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Capital Income Taxation and Specialization Patterns: Investment Tax vs. Saving Tax

by
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Capital Income Taxation and Specialization Patterns: Investment Tax vs. Saving Tax∗

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Abstract

Unless free international lending/borrowing is allowed, domestic saving equals domestic investment and hence saving and investment taxes have the identical effect, as is the case in a closed-economy context. However, if it is allowed, households can accumulate foreign assets besides domestic capital and hence saving and investment are separated, causing the two taxes to have different effects. Using a two-sector growth model, we show that the two taxes generate completely different effects on industrial structure. The investment tax always shrinks the capital-intensive sector whereas the saving tax may well expand it.

Keywords: saving tax, investment tax, two-sector growth model, industrial structure, financial asset trade

JEL Classification Numbers: F41, E62

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1 Introduction

In a closed-economy context domestic real capital is the only available asset when people accumulate wealth. Therefore, wealth accumulation (saving) always equals real capital accumulation (investment) although they are independently determined by different agents —i.e., the former is decided by households whereas the latter is by firms. It implies that saving taxation has the same policy implications as investment taxation.

The same property is valid even in an open economy context unless international asset trade is allowed. In this case a country’s current account has to be always balanced, and hence saving equals investment in the country. This is actually the case in most open-economy two-sector growth models such as Oniki and Uzawa (1965), Stiglitz (1970), Manning (1981), Chen (1992), Manning, Markusen and Melvin (1992), Mountford (1999) and Brecher, Chen and Choudhri (2002).1

If free international asset trade is allowed, however, saving is separated from investment since people can accumulate foreign asset along with domestic real capital. The gap between saving and investment equals the current account.2 Therefore, the effect of a saving tax may significantly differ from that of an investment tax. In fact, we shall show that their effects on industrial structure can be just opposite to each other. Judging from the recent expanding trend of international asset trade, it should be important to analyze the difference in the effects of the two taxes in the presence of international asset trade.3

In the literature on international taxation two taxation principles are con-

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1 Exceptions are Fischer and Frenkel (1972), Matsuyama (1988), Ono and Shibata (2005, 2006) and Futagami et al. (2006). They develop open-economy two-sector growth models with international trade of financial assets.

2 Even in this case world saving must equal world investment. This is because a country’s current account equals the minus of the other country’s current account in a two-country setting.

3 Since the influential work by Feldstein and Horioka (1980) a large body of researchers have investigated the degree of international capital mobility. Obstfeld and Taylor (2004), among them, persuasively showed that the degree of capital mobility was high before World War I, rapidly declined in the Great Depression period, then turned to increase after World War II and sharply accelerated in the final decades of the 20th century.
sidered; one is the residence principle and the other is the source principle.\textsuperscript{4} Under the former principle the home country’s government applies a uniform tax rate to residents’ income from capital regardless of where the capital is located. It however imposes no tax on nonresidents’ income from capital even if it is located in the home country. Under the latter principle a uniform tax rate is applied to income from capital installed in the home country regardless of the residency of the income recipients. These two tax principles can be regarded as saving and investment taxation respectively, as shown by Summers (1988) and Giovannini (1990).\textsuperscript{5}

Comparing these two principles in overlapping generations models of a small open economy, Iwamoto and Shibata (1991) and Bovenberg (1992) show that a rise in the residence-based tax rate leads the country to a current account deficit while an increase in the source-based tax rate makes the country run a current account surplus. Sorensen (1990) and Ihori (1991) use two-country models with capital mobility and show that a rise in the residence tax of the home country reduces capital stock in both countries while a rise in the source tax decreases capital stock in the home country but increases it in the foreign country. Thus, it is well recognized that the effect of an investment tax significantly differs from that of a saving tax in various respects.\textsuperscript{6}

However, these models cannot be used for analyzing industrial structure since they either employ a one-commodity model or assume that each country produces only one commodity. Most of the open-economy two-sector growth models can neither be used for the present analysis since they ignore international trade in financial assets, causing a saving tax to be equivalent to an investment tax.

\textsuperscript{4}See Chapter 2 of Frenkel, Razin and Sadka (1991) and Chapter 6 of Turnovsky (1997) for more detailed explanations on the economic implications of the residence and source principles. See also Iwamoto and Shibata (1999) for the deviations of the actual tax systems from the two principles.

\textsuperscript{5}Strictly speaking, the source tax would affect not only investment but also saving through changes in the wage rate.

\textsuperscript{6}See also Bianconi (1995) and Lejour and Verbon (1998), who analyze the international spillover effects of both source-based and residence-based taxation in one-commodity Ramsey type models.
To fill this void, we develop a dynamic two-sector model with free international asset trade and compare the effects of saving and investment taxes on industrial structure. It is found that an increase in an investment tax of a country moves labor from the capital-intensive sector to the labor-intensive sector in both countries. Particularly the country that imposes the tax eventually specializes in the labor-intensive sector while the other country produces both commodities. In contrast, a saving tax of the more patient country may move labor from the labor-intensive sector to the capital-intensive sector in both countries. It is indeed the case when the subsistence demand for the capital-intensive commodity is significantly larger than that for the other commodity. Moreover, a saving tax of the less patient country is found to have no effect on specialization patterns. Thus, the effect on specialization patterns of a saving tax is quite different from that of an investment tax.

2 The Model

We introduce endogenous capital accumulation into the standard Heckscher-Ohlin (HO) model. In each of the two countries, \( h \) and \( f \), there are two production sectors 1 and 2. Sector \( j \) \((j = 1, 2)\) hires labor and rents capital to produce commodity \( j \) unless it incurs negative profits. The two commodities are both tradeable whereas the two factors are immobile across countries but mobile within each country. There is another sector, called sector 3, which uses commodity 1 to accumulate capital and rents it to the two production sectors.\(^7\) Thus, each country’s capital varies over time.

The population of country \( i \) is \( L^i (i = h, f) \) and each household’s labor endowment is unity. Households consume both commodities and accumulate wealth so as to maximize their lifetime utility. The government of country \( h \) imposes investment tax \( s_I (> 0) \), saving tax (or equivalently asset-holding tax) \( s_A (> 0) \) and commodity tax \( \tau \) on commodity 2 whereas that of country

\(^7\)Even if we assume that sectors 1 and 2 themselves accumulate capital, we can obtain essentially the same results as derived below.
f imposes no tax, for simplicity.

