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OPTIMAL FISCAL OPERATION*

By Kazuo YOSHIDA**

I Fiscal Policy in Fiscal 1975–Fiscal 1990

Japanese economy which had enjoyed high growth for about twenty years greatly changed its growth pattern with the occurence of the first oil crisis, pointing toward low economic growth. With such conversion of economic growth pattern, the national public finance as it should be was forced into a great reform. Especially, contradiction of annual revenue and expenditure between increase in expenditure centered in natural increase due to the fiscal system established in the latter half of the period of high economic growth and decrease or slump in revenue due to slower growth led the fiscal operation to extremely difficult situation. The fiscal policy in fiscal 1975–fiscal 1990 was in the distress concerning adjustment of these two factors. This paper considers measures to solve today's fiscal issues by looking back on this period.

When considering the fiscal policy fiscal 1975–fiscal 1990, we can classify this period into four phases by types of the fiscal policy, which are called respectively Phase I (fiscal 1975–fiscal 1979), Phase II (fiscal 1980–fiscal 1982), Phase III (fiscal 1983–fiscal 1985) and Phase IV (fiscal 1986–fiscal 1990).

In Phase I, annual expenditure grew larger due to influence of inflation caused by the oil crisis in addition to increased expenditure owing to maturity of the fiscal system toward the welfare state which was established in the earlier half of 1970's, while tax revenue

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fell and continued slump. Therefore, this forced to issue exceptional national bonds not under the Financial Law and increased issue of such bonds continued. The proportion of revenue by bonds rose year by year due to positive fiscal policy by increased issue of construction bonds for escaping from stagnation in addition to issue a large quantity of exceptional bonds. Thus, the proportion rose to an abnormal level of 40 percent in the initial budget for fiscal 1979.

The average growth rate of the general expenditure (Total expenditure in general account − National debt service − Local allocation tax grants) was 18 percent during this phase. This was nearly equal to that during a period of high economic growth. In other words, the course of the fiscal policy in this period was for maintaining the already established fiscal system for constructing a so-called welfare state and, as well, to attain expansion of tax revenue through the natural increase of revenue by restoring the economic growth pattern thanks so the positive fiscal policy. It was influenced by the so-called Keynesian theory of expanding equilibrium and the fiscal authorities which are reluctant to increase issuance of bonds were exposed to strange blame to be "fiscal egoism." However, an idea, that introduction of a general consumption tax is indispensable to securing tax revenue necessary for fiscal reconstruction to which the natural increase of revenue only is not sufficient, was agreed within the Government. In concrete, it was agreed that the fiscal operation dependent on exceptional bonds would be terminated by fiscal 1984 and, therefore, exceptional bonds had been chosen just as a temporary and transitional measure of the fiscal operation until the above time.

In Phase II, while necessity of fiscal reconstruction was established as a consensus, efforts for the reconstruction were attempted in the manner of natural increase of revenue and increase of revenue thanks to a partial revision of the taxation system and restraint of annual expenditure because introduction of the general consumption tax had been shelved due to the defeat of the Ohira cabinet in the election which was held in the typhoon. Since the average growth rate of tax revenue up to the initial estimate for fiscal 1982 from fiscal 1979 was 15.6 percent and the average increase rate of annual expenditure was restrained to 8.8 percent with gradual decline of the rate, reduction in issue of bonds had been carried out at a pace of 2 trillion yen per annum and it had been considered that escape from exceptional bonds would become possible until fiscal 1984. During this period, the annual increase rate of general expenditure was as low as 3.7 percent. This means that, while thorough revision of the fiscal system obliging expenditure was not realized, effort to restrain expenditure centered on reduction of policy expenditure were made and increase in annual expenditure was restricted to amount nearly corresponding to the natural increase. On the other hand, construction bonds were also suppressed and kept on a nominally stable level.

As a result, the proportion of revenue by bonds lowered from 39.6 percent in the initial budget for fiscal 1979 to 21.0 percent in the initial budget for fiscal 1982. This shows that expenditure restraining policy was continued, though hard, toward the projected escape from dependence on exceptional bonds in fiscal 1984.

