京都大学防災研究所年報 第 52 号 B 平成 21 年 6 月
Annuals of Disas. Prev. Res. Inst., Kyoto Univ., No. 52 B, 2009

Temporal Variations in the Gutenberg-Richter distribution prior to the Kobe earthquake

Christine SMYTH and Jim MORI

Synopsis

We test if the parameters of the Gutenberg-Richter distribution vary on a short-term temporal scale in the seismically active Tamba region of Japan prior to the Kobe earthquake. Both the least squares and maximum likelihood estimates of the parameters are considered. To statistically test if the parameters differ temporally, ANOVAs and ANCOVAs are applied. The results show that the parameters vary significantly over the study region.

Keywords: Gutenberg-Richter, Kobe earthquake, Statistics

1. Introduction

The frequencies of earthquakes are linearly related to their size, with smaller magnitude earthquakes being more common. This relationship has been quantified with the Gutenberg-Richter formula:

$$\log_{10} N = a - bM. \quad (1)$$

The formula states that the logarithm of the number of earthquakes is linearly dependent on the magnitude of the earthquakes. The parameter *a* gives the total number of earthquakes and the parameter *b* generally takes a value close to one. The Gutenberg-Richter distribution applies to global catalogs of earthquakes or small geographical regions. The Gutenberg-Richter distribution estimates how many earthquakes greater than or equal to the magnitude *M* can be expected in some time period for a given region, if accurate values of the parameters *a* and *b* are known. Therefore, the Gutenberg-Richter distribution plays a major part in earthquake forecasting and subsequent earthquake hazards modeling.

The spatial variability of the parameters *a* and *b* has been studied extensively. The general consensus is that the parameters vary spatially, and that using a common value of the parameters over a large area will result in a poor fit of the model (Schorlemmer et al., 2004).

In contrast, the temporal variability of the parameters *a* and *b* has not been studied as rigorously. There are various reasons behind this incongruity. For example, some studies have shown that the temporal variability is not as great as the spatial variability (Wiemer and Wyss, 2002). Other studies have shown that the parameters' variations will “average out” over time and as earthquake forecasts are often specified for a long period of time, short-term fluctuations in the parameters are not of interest (Schorlemmer et al., 2004).

Here we statistically test if the parameters of the distribution change temporally from year to year in a specific region in Japan.

2. Data

The data used in this study came from the Tamba region of Japan. We use data collected over a 19 year period (1976-1994).The four corner vertices of the region are given in Table 1.

Table 1. Tamba region of Japan.

Latitude	Longitude
34.3691N	134.7391E
34.3691N	136.2609E
35.6310N	134.7272E
35.6310N	136.2728E

Fig. 1 shows the cumulative number of earthquakes in the region over the period of interest. There is an obvious change in slope after 1995 (following the Kobe earthquake). This change in slope may be indicative of a change in recording quality or simply the result of the Kobe earthquake triggering a greater number of earthquakes in the area. To avoid the complications resulting from this change in slope, the following analysis is performed on the data pre-1995.

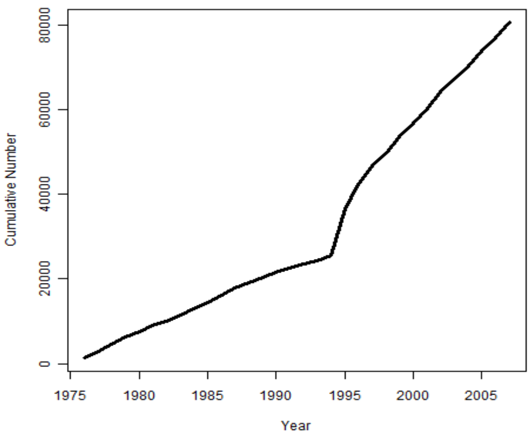

Fig. 1. Cumulative number of earthquakes.