2.1 Firms

The two countries have the same production functions that satisfy constant returns to scale:

\[ f_1(k_1^i)L_1^i \quad \text{and} \quad f_2(k_2^i)L_2^i, \quad i = h, f, \]

where \( k_j^i \) is the capital-labor ratio and \( L_j^i \) the labor input of sector \( j \) in country \( i \). Given capital rent \( r^i \), wage \( w^i \) and relative price \( p \), where commodity 1 is taken as the numeraire, each firm in country \( i \) maximizes profits and hence, if country \( i \) produces both commodities, \( k_1^i \) and \( k_2^i \) satisfy

\[ r^i = f_1'(k_1^i) = p f_2'(k_2^i), \quad (1) \]
\[ w^i = f_1(k_1^i) - f_1'(k_1^i)k_1^i = p [ f_2(k_2^i) - f_2'(k_2^i)k_2^i ]. \quad (2) \]

From these two equations, if both countries are imperfectly specialized, we have

\[ k_h^i = k_f^i = k_1(p), \quad k'_i(p) = \frac{f_2(k_2)}{f_1'(k_1)(k_2-k_1)}, \]
\[ k_h^i = k_f^i = k_2(p), \quad k'_i(p) = \frac{f_2(k_2)}{p^2 f_2'(k_2)(k_2-k_1)}, \]
\[ r^h = r^f = r(p), \quad r'(p) = \frac{f_2}{k_2-k_1}, \]
\[ w^h = w^f = w(p). \quad (3) \]

Since commodity 1 is used for investment as well as consumption, we naturally assume sector 1 to be more capital-intensive than sector 2:

\[ k_1(\cdot) > k_2(\cdot). \quad (4) \]

The optimal behavior of sector 3 is formalized to maximize

\[ V^i = \int_0^\infty \left[ r^i - (1 + s^i)h^i \right] K^i \exp(-\int_0^t Rds) dt \quad \text{s.t.} \quad \frac{\dot{K}^i}{K^i} = g(h^i), \quad (5) \]

where \( R \) is the equity rate of interest, \( h^i \) is the investment ratio that equals \( I^i/K^i \), \( s^i \) is the investment tax, and \( g(\cdot) \) is the inverse of the adjustment cost.

\[ ^8 \text{The case of asymmetric technologies is examined by Ono and Shibata (2006).} \]
function of investment that satisfies

\[ g' > 0, \quad g'' < 0, \quad g'(0) = 1, \quad g(0) = 0. \]  

(6)

Note that investment tax \( s^I_i \) can be regarded as a source-based tax, as proven in Appendix 1. Country \( f \) does not impose the investment tax and hence

\[ s^h_i = s^I_i > 0, \quad s^f_I = 0. \]  

(7)

Equity rate of interest \( R \) is internationally the same under free trade of international financial assets whereas capital rent \( r^i \) is not always the same across countries since real capital is internationally immobile and its rent is determined in each domestic market.\(^{10}\)

As the result of sector 3’s optimal behavior represented by (5), the dynamic path of real capital accumulation in country \( i \) is determined so that

\[ 1 + s^I_i = \lambda^i g'(h^i), \]  

(8)

\[ \dot{\lambda}^i = [R - \{g(h^i) - g'(h^i)h^i\}]\lambda^i - r^i, \]  

(9)

for given \( \{R(t)\}_0^\infty \) and \( \{r^i(t)\}_0^\infty \), where \( \lambda^i \) represents the co-state variable of \( K^i \).

### 2.2 Households

The government of country \( h \) imposes tax \( s_A > 0 \) on the household sector’s asset holding and tax \( \tau \) on its purchase of commodity 2, and gives lump-sum transfer \( z \) to it while that of country \( f \) neither imposes any tax nor gives any transfer. Under the balanced budget of the government of country \( h \)

\[ z = s_A a^h + p\tau C^h_2 + s^I_I h^h K^h, \]  

(10)

---

9 Without adjustment costs of investment the dynamic paths of \( K^h \) and \( K^f \) are not determined since the indeterminacy problem pointed out by Mundell (1957) arises.

10 When one of the two countries specializes in a sector, capital rents \( r^h \) and \( r^f \) generally differ from each other in transitional states, although they eventually converge to the same level. When both countries are imperfectly specialized, \( r^h \) and \( r^f \) take the same value because of the factor price equalization mechanism, as (3) shows. See also Niehans (1984, pp.130) for the difference in implication between real capital mobility and financial asset mobility.
where $C^i_j$ is country $i$’s consumption of commodity $j$ ($j = 1, 2$) and $a^i$ is its non-human wealth (= foreign assets + equities) whose interest rate is $R$.

The flow budget equation of each household is then

$$
\dot{a}^h = (R - s_A)a^h + w^h - C^h_1 - p(1 + \tau)C^h_2 + z, \quad (11)
$$

$$
\dot{a}^f = Ra^f + w^f - C^f_1 - pC^f_2. \quad (12)
$$

Note that saving tax $s_A$ can be regarded as a residence-based tax, as shown in Appendix 1.

Without loss of generality, households of country $i$ are assumed to have the following log-linear instantaneous utility:

$$
\alpha \ln(C^i_1 - \overline{C}^i_1) + (1 - \alpha) \ln(C^i_2 - \overline{C}^i_2),
$$

where $\overline{C}^i_j$ is the minimum level of commodity $j$ required by each household of country $i$ to survive. This formulation follows recent empirical evidence that supports the existence of the minimum requirement of consumption. Subject to flow budget equation (11) or (12) the representative household of country $i$ ($i = h, f$) maximizes

$$
U^i = \int_0^\infty [\alpha \ln(C^i_1 - \overline{C}^i_1) + (1 - \alpha) \ln(C^i_2 - \overline{C}^i_2)]e^{-\rho^i_t}dt, \quad (13)
$$

where $\rho^i$ is country $i$’s subjective discount rate.

