Two and Three Dimensional Slope Stability Analysis of Landslide Dam Failure due to Sliding

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Synopsis

Water flow in unsaturated porous media is water-air two-phase flow problem. To incorporate the effect of pore air flow in the soil seepage analysis a water-air two-phase flow model, based on the multi-phase flow theory is used. Slope stability analysis of landslide dam has been carried out using pore water pressure and moisture content calculated by only a conventional water-phase flow model as well as the water-air two-phase flow model so as to predict the failure of dam due to sudden sliding. Janbu’s simplified method as well as extended Spencer method based on limiting equilibrium considerations is used in slope stability analysis. Overall simulation results obtained by considering two-phase seepage flow combined with extended Spencer method of slope stability analysis are comparatively in good agreement with the experimentally observed results.

Keywords: two-phase flow, variably saturated soil, landslide dam, slope stability

1. Introduction

Landslide dams (i.e., the natural blockage of river channels by hill slope-derived mass movements, Costa and Schuster, 1988) are natural phenomena with great relevance on geo-morphological conditions and on the safety of people. In areas placed upstream, respect to the river dammed section, waters blocked by the dam may provoke floods spreading for kilometers, causing damage to human activities and interrupting communication lines. Catastrophic outburst floods and/or debris flows can be triggered by a rapid dam failure with exceptional rates of sediment erosion and deposition along the downstream part of the valley.

Landslide dams commonly form in mountainous areas of high relief (Costa and Schuster, 1988), where there is sufficient input from both tectonic (earthquakes, hill slope gradients, relief) and climatic (precipitation, snowmelt) controls. About 90% of some 390 landslide dams examined worldwide were triggered by either rainstorms/snowmelts or earthquakes (Schuster, 1993), although other less common causes, such as volcanic (e.g., Umbal and Rodolfo, 1996; Melekestsev et al., 1999) or even anthropogenic activity (e.g., Asanza et al., 1992), have been documented.

Landslide dams fail by a variety of processes including overtopping, piping, overtopping by a landslide-generated wave, slope failure of upstream or downstream face, and the effects of human activity, usually an attempt to excavate a spillway over the debris dam. Many landslide dams fail shortly after their formation (Costa and Schuster, 1988). Figure 1 shows the dam longevity curve constructed considering sample of 73 documented
landslide dam failures; 27 percent of the landslide dams failed less than one day after formation, while 85 per cent within a year. Schuster (1993, 1995) argued that about 55% of some 187 investigated examples worldwide had failed within one week of their formation, whereas 89% failed after one year.

Landslide dam failure has been frequently studied as an earthen dam failure despite of their differences in geometry, dimensions and material properties. In depth knowledge of the failure mechanisms of the landslide dam and measured data are still being lacked. Numerical models from international literature allow to roughly computing the hydrograph resulting from the dam failure, (Fread 1991; Giuseppetti and Molinaro, 1989; Macchione and Sirangelo, 1988) however not giving any indications regarding the whole dam stability. Very few models are developed for landslide dam failure that can treat both the sediment and debris flow. Most of the existing models are applicable to overtopping failure of landslide dam. A numerical model has been developed (Awal et al., 2008) that can predict the time at which landslide dam may fail and also detect the failure mode either due to overtopping or due to sliding as well as the resulting out flow hydrograph.

2.1 Seepage flow Model

Seepage analysis may be required in volume change prediction, ground water contamination control, slope stability analysis, and the design of earth structures such as dykes or dams. To develop the seepage flow model, the governing equations are solved by line successive over relaxation (LSOR) scheme used by Freeze (1971, 1978) by an implicit iterative finite difference scheme.
(1) Single-phase flow model

Awal (2008) has used the following two and three dimensional pressure based Richards’ equations to calculate the change in pore water pressure in variably saturated soil.

