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MARSHALL AND MARX

—“Waiting” and “Reproduction”—

by Kiichiro Yagi*

I Marshall’s “Waiting”

In the 1907 lecture titled “Social Possibilities of Economic Chivalry”, Alfred Marshall described himself forty years ago as a naive socialist who was impressed by the non-selfish spirit of social movements for the remedy of social distress. Encouraged by the hope of finding an effective measure for progress in social welfare, the young Marshall’s quest reached so far as F. Lassale and K. Marx.

Marshall might be one of very few British economists who read the first edition of *Das Kapital* in the original just after its publication. His comment on German socialism in the Appendix B of the *Principles of Economics* (PE: 769), which undoubtedly refers to Marx and his works, seems to retain some notes of Marshall’s past admiration.

The mature Marshall still kept his respect for socialists, but rejected their exploitation theory of profit and repeatedly warned against the risk accompanying collectivistic measures. His alternative lay in the hope of the growing public spirit among men of business. In this article, I will not deal with the socialist policies; nevertheless, the ideological element appears inevitably even in a theoretical discussion, because the topic, capital and interest, is one of the most delicate themes in economics in this respect. Theoretical views are interconnected with those related to the justification of one form of revenue. Marshall was well aware of this context in his criticism of Marx’s labour theory of value (PE: 586).

In the capitalist mode of production where capitalists acquire profits, prices of commodities are not in proportion to the labour directly and indirectly spent on them. This does not contradict the labour theory of value, because it can be argued that the way of price formation affects only the distribution of the produced value or surplus-value which represents the total social labour spent in this period or the total surplus-labour, respectively. Marx tried this in the theory of production price in the third volume of *Das Kapital*. Marshall shows no interest whatever in this kind of refinement of the labour theory. Instead, he simply offers another view of understanding interest of capital as the reward of “waiting”. According to him, “waiting” is an independent factor in addition to the labour. However, what does he mean by the term “waiting”?

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“If it be true that the postponement of gratifications involves in general a sacrifice on the part of him who postpones, just as additional effort does on the part of him who labours; and if it be true that this postponement enables man to use methods of production of which the first cost is great; but by which the aggregate of enjoyment is increased, as certainly as it would be by an increase of labour; then it cannot be true that the value of a thing depends simply on the amount of labour spent on it. Every attempt to establish this premiss has necessarily assumed implicitly that the service performed by capital is a “free” good, rendered without sacrifice, and therefore needing no interest as a reward to induce its continuance; and this is the very conclusion which the premiss is wanted to prove.” (PE: 587)

Marshall maintains that a supplier of capital endures the “sacrifice” of postponement of consumption and thus he needs a reward which covers this real cost. In brief, the interest is the reward for saving. However, one can say, the original reward of refraining from consumption is the keeping of the possession itself. The relation between “waiting” and the profit from the investment is not a direct one. Though Marshall argued that the effect of the “waiting” or saving reveals itself in the increase of productivity due to the “capital” stemming from the saving, Keynes could ridicule him by saying lots of accidents would happen between cup and lip. But to Marshall, whose mind was penetrated by typical Victorian ethics, the case of unproductive holding was excluded from the beginning. What is saved must be identical with what is invested.

Marx regarded N. Senior and others’ “abstinence” theory of profit as a typical vulgar theory. In his view, by saying that capital accumulation presupposes the negation of consumption, this theory only acertains a commonplace truth that every definition is in itself negation. The “abstinence” of capitalists means only that the function of acquiring surplus-value and of accumulating it as capital is assigned to capitalists (DK: I, 617ff.). Marshall, as well, admits the ridiculousness of the argument to seek for a huge amount of “abstinence” by Rothchild family as the grounds of their revenue (PE: 232n). By replacing “abstinence” with “waiting” he wanted a shift of stress in the explanation of saving from the refraining from consumption to the prospective expectation of the gain. In other words, Marshall’s “waiting” is a refined version of “abstinence” smoothed in the accumulation process flowing from saving to investment.