However, the corporate tax in fiscal 1981 sharply declined and crisis of revenue defects

where the initial estimate of revenue could not be attained happened. In such situation, the Government was forced to additionally issue exceptional bonds of three trillion yen by revising the initial budget for fiscal 1982. This was caused by the phenomenon that, as in the first oil crisis period, the corporate income tax revenue would sharply decline upon termination of rapid rise of wholesale prices. As a result, the proportion of revenue by bonds again reached 30 percent and, thus, the fiscal reconstruction process before then came to return its starting point.

In Phase III following such situation, fiscal reconstruction through review of the administrative and fiscal system under the banner of "fiscal reconstruction without new tax system introduction" clearly proposed by the Second Ad Hoc Administration Investigating Committee.

The "minus ceiling" system for demand for budgetary appropriations was introduced which required to demand appropriations amount to expenditure for the preceding fiscal year less that multiplied by a specific ratio. (Since the budget included some exceptional items, the total amount of budget was not always less than the preceding year.) Also for realizing such minus ceiling, drastic reforms were tried in the areas such as health insurance, public pension and subsidies which had ever been regarded as sanctuaries. In addition, there were taken measures such as the freeze of recommendations by the National Personnel Authority and suspension of commoditie price sliding of pension payment, etc., so that general expenditure during this period was kept on the level of the previous years. While expenditure of the character of natural increase was restrained by revision of the laws concerned, severe curtailment of policy expenditure was enforced for absorbing such increase. Public works were also restrained year by year while considering volume of works, and issue of construction bonds had been reduced. As a result, issue of bonds were reduced by one trillion yen per year and the proportion of revenue by bonds again declined from 30.2 percent in the revised budget for fiscal 1982 to 22.2 percent in the initial budget for fiscal 1985.

In Phase IV, the restriction of the general expenditure under the "Minus Ceiling" system continued, though the general expenditure increased slightly after fiscal 1988. And, by lower interest rate, national debt service did not increase comparatively with the increase of the outstandings of national bonds. On the other hand, the rapid economic recovery after fiscal 1982 brought big natural increase of tax revenue. The average rate of tax increase during fiscal 1985-fiscal 1990 (estimation) was 8.7 percent, though the tax reform which included 2 trillion yen net reduction of tax, was made. By the restriction of the expenditure and the natural increase of tax revenue, the proportion of revenue by bonds lowered from 22.2 percent in fiscal 1985 to 8.5 percent in the initial budget for fiscal 1990, and the exceptional bond issue was eliminated.

It cannot be overlooked as the background of this policy that national consensus had been reached on the course of administrative reform indicated by the Second Ad Hoc Administration Investigating Committee and reconsideration on the national policy of finance dependent type, such as review of welfare and attaching importance to private activities, had become a world trend. However, fiscal operation during this period was very severe

even though escape from exceptional bonds was deferred until fiscal 1990. Especially, national debt expenses came to occupy 20 percent of the fiscal expenditure due to accumulation of bonds, which caused fiscal rigidity.

The next basic task of fiscal policy is how to manage a large quantity of national bonds which have been accumulated as a matter of fact. In concrete, the national debt expenses on the outstanding national bonds of 164 trillion yen at the end of fiscal 1990 is nearly 14 trillion yen per annum.

It is necessary to consider how fiscal operation should be performed on the assumption of large burden of such national debt expenses is optimum. Therefore, this paper consider this issue from the realistic viewpoint how balance is kept between disadvantages to increase burden of later generation and acceptance of burden of present generation to realize fiscal reconstruction. In other words, it can be said to be a preferable figure that fiscal operation is performed to minimize burden through generations due to issue of national bonds.

II Burden of Public Bonds

It is necessary before entering discussion to arrange what burden arises following issue of public bonds. It is so-called Keynesian economics that broke common sense on social burden of public bonds that they disturb economy or leave burden for later generation.