Two dimensional equation:

\[ C \frac{\partial h}{\partial t} = \frac{\partial}{\partial x} \left( K_x \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left( K_y \frac{\partial h}{\partial y} \right) + \frac{\partial}{\partial z} \left( K_z \frac{\partial h}{\partial z} + 1 \right) \] (1)

Three dimensional equation:

\[ C \frac{\partial h}{\partial t} = \frac{\partial}{\partial x} \left( K_x \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left( K_y \frac{\partial h}{\partial y} \right) + \frac{\partial}{\partial z} \left( K_z \frac{\partial h}{\partial z} + 1 \right) \] (2)

where, \( h \) is the water pressure head; \( K_x, K_y \) and \( K_z \) are the hydraulic conductivity in x, y and z directions respectively; \( C = \partial \theta / \partial h \) is the specific moisture capacity, \( \theta \) is the soil volumetric water content; \( t \) is the time; \( x \) and \( y \) are the horizontal spatial coordinates; and \( z \) is the vertical spatial coordinate taken as positive upwards.

In order to solve the equations (1) and (2) following constitutive relationships proposed by Van Genuchten (1980) are used for establishing relationship of moisture content and water pressure head (\( \theta-h \)), and unsaturated hydraulic conductivity and moisture content(K-\( \theta \)):

\[ S_e = \left[ 1 + \left( \frac{\theta - \theta_r}{\theta_s - \theta_r} \right)^n \right]^{-m} \] (3)

\[ S_r = \frac{\alpha - \theta_r}{\theta_s - \theta_r} \] (4)

\[ K = K_s S_e^{0.5} \left[ 1 - \left( 1 - S_e^{1/m} \right)^2 \right] \] (5)

where, \( S_e \) is the effective saturation; \( \alpha \) and \( n \) are empirical parameters; \( \theta_s \) and \( \theta_r \) are saturated and residual moisture content respectively; \( K_s \) is the saturated hydraulic conductivity; and \( m = 1/\eta \).

(2) Two-phase flow model

Water flow in variably saturated soils is two-phase flow problem. The expedient approximation that air pressure is constant leads to the Richards’ equation. To fully and accurately model variably saturated soils, both the air and water phases should be treated separately, with pressure and flow of both of these phases tracked within the model. In this study following two and three-dimensional equations are derived for the simultaneous flow of water and air based on the one-dimensional flow equations derived by Touma et al. (1986):

Two dimensional equations can be described as follows.

Water-phase equation is

\[ C \left( \frac{\partial h_w}{\partial t} - \frac{\partial h_w}{\partial t} \right) = \frac{\partial}{\partial x} \left( K_{wx} \frac{\partial h_w}{\partial x} \right) \] (6)

Air-phase equation is

\[ \left( \alpha - \theta \right) \left( \frac{\partial \rho_a}{\partial h} \right) = \frac{\partial}{\partial x} \left( \rho_a K_{ax} \frac{\partial h_a}{\partial x} \right) + \frac{\partial}{\partial y} \left( \rho_a K_{ay} \left( \frac{\partial h_a}{\partial y} + \frac{\partial \rho_a}{\partial h} \right) \right) \] (7)

Three dimensional equations can be described as follows.

Water-phase equation is

\[ C \left( \frac{\partial h_w}{\partial t} - \frac{\partial h_w}{\partial t} \right) = \frac{\partial}{\partial x} \left( K_{wx} \frac{\partial h_w}{\partial x} \right) + \frac{\partial}{\partial y} \left( K_{wy} \frac{\partial h_w}{\partial y} \right) + \frac{\partial}{\partial z} \left( K_{wz} \frac{\partial h_w}{\partial z} + 1 \right) \] (8)

