

## Downscaling Spatial Rainfall Field from Global Scale to Local Scale Using Improved Multiplicative Random Cascade Method

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### Synopsis

Non-homogenous multiplicative random cascade method downscales spatial rainfall field from a coarse scale into a finer one. Currently, this kind of downscaling is less reliable even though it correctly produces a long term average spatial pattern. It fails reproducing the patterns in repeated trials; and there is a higher chance of magnitude fluctuation. These drawbacks are needed to overcome. In this study, a new method, named as random cascade Hierarchical and Statistical Adjustment (HSA) method, is introduced and tested to downscale 1.25 degree GAME Re-analysis data into 10-minute spatial resolution. The obtained results are highly improved, quite robust and reliable than the previous method.

**Keywords:** random cascade method, downscaling, GAME Re-analysis data, HSA method

### 1. INTRODUCTION

Accurate simulation of space time rainfall field is an important task in hydrology. It is an important binding forcing to understand the space time variability of hydrologic factors, and to drive small to large-scale, short to long-term simulations of runoff quantity and quality. There are numerous attempts to use products of global scale space time rainfall models, e.g. General Circulation Models (GCM), in local scale hydrological analysis for a number of reasons. This demands a reliable disaggregation of a coarse GCM scale rainfall field to a smaller scale of local catchments (Burlando and Rosso, 2002). This paper presents an improved method to disaggregate the spatial rainfall field using a non-homogenous multiplicative random cascade method.

There is a large scale difference between global scale (climate or atmospheric) models and regional or local hydrological models. Still these models are necessary to be coupled in order to understand and predict a clear scenario of local and regional impacts on hydrological cycle due to global changes. Coarse scale products of GCMs are an inadequate basis for assessing local / regional scale impacts as it is hardly able to resolve many important sub-grid scale processes (Hostetler, 1994; Wilby *et al.*, 1999). It is necessary to identify the sub-grid scale features for local or regional hydrological analysis, which is not seen in a coarser scale frame.

Current emphasis on the physical basis of rainfall representations (Eagleson, 1984; Gupta and Waymire, 1979; Smith and Karr, 1984) are developed after understanding a clear picture of rainfall field structure that a rained area of a given scale has one or several smaller-scale areas of more intense rain zones embedded within it (Waymire *et al.*, 1984). Investigation on the statistical fluctuations in space and time rainfall intensity and their mathematical representations has yielded two major stochastic space-time rainfall-modeling approaches. The first approach that focuses on cluster point process to reproduce the hierarchical spatial and temporal organization exhibited by observations of space-time rainfall (Austin and Houze, 1972; Gupta and Waymire, 1979; Waymire *et al.*, 1984; Kavvas *et al.*, 1987) has been criticized for its difficulty and unambiguity in parameter estimation (Sivapalan and Wood, 1987) and inability to fully describe the rainfall structure over a large range of scales (Foufoula-Georgiou and Krajewski, 1995). The second approach is based on the scaling invariance features of observed spatial rainfall fields (Schertzer and Lovejoy, 1987; Lovejoy and Schertzer, 1990; Gupta and Waymire, 1990) with extreme variability and strong intermittence (Georgakakos and Krajewski, 1996), which has yielded a multiplicative random cascade theory (Lovejoy and Schertzer, 1990; Gupta and Waymire, 1993). Due to the scaling invariance or self-similarity concept in this approach of space-time rainfall modeling, the parameterization

is parsimonious and valid over a wide range of scales (Lovejoy and Schertzer, 1990; Gupta and Waymire, 1993; Over and Gupta, 1994; Foufoula-Georgiou and Krajewski, 1995; Olsson, 1996).

In a large or global scale, the space-time rainfall fields obtained from the re-analysis of GCM outputs are now abundantly available with considerably acceptable accuracy (Prudhomme *et al.*, 2002). The advancement of global scale climate / atmospheric models has already achieved much higher resolution in temporal scale as short as 15 minutes unlike the spatial scale. Currently, GCM outputs at 6 hour, 12 hour or daily intervals are often being tested in hydrological simulations of large scale catchments. This range of data frequency approximately fulfills the general need of temporal data in hydrological modeling. On the other hand, there is a wide gap in the spatial scale between the availability of GCM or similar large-scale model outputs and the need of hydrological models (Burlando and Rosso, 2002). This is one of the major obstacles to apply the global scale observation in the assessments of local scale hydrological behavior.

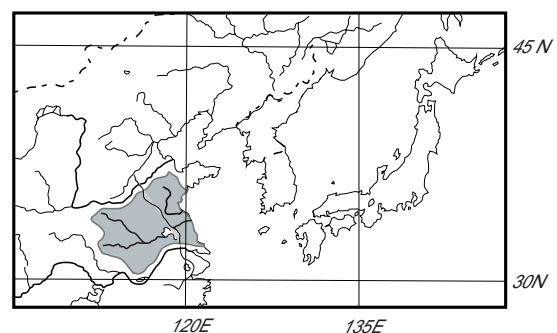
The need of reliable and accurate spatial disaggregation is pretty high to analyze real world problems by using current GCM scale outputs as the spatial rainfall structure induces significant effect in hydrological analysis of small to large-scale catchments (Shrestha *et al.*, 2002). Spatial rainfall field, which plays a significant role in any subsequent analyses involving the rainfall field as primary or secondary information, contains a higher degree of spatial variability that has to be modeled at the local scale. A multiplicative cascade treatment based on the statistical theory of turbulence (Mandelbrot, 1974) offers a concrete way of modeling these fields (Schertzer and Lovejoy, 1987) as the kinetic energy transfer is seen in the cascade of turbulent eddies from a large energy scale to smaller dissipation scales. Similarly, in the cascades of rainfall modeling, an area of higher intensity rainfall is embedded in larger areas of lower intensity rainfall, which are again a part of even larger areas but of even lower rain intensity. Debates are ongoing on suitability of approaches to form the multiplicative random cascade whether continuous or discrete. A continuous form of multiplicative random cascades has the major advantage of developing cascades over a continuous interval of scales instead of only a discrete set (Marsan *et al.*, 1996); however, a discrete form of multiplicative random cascade has ability to separate rainy and non-rainy area (Gupta and Waymire, 1993; Over and Gupta, 1994) and can be adopted to respect the discrete sub catchment partitioning of the landscape by the drainage network of a catchment (Gupta *et al.*, 1996; Over and Gupta, 1996).

The spatial rainfall modeling based on discrete multiplicative random cascades (Over and Gupta, 1994; Jothityangkoon *et al.*, 2000; Tachikawa *et al.*, 2003) has been tested for spatial disaggregation in

different places under different condition with similar conclusion that it is possible to capture long-term spatial variability of sub grid scale. The existing model is not applicable to generate rainfall field at shorter time scale and hence not useful to apply the outcomes as input to models of the rainfall-runoff process (Jothityangkoon *et al.*, 2000). To improve the modeling structure and / or algorithm is therefore very necessary and important to obtain accurate and reproducible disaggregated rainfall field, which may be useful for real world problems of short and long timescale.

The multiplicative random cascade method assumes the isotropic spatial statistics, which is not in line with the rainfall observation and fails to generate practically useful, especially short time scale rainfall field. As stated earlier, the need of spatial disaggregation is much crucial for the purpose of utilizing the disaggregated GCM scale spatial rainfall field; the focus in this paper is given on spatial structure of the rainfall field, leaving the part of study of temporal structure as a further research topic in random cascade method. We have found that inclusion of spatial correlation field in the multiplicative random cascade is successful to improve the disaggregation of the spatial rainfall field. This method is applied to a 560 km X 320 km region over eastern Chinese territory (Fig. 1) using hourly rainfall data of 1.25-degree resolution and disaggregated to 10-minute spatial resolution. The method is named as HSA method.

This paper is organized as follows: Section 2 reviews the existing discrete random cascade method. Section 3 describes the observation of rainfall spatial correlation. Section 4 discusses about the HSA method. Section 5 mentions about data and parameter estimations. Section 6 presents results and discussions. The conclusions are presented in section 7.



**Fig. 1** : Location of the study region.

## 2. Revision of existing stochastic spatial rainfall model

Spatial disaggregation is carried out using the discrete random cascade approach. In which, the cascade construction process successively divides a two-dimensional ( $d = 2$ ) bounded region into  $b$  equal

parts ( $b = 2^d$ ) at each step, and during each subdivision the mass (or volume) of rainfall over the region obtained at the previous disaggregation step is distributed into the  $b$  subdivisions by multiplying by a set of “cascade generators”  $W$ , as shown schematically in Fig. 2 (for the case of  $d = 2$  and  $b = 4$ ). The homogenous form of the discrete random cascade model is adapted as proposed by Over and Gupta (1994).