The first-order conditions for optimality are

$$
\frac{\dot{C}^h_1}{C^h_1 - \overline{C}^h_1} = R - s_A - \rho^h, \quad \frac{\dot{C}^f_1}{C^f_1 - \overline{C}^f_1} = R - \rho^f, \quad (14)
$$

11 As shown later in Proposition 1, the steady-state relative price takes a unique common value under all possible specialization patterns. This proposition is derived only from the properties of subjective discount rates and production functions.

12 In order to assure the existence of the equilibrium path, we assume that the initial levels of capital in the two countries are so large and the subsistence levels are so small that the worldwide production of each commodity always exceeds the worldwide subsistence level of it.

\[
\frac{C^h_1 - C^f_1}{C^h_2 - C^f_2} = (1 + \tau)\gamma p, \quad \frac{C^f_1 - C^f_1}{C^f_2 - C^f_2} = \gamma p, \quad \text{where} \quad \gamma = \frac{\alpha}{1 - \alpha},
\]
(15)
and the transversality condition is
\[
\lim_{t \to \infty} q^i(t)a^i(t)\exp(-\rho^i t) = 0,
\]
(16)
where \(q^i\) is the co-state variable of \(a^i\). Equations (11) - (16) give the dynamic paths of consumption and financial asset accumulation for given \(\{R(t)\}^\infty_0\), \(\{w^i(t)\}^\infty_0\) and \(\{p(t)\}^\infty_0\). Note that the path of financial asset \(a^i\) is determined as the result of households’ optimal saving behavior while the path of real capital \(K^i\) is determined as the result of sector 3’s optimal investment behavior represented by (8) and (9). The two paths generally differ from each other since households can hold foreign assets as well as domestic equities.

From (14) we obtain
\[
\frac{\dot{C}^h_1}{C^h_1 - C^f_1} \geq \frac{\dot{C}^f_1}{C^f_1 - C^f_1} \iff s_A + \rho^h \leq \rho^f.
\]
(17)
This implies that \(s_A\) works as if the subjective discount rate rose by \(s_A\) and that the country of which the sum of the subjective discount rate and the asset-holding tax rate is higher than the other’s gradually decreases its expenditure share in the world market and eventually consumes only its subsistence levels of the two commodities while the other country consumes all the rest.

### 2.3 Market Equilibrium Conditions

In the factor markets of country \(i\)
\[
K^i = k^i_1L^i_1 + k^i_2L^i_2,
\]
(18)
\[
L^i_1 + L^i_2 = L^i \quad \text{for} \quad i = h, \ f.
\]
(19)
Since \(K^i\) accumulates following (8) and (9), \(L^i_j\)’s change over time and eventually perfect specialization may arise. It is in sharp contrast to the standard Heckscher-Ohlin (HO) model since in the HO model neither \(K^i\) nor
$L^j_i$’s change over time and hence the two countries stay to be imperfectly specialized.

Since total demand for commodity 1 equals the sum of world consumption $C^h_1 L^h + C^f_1 L^f$ and world investment $h^k K^h + h^f K^f$, its market equilibrium condition is

$$C^h_1 L^h + C^f_1 L^f + h^k K^h + h^f K^f = f_1(k^h_1) L^h_1 + f_1(k^f_1) L^f_1. \quad (20)$$

The market equilibrium condition of commodity 2, which is used solely for consumption, is

$$C_2 (\equiv C^h_2 L^h + C^f_2 L^f) = f_2(k^h_2) L^h_2 + f_2(k^f_2) L^f_2. \quad (21)$$

### 3 Specialization Patterns

Since there is no investment in the steady state of the present dynamics,

$$h^i = 0, \quad (22)$$

and then (20) reduces to

$$C_1 (\equiv C^h_1 L^h + C^f_1 L^f) = f_1(k^h_1) L^h_1 + f_1(k^f_1) L^f_1. \quad (23)$$

From (6), (8), (9) and (22),

$$\lambda^h = 1 + s_I, \quad (24)$$

$$R = r^h/(1 + s_I) = r^f.$$  

As long as $s_I \neq 0$, the second equation of (24) is inconsistent with the condition of factor price equalization given by (3). Therefore we obtain\textsuperscript{14}

**Proposition 1.** As long as a non-zero investment tax-cum-subsidy is imposed, at least one country eventually leads to perfect specialization.

\textsuperscript{14}In the presence of an international difference in the relative productivity between the two sectors, no matter how small the difference is, imperfect specialization of both countries never obtains. See Baxter (1992) and Ono and Shibata (2006) for details.
Proposition 1 implies that the case of both countries’ imperfect specializa-
tion, viz. the standard HO world, never arises. Furthermore, in the steady
state real capital stays constant and hence production of each commodity
stays at each steady-state level. Thus, from (14) and (24) we obtain\footnote{15}{The equality between the steady-state interest rate and the subjective discount rate of the most patient country is shown by Becker (1980), Blanchard and Fischer (1989, pp.69-70) and Ikeda and Ono (1992) in a multi-country context. The present result is a simple extension of it to the case with saving and investment taxes and subsistence consumption.}

\[
R = \frac{r^h}{1 + s_I} = r^f = \rho^{\text{min}} \equiv \min(s_A + \rho^h, \rho^f).
\]  

(25)

Since we consider the case where investment tax \( s_I \) is positive, from (25) \( r^h > r^f \), which causes capital accumulation in country \( h \) to be less advantageous
than in country \( f \). Therefore, country \( h \) (or \( f \)) never specializes in sector 1 (or 2). Only the following three cases are possible, viz. (i) Country \( h \)’s specialization in sector 2 and country \( f \)’s imperfect specialization, (ii) country \( h \)’s specialization in sector 2 and country \( f \)’s specialization in sector 1, and (iii) country \( h \)’s imperfect specialization and country \( f \)’s specialization in sector 1.\footnote{16}{If \( s_I < 0 \), there are the following three cases: (i) country \( h \)’s perfect specialization in sector 1 and country \( f \)’s imperfect specialization, (ii) country \( h \)’s perfect specialization in sector 1 and country \( f \)’s perfect specialization in sector 2, and (iii) country \( h \)’s imperfect specialization and country \( f \)’s perfect specialization in sector 2.}

In the arguments below, for simplicity, we ignore commodity tax \( \tau \)

\[
\tau = 0
\]  

(26)

until we examine its effect in section 5. Thus, from (15) and (26), world
demand for each commodity, \( C_1 \) and \( C_2 \), has to satisfy

\[
\frac{C_1 - \overline{C}_1}{C_2 - \overline{C}_2} = \gamma p, \quad \text{where } \overline{C}_j \equiv \overline{C}_j^h L^h + \overline{C}_j^f L^f.
\]  

(27)

Using these properties we obtain steady-state conditions for each of the three
cases.