Air-phase equation is

\[ \left( \alpha - \theta \right) \left( \frac{\partial \rho_a}{\partial h} \right) = \frac{\partial}{\partial x} \left( \rho_a K_{ax} \frac{\partial h_a}{\partial x} \right) + \frac{\partial}{\partial y} \left( \rho_a K_{ay} \left( \frac{\partial h_a}{\partial y} + \frac{\partial \rho_a}{\partial h} \right) \right) + \frac{\partial}{\partial z} \left( \rho_a K_{az} \left( \frac{\partial h_a}{\partial z} + \frac{\partial \rho_a}{\partial h} \right) \right) \] (9)
where, $h_w$ is the water pressure head; $h_a$ is the air pressure head; $h_0$ is the atmospheric pressure expressed in terms of water column height; $C = \partial \theta / \partial h$ is the specific moisture capacity, $h_e = h_a - h_w$ is the; $\rho_a$ is density of air; $\rho_{oa}$ is density of air at the atmospheric pressure; $\rho_{ow}$ is density of water at the atmospheric pressure; $K_{wx}, K_{wy}$ and $K_{wz}$ are the hydraulic conductivity in $x$, $y$ and $z$ directions respectively; $K_{ax}, K_{ay}$ and $K_{az}$ are the air conductivity in $x$, $y$ and $z$ directions respectively; $C = \partial \theta / \partial h$ is the specific moisture capacity. $\theta$ is the porosity of soil; $\Omega$ is the soil volumetric water content; $t$ is the time; $x$ and $y$ are the horizontal spatial coordinates; and $z$ is the vertical spatial coordinate taken as positive upwards.

In order to solve the equations (6), (7), (8) and (9) following constitutive relationships proposed by Van Genuchten (1980) are used:

$$S_e = [1 + (\alpha_a h_c)^{\eta_a}]^{-m_a} \quad (10)$$

$$S_e = \frac{\theta - \theta_r}{\theta_s - \theta_r} \quad (11)$$

$$K_w = K_{wx} S_e^{0.5} \left[ 1 - \left( 1 - S_e \frac{1}{m_a} \right)^{m_a} \right]^2 \quad (12)$$

$$K_a = K_{ax} (1 - S_e)^{0.5} \left[ 1 - \left( 1 - S_e \frac{1}{m_a} \right)^{2m_a} \right] \quad (13)$$

where, $S_e$ is the effective saturation; $\alpha_a$ and $\eta_a$ are empirical parameters; $m_a = 1 - 1/\eta_a$; $\theta_s$ and $\theta_r$ are saturated and residual moisture content respectively; $K_{wx}$ is the saturated hydraulic conductivity; $K_{ax}$ is the saturated air conductivity.

$$K_{ax} = K_{wx} \frac{\mu_w}{\mu_a} \quad (14)$$

where, $\mu_w$ and $\mu_a$ are dynamic viscosity of water and air. $\mu_w = 1.002 \times 10^{-3}$ NS/m² and $\mu_a = 1.83 \times 10^{-5}$ NS/m² at 20°C.

2.2 Slope Stability Model

The stability of a slope depends on its geometry, soil properties and the forces to which it is subjected to internally and externally. The numerous methods currently available for slope stability analysis provide a procedure for assigning a factor of safety to a given slip surface, but do not consider the problem of identifying the critical conditions. Limit equilibrium method of slices is widely used for slope stability analysis due to its simplicity and applicability. In the method of slices, the soil mass above the slip surface is divided into a number of vertical slices and the equilibrium of each of these slices is considered. The actual number of the slices depends on the slope geometry and soil profile. The limiting equilibrium consideration usually involves two steps; one for the calculation of the factor of safety and the other for locating the most critical slip surface which yields the minimal factor of safety. Methods by Bishop, Janbu, Spencer and Morgenstern and Price are now well known.

In this study Janbu’s simplified method as well as Spencer method has been incorporated into an effective minimization procedure based on dynamic programming by which the minimal factor of safety and the corresponding critical non circular slip surface are determined simultaneously. Janbu’s simplified method only satisfies force equilibrium for the entire sliding mass and assumes resultant inter-slice forces horizontal where as Spencer/extended Spencer method satisfies both the force and moment equilibrium and assumes resultant inter-slice forces are at some angle to the horizontal. Fig. 2 shows the two dimensional general slip surface and forces acting on a typical slice and Fig. 3 shows the three dimensional general slip surface and forces acting on a typical column.

2-D and 3-D expressions of factor of safety $F_s$ for Janbu’s simplified method are as follows:

For 2-D Case;

$$F_s = \frac{1}{\sum_{i=1}^{n} W_i \tan \alpha_i} \times \sum_{i=1}^{n} \left( c l_i \cos \alpha_i + (W_i - u_i l_i \cos \alpha_i) \tan \theta \right) \cos^2 \alpha_i \left( 1 + \frac{1}{F_s} \tan \alpha_i \tan \theta \right) \quad (15)$$

where $W_i$ is the weight of each slice including surface water, $l_i$ is the length of the base of each slice, $u_i$ is the average pore water pressure on the base of the slice, $\alpha_i$ is the inclination of the base to the horizontal, $n$ is the total number of slices, and $c$ and $\theta$ are the Mohr-Coulomb strength parameters.