For an area at level 0, denoted by  $\Delta_0^0$ , has the outer length scale of  $L_0$  and average rain intensity of  $R_0$ . The initial rain volume  $\mu_0(\Delta_0^0)$  becomes  $R_0 L_0^d$ . At level 1, the rain volume  $\mu_0(\Delta_0^0)$  divides into  $b = 4$  sub-areas denoted as  $\Delta_1^i$ , ( $i = 1, 2, 3, 4$ ) and the sub-area rain volume  $\mu_1(\Delta_1^i)$  is  $R_0 L_0^d b^{-1} W_1^i$ , ( $i = 1, 2, 3, 4$ ). At level 2, each of the sub-area volume is further subdivided into  $b = 4$ , all together  $b^2 = 16$  sub sub-area, denoting them as  $\Delta_2^i$ , ( $i = 1, 2, \dots, 16$ ) and the corresponding volume  $\mu_2(\Delta_2^i)$  is  $R_0 L_0^d b^{-2} W_1^i W_2^i$ , ( $i = 1, 2, \dots, 16$ ). The process of sub-division is continued further until the  $n^{\text{th}}$  level up to  $b^n$  sub-areas, which are denoted as  $\Delta_n^i$ , ( $i = 1, 2, \dots, b^n$ ). At the  $n^{\text{th}}$  level, the volume in the sub-areas can be expressed as

$$\mu_n(\Delta_n^i) = R_0 L_0^d b^{-n} \prod_{j=1}^n W_j^i ; (i = 1, 2, \dots, b^n) \quad (1)$$

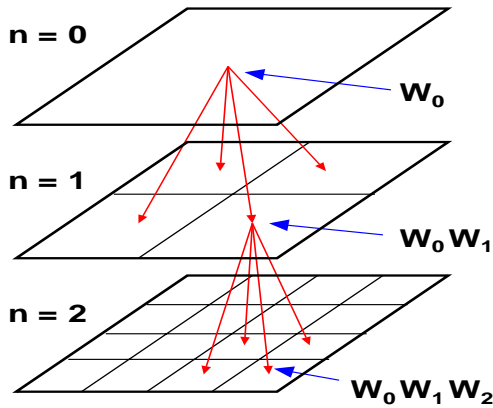


Fig. 2: Schematic of cascade branching

The cascade generators  $W$  are non-negative random values with  $E[W] = 1$ , which is imposed to ensure the mass conservation from one discretization level to the next (see Over and Gupta, 1996). To get the cascade generator  $W$  values, Over and Gupta (1994, 1996) has proposed a model called beta-lognormal model such that

$$W = BY \quad (2)$$

Here,  $B$  is a generator from the “beta model” that separates the rainy and non-rainy zone on the basis of discrete probability mass function and  $Y$  is obtained from lognormal distribution (Gupta and Waymire, 1993) in the form of  $Y = b^{\left(\frac{\sigma X - \frac{\sigma^2 \ln b}{2}}{\sigma}\right)}$ , where  $X$  is standard normal random variate and  $\sigma^2$  is a parameter equal to the variance of  $\log_b Y$ . The value of  $W$  is evaluated as

$$W = \begin{cases} 0 & \text{when } P(B=0) = 1 - b^{-\beta} \\ b^{\beta + \sigma X - \frac{\sigma^2 \ln b}{2}} & \text{when } P(B=b^\beta) = b^{-\beta} \end{cases} \quad (3)$$

This model consists of only two parameters,  $\beta$  and  $\sigma^2$ . The parameter estimation method is proposed by Over and Gupta (1994, 1996) by using the Mandelbrot-Kahane-Peyriere (MKP) function, named after Mandelbrot (1874) and Kahane and Peyriere (1976), which characterizes the fractal or scale-invariant behavior of the multiplicative cascade process. This method yields the following equations for the parameter  $\beta$  and  $\sigma^2$ ,

$$\beta = 1 + \frac{\tau^{(1)}(q)}{d} - \frac{\sigma^2 \ln b}{2} (2q - 1) \quad (4)$$

$$\sigma^2 = \frac{\tau^{(2)}(q)}{d} \ln b \quad (5)$$

Here,  $\tau^{(1)}(q)$  and  $\tau^{(2)}(q)$  are the first and second derivative of the slope  $\tau(q)$  with respect to  $q$ . The slope  $\tau(q)$  represents the scaling relationship for different exponent  $q$  in the process of obtaining statistical moment across different level of subdivision. Choosing the value of  $q = 2$  gives less variable estimates of  $\beta$  and  $\sigma^2$  without affecting the simulation (Jothityangkoon *et al.* 2000).

This model is preferred as a test disaggregation model for three main reasons. First, the number of parameter to be estimated is quite low. Second, the parameters  $\beta$  and  $\sigma^2$  values estimated by equation (4) and (5) exhibit spatial homogeneity and remain largely scale invariant across the wide range of regions (Jothityangkoon *et al.* 2000). Third, the spatial structure of rain field is based on the lognormal distribution, which has been observed as a good descriptor of the marginal distribution of rainfall intensities on field (Bell, 1987) and confirmed by Crane (1990) within a range of rain intensities. Thus, a combination of threshold setting for non-rainy zones by the use of “beta model” in rainfall field simulation of lognormally distributed rainy zones is a convincing approach. Works of Jothityangkoon *et al.*,

2000 and Tachikawa *et al.*, 2003 have presented additional examples of its ability to reproduce the spatial statistics of rainfall field using this model. However, some inconsistencies between the log-normality of rain and self similarity concept are limiting it to be a fully satisfying from theoretical analysis (Gupta and Waymire, 1993).

The beta lognormal model (Over and Gupta, 1994, 1996) of discrete random cascade theory is attempted to modify for describing the spatial statistics and generating practically applicable disaggregated rainfall field as well. The modified model tested by Jothityangkoon *et al.*, 2003 and Tachikawa *et al.*, 2003 have changed the equation (2) into

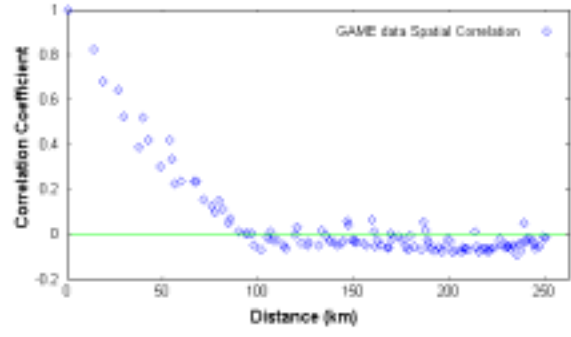
$$W = BYG \quad (6)$$

Here  $G$  is introduced to incorporate the observed spatial gradients in mean annual, monthly and daily rainfall as a ground truth forcing in terms of deterministic multiplier by Jothityangkoon *et al.*, 2003. Introduction of  $G$  has biased the beta lognormal model farther from its earlier form of theoretical agreement with self similarity concept; however, it has been justified for the need of considering the known ground information in the disaggregation process rather than depending just on mathematical numbers. The trial model succeeded to generate not only spatial patterns of long term mean daily, monthly, and annual rainfall but also to mimic spatial patchiness characteristic of daily rainfall, estimated in terms of a wet fraction. In the trial model of Tachikawa *et al.*, 2003, the  $G$  is introduced to incorporate the topographic effect in spatial distribution of rainfall, based on the findings of Nakakita *et al.*, 2001, in terms of deterministic multiplier as well. This trial also succeeded to generate long term spatial pattern of mean monthly rainfall. Both the trials are unable to generate rainfall fields at shorter time scales. This limits its use in many kinds of hydrological applications yet.

### 3. Spatial correlation of the rainfall field

Generally rain falling cloud clusters expand to few-hundred-kilometers scale but rainfall disaggregation is conducted up to smaller scale of a few kilometers because the rainfall events of hydrologic interest is of smaller spatial scale than the scale of cloud clusters. When the spatial scale of disaggregation target is much smaller, the neighboring cells inside or nearby a rainy zone possesses higher chance of rainfall around same time because they mostly would have encompassed within the same cloud cluster. In smaller spatial scale, the rain events are observed to have strong spatial correlation and shall not be treated as totally random event. The spatial correlation in rainfall field corresponds to the rain-cell coverage and movements

on its natural way along with the energy dissipation of the cloud mass. Existence of spatial correlation in the spatial rainfall fields can be described by a spatial correlation function as the observations have shown the same shape and approximately the same scale sizes (Crane, 1990). Fig. 3 shows the spatial correlation of the rainfall data over the study area.



**Fig. 3:** Spatial Correlation of rainfall data

The spatial correlation gradually decreases upon increase of distance. The decaying shape of the spatial correlation function may be represented by a logarithmic function (equation 7). Its parameters may be obtained simply by regression analysis of the observed spatial correlation data. A threshold distance may be set to omit spatial correlation at very far distance that may be out of interest in spatial rainfall disaggregation.

$$\rho_z = \begin{cases} \alpha + \kappa \log_{\lambda} Z & \text{if (+ve and } Z < Z_0) \\ 0 & \text{if (-ve or } Z > Z_0) \end{cases} \quad (7)$$

$$Z_0 = \text{anti log}_{\lambda} \frac{\rho_z - \alpha}{\kappa}, \text{ for } \rho_z = 0 \quad (8)$$

Where,  $Z$  is distance in kilometers;  $\rho_z$  is the spatial correlation value at  $Z$ ;  $Z_0$  is the threshold beyond which the spatial correlation remains zero by the use of the logarithmic spatial correlation function with  $\alpha$ ,  $\kappa$  and  $\lambda$  parameters.

