robot (constant).

[*Theory II-b*] In the social organization, where the disposition of funds for investments composing national income is not in the hands of individual enterpreneurs but is delegated as a whole to a public authority which is a single planning organ making activities for realizing certain objectives raised by the community, the investment ratio (I/Y) depends on the investment decision from time to time of such public authority. Thus, in such a society coefficient λ will, in principle, depend on the decision of the public authority. It might be said that the pattern of investments in modern Soviet economy is, among above-mentioned theoretical patterns, rather close to that of [*Theory II-b*], or at least principally so.

Optimum problem and coefficient λ. If coefficient λ can have significance in fact in the problem of optimal sector-composition, it will only be the case where λ is a parameter "operational" in policy.

In [*Theory I*], coefficient λ is determined simultaneously with other quantities in the competitive market, as a result of behaviours (aiming at the maximum) of individuals who are independent planners composing the system. In this meaning, the equilibrium value of λ is determined as an aggregated result of the game of that "free competition" in which individuals participate but which is beyond the might of individuals. Thus, coefficient λ of this type has the starkness to reject all the political interventions. Therefore, there seems to be no rationale in this model to extract coefficient λ as the parameter playing a strategic role for the optimum problem.

Things are different in [*Theory II-b*]. If we employ this theory, there seems to exist sufficiently convincing rationale to extract coefficient λ as a strategic parameter. As already discussed above, one of the reasons for this is that there is unavoidable, quantitative dependence between material composition λ of national income and sales of sectors, and the other reason is that coefficient λ depends principally on the investment plans of the public authority and therefore it is operational in policy. Thus, it becomes possible to operate coefficient λ strategically in selecting the national economic proportions estimated to be optimal in view of particular policy objectives.

It cannot be denied that [*Theory II-a*] has been tried and realized to some extent in the capitalistic planned economy which aims to achieve particular policy objectives by leading the investment ratio, namely coefficient λ to the desired value through the investment behaviour taken by the public authority itself or the control by it over private investments, on the other hand delegating investment planning itself to individual enterpreneurs (employing so-called free enterprise system). For example, this holds true for a full-employment plan. However, in such a plan, recently with the increase of the weight of government revenue and expenditure in national income, the role of direct and indirect investments by the government sector has become gradually unnegligible. That also holds true for the advent of so-called mixed economy. However, there seems to be hidden still in this type of economy a qualitative difference rather than a simple quantitative one (difference of extent) from the model where the public authority holds the disposition of substantial part of investment funds as in [*Theory II-b*].

The above explanation could throw a side light to the question why the key conception of the *Tableau Economique* has not been materialized in the capitalist economy but in some sense in Soviet economy which is in contrast to the former. It might also explain a part of the reason why those people (Marx and others), who learned deeply the source of Quesnay's conception and developed new analytical framework to elucidate the features of capitalist economy, neglected this concept (in a sense, focussing to coefficient λ). In my view, the parameter which plays a strategic role in the *Tableau Economique* is coefficient λ and the possibility to employ it on the rational and positive basis seems to exist not in the traditional capitalist economy but for the time being in the planning of development system in developing countries or rather typically nowhere but in socialistic planned economy. However, even in the capitalist economy in which we live, if we look not the past but the future, an occasion may come where the most fundamental in sights implied in the *Tableau Economique* come out in the world and are materialized in reality.

IV Price Determination in Reproduction System

Price determination model by Nemchinov. Nemchinov presents a price determination model of final products in the later part of this article.[17] The reproduction system underlying his model is not that of Quesnay but the expanded reproduction scheme of Marx formalized by him.[18] In the *Tableau Economique* of Quesnay, which stands on the assumption of *prix constants* of agricultural products, the determination mechanism of prices is, of course, not shown. Even in other works by Quesnay general and systematic studies on price theory cannot be found, although fragmentary or partial discussions on it are seen. In this section, taking into consideration the points how should be the price system consistent to the *Tableau Economique* of Quesnay and how should be the relation of the system with the price system of classical school, I will start the discussion at first with the examination of Nemchinov's price determination theory.