For 3-D Case;
respectively are as follows:

\[
F_s = \sum_{i=1}^{m} \sum_{j=1}^{n} \frac{[(c - u_j \tan \phi) \Delta x + (W_j + P_{ij}) \tan \phi]}{[(f + f_{sz} \tan \phi \cos \alpha_{szj})]^2} \sum_{j=1}^{n} (W_j + P_{ij}) \tan \alpha_{szj}
\]

(16)

where, \( J = \left(1 + \tan^2 \alpha_{szj} + \tan^2 \alpha_{zyj}\right)^{-\frac{1}{2}} \), \( W_{ij} \) is the weight of column, \( P_{ij} \) is the vertical external force acting on the top of the column, \( \Delta x \) and \( \Delta y \) are discretized widths of the columns in x and y directions respectively, \( \alpha_{szj} \) and \( \alpha_{zyj} \) are the inclination angles of the column base to the horizontal direction in the xz and yz planes respectively, \( m \) and \( n \) are the total number of columns in x and y directions respectively, and \( c \) and \( \phi \) are the Mohr-Coulomb strength parameters.

The factor of safety expressions for Spencer method (Bardet et al., 1989) and extended Spencer method (Jiang et al., 2004) in cases of 2-D and 3-D respectively are as follows:

For 2-D Case:

\[
F_f = \sum_{i=1}^{n} \frac{\{c_{ij} + (W_{i} \cos \alpha_i - u_{ij}) \tan \phi\}}{m_{\alpha}}
\]

\[
F_m = \sum_{i=1}^{n} \frac{\{c_{ij} + (W_{i} \cos \alpha_i - u_{ij}) \tan \phi\}}{m_{\alpha}}
\]

\[
\times \left(\frac{R_i \cos(\theta_i - \delta)}{m_{\alpha}}\right)
\]

(17)

(18)

where, \( F_f = \) factor of safety with respect to force equilibrium, \( F_m = \) factor of safety with respect to moment equilibrium, \( m_{\alpha} = \cos(\alpha_i - \delta) \left(1 + \frac{1}{f_i} \tan(\alpha_i - \delta) \tan \phi\right) \), \( \delta = \) inclination of inter-slice forces to the horizontal, \( R_i = \) distance from the base centre of the slice to an arbitrary reference point, and \( \theta_i = \) angle between the vertical direction and the \( R_i \) direction.

\( F_i \) and \( F_m \) can separately be computed from the equations (17) and (18) for several appropriately given values of \( \delta \). Then, two curves showing the relationships of \( F_i - \delta \) and \( F_m - \delta \) can be plotted so that the intersection of these two curves leads to a required value of factor of safety \( F_s \), satisfying both force and moment equilibrium.

where, \( m_{\alpha} = \left(1 + \tan \delta \tan \alpha_{szj}\right)/\left(1 + \sin \alpha_{szj} - \tan \delta \cos \alpha_{szj}\right) \tan \phi / F \) with \( F = F_f \) for Eq. (19) and \( F_m \) for Eq. (20), \( \delta = \) inclination of inter-slice forces to the x-axis, \( R_{ij} = \) distance from the axis of rotation to the base centre of a column in xz plane, and \( \theta_{ij} = \) angle between the horizontal direction and the \( R_{ij} \) direction in the xz plane.

Factor of safety \( F_s \), satisfying both force and moment equilibrium can be computed as mentioned in the case of 2-D.