### 4. Spatial correlation effect in random cascade method

The spatial rainfall field generated by multiplicative random cascade method is isotropic. Moreover, the repeated trial of rainfall generation has a rare possibility of yielding similar spatial rainfall field due to randomness of the cascade generator  $W$ . However, the observed rainfall is non-homogenous and anisotropy. The lower chance of reproduction and substantial difference from the observed scenario (discussed in section 6) has limited the popularity and practical application of the existing method. We sketch a method to improve current multiplicative random cascade method that attempts to include the

spatial correlation effect in order to incorporate the non-homogenous anisotropy and increase reliability. This method is named as HSA-method, which refers Hierarchical and Statistical Arrangement method. Fundamentally, this method learns from the neighboring region to understand the sub-grid scale local anisotropy by utilizing the multiple coarse grid scale information and their influence on small scale rainfall field in terms of the spatial correlation. The process description is as follows.

In the process of cascades of the downscaling a two-dimensional ( $d = 2$ ) spatial field, the  $b^n$  numbers of sub-areas, named as  $\Delta_n^i$ , ( $i = 1, 2, \dots, b^n$ ) are obtained at  $n^{\text{th}}$  level with the grid dimension  $\frac{L_0}{d^n}$ . For

each of these sub-areas, a spatial correlation reference index  $H$  is evaluated, which works as a spatial guide matrix later on. The reference index  $H$  is influenced by the average rain intensities  $R_m$  of the surrounding eight coarse scale grids ( $m = 1, 2, \dots, 8$ ) and corresponding distances of the sub-area from the referred neighbor grid  $Z^m$ . For  $n^{\text{th}}$  level, the reference index can be represented as,

$$H_n^{jk} = \sum_{m=1}^8 R_m \rho_{Z_n^m} \quad ; \quad \text{for } \begin{matrix} j=1,2,\dots,d^n \\ k=1,2,\dots,d^n \end{matrix} \quad (9)$$

$$Z_n^m = \sqrt{(j_m - j)^2 + (k_m - k)^2} \quad (10)$$

Here,  $H_n^{jk}$  is the non-negative reference index at  $n^{\text{th}}$  level for  $jk^{\text{th}}$  sub-area;  $R_m$  is the rainfall of  $m^{\text{th}}$  neighbor cell;  $\rho_{Z_n^m}$  is the spatial correlation with the  $m^{\text{th}}$  neighbor viewed from  $jk^{\text{th}}$  location at  $n^{\text{th}}$  level, from where the distance up to the  $m^{\text{th}}$  neighbor becomes  $Z_n^m$ . For the sub-area, the  $j$  and  $k$  represents

the central point of the sub-area; however, the  $j_m$  and  $k_m$  for coarse grid area are shifted to the nearest location of central grid area, as shown in Fig. 4, assuming that the average rainfall intensity of the coarse grid has no any specific rain centroid or dominant region inside it.

The random cascade generator  $W_n^i$ , associated with the sub-area  $\Delta_n^i$ , ( $i = 1, 2, \dots, b^n$ ), are expected to define proper magnitude of rainfall at proper location; however, there is no spatial order to identify a proper location for assigning a particular  $W_n^i$  value. Sequential assignment of the  $W_n^i$  values following the calculation order of current model leads to haphazard spatial arrangement due to dependence of the  $W_n^i$  values on the random variate  $X$ . A new form of

model is proposed here to improve the haphazard arrangement condition such that,

$$W[\bullet] = BY \quad (11)$$

Here,  $W[\bullet]$  represents the  $W$  with its spatial address  $[\bullet]$ , which is missing in equation (2). The reference index  $H_n^{jk}$  may be used to obtain the missing spatial address  $[\bullet]$  since it has a clear two dimensional spatial reference  $j$  and  $k$  inside the sub-dividing region.

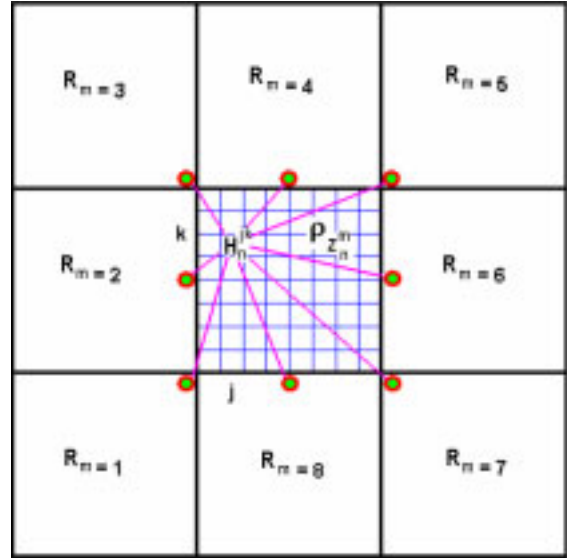


Fig. 4: Look up eight surrounding cells to evaluate reference matrix

At every additional level ( $n+1$ ), four more random cascade generators,  $W_{n+1}^{i'}$  ( $i' = b^{n+1}-3, b^{n+1}-2, b^{n+1}-1, b^{n+1}$ ) appear for newly disaggregated sub-area  $\Delta_{n+1}^{i'}$  from  $\Delta_n^i$ , ( $i = b^n$ ) as usual. In the new model, their spatial address  $[\bullet]$  is determined on the basis of comparison between the reference indexes  $H_{n+1}^{j'k'}$  and the random cascade generators  $W_{n+1}^{i'}$ . The  $W_{n+1}^{i'}$  may need to reshuffle its location within  $\Delta_{n+1}^{i'}$  in order to attain same hierarchy of  $H_{n+1}^{j'k'}$  locations. This is called as the hierarchical adjustment that minimizes the chances of haphazard spatial allocation of the cascade generators in successive progress of disaggregation. In other word, it is a forcing mechanism to control the random spatial location by assigning address to the homeless random generators. This process principally does not introduce arbitrary bias to the theoretical consideration of current random cascade method, because the generators  $W_{n+1}^{i'}$  are mathematically independent to their spatial locations

within the boundary sub-areas  $\Delta_n^i$  at one level back until the  $W_{n+1}^i$  values are generated by equation (2). One relocation operation at this case involves only four  $W$  values. In the case of testing re-allocation after two or more level ahead, a strong influence of spatial correlation may dominate the process due to overlapped assignment of  $[\bullet]$ , which may generate a risky condition of disappearing random nature and essence of turbulence theory.

The spatial  $H_n$  field is a smooth gradient surface and its shape is based on the surrounding coarse grid average rainfall. The lowest and highest zones of the  $H_n$  field present valuable information as these are most possibly non-rainy and rainy zones respectively. Most of cases, the peak rainy cells of the  $\mu_n(\Delta_n)$  field may not be in accordance with the possible rainy zones of  $H_n$  field and / or non-rainy cells of the  $\mu_n(\Delta_n)$  field may not be in accordance with the possible non-rainy zones of  $H_n$  field. Though these extreme high and low value cell numbers may not be significant to influence spatial statistics, they might have practical significance. Therefore at the  $n^{\text{th}}$  level, the spatial locations of the extreme  $\mu_n(\Delta_n)$  values are re-adjusted following hierarchical order after statistical separation of extreme high and low zones for both fields. This process is called as the statistical adjustment. This process might induce some bias, however, it may be considered as a compromise in theory to obtain improved result. The statistical adjustment is omitted if the correlation of  $H_n$  field and  $\mu_n(\Delta_n)$  field is found higher than a target correlation, which is adapted 80% arbitrarily in this case. The extreme value separation, which takes part in statistical adjustment, is done by statistical measures of the entire  $b^n$  rainfall values (See Appendix C). The flowchart of the HSA method is given in Appendix A.