3. Method

3.1 Minimum Magnitude of Completeness

Here, we use the method of Wiemer and Wyss (2000) to calculate the minimum magnitude of completeness of the data. Briefly, the method assumes a magnitude of completeness and calculates the maximum likelihood estimates of the Gutenberg-Richter distribution. Then, a simulated distribution is created using these estimates. Finally, the difference between the simulated number of earthquakes and the observed number of earthquakes is obtained. This information is summarized in the value R:

$$R = 100 - \left(\frac{\sum_{i=M_c^{assumed}}^{M} \left| O_i - S_i \right| \times 100}{\sum_{i=M_c^{assumed}}^{M} O_i} \right) \quad (2)$$

where O_i and S_i are the observed and simulated number of earthquakes in each magnitude bin. Here we use a bin size of 0.1. Different values of R are obtained by varying the assumed magnitude of completeness.

3.2 Gutenberg-Richter Distribution

After deciding upon the minimum magnitude of completeness, we then obtain the estimates of the parameters of the Gutenberg-Richter distribution. We use both the usual method of maximum likelihood and the relatively non-favored method of least squares. Maximum likelihood weights all earthquakes equally in the determination of the parameter values however the least squares estimates are biased towards the ends of the distribution. We believe there is merit in using the least squares estimates. Therefore, we trial both estimates here. The maximum likelihood estimate of b is given by:

$$\hat{b}_{ML} = \frac{\log(e)}{\bar{m} - m_{\min}} \quad (3)$$

where $\bar{m}$ is the mean magnitude of the data and m_{min} is the minimum magnitude of the data, that is the magnitude for which the data can be considered complete (Aki, 1965). The value m_{min} is given by m_c- Δm where Δm is the resolution of the earthquake catalogue (Guo and Ogata, 1997).

The least squares estimate of b is given by:

$$\hat{b}_{LS} = \left(\underset{\sim}{m}^T \underset{\sim}{m} \right)^{-1} \underset{\sim}{m}^T \log \underset{\sim}{n} \quad (4)$$

where the vector $\underset{\sim}{m}$ contains the magnitudes of interest, $\underset{\sim}{m} = \left(m_c, m_c + \Delta m, m_c + 2\Delta m, \ldots, m_{\max} \right)$, and the vector $\log \underset{\sim}{n}$ gives the respective log number of earthquakes in the dataset greater than or equal to these magnitudes.

The estimates, $\hat{b}_{ML}$ and $\hat{b}_{LS}$ are obtained for each year in the data to give $\hat{b}^i_{ML}$ and $\hat{b}^i_{ML}$ where $i \in 1,\ldots,19$. We perform a two-sided paired t-test to assess the equality of the maximum likelihood and least squares estimates for each year.

It is the main intent of this work to test if these estimates vary year to year. The manner in which this hypothesis is tested is different for each parameter estimation technique.

3.3 ANCOVA Tests

To test if the least squares estimates differ, an analysis of covariance (ANCOVA) is applied. The ANCOVA is a statistical test employed to test for significant differences between slopes of linear regression lines. Here, we have a regression line and associated parameters for each year of the data. The ANCOVA considers if the data should be modeled by a different regression line each year or if a common regression line over all years can be fit to the data. We present the results of fixed effects ANCOVAs here – readers interested in the difference between random effects and fixed effects are directed to Pinheiro and Bates (2004).

3.4 Likelihood Ratio Tests

To test if the maximum likelihood estimates vary, a different approach needs to be taken. A different approach is necessary because the ANCOVA tests for differences between regression lines, however we do not obtain the necessary intercept with the maximum likelihood approach. Therefore, a likelihood ratio test between the null hypothesis (a common b models the data for each year), and the alternative hypothesis (a common b cannot be used to model the data) is performed.

The test statistic can be simplified to:

$$-2\log\Lambda = 2\left(\sum_i n_i \log(b_i^*)\right) - 2n_{total}\log(b_{total}^*) \quad (5)$$

where b_i^* is the value $\hat{b}^i_{ML}/\log(e)$ for the i^{th} year in the series, and the test statistic is distributed asymptotically as chi-square. For further explanation of the derivation of general test statistics, the reader is directed to Hogg et al. (2005). The likelihood ratio test is designed to test if the simple model, with one common b across years, is sufficient to model the data, and its intent is exactly the same as the ANCOVA's.

4. Results

The estimates for the minimum magnitude of completeness are shown in Fig. 2. A line is drawn at the value 10, below this line more than 90% of the data are explained by the assumed magnitude of completeness. We stress that this is purely a subjective value, some investigators may believe a value of 90% is too stringent, some may believe it is too low. Due to these differing opinions, and as it is the major parameter of this study, we trial other minimum magnitude of completeness values in our analysis.