**Case (i): Country \( h \)’s imperfect specialization and country \( f \)’s per-
fected specialization in sector 1**
In country $h$, (1) and (2) are valid and hence from (25) $p$ equals $p^h$ that satisfies

\[(1 + s_I)^\rho^{\text{min}} = f_1'(k_1^h) = p^h f_2'(k_2^h),
\]
\[f_1(k_1^h) - f_1'(k_1^h)k_1^h = p^h \left[ f_2(k_2^h) - f_2'(k_2^h)k_2^h \right]. \tag{28}\]

Since country $f$ specializes in sector 1 and $r^f = \rho^{\text{min}}$ from (25),

\[L_1^f = L^f, \quad L_2^f = 0,
\]
\[\rho^{\text{min}} = f_1'(k_1^f). \tag{29}\]

Therefore, from (18), (19), (21), (23) and (27),

\[L_1^h = L_1^{h(1)} = \frac{\gamma p^h [f_2(k_2^h)L^h - \overline{C}_2] - [f_1(k_1^f)L^f - \overline{C}_1]}{f_1(k_1^h) + \gamma p^h f_2(k_2^h)},
\]
\[L_2^h = \frac{f_1(k_1^h)L^h + f_1(k_1^f)L^f - \overline{C}_1 + \gamma p^h \overline{C}_2}{f_1(k_1^h) + \gamma p^h f_2(k_2^h)},
\]
\[K^h = k_1^h L_1^h + k_2^h L_2^h,
\]
\[K^f = k_1^f L_1^f. \tag{30}\]

**Case (ii): Country $h$’s perfect specialization in sector 2 and country $f$’s perfect specialization in sector 1**

In this case

\[L_1^h = 0, \quad L_2^h = L^h,
\]
\[L_1^f = L^f, \quad L_2^f = 0. \tag{31}\]

In country $h$ capital rent $r^h$ equals the marginal productivity of capital in sector 2 while in country $f$ capital rent $r^f$ equals the marginal productivity of capital in sector 1. Therefore, from (25) we obtain

\[(1 + s_I)^\rho^{\text{min}} = p^f f_2'(k_2^h),
\]
\[\rho^{\text{min}} = f_1'(k_1^f), \tag{32}\]
where $p^s$ is determined so that the production of commodity 1 (which equals $C_1$) and that of commodity 2 (which equals $C_2$) satisfy (27):

$$f_1(k^f_1)L^f - C_1 = \gamma p^s [f_2(k^h_2)L^h - C_2]. \quad (33)$$

From (18) and (31),

$$K^h = k^h L^h, \quad K^f = k^f L^f. \quad (34)$$

**Case (iii): Country $h$’s perfect specialization in sector 2 and country $f$’s imperfect specialization**

Country $f$ is imperfectly specialized and hence (1) and (2) are valid. Therefore, from (25) $p$ equals $p^f$ that satisfies

$$\rho^\text{min} = f_1(k^f_1) = p^f f'_2(k^h_2),$$

$$f_1(k^f_1) - f'_1(k^f_1)k^f_1 = p^f \left[ f_2(k^f_2) - f'_2(k^f_2)k^f_2 \right]. \quad (35)$$

In country $h$ only sector 2 operates and thus

$$L^h_1 = 0, \quad L^h_2 = L^h,$$

$$(1 + s_I)\rho^\text{min} = p^f f'_2(k^h_2), \quad (36)$$

where $p^f$ is given by (35). Therefore, from (18), (19), (21), (23) and (27),

$$L^f_1 = \frac{\gamma p^f f_2(k^h_2)L^h + f_2(k^h_2)L^f - C_2 + C_1}{f_1(k^f_1) + \gamma p^f f_2(k^h_2)},$$

$$L^f_2 = L^f_{2,(iii)} = \frac{[f_1(k^f_1)L^f - C_1] - \gamma p^f [f_2(k^h_2)L^h - C_2]}{f_1(k^f_1) + \gamma p^f f_2(k^h_2)},$$

$$K^h = k^h L^h,$$

$$K^f = k^f L^f_1 + k^f L^f_2. \quad (37)$$

### 4 Investment Tax vs. Saving Tax

Using the steady-state conditions for the three cases obtained in the previous section, we examine the effect of investment tax $s_I$ and that of saving tax $s_A$ on specialization patterns.
4.1 Investment Tax

By comparing $L_{1,(i)}^h$ in (30) with (33), we find that (33) is valid when $L_{1,(i)}^h$ equals zero. Also, since (32) in case (ii) is valid if (28) and (29) in case (i) are valid, we find $p^f = p^s$ when $L_{1,(i)}^h$ equals zero. Therefore, the two cases coincide with each other when $L_{1,(i)}^h$ equals zero. Analogously, when $L_{2,(iii)}^f$ in (37) equals zero, (33) is valid, implying $p^h$ equal to $p^s$, and then cases (ii) and (iii) coincide with each other. Therefore, if a change in $s_I$ makes $L_{1,(i)}^h$ (or $L_{2,(iii)}^f$) zero, it changes specialization patterns from case (i) to (ii) (or from case (iii) to (ii)). We shall below show this property.