Nemchinov classifies prices into three kinds: price vector $\boldsymbol{P}$ of intermediate products; price vector $\boldsymbol{P}''$ of final products; and price vector $\boldsymbol{P}'$ of total products (or practical average price). However, as far as the price determination of final products concerned, his model is essentially equivalent to a dual price system for the open Leontief system of physical quantities. Now, the system of equations of n prices are shown as in the following (1).

$$\left.\begin{aligned} p_1a_{11}+p_2a_{21}+\cdots+p_na_{n1}+v_1 &= p_1 \\ p_1a_{12}+p_2a_{22}+\cdots+p_na_{n2}+v_2 &= p_2 \\ \vdots \qquad\qquad \vdots \quad\;\; &\;\;\vdots \\ p_1a_{1n}+p_2a_{2n}+\cdots+p_na_{nn}+v_n &= p_n \end{aligned}\right\} \tag{1}$$

where, $\boldsymbol{A}=[a_{ij}]$ is regarded as the square matrix of technical coefficients of order n, $\boldsymbol{P}=[p_1, p_2, \cdots, p_n]$ as the vector of n prices of final products, and $\boldsymbol{V}=[v_1, v_2, \cdots, v_n]$ as the vector of n values added (surplus values) per unit of production. Then, (1) can be changed into the following equations.

$$\left.\begin{aligned} \boldsymbol{PA}+\boldsymbol{V} &= \boldsymbol{P} \\ \boldsymbol{P}(\boldsymbol{I}-\boldsymbol{A}) &= \boldsymbol{V} \\ \boldsymbol{P} &= \boldsymbol{V}(\boldsymbol{I}-\boldsymbol{A})^{-1} \end{aligned}\right\} \tag{2}$$

17) See the section having the title of "Interrelation between quantitative and value elements of (reproduction) scheme" (Взаимоотношение обьемных и стоимостных элементов схемы). II, pp. 108-11.

18) According to Nemchinov, if the quantity in value expression, which is the component of the reproduction scheme by Marx, is denoted as X_{ij}, X_{ij} is the value of i products inputted into production of j products, and $X_{ij}=q_{ij}\ P_{ij}$ can be obtained. In this equation, q_{ij} is the quantity of i products inputted into production of j products, and P_{ij} is the unit price of these products. In this way, it is supposed that the numerical values composing the scheme can all be shown by quantity index q and price index P. cf., II, p. 108.

where, $\boldsymbol{I}$ is the identity matrix of order n.

Equations (2) have the same construction with the price determination model of final products by Nemchinov (II, p. 110). It will be clear at a glance that (2) constitutes a dual system for the open Leontief system concerning the determination of outputs.[19)]

Price determination model by Quesnay. The remarkable characteristic of Quesnay's physiocratic system is in that it considers only agriculture to produce produit net, namely surplus products and manufacture to be sterile in the sense not to create produit net. If we explain this thesis by using model (1) above-mentioned and consider the first equation to show the price equation of agriculture and the second and following equations to show that of manufacture, it comes out that only v_1 in the first equation is positive and $v_2 \cdots v_n$ are all zero, namely $\boldsymbol{V}=[v_1, o, o, \cdots o]$. Therefore, if agricultural produce is selected as numéraire and its price p_1 is considered to be 1, the price model shown by the system of equations (1) will become a determinate system that determines the surplus value per unit of agricultural produce, namely v_1 of produit net and the relative prices $p_2, p_3, \cdots p_n$ in terms of agricultural products.

In order to understand these points clearly, let us examine the following price model consisting of two sectors of agriculture and manufacture:

$$\left.\begin{aligned} p_1a_{11}+p_2a_{21}+v_1 &= p_1 \\ p_1a_{12}+p_2a_{22}+v_2 &= p_2 \end{aligned}\right\} \qquad (3)$$

As described above, according to the physiocratic theory, manufacture is sterile, different from agriculture in the sense that it does not create produit net. Accordingly, v_2 equals zero. That is:

$$\left.\begin{aligned} p_1a_{11}+p_2a_{21}+v_1 &= p_1 \\ p_1a_{12}+p_2a_{22} \qquad &= p_2 \end{aligned}\right\} \qquad (3')$$

The system of equations (3′) is composed of two equations which have three unknown quantities of v_1, p_1 and p_3. Now, if agricultural produce is selected as numéraire and the surplus and prices are expressed by it, p_1 will equal 1. Then, unknown quantities will become two of v_1 and p_2, and this system will be composed of two independent equations which determine one price and one net product (produit net). Thus, the system would become determinate.