We also show in Fig. 3 an estimate of minimum magnitude of completeness for a window of 500 earthquakes (grey line). The magnitude of completeness is calculated for the first 500 earthquakes in the dataset, then the window is shifted by 10 earthquakes, and the magnitude of completeness calculated and so on (grey line). The darker line shows a moving average of the lighter grey line (Fig. 3). The average always stays below 1.1. Therefore, we trial magnitudes of completeness of 1.1, 1.3, and a more common value of 2 for this analysis.

The results of interest are shown in Table 2. The first column shows the trialed minimum magnitude of completeness. For each minimum magnitude of completeness of the data we also trial three values of $m_{\max}$ in (4). The least squares estimates are tested with a fixed effects model. The p-value within the table shows the result of the test of the null hypothesis: the simple model (with a common b across years) is sufficient; versus the alternative hypothesis: the complex model (with different b for each year) better fits the data. A p-value of less than 0.05 is considered significant. It may be argued that because we are testing numerous hypotheses on the data, we should employ a Bonferroni correction. The Bonferroni correction requires that we test at α / n significance level, where n is the number of tests being performed. Here we indicate if the result of the test changes (becomes insignificant) after taking into account the Bonferroni correction with an asterisk and displaying the p-value. In the text however, we do not consider the Bonferroni correction results.

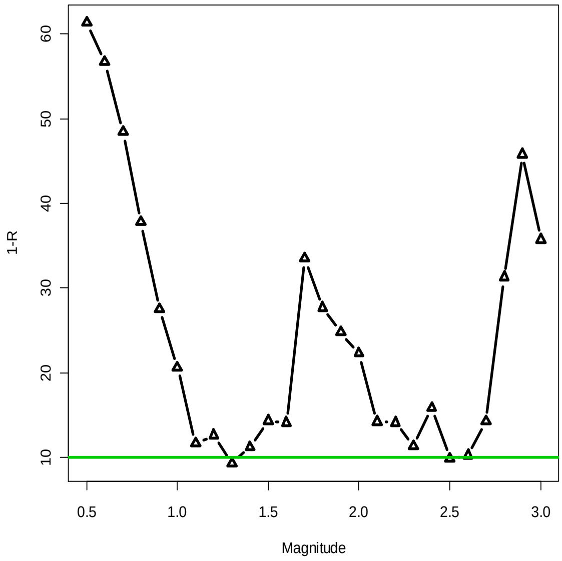

Fig. 2. Minimum magnitude of completeness of the data pre-1995

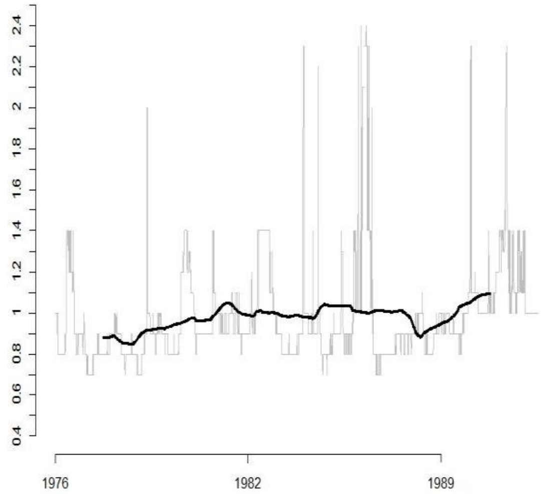

Fig. 3. Moving average of the minimum magnitude of completeness for the data pre-1995

The results show that irrespective of minimum magnitude of completeness and maximum magnitude, the data are better represented by different parameters at each year, when considering the least squares estimates.

To further illustrate the differences in the least squares estimates of the parameters over the years we show the fit to the data with a single *b* value (the same for every year) (Fig. 4) versus a different *b* value for each year of the data (Fig. 5). Allowing the *b* value to vary over the years obviously gives a better fit to the data.

Table 2. Results of pre-1995 data.