## 5. Experiment data and parameter estimation

### 5.1 Experimental data preparation

The rainfall data is obtained from the GEWEX Asia Monsoon Experiment (GAME) Re-analysis, Version 1.1 for the period of May 1<sup>st</sup>, 1998 to August 31<sup>st</sup>, 1998. This data is available for 6 hour interval in two spatial resolutions - 1.25 degree and 2.5 degree. We have taken GAME 1.25 degree data in this study to use in spatial downscaling experiment to obtain a 10-minute resolution data. Since such finer resolution data is not available, there rises up a problem to verify the obtained result of downscaling afterward. To fill up this gap, the GAME 1.25 degree data is converted to 10 minute spatial resolution data by

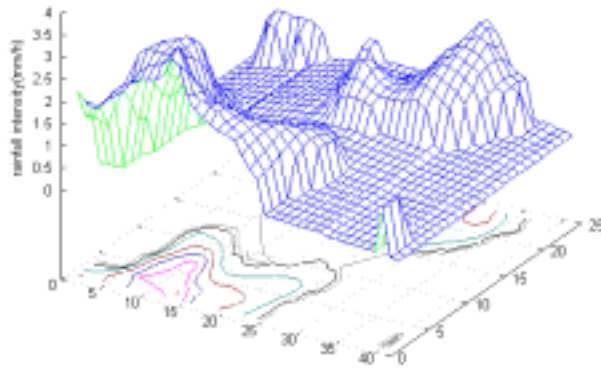
forcing a known spatial pattern from HUBEX-IOP EEWB data (abbreviated from the ‘‘Huaihe River Basin Experiment – Intense observation period – Estimation of Energy and Water Budget’’ termed ‘EEWB data’ here after). The precipitation field of EEWB data is generated from ground based observation with time and space (distance and direction) interpolation technique (Kozan *et al.*, 2001) at hourly time step. A number of various methods may be adapted to transfer the spatial pattern from one data to another data. In this experiment, the spatial rainfall pattern of source EEWB data is considered as a fluctuation from its mean value, which is transferred to the target GAME data over the same domain simply using arithmetic approach. The spatial pattern transfer relation may be written as,

$$P_{4t} = \begin{cases} P_{1t@s} + \xi(P_{2t} - P_{3t@s}) & ; \text{if } P_{4t} \geq 0 \\ 0 & ; \text{otherwise} \end{cases}, \quad \text{for time } t = 1, 2, 3 \dots T \quad (12)$$

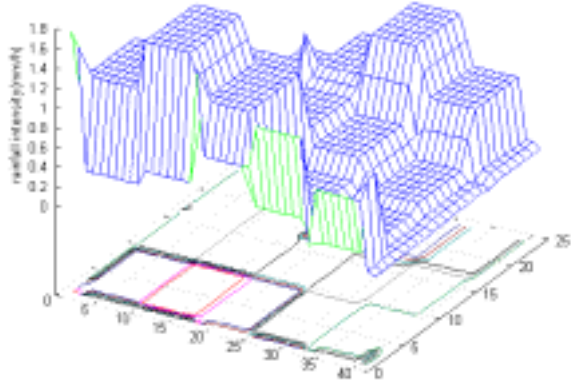
$$P_{5t} = P_{4t} \frac{\sum_{t=1}^T P_{1t}}{\sum_{t=1}^T P_{4t}}, \quad \text{for time } t = 1, 2, 3 \dots T \quad (13)$$

Here,  $P_{4t}$  is experimental 10-minute data;  $P_{1t@s}$  is original GAME 1.25 degree data at  $s$  – reference resolution;  $P_{2t}$  is EEWB 10 minute data; and  $P_{3t@s}$  is reference coarse data produced by averaging the EEWB data at the reference resolution  $s$ . The reference resolution  $s$  is set 1.25 degree and coefficient  $\xi = 1$ . In some cases,  $P_{4t}$  values may appear negative, which is forced to make zero. The accumulated value of the new data then appears to be slightly different from the accumulated value of the original GAME1.25 data due to forcing the negative values to zero. A factor of ratio between the new accumulated value and the accumulated value of original data is multiplied to all the data (equation 13) for maintaining the total accumulated input as same as that of the original data. Thus the  $P_{5t}$  is the final experimental data (Fig. 5), which is utilized here equivalent to the observed data.

The ‘experimental 10 minute data’ equivalent to ‘observed fine resolution data’ is aggregated to 1.25-degree resolution to obtain the original level of coarse resolution. The aggregated form of 1.25-degree data is employed in the downscaling process as the input coarse resolution data source and then the disaggregated 10 minute spatial data is obtained as the ‘simulated fine resolution data’. The fine resolution observed and simulated data is then compared with each other to check the model’s performance. (See Appendix B).



(a) Experimental 10 min Data



(b) GAME 1.25 degree Data

**Fig. 5:** Typical example of spatial patterns of Experimental data and GAME1.25 data

### 5.2 Sub-grid organization

There is an inconsistency between the grid system of coarse resolution and finest target resolution. The 1.25-degree data is 75-minute, which is 7 and half grids at 10-minute resolution. In downscaling process, 3 levels succession of cascading needs 80-minutes coarse data to obtain 10-minute data as the final product. In this experiment, it is necessary to stick on 10-minute resolution to compare the obtained results with the available finest resolution data. Thus, to compensate the half grid mismatch between the source data and downscaling product, the sub-grid organization is employed as follows.

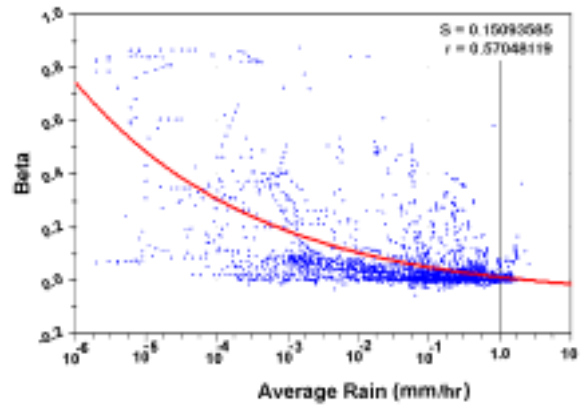
The coarse grid cell rainfall is multiplied by 7.5/8.0 before beginning the downscaling. This is treated as level 0 rainfall intensity and proceeded to downscale into level 1, 2 and 3. At 3<sup>rd</sup> level, 8 sub-grid cells are developed. Rainfall intensities of the sub-grid cells are multiplied by 8.0/7.5 after the downscaling and treated them as 10-minute sub-grid cell rainfall intensities. The 8<sup>th</sup> sub-grid cell value is memorized to average with the 1<sup>st</sup> sub-grid cell value of adjacent coarse grid. In this way, one sub-grid cell plays role of common sub-grid cell in between two coarse grid cells both in rows and columns. There is

no common sub-grid between 2<sup>nd</sup> and 3<sup>rd</sup> coarse grid cells. Next common sub-grid cells appear in between 3<sup>rd</sup> and 4<sup>th</sup> coarse grids and so on.

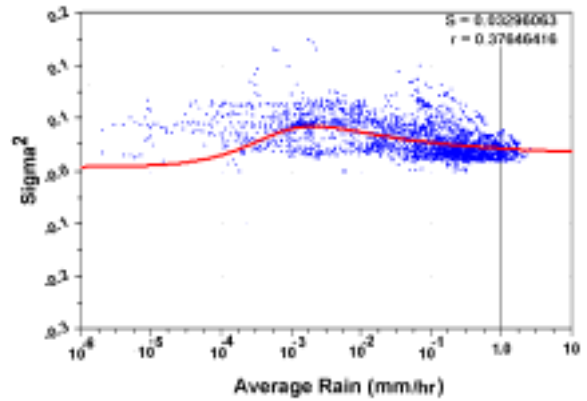
### 5.3 Parameter estimation of “beta-lognormal” model

The parameters  $\beta$  and  $\sigma^2$  values are estimated using the data of fifteen coarse cells for 2952 time steps following method advised by Over and Gupta, 1994. The  $\beta$  values are found more diverse in case of low rain intensity (Fig. 6). The  $\sigma^2$  values are less sensitive (Fig. 7) to the rain intensity variation. After trying several forms of equations, the best-fit curve is given by following parametric relation (equation 14 and 15) with the coarse grid rainfall.

$$\beta = -0.078(0.845 \exp^{0.1692 \exp^R}) \quad (14)$$



**Fig. 6:** Distribution of  $\beta$  versus rain



**Fig. 7:** Distribution of  $\sigma^2$  versus rain

$$\sigma^2 = \frac{0.0194 + 0.002 \exp^R}{1 + 0.241 \exp^R + 0.016566 \exp^{2R}} \quad (15)$$

Here, R is the rainfall intensity at level 0. These equations are used to evaluate the  $\beta$  and  $\sigma^2$  in the process of obtaining random cascade generator as described in previous section.