From $L_{1,(i)}^h$ in (30) and $L_{2,(iii)}^f$ in (37), case (i) arises when $L_{f}^f/L_{h}^h$ is small enough and case (iii) does when $L_{f}^f/L_{h}^h$ is large enough. It implies that the country with a much larger population tends to be imperfectly specialized and that the other tends to specialize in a sector. It is naturally understood since a supply shortage of the other commodity occurs if the country with a much larger population specializes in a sector.\(^{17}\)

Now suppose that $L_{f}^f/L_{h}^h$ is so small that case (i) arises and that the government of country $h$ raises the investment tax. From (28) and (29), $p^h$, $k_2^h$ and $k_1^f$ are all independent of $L^i$’s or $\gamma$ and satisfy

\[
\frac{dp^h}{ds_I} < 0, \quad \frac{dk_1^f}{ds_I} = 0, \quad \frac{dk_2^h}{ds_I} < 0,
\]

which implies that an increase in $s_I$ reduces the numerator of $L_{1,(i)}^h$ in (30). Therefore, if $L_{1}^h$ is small enough, an increase in $s_I$ makes it zero, causing specialization patterns to move from case (i) to case (ii).

Suppose next that $L_{f}^f/L_{h}^h$ is so large that case (iii) arises. From (35) and (36), $p^f$, $k_2^f$ and $k_1^f$ are all independent of $L^i$’s or $\gamma$ and satisfy

\[
\frac{dp^f}{ds_I} = 0, \quad \frac{dk_1^f}{ds_I} = 0, \quad \frac{dk_2^h}{ds_I} < 0,
\]

which implies that a decrease in $s_I$ reduces the numerator of $L_{2,(iii)}^f$ given in (37). Therefore, if $L_{2,(iii)}^f$ is small enough, a decrease in $s_I$ makes it zero —i.e., an increase in $s_I$ moves specialization patterns from case (ii) to case (iii).

\(^{17}\)See Ono and Shibata (2006) for this point.
In sum, an increase in $s_I$ varies specialization patterns as stated below.\footnote{Dependent upon the values of technological, preference and population parameters, the changes in specialization patterns mentioned in proposition 2 may not occur even under a sufficiently large change in the investment tax. For example, if $L^f / L^h$ is very large, country $f$ cannot perfectly specialize in the capital-intensive sector and hence only case (iii) is valid. In fact, from (35), (36), and $L^f_{2(iii)}$ in (37) we find $L^f_{2(iii)}$ to stay within $(0, L^f)$ for any positive $s_I$. However, we can definitely say that the shifts in specialization patterns opposite to what proposition 2 states never arise.}

**Proposition 2.** An increase in an investment tax of country $h$ leads the case of country $h$’s imperfect specialization and country $f$’s specialization in the capital-intensive sector to the case of country $h$’s specialization in the labor-intensive sector and country $f$’s specialization in the capital-intensive sector. A further increase in the tax leads to the case of country $h$’s perfect specialization in the labor-intensive sector and country $f$’s imperfect specialization.

Intuitively, in country $h$ an increase in $s_I$ raises capital rent $r^h (= (1 + s_I)\rho_{\text{min}})$ and makes the capital-intensive sector less advantageous whereas in country $f$ capital rent $r^f$ remains equal to $\rho_{\text{min}}$. Therefore, if initially country $h$ is imperfectly specialized and country $f$ specializes in the capital-intensive sector, country $h$ reduces, and eventually stops, production of the capital-intensive commodity as $s_I$ rises. Then, country $h$ specializes in the labor-intensive sector while country $f$ does in the capital-intensive sector. A further increase in $s_I$ reduces country $h$’s capital stock more and decreases production of the labor-intensive commodity, the only commodity that country $h$ produces. Therefore, country $f$ starts producing the labor-intensive commodity as well. Figure 1 depicts a typical case of the investment-tax effect on specialization patterns.

### 4.2 Saving Tax

In the case where foreign-asset holding is not allowed, including the closed-economy case, a saving tax is equivalent to an investment tax since people as
a whole has no other choice than domestic real capital when saving. However, unless foreign-asset holding is restricted, saving and investment are separated from each other. Therefore, the effect of a saving tax should differ from that of an investment tax. This section examines the effect of a saving tax and compare it with that of an investment tax obtained in the previous section.

Whether the country that imposes saving tax $s_A$ is more time-patient or less results in a quite different effect on production and specialization patterns of the two countries. It is in sharp contrast to the effect of investment tax $s_I$ summarized by proposition 2, which is independent of which country is more patient.

First, we analyze the case where the less patient country imposes $s_A$ and obtain the following proposition:

**Proposition 3.** An increase in the less patient country’s saving tax affects neither specialization patterns nor demand patterns in the steady state.

Proof. Suppose that country $h$ is the less patient country, i.e., $s_A + \rho^h > \rho^f$. Then, an increase in $s_A$ does not affect $\rho^{\text{min}}(= \min(s_A + \rho^h, \rho^f))$. Therefore, the steady-state conditions in the three cases presented in section 3 are all left unaffected. Q.E.D.

If $s_A + \rho^h < \rho^f$ and thus country $h$ is more patient than the other, an increase in its saving tax raises the steady-state world interest rate $\rho^{\text{min}}(= s_A + \rho^h)$ and harms the capital-intensive sector in both countries. In this case we obtain the following proposition:

**Proposition 4.** Suppose that country $h$ imposes an investment tax so that it never specializes in the capital-intensive sector. Under Cobb-Douglas production technology, if the worldwide subsistence level of the labor-intensive sector

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19 As mentioned in footnote 18, dependent upon the values of technological, preference and population parameters, the changes in specialization patterns mentioned in proposition 4 may not occur even under a sufficiently large change in the investment tax. For example, if $L^f/L^h$ is very large, country $f$ cannot perfectly specialize in a sector. In fact, $L^f_{2(iii)}$ in (37) stays within $(0, L^f)$ even if $s_A$ (and hence $\rho^{\text{min}}$) is very large. However, we can definitely say that the shifts in specialization patterns opposite to what proposition 4 states never arise.
commodity is sufficiently small while that of the other commodity is not, an increase in a saving tax of the more patient country changes the case of country h’s perfect specialization in the labor-intensive sector and country f’s imperfect specialization to the case of country h’s specialization in the labor-intensive sector and country f’s specialization in the capital-intensive sector. A further increase in the tax leads to the case of country h’s imperfect specialization and country f’s specialization in the capital-intensive sector. If the subsistence level of the capital-intensive commodity is sufficiently small while that of the other commodity is not, an increase in a saving tax of the more patient country generates just the opposite transition in specialization patterns. Note that the results hold true regardless of which country is more patient.