Minimum Magnitude of Completeness	Least Squares		Maximum Likelihood		T-test
	M_{max}	Fixed Effects ANCOVA	Ratio Test	Fixed Effects ANOVA	
1.1	5	$p<0.05$	$p<0.05$	$p<0.05$	Different $p<0.05$
	6	$p<0.05$			Different $p<0.05$
	7	$p<0.05$			Different $p<0.05$
1.3	5	$p<0.05$	$p<0.05$	$p<0.05$	Different $p<0.05$
	6	$p<0.05$			Different $p<0.05$
	7	$p<0.05$			Different $p<0.05$
2.0	5	$p<0.05$	$p>0.05$	$p<0.05$	Not Different $p>0.05$
	6	$p<0.05$			Not Different $p>0.05$
	7	$p<0.05$			Not Different $p>0.05$

The maximum likelihood estimates tell a similar story. The likelihood ratio statistic tests if a common b can be used to model the data. Here the results show evidence that a common b cannot be used to model the data for lower minimum magnitudes of completeness.

We investigate the maximum likelihood estimates further using an ANOVA. To do so, we obtain a bootstrap distribution of b for each year (Schorlemmer *et al.*, 2003). The distribution is obtained by sampling, with replacement, from the data at each year to create a bootstrap dataset the same size as the original dataset. The maximum likelihood estimate of this bootstrap dataset is obtained. The process is repeated here 1000 times. We then possess 1000 bootstrap estimates for the maximum likelihood estimate of b at every year. We can obtain confidence intervals of these estimates using the quantiles of the distribution to depict graphically the differences in parameter values. The confidence intervals are shown in Fig. 6 (for a minimum magnitude of completeness of 1.3). The green line depicts the common b over all years.

We then perform an ANOVA over the bootstrap estimates across all years. An ANOVA tests for significant differences between means, here we test for differences in mean b across all years. As expected, the ANOVA shows that there are in fact differences in the mean b across years.

Interestingly, the least squares and maximum likelihood estimates differ for each year for the lower minimum magnitudes of completeness. However, as the minimum magnitude of completeness increases to 2, the least squares estimates and maximum likelihood estimates are identical. If the minimum magnitude of completeness is low, there are more data to use to fit the linear relationship. Therefore, the least squares estimates, strongly influenced by the extremes of the data are likely to be different from the maximum likelihood estimates. However, if the minimum magnitude of completeness is raised, there are less data, and the extremes of the magnitudes are not as varied and therefore the least squares estimates are likely to be indistinguishable from the maximum likelihood estimates.

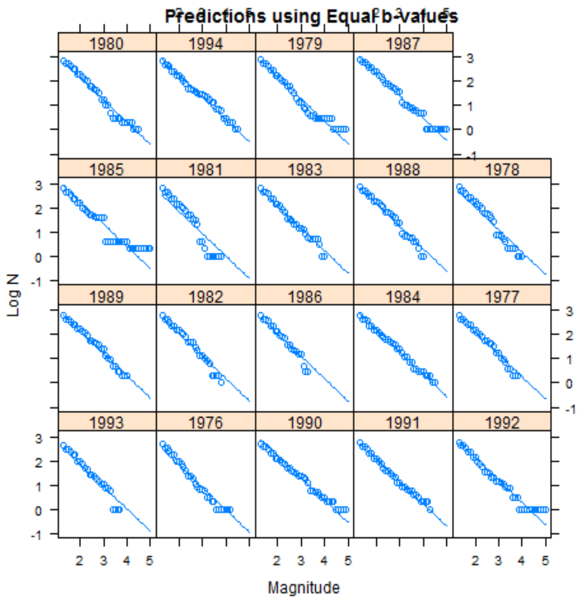

Fig. 4. Linear fit using equal parameter values.

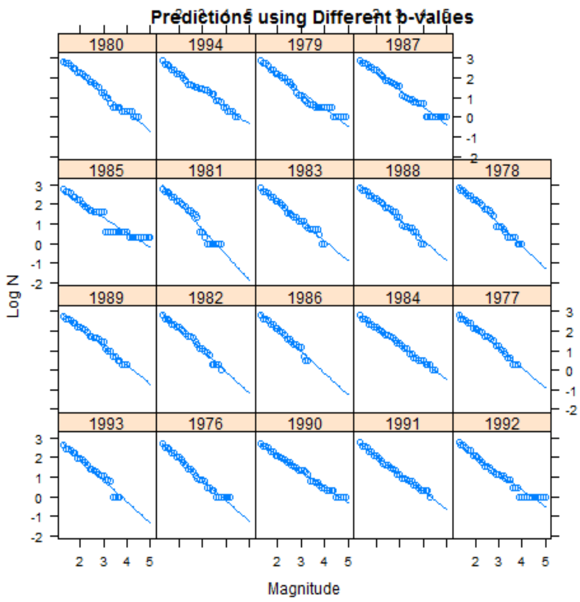

Fig. 5. Linear fit using different parameter values.