Keynes[6] maintained to the financial authorities and statesmen that issue of public bonds is indispensable to control of the economic activity level. This notion has become the mainstream of postwar economics, and Lerner[7] gave clear solution on the burden of public bonds from the above standpoint and afford influencial guides to economists criticizing financial authorities which are timid about issuing public bonds. By these guide, when such public bonds were issued as foreign bonds, quantity of available resources expands on the occasion of issue and reclines on the occasion of redemption, so burden of the later generation will arise. In case of domestic bonds, however, transfer of resource between different periods cannot occur, so neither generation will receive benefit nor bear loss. Rather, merits of utilization of unused resources such as unemployment will be available and no increase of burden imposed on either generation. Such notion has established the status of the "new orthodox school." For these functional fiscalists, in case there is any underemployment in private sectors, it may be preferable that public bonds enough to absorb such underemployment will be issued. Therefore, the optimum level of issue of public bonds will be dependent on the balance between saving and investment and a notion that the optimum level may be found in the trade-off between burden and benefit is not likely.

Against such notion of functional fiscalists, another notion on the burden of public bonds has been proposed from the standpoint of the new classical school.

The first proposition has been made by Modigliani[8], maintaining that issue of public bonds needs to raise much fund in the capital market and, therefore, may cause to eject fund raising for private plant investment. The reduction of the private plant investment

owing to such issue of public bonds, will make loss of income for the future generation, which means burden by the issue of public bonds for the later generation. Since, however, taxation also decreases private plant investment through reduction in saving, difference of effects between issue of bonds and taxation will be the net worse of public bonds.

The second proposition has been made by Bowen-Davis-Kopf[2], maintaining that, while generation at the time of issue of public bonds bears no burden because it voluntarily purchases the bonds and receive their redemption and the total quantity of its lifetime consumption is not influenced by the issue of bonds, later generation bears burden because it connot recover the tax burden imposed at the redemption of the bonds and the total quantity of its lifetime consumption decreases. From this standpoint, the notion of optimum issue of public bonds may arise from the viewpoint how the burden should be shared between the present generation and the later generation.

It is an argument from the hypothesis of the neutrality by Barro[1] that is now establishing the status of the "extra new authodox school". By assuming rational expectation and considering also inheritance effects, the present value of future tax burden for future payment of the principal and interest of public bonds becomes the burden of the present generation, so difference in burden by methods of fund raising becomes disappeared whether they may be issue of public bonds or taxation. Accordingly, "public bonds" will never impose new burden and, therefore, the notion of tax-free state such as Noguchi[10] may appear. In this case, argument on the optimum issue of public bonds will not occur.

When considering issues on actual fiscal reforms on the assumption of repayment of debts, it is also necessary to consider an issue who bears burden of fiscal reconstruction rather than an issue of loss of future opportunity income. Concerning this issue, it would be necessary to consider with a naive concept of burden that issue of national bonds causes burden of future increase of taxes as in Buchanan[3]. In this case, burden of interest in the budget since Domar[4] becomes an important element. Considering these elements, it can be said to be the optimum fiscal policy to develop fiscal reforms in the manner to minimize social costs including burden of later generation. Among the fiscal policy in fiscal 1975–1990, an issue of burden by later generation has always been regarded as important and should be considered for developing fiscal reforms. Following chapters will consider optimal fiscal operation from this point of view.

III Character of Optimum Burden of Later Generation

From the concept of burden of public bonds by Bowen et al.[2], an outline of issues on optimum character including that of burden of later generation. Rearrangement of these arguments are as follows:

(1) Generation at the time of issue of public bonds voluntarily purchases the bonds and receives payment of principal and interest, so its lifetime income does not change and its scheduled lifetime consumption level is not cut down.

This means that no burden is imposed on this generation.

(2) Generation at the time of redemption of public bonds bears tax burden for the

redemption. Since this burden is not compensated, its lifetime consumption is decreased. This means that actual burden is imposed on this generation.

Therefore, issues on issuance of public bonds are essentially those how the financial burden should be allocated between the generation concerned. Given a standard of evaluation of income distribution, the optimum quantity of issuance of public bonds can be understood to be that of maxmizing social welfare function concerning both generations. Then, a study of the optimum dependence on public bonds from the viewpoint of burden by later generation is given by means of the following simple model.

The lifetime consumption of the first generation at the time of issuance of public bonds (C_1) is equal to disposable income (w (wage) $-$ T (tax)) on the asumption that its lifetime income is spent up without leaving inheritance. (While amount of issue of public bonds is equal to the difference between government expenditure (G) and tax (T), interest on the bonds need not be considered, because the present value of future payment of principal and interest of the issued bonds is equal to amount of issue of the bonds).