Proof. See Appendix 2.

Propositions 2, 3 and 4 show that the effect on specialization patterns of a saving tax completely differs from that of an investment tax. Particularly if the subsistence level of the labor-intensive commodity is sufficiently small while that of the other commodity is not, under Cobb-Douglas production functions the more patient country’s saving tax generates an effect opposite to what its investment tax does —i.e., the saving tax moves labor to the capital-intensive sector and hence changes specialization patterns from case (iii) to case (ii) and from case (ii) to case (i) whereas an increase in the investment tax moves labor to the labor-intensive sector and hence changes specialization patterns from case (i) to case (ii) and from case (ii) to case (iii). Figure 2 illustrates the saving-tax effect on specialization patterns in this case.

An investment tax raises the capital rent and makes the capital-intensive sector less advantageous only in the country that imposes it. Therefore, changes in specialization patterns are generated by the labor movement from the capital-intensive sector to the other only in the country. In contrast, an increase in the more patient country’s saving tax globally raises the capital rent and makes the capital-intensive sector less advantageous in both
countries. It also reduces the steady-state level of capital stock. Therefore, if prices were fixed, labor would move from the capital-intensive sector to the labor-intensive sector in both countries (the Rybczynski effect). As the capital-intensive commodity is less produced, however, the price elasticity of demand for it decreases in the presence of the subsistence level, and thus the magnitude of the price rise increases, attracting more labor to the capital-intensive sector. Specialization patterns depend on the relative strength of the two opposite effects on labor movement.

Particularly if $C_1$ is much larger than $C_2$, demand for the capital-intensive commodity remains large and hence the rise in its price is so high that the latter effect dominates the former, causing labor to move from the labor-intensive sector to the capital-intensive sector. Thus, in the case where country $h$ specializes in the labor-intensive sector and country $f$ is imperfectly specialized, labor moves to the capital-intensive sector in country $f$ and eventually country $f$ specializes in the capital-intensive sector. Note that country $f$ has comparative advantage on the capital-intensive sector when country $h$ imposes an investment tax and thus country $f$ reallocates labor to the capital-intensive sector before country $h$ does so. Once all labor moves to the capital-intensive sector in country $f$, a further increase in the saving tax reduces the country $f$’s production of the capital-intensive commodity beyond worldwide demand for it, and thus country $h$ starts producing the capital-intensive commodity, which leads the country to imperfect specialization.

If $C_2$ is much larger than $C_1$, on the contrary, demand for the labor-intensive commodity remains large so that the price rise of the capital-intensive commodity raises demand for the labor-intensive commodity. It moves labor from the capital-intensive sector to the labor-intensive sector. Thus, in this case a saving tax changes specialization patterns in the same direction as an investment tax does.
5 Commodity Tax

In steady state the representative household of the less patient country consumes the subsistence levels of the two commodities regardless of whether a commodity tax is imposed or not in the country. Since the tax does not affect the domestic prices in the other country, there is no international spillover effect. Thus, a commodity tax imposed by the less patient country generates no effect.

We next examine the case where the more patient country imposes commodity tax $\tau$. Naturally, it will be found that an increase in $\tau$ stimulates production of commodity 1 and harms production of commodity 2 if both commodities are produced.

Without loss of generality, suppose that country $h$ is more patient and imposes commodity tax $\tau$ on commodity 2. Then, $\gamma$ in (15) is replaced by $\gamma(1 + \tau)$, implying that in country $h$

$$
\frac{C_1^h - \bar{C}_1^h}{C_2^h - \bar{C}_2^h} = \gamma(1 + \tau)p,
$$

while in country $f$

$$
C_1^f = \bar{C}_1^f, \quad C_2^f = \bar{C}_2^f.
$$

Therefore, $\gamma(1 + \tau)$ replaces $\gamma$ in (30) and (37). All equations that represent firms’ behavior remain unaffected, implying that $k_2^h, k_1^f, k_2^f$ and $p^h$ given by (35) and (36) and $k_1^h, k_2^h, k_1^f$ and $p^h$ given by (28) and (29) are all unchanged.

Replacing $\gamma$ by $\gamma(1 + \tau)$ in (30) and (37), differentiating them with respect to $\tau$ and applying (21), (23), (29) and (36) to the results yield

$$
dL^h_{1-1(i)}/d\tau = \frac{p^h \gamma \left[(C_2 - \bar{C}_2) f_1(k_1^h) + (C_1 - \bar{C}_1) f_2(k_2^h)\right]}{[f_1(k_1^h) + \gamma^h p^h f_2(k_2^h)]^2} > 0,
$$

$$
dL^f_{2-1}(iii)/d\tau = -\frac{p^f \gamma \left[(C_2 - \bar{C}_2) f_1(k_1^f) + (C_1 - \bar{C}_1) f_2(k_2^f)\right]}{[f_1(k_1^f) + \gamma^f p^f f_2(k_2^f)]^2} < 0.
$$

This implies the following property:
Proposition 5. Suppose that the more patient country (which we call country h) imposes an investment tax so that it never specializes in the capital-intensive sector. An increase in its commodity tax on the labor-intensive commodity changes the case of country h’s perfect specialization in the labor-intensive sector and country f’s imperfect specialization to the case of country h’s perfect specialization in the labor-intensive sector and country f’s perfect specialization in the capital-intensive sector. A further such change eventually results in country h’s imperfect specialization and country f’s perfect specialization in the capital-intensive sector.

6 Conclusions

Unless international asset trade is allowed, saving always has to equal investment within a country, as in the case of a closed economy. Therefore, a saving tax has the same effect on each country as an investment tax does, although households’ saving decisions are determined independently of firms’ investment decisions. Once free international asset trade is allowed, households have a choice of asset accumulation between real capital and foreign asset holdings—i.e., domestic saving is used for not only domestic investment but also foreign asset holdings. Therefore, the effect of a saving tax may differ from that of an investment tax. This paper shows that the effect of a saving tax on specialization patterns in fact significantly differs from that of an investment tax. They can even be opposite to each other.