#### 5.4 Parameters estimation of spatial correlation function

The spatial correlations are evaluated for all four-month rainfall data on hourly interval from May through August, which include both dry and rainy season. The decreasing trend of rainfall spatial correlation with increase in distance is found consistently within approximately 100 kilometer radius region. Beyond that distance, the correlation appears to be negative or zero or fluctuating between small negative and positive values. The rain events beyond 100 kilometer radial range rarely find their mutual spatial correlation means that the rainfall field at this spatial scale is governed by separate processes; possibly separate dominant cloud cluster. The parametric values of  $\alpha$ ,  $\kappa$  and  $\lambda$  are found to be 2.0, -0.3 and 2. The  $\alpha$  and  $\kappa$  has exhibited negligible sensitivity (not shown here) in overall result of the experiment at  $\pm 25\%$  fluctuation in the evaluated values. However,  $\lambda$  has exhibited high

sensitivity, as it is prime parameter to determine the shape of logarithmic function

## 6. Results and discussions

In this section, we present and compare the results of current disaggregation model and modified new model by viewing their ability to generate correct and reliable rainfall field. Current multiplicative random cascade method using “beta-lognormal model” has a major weakness that it does not reproduce similar patterns in separate realizations from the same source data. Completely different results in terms of spatial pattern and mass conservation may outcome in repeated trials. Fig. 8 illustrates this and clearly displays the limitation of the current model. Clear split of rainy and non-rainy bands are visible to appear at different locations in different trials. This kind of random or unreliable output is mainly responsible for their inappropriateness in practical applications, even though it helps to understand the rain statistics.

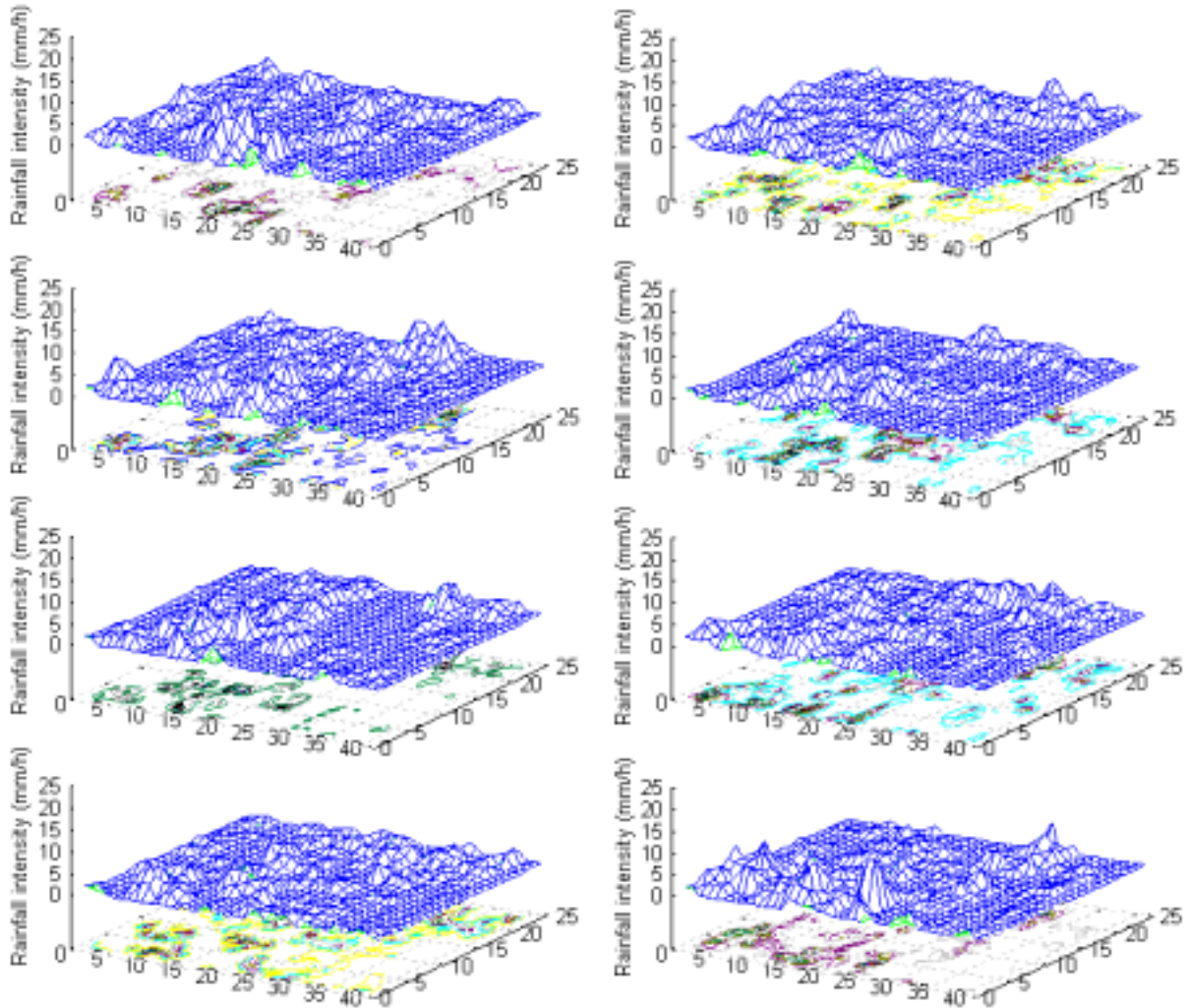


Fig. 8: Repeated realizations from the random cascade method



Unpredictable and random spatial arrangement of the disaggregation model output corresponds to the absence of spatial guidance in the multiplicative random cascade theory. Failure to model peak rainfall, numbers of rainy / non-rainy cells and rainfall statistic corresponds to the wrong selection of statistical distribution considered in developing the multiplicative cascades and thresholds. The weaknesses of the current model can also be viewed in the same way. Basically its ability to simulate rain statistics (Jothityangkoon *et al.*, 2000; Tachikawa *et al.*, 2003) gives less ground to debate on statistical limitations of cascade building and threshold setting “beta model”. The observed differences in spatial structure of multiple realizations clearly show that spatial modeling part may be criticized more sharply.

On the other hand, the different realizations obtained from the introduction of HSA method, discussed in section 4, in random cascade

downscaling technique (called hereinafter “random cascade HSA method”) has displayed a promising ability to fix the problem of unpredictable random spatial pattern (see Fig. 9). Random spatial structures on multiple realizations are disappeared from the simulated set of results by this modified method and the outcomes are closely similar to the spatial structure of observed data (see Fig. 10). The rainy and non-rainy bands do not mix up randomly on the repeated trials and the separations of zone are found in proper location as that exist in observed data. We have found the same kind of result in all 2952 realizations of hourly time steps. This may be considered as a prominent signal of reproducibility and accuracy of the spatial rainfall modeling. The random cascade HSA method seems able to model the spatial rainfall pattern without ensemble of realizations or much fewer than what is needed before.