Fig. 2 Two dimensional general slip surface and forces acting on a typical slice

(a) Sliding mass and vertically divided slices

(b) Forces acting on the slice i
Fig. 3 Three dimensional general slip surface and forces acting on a typical column

For 3-D Case:

\[ F_f = \sum_{i=1}^{m} \sum_{j=1}^{n} \left( c - u_{ij} \tan \theta \right) \sec \alpha_{xij} \Delta x \Delta y + \left( W_{ij} + P_j \right) \tan \phi + F_f \tan ^2 \alpha_{xij} \right] / m_n \]

\[ F_m = \sum_{i=1}^{m} \sum_{j=1}^{n} \left( c - u_{ij} \tan \theta \right) \sec \alpha_{xij} \Delta x \Delta y \left( \sin (\Theta_i + \delta) / \cos \delta + (W_{ij} + P_j) \right) \cos (\Theta_i + \alpha_{xij}) \left\{ \tan \phi \tan (\Theta_i + \alpha_{xij}) + F_m \sec \alpha_{xij} / J \right\} / m_n \]

\[ \sum_{i=1}^{m} \sum_{j=1}^{n} \left( W_{ij} + P_j \right) R_i \cos \Theta_i \]

(19)

(20)

3. Experimental Test

Experimental results obtained by Awal (2008) are compared with the simulation results so as to evaluate the capability of the model. In his study (Awal, 2008), he considered constant water level and the steady discharge in the upstream reservoir in case of two-dimensional experiments and steady discharge in the upstream reservoir in case of three-dimensional experiments.

In case of two-dimensional experiments a rectangular flume of length 5m, width 20cm and depth 21cm was used. The slope of the flume was set at 17°. A rectangular flume of 5m long, 30cm wide and 50cm deep was used in case of three-dimensional experiments. The slope of the flume was set at 20°. The rectangular shape of the flume was modified to make cross slope of 20° so that the height of the dam is 30cm in one side and decreased uniformly towards other side to 19.08cm.

Triangular dam was prepared on the rigid bed of flume by placing mixed silica sand (Mix 1-7) on the flume. Water content reflectometers (WCRs) were used to measure the temporal variation of moisture content during seepage process. Red colored sediment strips were placed in the dam body at the flume wall face so as to measure the movement of the dam slope during sliding. The two-dimensional and three-dimensional landslide dams with the arrangement of WCRs are schematically shown in Figure 4 and Figure 5 respectively.

Fig. 4 Two dimensional dam body shape and size with the arrangements of WCRs (1-9)
4. Results and Discussions

In two dimensional analysis numerical simulation has been carried out with space step of 2 mm and time step of 0.004 second in seepage flow model and longitudinal space step (parallel to dam base) of 6 cm and time step of 1 second in slope stability model. In three dimensional analysis numerical simulation has been carried out with space step of 1 cm and time step of 0.01 second in seepage flow model and longitudinal space step of 5 cm, lateral space step of 3 cm and time step of 1 second in slope stability model.

In case of 2-D experiment with constant water level in the upstream reservoir, water level at 16 mm below the crest level of the dam was maintained in 25 seconds and the sudden sliding of the dam was observed at 255 seconds. The simulated failure time using Janbu’s simplified method of slope stability analysis is 222 and 231 seconds in conventional and water-air two-phase seepage flow considerations respectively where as the failure time using Spencer method of slope stability analysis is 323 and 322 seconds in conventional and water-air two-phase seepage flow considerations respectively. Figure 6 shows the simulated and experimental moisture content profile at WCR-4, WCR-6 and WCR-8 and Figure 9 shows the simulated and experimental slip surfaces.

In case of 3-D steady discharge in the upstream reservoir, 29.8 cm³/sec discharge was supplied and the sudden sliding of the dam was observed at 930 seconds. The simulated failure time using Janbu’s simplified method of slope stability analysis is 768 and 777 seconds in conventional and water-air two-phase seepage flow considerations respectively where as the failure time using extended Spencer method of slope stability analysis is 793 and 802 seconds in conventional and water-air two-phase seepage flow considerations respectively. Figure 10 shows the simulated and experimental moisture content profile at WCR-2, WCR-4 and WCR-8 and Figure 11 shows the simulated and experimental slip surfaces.