An investment tax raises the capital rent and reduces capital accumulation only in the country that imposes the tax. It makes the capital-intensive sector less advantageous than the labor-intensive sector. Therefore, possible specialization patterns are only the following: (i) the country’s imperfect specialization and the other country’s specialization in the capital-intensive sector, (ii) the country’s specialization in the labor-intensive sector and the other’s specialization in the capital-intensive sector, and (iii) the country’s specialization in the labor-intensive sector and the other’s imperfect specialization.
If the country raises the investment tax in case (i), labor moves from the capital-intensive sector to the other and eventually makes the country specialize in the labor-intensive sector, which is case (ii). Since the other country faces a lower rent of capital, it specializes in the capital-intensive sector. After the country that imposes the investment tax specializes in the labor-intensive sector, a further increase in the tax reduces production in the labor-intensive sector and generates a global shortage of its supply. It leads the other country to start producing the labor-intensive commodity, which is case (iii).

A saving tax has a completely different effect from that of an investment tax. Its effect is equivalent to that of an increase in the subjective discount rate of the country that imposes the tax. Since the steady-state interest rate equals the more patient country’s subjective discount rate, a saving tax of the less patient country has no effect. However, a saving tax of the more patient country globally raises the capital rent in the two countries and affects labor allocation between the two sectors. Consequently, specialization patterns change. We find that the effect on specialization patterns depends on the subsistence levels of the two commodities and the shapes of production functions.

For example, suppose that production functions are of the Cobb-Douglas type and that the subsistence level of the labor-intensive commodity is sufficiently small while that of the other commodity is not. Suppose also that the more patient country imposes an investment tax and perfectly specializes in the labor-intensive sector while the other country is imperfectly specialized, which is case (iii). Then, an increase in a saving tax by the more patient country leads both countries to perfect specialization, which is case (ii), and eventually to the country’s imperfect specialization and the other’s specialization in the capital-intensive sector, which is case (i). Thus, it generates just the opposite effect on specialization patterns to what is generated by an increase in an investment tax. In contrast, if the subsistence level of the capital-intensive commodity is sufficiently small while that of the other is not, it generates the same transition in specialization patterns as generated
by an increase in an investment tax.
Appendix 1: Residence-based and Source-based Taxation

In this appendix we show that the saving and investment taxes formalized in the text are respectively equivalent to the source-based and residence-based taxes in the literature on international taxation.

Let us assume that in country \( f \) no tax is imposed and the financial interest rate is \( R \). In country \( h \) residence-based tax \( \theta^h \) and source-based tax \( \eta^h \) are imposed and the financial interest rate is \( R^h \). From the no-arbitrage condition in country \( h \):

\[
(1 - \theta^h)R = (1 - \theta^h)(1 - \eta^h)R^h = (1 - \theta^h)(1 - \eta^h) \left[ \frac{\dot{V}^h + Div^h}{V^h} \right],
\]

where \( V^h \) and \( Div^h \) are respectively the firm value of the investment sector and its dividends:

\[
Div^h = [r^h(t) - h^h(t)]K^h(t), \quad (A1)
\]

we derive

\[
R = (1 - \eta^h)R^h, \quad (A2)
\]

\[
\frac{R}{1 - \eta^h} = \frac{\dot{V}^h + Div^h}{V^h}. \quad (A3)
\]

Note that (A1) also implies the no-arbitrage condition for foreign investors between the two countries’ financial assets.

From (A1) and (A3), the investment sector’s behavior is formalized to maximize

\[
V^h = \int_0^\infty [r^h(t) - h^h(t)]K^h(t) \exp \left( - \int_0^t \frac{R(s)}{1 - \eta^h} ds \right) dt
\]

s.t. \( \frac{\dot{K}^h(t)}{K^h(t)} = g \left( h^h(t) \right) \),

of which the optimal conditions are

\[
1 = \lambda^h g'(h^h),
\]
\[ \lambda^h = \left[ \frac{R}{1 - \eta^h} - \left\{ g(h^h) - g'(h^h)h^h \right\} \right] \lambda^h - r^h. \]

In steady state they reduce to

\[ r^h = \frac{R}{1 - \eta^h}. \]

We find it to be equivalent to (24) by replacing \( s_I \) by \( \eta^h/(1 - \eta^h) \) in (24) —i.e., investment tax \( s_I \) is equivalent to source-based tax \( \eta^h \).

In the presence of residence-based tax \( \theta^h \) and commodity tax \( \tau^h \) the flow budget constraint of the representative household is

\[ \dot{a}^h = (1 - \theta^h)Ra^h + w^h L^h - C_1^h - p(1 + \tau^h)C_2^h + z, \]

which is equivalent to (11) where \( s_A \) is replaced by \( \theta^h R \). Thus, saving tax \( s_A \) is found to be equivalent to residence-based tax \( \theta^h \).

Appendix 2: Proof of Proposition 4

We prove Proposition 4 by assuming production functions to be of the following Cobb-Douglas type:

\[ f_1(k_1) = A(k_1)^\theta, \quad f_2(k_2) = B(k_2)^\nu, \tag{A4} \]

where \( A \) and \( B \) are positive constants. Factor-intensity condition (4) reduces to

\[ \theta > \nu. \]

Under production functions (A4) \( k_1^f, k_2^h \) and \( p^h \) in case (i) are

\[ k_1^f = \left( \frac{A\theta}{\rho_{\text{min}}} \right)^{1 - \nu} \theta, \quad k_2^h = \left( \frac{A\theta}{(1 + s_I)\rho_{\text{min}}} \right)^{1 - \nu} \theta \left( \frac{\nu}{1 - \nu} \cdot \frac{1 - \theta}{\theta} \right), \]

\[ p^h = \frac{(A\theta)^{1 - \nu}}{B\nu[(1 + s_I)\rho_{\text{min}}]^{\frac{\theta - \nu}{\nu}}} \left( \frac{\nu}{1 - \nu} \cdot \frac{1 - \theta}{\theta} \right)^{1 - \nu}. \tag{A5} \]