Therefore,

$$C_1 = w - T, \tag{1}$$

is given. The present value of lifetime consumption of the second generation (C_2) is also the same as the above. However, it should be also considered that wage will expand with economic growth (growth rate: r) and the second generation must bear extra costs for payment of principal and interest of the public bonds issued at the age of and purchased by the first generation (B) in addition to tax necessary for government expenditure to the second generation. (Rice of commodity prices is disregarded in consideration of net value). The present value of consumption by the second generation (C_2) is shown by

$$C_2 = \{w(1+r) - G - (1+i)(G-T)\}/(1+i). \tag{2}$$

Here, consumption by the first generation will decrease by increasing tax (T) and consumption by the second generation will increase by decreasing tax (T).

If social welfare function (W) is shown by the following linear homogeneous Cobb-Douglas type function of consumption by both generation:

$$W = AC_1^{\alpha}C_2^{1-\alpha}, \tag{3}$$

and the natural logarithm value of expression (3) (π) is given by

$$\pi = \log W = \log A + \alpha \log C_1 + (1-\alpha)\log C_2. \tag{4}$$

Then taxation to maximize π will be the optimum taxation and the dependence on public bonds at that time the optimum dependence on public bonds. The necessary condition of such maximization is shown by

$$\frac{\partial \pi}{\partial T} = \frac{-\alpha}{w-T} + \frac{(1-\alpha)(1+i)}{w(1+r) - G - (1+i)(G-T)}. \tag{5}$$

Tax at that time (T) is shown by

$$T=w\left(1-\alpha-\alpha\frac{1+r}{1+i}\right)+\alpha\frac{2+r}{1+i}G. \tag{6}$$

If the distribution ratios of the first generation and the second generation are equal and $\alpha=0.5$, then

$$T=(1-0.5i)G-(r-i)w. \tag{7}$$

The optimum dependence on public bonds (η) is shown by

$$\eta=0.5i+(r-i)\frac{1}{g}. \tag{8}$$

(where g is the ratio of government expenditure to income (G/w))

If economy is on the so-called golden route and the interest rate and the growth rate are equal, dependence on public bonds equal to half of the interest rate (growth rate) will be the optimum dependence on public bonds.

Here, if growth rate is higher than interest rate, the optimum dependence on public bonds will be higher than the above. In this case, if g is small, η should be much higher. On the contrary, if interest rate becomes higher than growth rate, the dependence on public bonds should be lowered than 0.5i. In this case, if g is small, η should be lowered much more. When economic growth rate is high, it may be justified, from the fact that high economic growth is results of efforts by the first generation, that part of the current fiscal expenditure should be shared with later generation to receive benefit of high economic growth. However, issue of public bonds under low growth economy causes to lower the consumption level of the later generation and produces a great problem viewing from a burden concept by Bowen et al. If tax revenue falls down when economic growth rate declines, the dependence on public bonds would rise even under the fall of the optimum dependence on public bonds. This would leave a large burden to the later generation.

IV Fiscal Operation under Burden of Later Generation

If national bonds are continuously borrowed with compound interest without the redemption, outstanding national bonds will become larger endlessly, but national income as source of its burden will grow to infinity at the same time. Therefore, it may be one choice to make a generation after infinite years with infinite ability of burden accept national bonds. In other words, if the ratio of outstanding national bonds to national income is converging to zero, this choice may be appropriate balance with one later generation. If government expenditure other than costs of national bonds (G) is equal to tax and other non-debt revenue (T), the condition under which the rate of outstanding national bonds (B) to national income (b) becomes zero can easily be obtained. The constraint formula of government budget is

$$\dot{B}=iB+G-T, \tag{9}$$

(where, i: interest rate),

and by dividing both sides by national income (Y)

$$\dot{b}=(i-r)b+g-\tau, \qquad (10)$$

$$\left(\text{where, r: economic growth rate, } g=\frac{G}{Y},\ \tau=\frac{T}{Y}\right)$$