If the ideological aspect is put aside, Marshall’s capital theory is characterized first by his stress on the time aspect of capital formation and second by his view of saving and investment as making a macro-capital market. To show the first point in pure form Marshall has recourse to the subjective calculation of a farmer living in isolation who decides to build a house by his effort, using gratis endowed natural resources (PE: 351ff.). In the estimation of the cost he will not simply add up all the labour spent during the construction period, but accumulate this with a compound rate of weight according to the time span between the point of input and the point of completion of the house. On the other side, though the future utility generating from the new house will be larger by each additional involvement of labour, he will make a discount in the estimation of the future utility in accordance with the time interval up to the point of the utility
realization. The former is the motive for the suppression of the construction, the latter that for the promotion. He will decide on construction as long as the latter exceeds the former and will be perplexed where both come to a balance. In other words, he would decide to work up to the point where the increase in the utility generating from the refinement of the house becomes equal to the extra labour cost added the time element between the labour input and the generation of the utility.

In this respect the "waiting" is reduced literally to the lapse of time. Whether the time element should be positively estimated (accumulation) or negatively (discount) depends on the choice of the referential time point. Following Ch. Bliss, we consider the case in which the time pattern is very simplified.1)

The total labour amount in the house construction, \( L \), is spent at one time and the time interval from it, \( T \), also contributes to the increase in the degree of comfort, which will be \( U(X, T) \) at the time of completion. If we should choose the point of labour input as the reference point, the utility of the house must be discounted by a continuous discount rate \( v \). The builder of the house will then maximize the difference between the discounted utility and the cost which is linear to the labour spent.

Max \( U(L, T)e^{-vT} - L \) ................................................................. (1)

The \( X \) and \( T \) which fit the maximization are

\[
U_L e^{-vT} - 1 = 0 \quad \text{(2)}
\]
\[
-v U(L, T) + U_T = 0 \quad \text{(3)}
\]

The suffix signifies partial differential. These can be rewritten further in the following:

\[
v = \log(U_L) / T \quad \text{(4)}
\]
\[
\frac{v}{v} = U_T / U(L, T) \quad \text{(5)}
\]

Equation (4) or in its original form Eq. (2) shows that the discounted value of extra utility stemming from the extra labour becomes equal to the present cost of labour which is made 1 in Eq. (2), or that the discount rate is equalized to the marginal productivity of labour \( = \) capital modified by \( T \). Equation (5) shows that the discount rate is equalized to the marginal productivity of time ("waiting"). This is the implication of the Marshallian Robinsonade.

II Supply and Demand of "Capital"

As is seen above, Marshall stressed the dual aspects of capital, namely the aspects of the total cost of labour and saving which are involved in production as well as that of the total benefits stemming from the products. In the case of a national economy, in Marshall's view, they ap-

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1) Bliss, p. 229f. Confer also Marshall's original formulation in the mathematical appendix XIII to the Principles.
pears on the supply side and demand side, respectively. In his words, “prospectiveness and productiveness control the demand for capital and the supply of it” (PE: 82). The natural outcome of this thought is that the rate of interest is the price in the market where supply and demand of capital meet and that it has the function of attaining the equilibrium of supply and demand of capital. However, as Keynes found, Marshall’s view in this respect remained rather vague.

Marshall’s ambiguity lies in the fact that what he means by “capital” shifts each time. He asserts that the concept of interest is not applicable to the existing capital (old investment) if that concept is taken seriously. The revenue stemming from the existing capital, which is neither homogenous nor movable, is more appropriately viewed as the kind of income similar to the rent of the land, “quasi-rent” (PE: 412). The ‘capital’ most appropriate for the market where the rate of interest emerges as the price is the “free or floating capital” (PE: 412) which flows to the new investment. It is without doubt that Marshall meant this in most cases when he mentions supply and demand of ‘capital’. But Marshall adds, in very peculiar Marshallian style, that the border between free and fixed capital makes a continuous zone where the principle of substitution is in operation. The following text mentions the possibility of the withdrawal of capital from a poorly profitable investment as the effect of the increase in the rate of interest, though at the same time it concludes that the coordination of supply and demand by means of interest rate works very gradually.