Substituting (A5) into \( L_{1,(i)}^h \) given in (30) gives

\[ L_{1,(i)}^h = \frac{p^h f_2(k_2^h) L^h - C_2^h}{f_1(k_1^h) + \gamma p^h f_2(k_2^h)} \left( \gamma - \left( \frac{(1 + s_I)^{\theta - \nu}(1 - \nu) L^f}{(1 - \theta)L^h} \right) \Phi(\rho_{\text{min}}) \right), \]
Substituting (A7) into

\[ \Phi(\rho^{\text{min}}) = \frac{(\rho^{\text{min}})^{\frac{1}{\nu}} - C_1}{(\rho^{\text{min}})^{\frac{1}{\nu}} - (1 + s_f)^{\frac{1}{\nu}}(\rho^{\text{min}})^{\frac{1}{\nu}} C_2}, \]

\[ C_1 = \frac{\mathcal{C}_1}{A^{\frac{1}{\nu}} - \frac{\rho^{\text{min}}}{\nu} L^f}, \quad \hat{C}_2 = \frac{\mathcal{C}_2}{B(A\theta)^{\frac{1}{\nu}}(\frac{\nu}{1 - \frac{\nu}{\theta}})^{\nu} L^h}. \]

Differentiating \( \Phi(\rho^{\text{min}}) \) with respect to \( \rho^{\text{min}} \) yields

\[ \Phi'(\rho^{\text{min}}) = \frac{\nu(\rho^{\text{min}})^{\frac{\nu}{\theta}}(1 + s_f)^{\frac{1}{\nu}}(\rho^{\text{min}})^{\frac{1}{\nu}} - (1 - \nu)\hat{C}_1 \hat{C}_2(\rho^{\text{min}})^{\frac{1}{\nu}}}{(1 + s_f)^{\frac{1}{\nu}}(1 - \theta)(\rho^{\text{min}})^{\frac{1}{\nu}} - (1 + s_f)^{\frac{1}{\nu}}(\rho^{\text{min}})^{\frac{1}{\nu}} \hat{C}_2}. \]

Thus, when \( L_{1-\text{}}^h \) is small enough,

\[ \frac{dL_{1-\text{}}^h}{d\rho_{\text{min}}} (\frac{\nu}{d_{SA}}) > 0 \] if \( \mathcal{C}_1 > 0 \) and \( \mathcal{C}_2 = 0 \),

\[ \frac{dL_{1-\text{}}^h}{d\rho_{\text{min}}} (\frac{\nu}{d_{SA}}) < 0 \] if \( \mathcal{C}_1 = 0 \) and \( \mathcal{C}_2 > 0 \). \hspace{1cm} (A6)

Under (A4) \( k_1^f, k_2^h \) and \( p^f \) in case (iii) are

\[ k_1^f = \frac{(A\theta)^{\frac{1}{\nu}}}{(\rho^{\text{min}})^{\frac{1}{\nu}}}, \quad k_2^h = \frac{(A\theta/\rho^{\text{min}})^{\frac{1}{\nu}}}{(1 + s_f)^{\frac{1}{\nu}}}, \quad \frac{\nu}{1 - \theta} \]

\[ p^f = \frac{(A\theta)^{\frac{1}{\nu}}}{B(\rho^{\text{min}})^{\frac{\nu}{\theta}}}, \quad \left( \frac{\nu}{1 - \theta} \right)^{\nu}. \hspace{1cm} (A7) \]

Substituting (A7) into \( L_{2-\text{}}^f \) given in (37) gives

\[ L_{2-\text{}}^f = \frac{p^f[f_2(k_2^h) L^h - \hat{C}_2]}{f_1(k_1^f) + \gamma p^f f_2(k_2^h)} \left( \frac{(1 + s_f)^{\frac{1}{\nu}} - (1 - \nu) L^f}{(1 - \theta) L^h + \Omega(\rho^{\text{min}}) - \gamma} \right), \]

\[ \Omega(\rho^{\text{min}}) = \frac{(\rho^{\text{min}})^{\frac{1}{\nu}} - \hat{C}_1}{(\rho^{\text{min}})^{\frac{1}{\nu}} - (1 + s_f)^{\frac{1}{\nu}}(\rho^{\text{min}})^{\frac{1}{\nu}} \hat{C}_2}. \]

Differentiating \( \Omega(\rho^{\text{min}}) \) with respect to \( \rho^{\text{min}} \) yields

\[ \Omega'(\rho^{\text{min}}) = \frac{\nu(\rho^{\text{min}})^{\frac{\nu}{\theta}}(\rho^{\text{min}})^{\frac{1}{\nu}} - (1 + s_f)^{\frac{1}{\nu}}(\rho^{\text{min}})^{\frac{1}{\nu}} \hat{C}_1 \hat{C}_2(\rho^{\text{min}})^{\frac{1}{\nu}}}{(1 + s_f)^{\frac{1}{\nu}}(1 - \theta)(\rho^{\text{min}})^{\frac{1}{\nu}} - (1 + s_f)^{\frac{1}{\nu}}(\rho^{\text{min}})^{\frac{1}{\nu}} \hat{C}_2}. \]
Thus, when $L_1^{h(i)}$ is small enough,

$$\frac{dL_2^{I(iii)}}{d\rho_{\min}} = \frac{dL_1^{h(i)}}{ds_A} < 0 \quad \text{if } C_1 > 0 \text{ and } C_2 = 0,$$

$$\frac{dL_2^{I(iii)}}{d\rho_{\min}} = \frac{dL_1^{h(i)}}{ds_A} > 0 \quad \text{if } C_1 = 0 \text{ and } C_2 > 0. \quad \text{(A8)}$$

(A6) and (A8) imply the changes in specialization patterns mentioned in proposition 4.

In the above analysis we assume that the country that imposes investment tax $s_I$ also imposes saving tax $s_A$. However, even if the country that does not impose an investment tax imposes a saving tax, it raises $\rho_{\min}$ in the same way as long as the country is more patient. Thus, as long as the more patient country imposes a saving tax, all of the above results hold true regardless of which country is more patient.
References


Figure 1

An Illustration of the effect of $s_i^h$ on specialization pattern
Figure 2

An Illustration of the effect of \( s_A \) on specialization pattern under CES technology (when \( \overline{C}_2 \) is sufficiently small and \( \overline{C}_1 \) is relatively large)