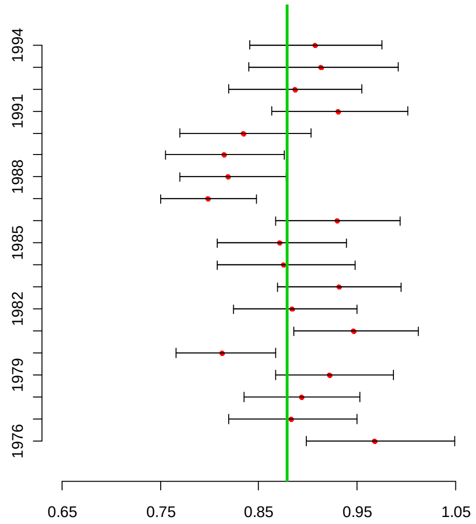

Fig. 6. Confidence intervals for the maximum likelihood estimates for the data pre-1995.

5. Conclusions

We have shown that the parameters of the Gutenberg-Richter distribution vary temporally in the Tamba region of Japan. The models with temporally-variant parameter values described the data significantly better than a temporally-invariant model, irrespective of reasonable minimum magnitudes of completeness values of the data. It is not always true that the maximum likelihood estimates and the least squares estimates agree from year to year. This is important, as it suggests that there may be valuable information in the least squares estimates of the parameters, particularly if the investigator is interested in modeling the extremes of the distribution.

Acknowledgements

The authors are grateful to Professor Katao (DPRI, Kyoto University), who made the data available.

Christine Smyth is a recipient of a Japan Society for the Promotion of Science Post Doctoral Research Fellowship.

References

Aki, K. (1965): Maximum Likelihood Estimate of b in the Formula log N = a-bM and its Confidence Limits, Bulletin of the Earthquake Research Institute, Vol. 43, pp. 237-239.

Guo, Z. and Ogata, Y. (1997): Statistical relations between the parameters of aftershocks in time, space and magnitude, Journal of Geophysical Research, Vol. 102, pp. 2857-2873.

Hogg, R. V., McKean J. W. and Craig, A. T. (2005): Introduction to Mathematical Statistics, Upper Saddle River, Pearson Prentice Hall.

Pinheiro, J. C. and Bates, D. M. (2004): Mixed-effects models in S and S-PLUS, Statistics and Computing, New York, Springer Science.

Schorlemmer, D., Neri, G., Wiemer, S. and Mostaccio, A. (2003): Stability and significance tests for b-value anomalies: Example from the Tyrrhenian Sea, Geophysical Research Letters, Vol. 30, No. 16, pp. 1835.

Schorlemmer, D., Wiemer, S. and Wyss, M. (2004): Earthquake statistics at Parkfield: 1. Stationarity of b values, Journal of Geophysical Research Vol. 109, pp. B12307.

Wiemer, S. and Wyss M. (2000): Minimum Magnitude of Completeness in Earthquake Catalogs: Examples from Alaska, the Western United States, and Japan, Bulletin of the Seismological Society of America, Vol. 90, No. 4, pp. 859-869.

Wiemer, S. and Wyss M. (2002): Mapping spatial variability of the frequency-magnitude distribution of earthquakes, Advances in Geophysics, Vol. 45, pp. 259-301.

兵庫県南部地震の前にGutenberg-Richter の地震の頻度分布の変異

Christine SMYTH・Jim MORI

要　旨

兵庫県南部地震の前、丹波地方に焦点を合わせ地震活動の変異を調査した。最尤推定と最小２乗を使用し，Gutenberg-Richterの地震の頻度分布のパラメーターを推定した。分散分析と共分散分析を適用し，これらのパラメーターは毎年変わるという有意な結果が出た。

キーワード：兵庫県南部地震，頻度分布，統計