“If we are considering the whole world, or even the whole of a large country as one market for capital, we cannot regard the aggregate supply of it as altered quickly and to a considerable extent by a change in the rate of interest. For the general fund of capital is the product of labour and waiting; and the extra work, and the extra waiting, to which a rise in the rate of interest would act as an incentive, would not quickly amount to much as compared with the work and waiting, of which the total existing stock of capital is the result. An extensive increase in the demand for capital in general will therefore be met for a time not so much by an increase of supply, as by a rise in the rate of interest; which will cause capital to withdraw itself partially from those uses in which marginal utility is lowest. It is only slowly and gradually that the rise in the rate of interest will increase the total stock of capital.” (PE: 534)

The rate of interest is determined by the supply-demand relation of the “free capital” which occupies only a small part of the larger frame of his view of capital. But this concept of “free capital” itself is still unclear as to whether it is conceived in monetary terms or in natural terms, and stock in character or flow, and so on. Several seem to have interpreted “free capital” as the “loanable fund”, but E. Eshag judged this as real capital (stock of capital goods) which is produced already but still not fixed by the investment. But with doubt about the consistency of such a concept of capital the question will appear as to whether the price of capital goods, not the rate of interest, should be the coordinator in the market so long as it is related to the supply and demand of capital goods.

Modern businessmen will make in money terms the calculation which the lonely farmer has
done by his decision making on the house building (PE: 352). The supply price of a capital good will be attained by accumulating the costs by the prevailing rate of interest. If the present value of the total gains from that capital good is equalized to its supply price by some well chosen discount rate, this is in itself the rate of return which Marshall named “marginal utility of capital” (PE: 520). So long as this rate exceeds the interest rate, the investment is favourable and reaches the point where both rates become equal. Therefore, the rise in the rate of interest will reduce the number of favourable investments, and the fall will make more investments favourable. In short, the amount of investment or demand of new capital will move in reversed direction to that of interest rate (PE: 520f.).

However, regarding the supply side of capital, Marshall did not consider the increase (decrease) of saving due to the rise (fall) in the rate of interest as an obvious fact. While many will be stimulated to save by the rise of interest rate, those who will slow down the saving cannot be ruled out. Marshall wrote, “it is a nearly universal rule that a rise in the rate [of interest] increases the desire to save” (PE: 236). It seems, however, he did not think the elasticity of saving to the interest rate so large as to be able to catch up with the increased demand of capital in a shorter period. 2)

III "Reproduction" View of Value Theory

Contrary to Marshall, Marx did not hesitate to admit that his theory was imbued with his socialist convictions. However, as Joan Robinson suggests, “The best way to separate out scientific ideas from ideology is to stand the ideology on its head and see how the ideas look the other way up. If they disintegrate with the ideology they have no validity of their own. If they still make sense as a description of reality, then there is something to be learned from them, whether we like the ideology or not” (Robinson: 26), our task is to examine whether Marx's value theory has a solid base independent of his ideology.

First, we must notice that both Marx and Marshall considered themselves as pupils of Ricardo in the value theory. In his Principles Ricardo summarized his position in the following way: "Not only the labour applied immediately to commodities affect their value, but the labour also which is bestowed on the implements, tools, and buildings, with which such labour is assisted"

2) Another link between the rise (fall) of interest rate and the saving which Marshall presents is that the high (low) interest rate might signify the high (low) capacity to save. Because “the power to save depends on an excess of income over necessary expenditure” (PE: 229), and this excess is the greatest among the wealthy "commercial class" whose income depends largely on their possession of capital. If the propensity to save of this class is constant, the amount of saving will increase due to the increase in income as the interest rate goes up. This classical concept of income as "the power to save" will, if the propensity to save is stable, lead easily to the theory of balancing saving and investment by a change in the volume of income rather than by changing the rate of interest. Marxists, too, might be interested in this link, because it reveals Marshall himself could not rely much on his refined “abstinence” theory and approved the discouraging fact that the ground of interest is the possession of capital.
Two directions in the value theory were open from this position which considers the labour embodied in the means of production as well as the directly spent labour affecting the value of commodities.