Price determination model by Ricardo. In the system of classical school, especially in that of Ricardo, the physiocratic assumption considering manufacture to be

19) Because the open Leontief system can be shown by the following system of equations (2A).

$$\begin{aligned} &\boldsymbol{QA}+\boldsymbol{Y}=\boldsymbol{Q} \\ &(\boldsymbol{I}-\boldsymbol{A})\boldsymbol{Q}=\boldsymbol{Y} \qquad (2A) \\ &\boldsymbol{Q}=(\boldsymbol{I}-\boldsymbol{A})^{-1}\boldsymbol{Y} \end{aligned}$$

where, $\boldsymbol{Q}$ is the vector of outputs of n commodities and $\boldsymbol{Y}$ is the vector of n final demands (final products).

sterile is rejected and both of agriculture and manufacture are regarded to create surplus product, namely produit net. Moreover, it is considered that through the free competition among capitals a uniform, general rate of profits for advanced capitals is established eventually in every industry. Therefore, if we employ two-sector model, following equations can be obtained:

$$\left.\begin{aligned} v_1 &= r(p_1a_{11}+p_2a_{21}) \\ v_2 &= r(p_1a_{12}+p_2a_{22}) \end{aligned}\right\} \tag{4}$$

Namely, it is considered that the uniform rate of profits r shown in (4) is established. Thus, by substituting (4) for v_1 and v_2 in (3), the Ricardian system can simply be shown with the following equations:

$$\left.\begin{aligned} (p_1a_{11}+p_2a_{21})\ (1+r) &= p_1 \\ (p_1a_{12}+p_2a_{22})\ (1+r) &= p_2 \end{aligned}\right\} \tag{5}$$

Now, in the same way as that in the above, if agricultural produce is selected as numéraire, a determinate system will be obtained that includes two independent equations determining one price (p_2) and the rate of profits (r).

A general case consisting of n commodities and n sectors can be obtained easily as in the following.

From the assumption of classical school concerning the uniform rate of profits,

$$\boldsymbol{V} = r\boldsymbol{PA} \tag{4A}$$

can be obtained. From this equation, the price model by Ricardo can be shown with the following equations:

$$\left.\begin{aligned} \boldsymbol{PA}+r\boldsymbol{PA} &= \boldsymbol{P} \\ \boldsymbol{PA}(1+r) &= \boldsymbol{P} \end{aligned}\right\} \tag{5A}$$

As this system, in case any one commodity is selected as numenaire, includes n independent equations determining $n-1$ prices and the rate of profits, it represents a determinate model. It is worth to pay attention that this model is nothing but the system shown in Section 4 of Chapter 2 "Production with a Surplus" in the above-mentioned Sraffa's classic work.[20]

Summary. In retrospect, both Quesnay's and Ricardo's theory are, as mentioned above, based on framework which consists in separating price determination from output determination. Moreover, it may be noticed that Quesnay's *Tableau Economique* is, as seen above, concerned with the determination of sales by each industry and therefore the determination of their outputs (only if the internal transactions are taken into account) in case of *prix constants,* while the determination of prices could be treated under the assumption of constant returns, or rather of given outputs in

20) Sraffa, *op. cit.*, p. 6.

the physiocratic and classical conceptual framework.[21]

21) It appears to be Léon Walras who has raised the problem of price determination generally (not partially) as a subject and tried to solve it, although he stands on the position in contrast with that of Sraffa. For he declares in the introduction of the 4th edition (1900) of his "*Elements d'économie politique pure ou théorie de la richesse sociale*" as follows: "*Pure economics* is, in essence, the theory of the determination of price under a hypothetical régime of perfectly free competition" (L. Walras [translated by W. Jaffé], *Elements of Pure Economics or the Theory of Social Wealth*, 1954, p. 40). As the forerunners of Sraffa's theory, except those several economists who are listed in Appendix D, References to the Literature in his own book, V.K. Dmitriev and L. von Bortkiewicz should be mentioned. Marx pays, in the case of formation of the average profit through competition among capitals, attention to the tendency of production prices of commodities to deviate from values of themselves, but seems not to clarify the mechanism of determination of their prices. While, Sraffa, different from the Marginalists such as Walras, is based on the production system, namely the system of production and consumption as a circular process with the *produit net* which is formalized by Quesnay for the first time, he elucidates the mechanism of general price-determination and puts it as the starting point of his own analysis. For understanding the position of Sraffa's theory in modern economics, the following works by Prof. Pasinetti are an excellent reference material: L. Pasinetti, *Lectures on the Theory of Production*, 1977, and "Piero Sraffa", *International Encyclopedia of the Social Sciences Biographical Supplement*, Vol. 18, 1979, pp. 736–39. On the same matter, I should like to mention here only one amongs many reference books: A. Roncaglia, *Sraffa and the Theory of Prices*, 1978. As described above (see note 1), price determination in the reproduction system discussed briefly in this section is the point which I have suggested but not developed in my article in Japanese published in 1965. After that, Prof. G. Candela of the University of Bologna has described this point in detail in the "Introduction" of a work concerning physiocracy edited by him. This book, shown in the following, is the most appropriate reference material for this section: G. Candela and M. Palazzi (ed.), *Dibattito sulla Fisiocrazia*, 1979, especially, see pp. xxxiv–lv in it.