is obtained. The converging condition of a differential equation when g and τ are equal is that economic growth rate exceeds interest rate. When this condition is satisfied, the rate of outstanding national bonds to national income will converge on zero in due course and the burden of bonds are transferred to a generation after infinite years with income of infinity without any burden. In concrete, the possible way is to raise all funds for payment of interest and repayment of principal of national bonds by means of new national bonds issued by the Debt Consolidation Fund and not to impose any fund the general account. Reviewing balance between benefit (g) and burden (τ) of the nation, in the general account, it is possible to increase expenditure upon judging to increase benefit and, on the contrary, to curtail expenditure upon judging to reduce burden. In other words, fiscal operation can be developed while maintaining the function of balanced finance as the necessary condition of realizing the optimum scale of finance. In case of reverse relation between interest rate and economic growth rate, there would be contradiction in the general account between the method of judging the optimum scale of finance through balancing benefit with burden and the method of converging to zero the rate of outstanding national bonds to national income. In concrete, the rate of outstanding national bonds to national income will rise one-sidedly even though $g=\tau$ is maintained. Therefore, it must be decided to repay or pay principal or interest through a tax increase sometime or to choose inflation instead of raising money. If the rate of outstanding national bonds to national income should be lowered every year, taxes must be maintained the level of

$$\tau>g+(i-r)b. \qquad (11)$$

If on the condition that the growth rate is greater than the interest rate, the level of taxes is maintained as expression (11) is contineued, it becomes possible to lower the rate of outstanding national bonds to national income and bring it to zero while continuing the status where burden is less than benefit.

V Optimal Fiscal Operation

While the above method forces burden one-sidely on the later generation with infinite ability, it does not guarantee the optimal balance between present and future. If the present generation feels burdensome for imposing burden to a later generation, it should be appreciated as a social cost. Thus, we always appreciate the excessive burden of the present generation (burden exceeding benefit of the present generation: $h=\tau-g$) and the burden left for the future (rate of outstanding national bonds to national income (b)) and consider fiscal operation in case the total burden should be as small as possible.

Suppose that appreciation of the excessive burden (h) and the rate of outstanding na-

tional bonds to national income (b) (social cost function) is now expressed simply by the quadric function. (Suppose that, in the initial status, outstanding national bonds are positive and h and b are considered only in the positive region.) By integrating this function by time to the period of planning (Γ), the objective function of fiscal operation can be obtained: that is,

$$\int_0^{\Gamma} \frac{1}{2}(\alpha h^2+\beta b^2)dt. \tag{12}$$

Obtain the path of rate of excessive burden (h) to minimize the value of this objective function under the constraint of expression (10). In this case, however, the rate of outstanding national bonds to national income must be zero at the last stage of the period of planning. By arranging the above, the following problem is given:

$$\min \int_0^{\Gamma} \frac{1}{2}(\alpha h^2+\beta b^2)dt, \tag{13}$$

$$\text{s.t. } \dot{b}=(i-r)b-h, \tag{14}$$

$$b(\Gamma)=0. \tag{15}$$

From this problem, the necessary condition of optimization is obtained by Pontryagin's minimum principle.

In order to obtain the necessary condition of the optimal path, by introducing an auxiliary variable λ and defining Hamiltonian H as follows:

$$H=\frac{1}{2}(\alpha h^2+\beta^2)+\lambda\{(i-r)b-h\}. \tag{16}$$

The necessary condition for the path to minimize the objective function is shown by

$$\frac{\partial H}{\partial h}=\alpha h-\lambda=0, \tag{17}$$

$$\dot{\lambda}=-(i-r)\lambda-\beta b. \tag{18}$$

By substituting expression (17) for expression (18),

$$\dot{h}=-(i-r)h-\beta b/\alpha, \tag{19}$$

is obtained and the optimal path of the excessive burden rate (h) and the rate of outstanding national bonds to national income (b) for each fiscal year is shown as the solution of the simultaneous differential equations of expressions (14) and (19).

These simultaneous equations are rewritten as

$$\begin{bmatrix}\dot{b}\\ \dot{h}\end{bmatrix}=\begin{bmatrix}i-r & -1\\ -\beta/\alpha & -(i-r)\end{bmatrix}\begin{bmatrix}b\\ h\end{bmatrix} \tag{20}$$

and, since the values of both equations must be zero at the last stage of the period of planning, the optimum path of fiscal operation is obtained by solving these simultaneous differen-

tial equations by reverse time.