The one which was taken by Marshall (and by Böhm-Bawerk) is to arrange the indirectly spent labour on the time axis. Marshall paid attention to the part of Ricardo's value theory where Ricardo dealt with the influence of the different investment period to the relative value and concluded that Ricardo emphasized "the fact that time or waiting as well as labour is an element of cost of production" (PE: 816).

Another direction has a synchronic perspective and considers different kinds of labour embodied in different commodities as "coexisting" in the production system composed of different industries as a whole. In other words, this is to consider the value of commodities not individually but as a system in which all commodity values are determined simultaneously. Marx seems to have held this idea intuitively, though he could not formulate this idea in a clear mathematical form. This synchronic system corresponds to his concept of "reproduction" which views production in a continuing condition in which not only final goods but also the means of production consumed in the production process are also produced. From this viewpoint, the labour added to the value of final goods indirectly through the consumption of means of production is considered not as past labour but as "simultaneously coexisting labour."

"Baumwolle, Garn und Gewebe werden nicht nur eine nach dem andren und aus dem andren produziert, sondern sie werden gleichzeitig nebeneinander produziert und reproduziert. Was sich als effect der antecedent labour darstellt, wenn ich den Produktionsprozeß der einzelnen Ware betrachte, stellt sich zugleich als Wirkung der coexisting labour dar, wenn ich ihren Reproduktionsprozeß betrachte, also ihren Produktionsprozeß in seinem Fluß und der Breite seiner Bedingungen, nicht nur in einem isolierten Akt betrachte oder in beschränktem Raum." (MW: I, 275)

In the third volume of Das Kapital Marx presented the theory of production price which contains the general rate of profit as a standard principle of price under the capitalist production. Marx is Ricardo's pupil also in his recognition that the profit in proportion to capital enters in the supplier price of the capitalist producer. Nevertheless, the "reproduction" viewpoint is retained in the production price, too. In the base of production price Marx supposed the value system in which the time element does not appear. Then, can we conclude that the system of production price also has no time axis? But we had better avoid a hasty conclusion.

Marx regarded the essence of profit which remains even after the abstraction of the time axis as the "exploited surplus-value." In a more neutral expression, the rate of profit represents above all the distributive relation of the capitalist production. But as we have seen it before, the profit rate in the production price has another aspect in the marginal productivity of capital as well as of time or "waiting". In the value system this aspect disappears and the distributive aspect appears in a simple zero-sum division of the coexisting labour between necessary labour and the surplus labour. This is the reason why Marx began with the labour value theory.
Marx discusses the relation between the exploitation rate and the profit rate before entering the section of the production price. Since capital is composed not only of the "variable capital" (V), wages the employed are paid, but also of the "constant capital" (C), namely physical capital, the profit rate (r) is determined by the rate of organic composition of capital (ω) in addition to the exploitation rate (e). In the production with no fixed capital,

\[
\frac{r}{\omega + 1} = \frac{M}{C+V} \left( \frac{C}{V} + 1 \right) = \frac{e}{\omega + 1}
\]

If every industry of a society has the same rate of organic composition of capital, the rate of profit and that of surplus-value are combined directly by Equation (6) mentioned above. Otherwise, the amount of profit that each industry demands in proportion to its capital is disconnected from the surplus-labour (surplus-value) exploited (created) from workers in each industry. If profit should be considered as transformed surplus-value, there must be a redistribution of surplus-value among industries under the domination of the general profit rate. Marx thought, therefore the determination of profit rate by exploitation rate, which is essentially a relation in the macro level, is explained in advance of the transformation of value to the production price.