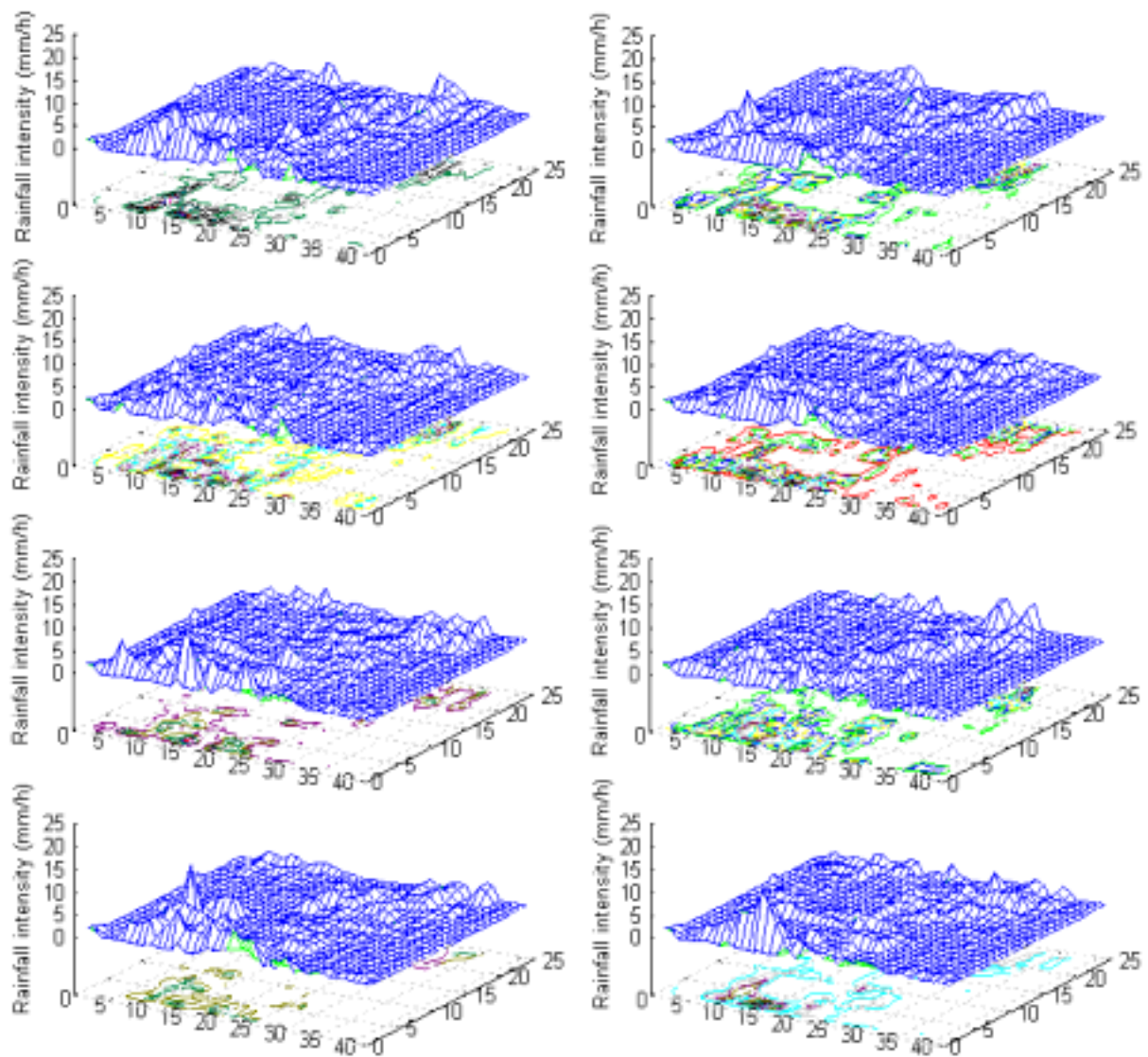
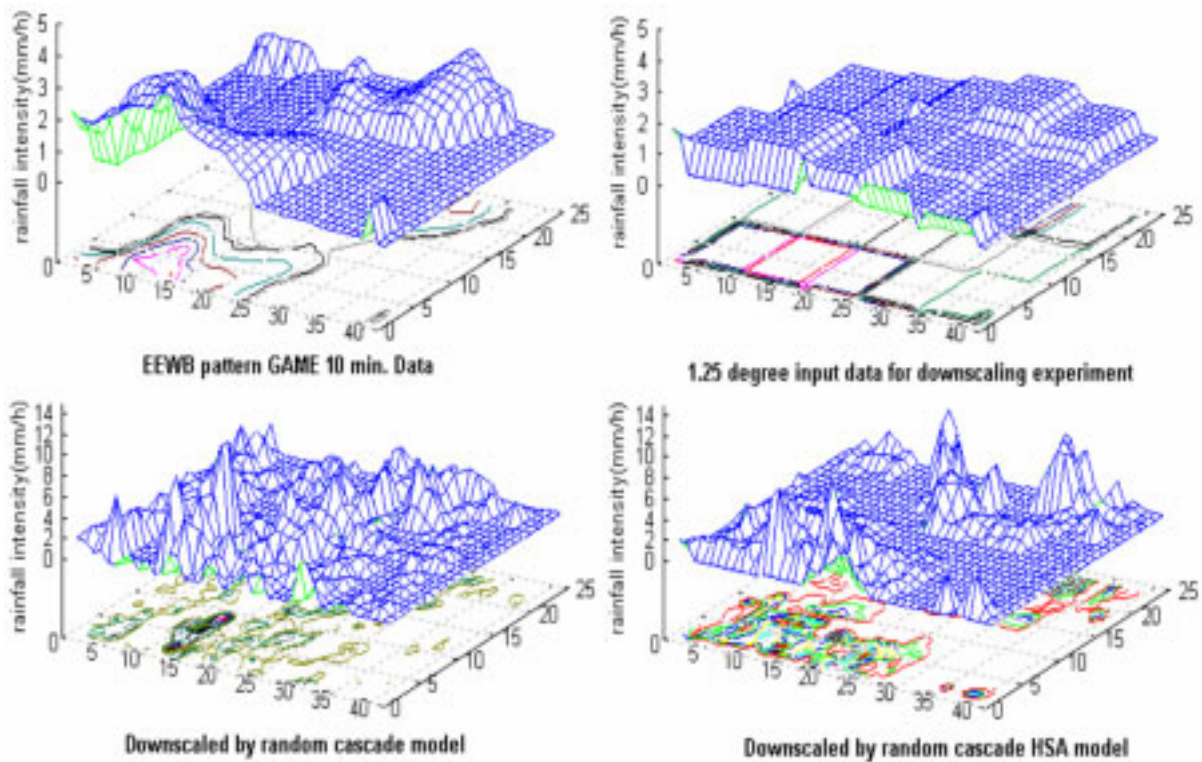


Fig. 9: Repeated realizations from the random cascade HSA method

**Table 1** Spatial correlation statistics with different disaggregation methods

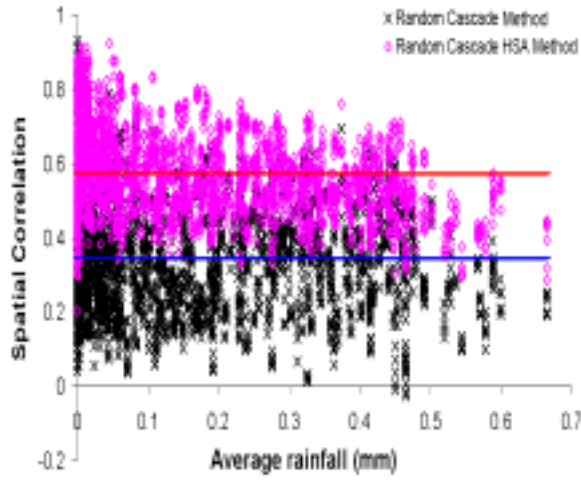
	RC	RCV	RCH	RCHV	RCHSA	RCHSV
Average	0.3447	0.3089	0.5486	0.5734	0.5717	0.6000
Maximum	0.9303	0.8937	0.9069	0.9246	0.9250	0.9255
Minimum	-0.0252	-0.0081	0.2218	0.2265	0.2026	0.2382
Std. dev.	0.1402	0.1252	0.1019	0.1035	0.1093	0.1072

RC : random cascade method; RCV : random cascade method with mass conservation; RCH : random cascade method with hierarchical adjustment; RCHV : random cascade method with mass conservation and hierarchical adjustment; RCHSA : random cascade HSA method; RCHSV : random cascade HSA method with mass conservation

**Fig. 10:** Snapshots of Input data and output data in downscaling experiment

There are 15 coarse grid spatial blocks of 1.25 degree resolution, which are downscaled to (42 X 25) grids of 10 minute resolution (see Fig. 10). Each block is downscaled separately to obtain the entire disaggregated rainfall field. The hierarchical adjustments of the cascade generators are done at every additional level and the statistical adjustment at the end level. To evaluate the performance of the generated rainfall field, a two-dimensional matrix correlation coefficient is calculated between the simulated rainfall fields and observed fields (discussed in section 4). The average of 2952 correlation coefficient between the disaggregated data

and the observed data is found to be 0.34. The realization with very low correlation coefficient, for example 0 or negative, is considered as a complete failure case to simulate the rainfall field, however very high correlation coefficient such as 0.93 is experienced as a rare success case. The random cascade HSA method is successful to increase the average correlation value level high up to 0.57 in this simulation (See Fig. 11). The hierarchical adjustment is found much effective to improve the results than the statistical adjustments and both of them are found always to improve the result (See Table 1).



**Fig. 11:** Comparison of spatial correlations between disaggregated and experimental 10 minute data before and after introduction of HSA method

### 6.1 Mass conservation in disaggregation

Though the spatial structures are reproducible by random cascade HSA method, the magnitudes of the downscaled rainfall field are variable due to presence of random generators and failing the mass to conserve. The recursive equation of limiting mass  $\mu_\infty$  by letting  $n \rightarrow \infty$  is given by Gupta and Waymire (1993),

$$\mu_\infty(\Delta_n^i) = \mu_n(\Delta_n^i) D_\infty(i), \quad (i = 1, 2, \dots, b^n), \quad (16)$$

where,  $D_\infty(i)$  is statistically independent of  $\mu_n(\Delta_n^i)$  and identically distributed as,

$$D_\infty(i) = \frac{\mu_\infty([0, L_0]^d)}{R_0 L_0^d}, \quad \text{for all } i \quad (17)$$

With the constraint  $E[W] = 1$ , it can be shown  $E[D_\infty] = 1$  with conjunction of equation (1) and (16) that verifies the interpretation of mass conservation (Over and Gupta, 1994). However, this condition does not guarantee the conservation of mass for each realization of the cascade, which is appeared in Fig. 9. It is practically impossible to attain mass conservation in every realization with present cascade formulation. Nevertheless, the total mass accumulated in space, time has significant importance in hydrological analysis, and the mass fluctuation is not preferred as it may confuse further investigation processes. The practical applications based on random cascade downscaled data may have a risk of losing ground to believe especially in rainfall quantity driven analysis if its mass fails to conserve in practice.