VI Rule of Optimal Fiscal Operation

As the purpose of this chapter is to seek the rule of optimal fiscal operation, we seek the formalized rule of optimal fiscal operation expressing the excessive burden rate (h) necessary for each year as a variable of the rate of outstanding public bonds to national income (b) to be a state variable by seeking an optimal regulator shown by Kalman[5] et al. By defining an auxiliary variable λ as a linear function of a state variable

$$\lambda = pb, \tag{21}$$

(p is positive real variable) the rule of opimum fiscal operation can be shown from expression (17) in the manner of feedback control as follows:

$$h = pb/\alpha. \tag{22}$$

Since this relation must simultaneously satisfy expression (18), p must satisfy the following Riccati type differential equation:

$$\dot{p} = p^2/\alpha - 2(i-r)p - \beta. \tag{23}$$

Therefore, a feedback coefficient is obtained by solving this differential equation by reverse time.

If the period of planning of the initial problem is infinitely remote, p becomes a constant and, by solving the quadratic equation obtained from expression (23) with the left side of zero,

$$p = \alpha\{(i-r) + \sqrt{(i-r)^2 + \beta/\alpha}\}, \tag{24}$$

is obtained. Therefore, the optimum fiscal operation which has been sought can be shown in the form of the following simple linear feedback control:

$$h = \{(i-r) + \sqrt{(i-r)^2 + \beta/\alpha}\}b. \tag{25}$$

In other words, the method of fiscal operation to minimize the total social costs on the time path with a target of making the rate of outstanding national bonds to national income zero at the time of infinite remoteness is only to transfer from the general account to the Special Account for Debt Consolidation Fund an amount obtained by multiplying the rate of outstanding national bonds to national income by a coefficient with variables of the difference between interest rate and economic growth rate and the ratio of the evaluation parameters of the excessive burden rate and the rate of outstanding national bonds. (However, the general account must be balanced inclusively of an amount of the transfer.) In this case, if the growth rate is higher than the interest rate, value in parentheses of expression (25) is negative for the antecedent and positive for the latter term, so the value of the equation becomes small due to partial offset and anual transfer may be a few. On the contrary, if interest rate is higher than growth rate, the coefficient becomes

large and annual transfer must become huge.

If the ratio of parameters (β/α) is large or there is stronger fear for the burden left in the future by national bonds in relation with the present burden, it is, of course, necessary to sooner repay national bonds and to increase transfer to the Debt Consolidation Fund due to expansion of the coefficient of excessive burden in expression (25).

On the assumption of such fiscal operation, the rate of outstanding national bonds to national income will be subject to the trend of the following differential equation:

$$\dot{b} = -\sqrt{(i-r)^2+\frac{\beta}{\alpha}}\, b \tag{26}$$

Thus, the rate of outstanding national bonds to national income will follow the path shown by

$$b = b_0\, e^{-\sqrt{(i-r)^2+\beta/\alpha}\, t} \tag{27}$$

and finally will converge to zero.

In case with β of zero optimum fiscal operation, if interest rate is lower than growth rate, the value in the parentheses of expression (25) will be zero and excessive burden will not be required. The method of imposing burden on later generation after infinite years as mentioned in the preceding chapter is shown to be a special case of the optimal fiscal operation. On the other hand, when β is zero as in the above and interest rate is higher than growth rate, the optimum excessive burden rate is shown in the following simple rule:

$$h = 2(i-r)\, b. \tag{28}$$

VII Influence on Economy

The above analysis has been made on the assumption that interest rate and growth rate are given. However, issue of national bonds will give great influence on both rates. Repayment of national bonds will cause corresponding increase in saving in the private sector, which will raise capital stock and contribute to rise of economic growth rate. On the other hand, interest rate will decline in the capital market and this will reduce interest burden of national bonds. Thus, repayment of national bonds will make fiscal operation easier. Here, the optimum fiscal operation will be considered by means of a model considering about a simple case mutual relation between finance and economy.