Now, we should remember that the "reproduction" of economy has the the quantitative aspect as well as the value (price) aspect. In this respect A. Medio's solution of the transformation process is suggestive. 3)

Suppose the economy is composed of single product industries (i=1, ..., n) with no fixed capital and signify the input-coefficients and labour coefficients by \( a_{ij} \) and \( r_i \) respectively. Then, the value of the product of each industry \( \lambda_i \), is shown by the following equation:

\[
\lambda = A\lambda + \tau
\]

where \( \lambda \) and \( \tau \) are the column vectors \( (\lambda_1, ..., \lambda_n) \) and \( (\tau_1, ..., \tau_n) \), respectively, and \( A = (a_{ij}) \) is a indecomposable productive \( n \times n \) matrix.

On the other hand, by fixing the real wage or consumption vector of the worker's household per unit labour as \( b = (b_1, ..., b_n) \), the equation system of production price with general profit rate is shown by the following:

\[
p = (1 + r) (A + \tau b)p
\]

where \( p \) is column vector \( (p_1, ..., p_n) \).

With \( b \) and exploitation rate \( e \) the value system can be written as follows:

\[
\lambda = A\lambda + \tau bh + e\tau bh
\]

The three terms on the right side are constant capital, variable capital and surplus value, respectively. If the output of each industry is signified by \( x = (x_1, ..., x_n) \), signifying the unit matrix

3) Since Medio's formulation is complicated I borrow here its summary in Sato. However, I owe the interpretation of Medio's solution to Takasuga's survey article as well.
by \( I \) the profit rate which Marx assumed before the transformation of value to production price is
\[
r_* = \frac{x(I-A-\tau b)\lambda}{x(A+\tau b)\lambda} \tag{10}\]

If we suppose a special output vector \( x^* = (x_1^*, ..., x_n^*) \) which satisfies following condition
\[
\begin{align*}
  &x^* (A+\tau b) = kx^* \\
  &x^* \tau = xt
\end{align*} \tag{11}
\]
then,
\[
r_*^* = (1-k)/k \tag{12}
\]

The profit rate under the production price which is independent from the proportion of output takes the same rate:
\[
r = \frac{x^* (I-A-\tau b)p}{x^* (A+\tau b)p} = \frac{1-k}{k} \tag{13}
\]

In this proportion of output, if we give the absolute price by choosing one of two assumptions that Marx made, "total price = total value" and "total profit = total surplus-value", another holds at the same time.
\[
\begin{align*}
  x^* p &= x^* \lambda \\
  x^* (I-A-\tau b)p &= x^*(I-A-\tau b)\lambda
\end{align*} \tag{14, 15}
\]

This proportion of output which satisfies Marx's supposition in his transformation procedure is such under which the ratio of this period's consumption of products to the total output takes a unique rate \( k \) in every industry. Thus it is the proportion that enables the expansion of the total system with the rate of \( 1/k \). By rewriting \( 1/k = 1+g^* \), we get
\[
r = g^* \tag{16}
\]

As seen at once from the Eq. (11), \( g^* \) is the largest growth rate under the matrix \( (A+\tau b) \) which signifies the condition of production.

**IV Value in an Expanding Economy**

Ironically enough the reflection above brings us back to the problem of time axis just amid the linkage which combines the value system with that of the production system. The output proportion \( x^* \) corresponds to the steady state growth path known under the name of von Neumann's "golden age" on which every industry expands with the same growth rate of \( g^* \).

---

4) The proportion of output on the von Neumann growth path was first introduced to the discussion of the transformation problem by Morishima=Seton. See also Morishima.
The general profit rate which emerges in the production price system is equal to this growth rate which is theoretically the maximum under given production conditions. This urges us to reconsider the view of "reproduction" by which I characterized Marx's approach.

It was Marx's valuable recognition which he acquired from the analysis of reproduction that an appropriate proportion of sectors must exist so as to secure the balance in reproduction. This is true in expanded reproduction as well as simple reproduction. In an expanding economy, industry is producing not only the amount that is consumed in this period but also an extra amount of means of production which corresponds to the expanded production of the next period. The proportion of industries in the expanded reproduction will thus differ from that in the simple reproduction. In this expanding system, the term "reproduction" cannot be applied to signify the same size of production as before. It is clear that if industries should refuse expansion, then that part of extra products of this period would become useless, so even the production of this period could not be maintained.