Alternatively, additional multiplier can be introduced to obtain path-wise mass conservation in

the form of “microcanonical” cascades (Mandelbrot, 1974). A forced mass balance is tested by introducing independent multiplier after implying cascade generators in every level. The multiplier is given by,

$$M_n^i = \frac{\mu_{n-1}(\Delta_{n-1}^i)}{\sum_{p=1}^4 \mu_n(\Delta_n^p)} \quad (18)$$

Here  $M_n^i$  is multiplier for sub-area  $\Delta_{n-1}^i$  at  $(n-1)^{\text{th}}$  level. The  $\Delta_n^p$  are further divided sub-areas inside  $\Delta_{n-1}^i$ . This forcing assures the mass conservation. A summary of result-comparison is presented in Table 1.

The mass conservation forcing has deteriorated the random cascade method in average performance but it has less standard deviation in the result that indicates a more stable disaggregation. The hierarchical adjustment is seen more effective than the statistical adjustment (see RC, RCH and RCHS values in table 1) to improve the spatial pattern and reliability. The mass conservation forcing also improves the disaggregated result significantly when it is coupled with random cascade HSA method or random cascade method with hierarchical adjustment. This may be an indication that the hierarchical adjustment is synchronous with microcanonical cascades but the mass conservation forcing may not work in other conditions.

### 6.2 Influence of downscaling on catchment scale rainfall data

The total rainfall amount inside the catchment is likely to be changed as the progression of the downscaling process. Multiplication of the random cascade generators and the initial rainfall mass successively disorders the total rainfall amount at  $n^{\text{th}}$  level of disaggregation. This phenomenon is investigated in both random cascade method and random cascade HSA method by evaluating the accumulated rainfall mass over four month period of analysis in three different sized catchments inside the study region. The catchments considered for this investigation are respectively Suiping (2,093 km<sup>2</sup>), Wangjiaba (29,844 km<sup>2</sup>) and Bengbu (132,350 km<sup>2</sup>).

To obtain the accumulated rainfall mass, the disaggregation experiment is repeated 30 times. Every realization yields a different mass of the total accumulated rainfall inside the catchments boundary even after forcing the mass conservation in both random cascade method and random cascade HSA method. The fluctuation of the accumulated rainfall over the entire analysis period is evaluated in terms of the standard deviation of the accumulated rain within the catchments coverage. The results are presented in Table 2.

**Table 2** Statistics of accumulated rainfall mass over four month period from 30 disaggregation realizations

		<b>Bengbu</b> (132,350 km <sup>2</sup> )		<b>Wangjiaba</b> (29,844 km <sup>2</sup> )		<b>Suiping</b> (2,093 km <sup>2</sup> )	
<b>Mean of accumulated rainfall in 4 months (mm)</b>							
<b>Level</b>	<b>RC</b>	<b>RCHSA</b>	<b>RC</b>	<b>RCHSA</b>	<b>RC</b>	<b>RCHSA</b>	<b>RCHSA</b>
0	807.2	807.2	901.7	901.7	901.0	901.0	
1	805.9	803.4	900.7	893.6	905.7	1019.6	
2	807.7	783.2	899.8	882.3	905.5	1011.6	
3	807.0	728.6	899.0	833.6	904.8	961.9	
<b>Standard deviation of accumulated rainfall (mm)</b>							
<b>Level</b>	<b>RC</b>	<b>RCHSA</b>	<b>RC</b>	<b>RCHSA</b>	<b>RC</b>	<b>RCHSA</b>	<b>RCHSA</b>
0	0.000	0000	0.000	0.000	0.000	0.000	
1	8.891	1.442	16.168	4.944	21.086	9.068	
2	10.264	1.550	17.391	4.110	38.380	24.921	
3	11.415	1.647	19.944	4.257	41.455	25.407	

RC : random cascade method; RCHSA : random cascade HSA method;

The obtained results in Table 2 present an interesting scenario of differences between the assumptions and reality. The random cascade method is almost successful to conserve the mean rainfall mass throughout the progress of disaggregation level displaying its strength of homogenous distribution of rainfall field in all regions because the mass is found conserved in all different sized catchments. This may not represent the ground truth. The boundaries of the catchments do not follow the rectangular grid boundaries of the current modeling. Therefore there is expected to appear changed accumulation of rainfall mass depending on the size of grids and the rainfall falling into it. Surrounding rainy or non-rainy zones, which is not included in the same catchment but included in the same grid covering over the catchment, should have shown their influence in the accumulated rainfall mass when the size of the grids are subjected to change. Since, the rainfall field is non-homogenous and anisotropy in nature, the change in accumulation of rainfall mass should appear more clearly within the catchment boundary unlike the results produced by the random cascade method. The random cascade HSA method seems able to produce the effect of anisotropy and non-homogenous rain in the accumulated mass within the catchments. There is both increase and decrease of mean accumulation, which might occur due to the changed system of grid size when viewed within a boundary of particular catchment.

The increase of standard deviation value (Table 2) as it grows the level of disaggregation indicates

that the disaggregated outputs are more uncertain at higher disaggregation level. Rapid gain of standard deviation in random cascade method depicts a lower chance of getting reliable disaggregated result. The random cascade HSA model seems to perform much better reliability because it has prevented to swell the standard deviation value drastically, which maintains very small deviation in the disaggregated results than that of the random cascade method. This phenomenon is appeared in all three catchments of different sizes.

The different standard deviation values on different size catchments represent the uncertainty level in rainfall field generation with respect to the size of the catchments. A higher value of standard deviation in smaller catchments size (see Table 2) is a clear indication of the higher uncertainty associated with the rainfall field generation in the smaller scale of catchments.

## 7. Conclusions

Existing spatial rainfall disaggregation method based on multiplicative random cascade theory is attempted to modify in order to control the random realizations. Current ability of those models to describe the spatial statistics of disaggregated rainfall field from GCM output scale to finer scales are not enough to produce practically useful results for short time scale, especially to use them in hydrologic analysis. This limitation is improved drastically by the proposed modification in the random cascade method, which incorporates the spatial correlation of

rainfall field that strongly exists at finer scale. In the proposed HSA method, the cascade of disaggregation is devised to incorporate the spatial correlation effect by assigning the cascade generators' spatial location. The controlled spatial assignment of cascade generators on the basis of correlated reference index re-arranges spatial pattern such that it is found promisingly successful to preserve the existence of hierarchical structure of rain band, which is one major component of meso scale rain cells.

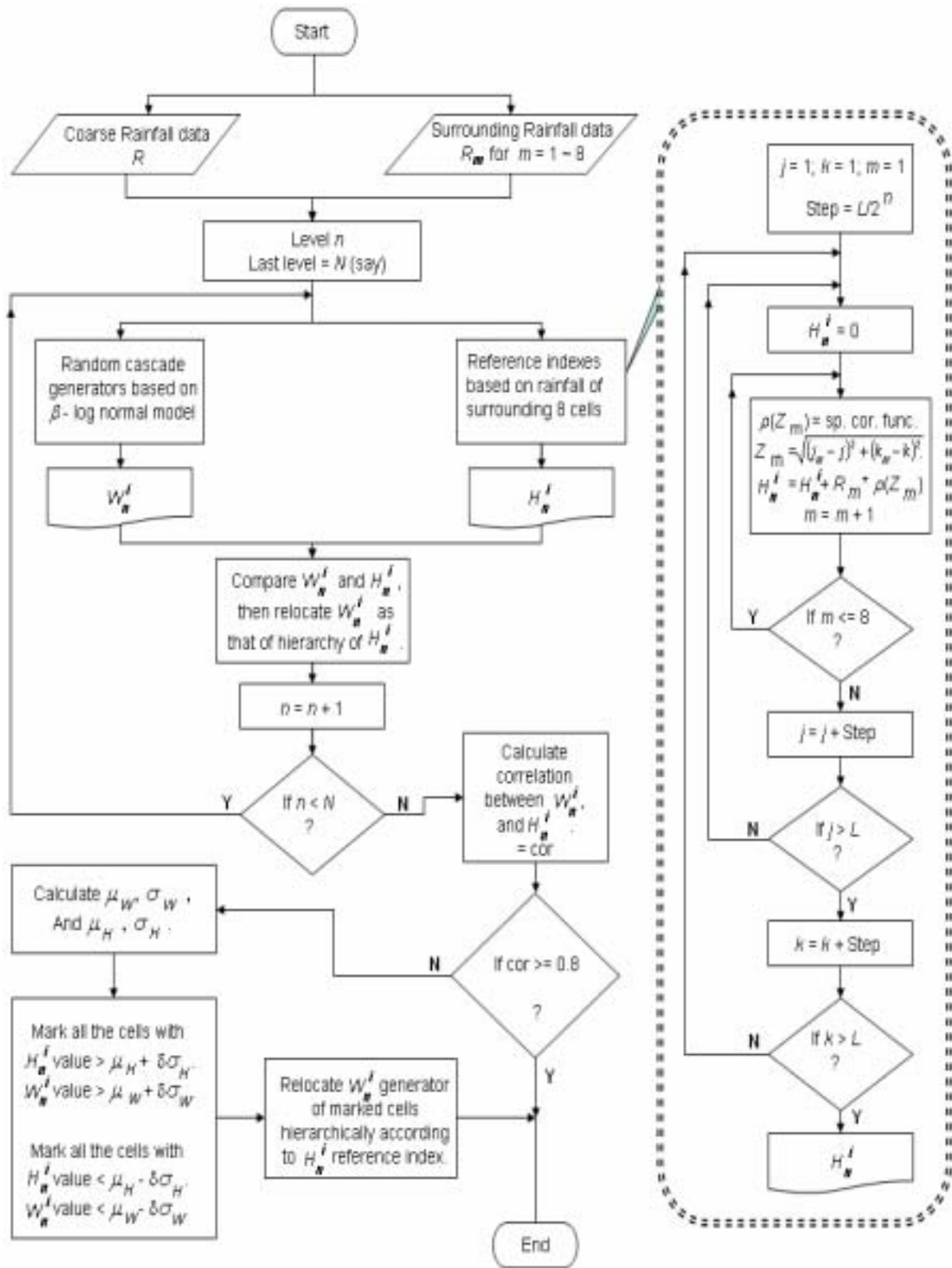
Synthetic rainfall field generation is tested over eastern Chinese region covering 560 km X 320 km area. The simulation has used 15 grids of 1.25 degree resolution to generate 10-minute spatial data for four month period at hourly time step. The random cascade HSA method is found quite successful to reproduce exactly similar spatial rainfall field pattern as that of observed pattern in every realizations separating rainy and non-rainy zones properly. The average correlation coefficient between the observed and simulated rainfall field is improved to 0.57 from 0.34 after introducing the HSA method. The hierarchical adjustment of HSA is found more effective than the statistical adjustment. The statistical adjustment is necessary to fine-tune the proper location of extreme values. A mass conservation forcing to form microcanonical cascade is found working well with the hierarchical adjustment of the cascade generator, which has further improved the average correlation coefficient to 0.60. The ability of reproducibility and accuracy of the spatial rainfall modeling opens up chances to use the synthetically generated rainfall data in practical application of shorter time scale such as rainfall-runoff analysis.