In order to study this problem, consideration on the assumption of mutual relation between finance and economy will be made by means of a very simple model. Since saving is consumed due to fiscal deficit, the rate of capital accumulation in the private sector declines. On the contrary, financial surplus heightens growth rate. For simplification, all income by payment of interest of national bonds is to be saved and all issue of national bonds for payment of interest financed by itself. Therefore, capital acculation rate available in the private sector is equal to the saving rate in the private sector (s) plus the excessive burden rate (h). (The social saving rate is expressed as $s(1-\tau)-(g-\tau)$, but $s\tau$ is disregarded because of the quadratic minimum). Considering Harrod-Domar type

growth models, economic growth rate is shown by

$$r=(s+h)/v, \tag{29}$$

(where v is capital coefficient).

On the other hand, when repayment of national bonds becomes larger, interest rate will decline. As supposed earlier, income by payment of interest for national bonds is saved, so interest rate will be a function of the excessive burden rate. Supposing the interest rate when the excessive burden rate is zero as $\bar{i}$, interest rate (i) is shown by

$$i=\bar{i}+\theta h, \tag{30}$$

(where θ is a negative constant).

By substituting this equation for expression (14),

$$\dot{b}=\left(\theta-\frac{1}{v}\right)bh+\left(\bar{i}-\frac{s}{v}\right)b-h, \tag{31}$$

can be obtained. The solution of a problem, to minimize the value of expression (13) having as the calculation term infinite remoteness which have as a constraint equation this non linear differential equation and zero as b at the right terminal point, will be the optimum fiscal operation.

However, this problem is an optimization problem having as a constraint nonlinear differential equation, and strictly it is not easily assured existence and uniqueness of any optimum solution and cannot be solved in a easy form. Then, as problems such as existence of any optimum solution are satisfied, an optimum solution is sought by replacing the rate of outstanding national bonds to national income (b) as follows:

$$b=\frac{1}{\theta-\frac{1}{v}}\left(e^{(\theta-1/v)x}+1\right). \tag{32}$$

By substituting this equation for expression (31) and arranging it,

$$x=\frac{\bar{i}-\frac{s}{v}}{\theta-\frac{1}{v}}\left(1+e^{-(\theta-1/v)x}\right)+h, \tag{33}$$

is obtained. On the other hand, the objective function becomes

$$\int_0^\infty \frac{1}{2}\left\{\alpha h^2+\beta\left(\theta-\frac{1}{v}\right)^{-2}\left(e^{(\theta-1/v)x}+1\right)^2\right\}dt, \tag{34}$$

so, defining Hamiltonian H as follows:

$$H=\frac{1}{2}\left\{\alpha h^2+\beta\left(\theta-\frac{1}{v}\right)^{-2}\left(e^{(\theta-1/v)x}+1\right)^2\right\}$$
$$+\mu\left\{\frac{\bar{i}-\frac{s}{v}}{\theta-\frac{1}{v}}\left(1+e^{-(\theta-1/v)x}\right)+h\right\}, \tag{35}$$

the necessary condition of optimization is shown by

$$\frac{\partial H}{\partial h}=\alpha h+\mu=0 \tag{36}$$

$$\dot{\mu}=-\frac{\partial H}{\partial x}$$

$$=-\beta\left(\theta-\frac{1}{v}\right)^{-1}(e^{(\theta-1/v)x}+1)e^{(\theta-1/v)x}$$

$$+\mu\left(\bar{i}-\frac{s}{v}\right)e^{-(\theta-1/v)x} \tag{37}$$

Then, by substituting expression (36) for expression (37) and further substituting expression (32), the path of the optimal excessive burden rate (h) is obtained as follows:

$$\dot{h}=\left(\bar{i}-\frac{s}{v}\right)\left\{\left(\theta-\frac{1}{v}\right)b-1\right\}^{-1}h$$

$$+\frac{\beta}{\alpha}\left\{\left(\theta-\frac{1}{v}\right)b^2-b\right\} \tag{38}$$

The optimal path of this problem is situated within the area partitioned by curves obtained by setting zero for respective left sides of equations (31) and (38), and the origin to be the point of intersection of the following simultaneous equations is the point of long-term equilibrium.