In Marx's two-departments-model with no fixed capital, the scheme of expanded reproduction can be written as follows:

\[
\begin{align*}
X_1 &= C_1 + C_2 + DC_1 + DC_2 \\
X_2 &= V_1 + V_2 + DV_1 + DV_2 + Mk_1 + Mk_2
\end{align*}
\]

The suffix 1 and 2 signifies the production of capital goods and consumption goods, respectively. Mk means consumption from surplus-value. If we assume that capitalists need not consume and the whole profits are converted to new capital, due to the existence of a general profit rate, both departments will expand with the same growth rate \( g^* = r \). The production price is, thus, the exchange value which secures the reproduction of this expanding economy.

It does not mean, of course, that a production system in the state of simple reproduction lacks the capacity to grow, so long as it is producing surplus. The diversion of goods or labour which are now devoted to unproductive consumption for the purpose of growth is not ruled out. But such change is inevitably accompanied by the imbalance of the supply and demand in many industries. After some adjustment process the system will recover the balancing proportion which secures growth. If unstable factors such as capitalists' consumption or the flexible part of workers' consumption are negligible, this proportion is nothing other than the proportion of von Neumann's "golden rule path". The real proportion differs from it, still, the production price has its corresponding quantity system in this (imaginative) expanding economy.

In the replacement of fixed capital the problem of proportion emerges exactly in the vintage structure of equipment. Capitalists accumulate depreciation of fixed capital in each period so as to be able to finance its replacement at the end of a durable length. While the accumulation of depreciation itself makes no demand for capital goods, capitalists whose equipment needs replacement make investment which cannot be covered by a single accounting period. It is, however, unrealistic to suppose that even in the economy as a whole such a proportional distribution in the vintage of equipment should exist as to equalize this replacement to the total depreciation. Marx envisaged here a seed of crisis which is persistent even under the condition of simple reproduc-
More interesting is his brief comment on the inevitable discrepancy between depreciation and replacement in the expanding reproduction (DK: II, 462). Marxian economists interpreted it to suggest the fact that in a period on the path of expanded reproduction the total amount of depreciation \( D \) is larger than that of replacement \( R \). \(^5\)

But as we affirmed before, the "reproduction" of the expanding economy implies growth, to which the concept of mere replacement does not fit. If the economy is expanding at rate \( g \), the gross investment on equipment \( J \) must also grow at the same rate. To understand the relation of these three concepts we should turn to the value system corresponding to the expanding economy to recover the time axis.

In an economy where the interest rate is only a monetary expression of the general profit rate which is equal to the growth rate, capitalists can adopt the annuity method of depreciation to finance the replacement of their gross investment. Suppose the equipment survives \( n \) periods, then, the rate of depreciation by this method is 
\[
\frac{r(1+r)^n}{(1+r)^n-1}.
\]
The total amount of depreciation by this method in one single period, is \( (1+g)^n R \) or \( I \). However, in the case where investment is financed by borrowing, the depreciation amounts just to clear the borrowing made for the purchase of old equipment \( D=R \). The purchase of new equipment must be performed again through borrowing of increased scale.

V Concluding Remarks

Referring Ricardo's value theory, Marshall remarked, "the element of time, which he had carefully kept in the background at starting, was necessarily introduced" (PE: 815). Our discussion on Marx might give a similar impression. The typical Marshallian theme of investment in the time perspective, too, began to appear though in a different frame. Marxian value theory requires reconsideration from the view outlined above.

On the other hand, the emergence of the time element with the growth provides suggestions related to the Marshall's definition of interest as the reward of "waiting". Under the assumption of the constant consumption of workers (=real wage), the general profit rate is the potential maximum of the growth rate; the difference with the actual growth rate is due to the unproductive consumption from profit. This reminds me of Marx's view that regards accumulation as the historical mission of capitalists (DK: I, 621).

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\(^5\) See Nihe. The discrepancy of \( D \) and \( R \) was noticed also by E. Domar.


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