The rainfall variability of the disaggregated rainfall field within the boundary of hydrologic

catchment is found growing as the growth of disaggregation level, which is more serious for smaller sized catchments than a larger one. From the evaluation of the statistics of 30 realizations, it is found that the random cascade method is able to conserve the accumulation mass according to its initial assumption. The accumulated mass conservation of generated rainfall field is erroneous while viewed for a particular catchment in different grid size system. For the catchment, the accumulation mass is expected to fluctuate as a response of non-homogenous and anisotropic rainfall field in the course of disaggregation process, which is appeared in random cascade HSA method. In addition, the random cascade HSA method is found largely successful to control the swelling uncertainty of rainfall accumulation in small to large size catchments.

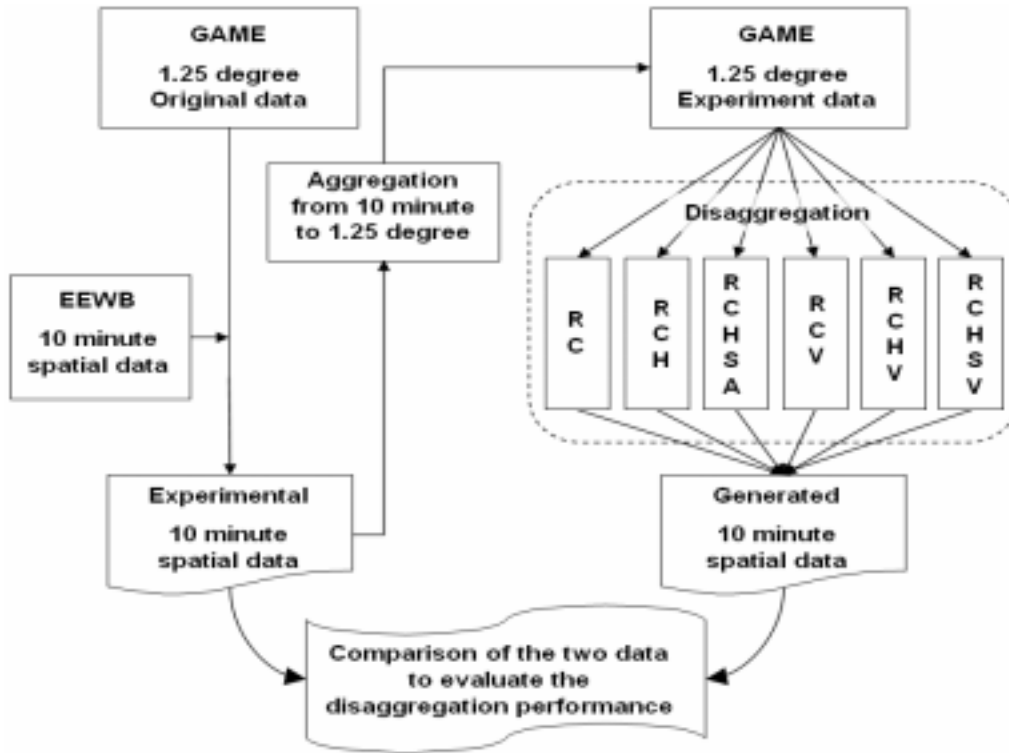
The presented method contains some limitations, for example, it can be applicable only in continuous coarse grid structure such as GCM outputs because it uses the spatial correlation information from neighboring coarse grids. So, this method may find difficulty to apply in single coarse grid disaggregation having no surrounding neighbor. Also, this method does not consider temporal correlation; however, there are many evidences of the existence of temporal correlation of rainfall field. The adaptation of beta-lognormal model may also be questioned for proper statistical representation of the rainfall field as there is no evaluation of alternative distribution even though the beta-lognormal model has produced satisfactory results in this case. Perhaps this model is suitable for the study region in particular.

# Appendix A



Flow chart of random cascade HSA method

## Appendix B



Flow chart of experimental data generation and disaggregation of rainfall fields

## Appendix C

In the statistical adjustment, the performance often depends on extreme values selection criteria. The statistical measure  $(\mu_B \pm \delta\sigma_B)$  is used as the margin to separate extreme high and low values of the distribution of  $H_n$  and  $\mu_n(\Delta_n)$  field, where,  $B$  stands for either  $H_n$  or  $\mu_n(\Delta_n)$ ;  $\mu_B$  is the mean of  $B$  field and  $\sigma_B$  is the standard deviation of  $B$  field;  $\delta$  is a coefficient to widen and narrow down the selection. Equal number of extreme high value cells filtered by a “greater than  $\mu_B + \delta\sigma_B$  condition” and equal number of extreme low value cells filtered by a “lower than  $\mu_B - \delta\sigma_B$  condition” are allowed to take part in statistical adjustment. The results are appeared well when  $\delta = 1.2$  is taken. The statistical adjustment is found less effective when the value of  $\delta$  is  $0.8 > \delta > 1.6$ . Nonetheless, the overall results are not much affected while the  $\delta$  value is ranged within  $1.0 \leq \delta \leq 1.4$  (see Table 3).

**Table 3** Sensitivity of  $\delta$  value in the performance of random cascade HSA method

	$\delta$ value used in $(\mu_B \pm \delta\sigma_B)$ condition						
	0.9	1.0	1.1	1.2	1.3	1.4	1.5
Average	0.5583	0.5677	0.5712	0.5717	0.5702	0.5698	0.5572
Maximum	0.9059	0.9289	0.9150	0.9250	0.9245	0.9170	0.9269
Minimum	0.2238	0.2126	0.2137	0.2026	0.2054	0.2133	0.2118
Std. dev.	0.1079	0.1067	0.1102	0.1093	0.1099	0.1042	0.1137

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# ランダムカスケードモデルの改良とそれを用いたグローバルスケール 降雨推定量のダウンスケーリング

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## 要 旨

非等方性を考慮したランダムカスケードモデルを開発し、それを用いてマクロスケールの降雨の空間場からより信頼性の高い詳細スケールの降雨場を生成することを目的とする。これまで、ランダムカスケードモデルを用いたダウンスケーリング手法は、長期間の平均的な降雨の空間パターンの再現には成功しているものの、ある時間の平均空間降雨強度をダウンスケールした結果は、その時間の実際の降雨の空間分布とは必ずしも対応しない。これらの欠点を克服するために、本研究では階層統計的適合ランダムカスケードモデル(Random Cascade Hierarchical and Statistical Adjustment Method)を新たに提案し、それを用いて GAME 再解析データ(1.25 度空間分解能)を 10 分空間分解能にダウンスケールすることを試みた。本手法によって得られた詳細スケールの降雨空間場は、これまでのダウンスケール手法と比べてより現実の降雨場に近く、生成される降雨場はシミュレーションごとに大きくばらつかないことを示した。

**キーワード:** ランダムカスケードモデル, ダウンスケーリング, GAME 再解析データ, HSA 法