$$h=\frac{-\left(\bar{i}-\frac{s}{v}\right)b}{\left(\theta-\frac{1}{v}\right)b-1} \tag{39}$$

$$h=\frac{-\beta b}{\alpha\left(\bar{i}-\frac{s}{v}\right)}\left\{\left(\theta-\frac{1}{v}\right)b-1\right\}^2 \tag{40}$$

The optimal path is obtained by solving these equations by reverse time concerning h with the right terminal point of this point. Illustrating the optimum path, it is as shown in Fig. 1, in case $\frac{s}{v}$ is larger than $\bar{i}$. Here, when the initial point of the optimal path coresponding to the initial value of the rate of outstanding national bonds (b_0) is A, the optimal path is shown by the broken line toward the origin or the long-term equilibrium point from the point A. In other words, when the initial value of the optimum path (h_o) is obtained, the method of the optimum fiscal operation can be obtained by solving the simultaneous equations of expressions (31) and (38) with the initial values (b_o, h_o).

On the contrary, if $\bar{i}$ is larger than $\frac{s}{v}$, the path which gradually approches from the initial point (A) corresponding to the initial value of the rate of outstanding national bonds to national income (b_o) toward the origin along the broken line as shown in Fig. 2 constitutes

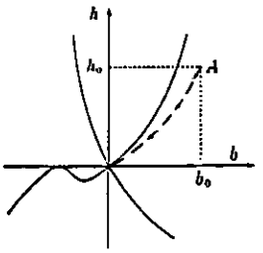

Fig. 1

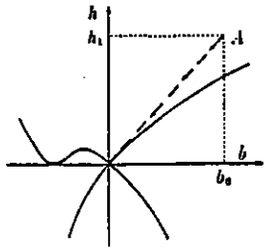

Fig. 2

the optimum process. As mentioned in the above case, the method of the optimum fiscal operation may be obtained by solving the simultaneous equations of expressions (31) and (38).

In order to concretely obtain the optimum path, methods to solve approximately are available by means of semi-linearization, optimization, etc., all of which require considerably complicated calculation systems. (As to technique of optimization by semi-linearization, refer to Goto[9], etc.) In a simple case with β of zero, optimization can be obtained by making excessive burden zero in case $\frac{s}{v}$ is larger than $\bar{i}$. While this is the case without consideration to economy, transfer curtailed in some degree from the level of expression (28) in a reverse case. It may be optimum to transfer the following amount for repayment and payment of principal and interest of national bonds:

$$h=qb\left\{\left(\theta-\frac{1}{v}\right)b-1\right\}^{-1} \tag{41}$$

In this equation, q is a variable to satisfy

$$\dot{q}=\frac{q^2+2q\left(\bar{i}-\frac{s}{v}\right)}{\left(\theta-\frac{1}{v}\right)b-1}, \tag{42}$$

and can be obtained by solving by reverse time from the right terminal.

In any case, considering influence on economy, the excessive burden by the current generation may be smaller than that of the rule of fiscal operation mentioned in the preceding chapter. This is natural because repayment of national bonds will raise growth rate and lower interest.

VIII Conclusion

When considering how the fiscal operation in Japan should be from now on, it is necessary to make the optimum burden regulation between generations. From this point of view, an issue of outstanding national bonds must be managed. Unless repayment of national bonds has a great influence on economy, it may be optimum to make financial

operation based on the rules considered in Chapter 6. When growth rate is higher than interest rate, fiscal operation which finances interest and principal payment by bonds issue, would be easy and appropriate. In economy where interest rate is higher than growth rate, considerable burden may be required. Further, considering relation with economy, repayment of national bonds raises economic growth rate and lowers interest rate, so better environment for fiscal operation is created and agreeable fiscal operation can be expected. Especialy, when interest rate is higher than growth rate, with excess burden of finance it can be said to be appropriate fiscal operation to redempt national bonds actively to make the rate of outstanding national bonds to national income lower and to raise growth rate.

Considering mutual relation between issue of national bonds and economy, it is very dangerous to disregard contradiction that issue of national bonds aggravates environment of finance, even though national bonds are issued as economy stabilization policy. Especially, additional issue of national bonds under the environment near to full employment is nothing but coercion of high burden of national bonds to later generations.

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