Simulated and experimental moisture profiles are satisfactorily in agreement although results comparison shows some simulated results have faster moisture movement than that of experimental one and some have slower movement. Also the simulated failure shapes are matching satisfactorily with experimental failure shapes of all considered cases. From the view point of safety before and
Saturated hydraulic conductivity $K_s$ is a key parameter for guiding the moisture profile and failure time of dam body which depends on the sand mix and its compaction. It is difficult to ensure uniformity of sand mix and its compaction during $K_s$ test and formation of experimental dam body. So it is better to have record the temporal variation of upstream reservoir head during experiment so that $K_s$ value can be played during simulation to get more accurate simulated reservoir head that may lead simulated moisture movement and dam failure time quite closer to experimental one.

Actually air become trapped in the voids by the infiltrating water from the upstream reservoir, initially causing compression of the air phase, leading to a reduction in the rate of water infiltration. The air pressure will increase until it reaches a sufficient value for the air to escape by bubbling. This phenomenon has not been
Fig. 10 Simulated and experimental moisture content profiles for steady discharge in upstream reservoir (3D case)

Fig. 11 Simulated and experimental critical slip surfaces for steady discharge in upstream reservoir (3D case)

Fig. 12 Typical relationships of $F_m$-$\delta$ and $F_f$-$\delta$

considered in conventional seepage flow model but the water-air two-phase flow model has been considered it. Hence simulated moisture profiles obtained by the two-phase seepage flow model are slightly delayed than that of conventional seepage flow model. Ultimately, failure time of the dam body has also found to be delayed while considering two-phase seepage flow in Janbu’s simplified method as well as Spencer/extended Spencer method in the cases of 2-D constant head and 3-D steady discharge. Since there was flow path for the air phase in the dam body laterally.
toward downstream and vertically up beyond the reservoir water ponding reach, the difference of predicted failure times considering only one-phase and two-phase seepage flow is not so much large.

It is already mentioned in section 2.2 that Janbu’s simplified method only satisfies force equilibrium for the entire sliding mass and assumes resultant interslice forces horizontal where as Spencer/extended Spencer method satisfies both the force and moment equilibrium and assumes resultant interslice forces are at some angle $\delta$ to the horizontal. Hence the factor of safety calculated by the Janbu’s simplified method is less than that of Spencer/extended Spencer method in a given condition, resulting failure time of the dam body earlier in case of Janbu’s simplified method. Choosing $\delta$ values from $0^\circ$~$40^\circ$ at $5^\circ$ intervals, required $\delta_a$ value was evaluated for calculating factor of safety $F_s$ using Spencer/extended Spencer method as mentioned in section 2.2. The $F_m-\delta$ and $F_f-\delta$ relationships obtained during simulation of 3D case, considering two-phase flow seepage analysis, are typically shown in Fig. 12. Required $\delta_a$ along with the corresponding $F_s$ is also presented in the same figure.

5. Conclusions

The comparison of experimental and simulated results shows that the results obtained by water-air two-phase flow model and extended Spencer method of slope stability analysis is more closer to the experimental results than that of conventional seepage flow model and Janbu’s simplified method of slope stability analysis even though simulated failure time is much earlier than that of experimental in steady discharge cases of 2-D and 3-D. More experimental studies as well as sensitivity analysis are necessary to get experimental and simulated results quite close. The performance of the model can further be improved by incorporating more rigorous method of slope stability analysis that also considers the influence of pore air pressure in the soil slope stability. It is recommended to carry out the experimental measurement of air pressure at different points inside the dam body so that the result that may be obtained by the simulation can be verified.

References


地すべりによる天然ダム決壊の二次元および三次元斜面安定解析

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要 旨
不飽和多孔質媒体内の流れは、水と空気の2相流れの問題となる。本研究では間隙内空気の流れの効果を考慮するために、多相流理論に基づき水と空気の2相流モデルの導入を行った。これまで天然ダムの斜面の安定解析では、突然の決壊を予測するために、間隙水圧と含水率を解く従来の水相流モデル、水と空気の2相流モデルによって解析されており、そのモデル内はJanbuの簡易的な方法だけでなく、平衡状態に基づいた拡張Spencer法などが用いられている。本研究では、拡張Spencer法を用いて2相流れを解析しており、実験結果とも比較的良く一致した結果が得られている。

キーワード：2相流、飽和土、天然ダム、斜面安定