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"Development of Interactive Support Systems for Multiobjective
Decision Analysis under Uncertainty: MIDASS"

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# Development of Interactive Support Systems for Multiobjective Decision Analysis under Uncertainty: MIDASS

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**Abstract**

This paper presents interactive multiobjective decision analysis support systems, called MIDASS, which is a newly developed interactive computer program for strategic use of expected utility theory. Decision analysis based on expected utility hypothesis is an established prescriptive approach for supporting business decisions under uncertainty, which embodies an effective procedure for seeking the best choice among alternatives. It is usually difficult, however, for the decision maker (DM) to apply it for the strategic use in the realistic business situations. MIDASS provides an integrated interactive computer system for supporting multiobjective decision analysis under uncertainty, which assists to derive an acceptable business solution for DM with the construction of his/her expected multiattribute utility function (EMUF).

## 1. Introduction

This paper presents a newly developed interactive computer program for supporting multiobjective decision analysis under uncertainty along with the methodological discussions.

The methodology is decision analysis, which is founded on the von Neumann-Morgenstern type expected utility hypothesis. The program is called MIDASS (Multiobjective Interactive Decision Analysis Support Systems), which intends to support the expected multiobjective decision analysis in practical use as an extension of multiobjective decision analysis to uncertain environments. In section 2, methodological backgrounds of the expected utility hypothesis are discussed along with the criticisms against it. The characteristics of decision analysis as a practical tool and its multiobjective extensions are discussed. In Section3, the precedent development of computer programs for supporting decision analysis is examined. Section 4 concerns the development of a new computer program MIDASS for multiobjective interactive decision analysis under uncertainty. Necessity of construction of the intelligent decision support systems (IDSS) for executing decision analysis is discussed and the main characteristics of MIDASS as the computer program are discussed. In Section 5, the operations of MIDASS program are explained in some detail. This section can be used as the manual of MIDASS in practical use. Concluding remarks for a new era of decision analysis are presented in Section 6.

## 2. Methodological discussions

### 2.1 Expected utility hypothesis

The main characteristic of von Neumann-Morgenstern type expected utility hypothesis (von Neumann and Morgenstern 1944, 1947) is to provide an integration of the evaluation of the decision maker (DM). Generally making decisions in business environments includes two phases: One is controllable and the other is uncontrollable to DM. Alternative actions chosen by DM in business management are

usually controllable. Uncertain events to occur in the decision environments are uncontrollable. Therefore the evaluations for decision results should be twofold: One is the evaluation assigned to a consequence of an action of DM and the other is the evaluation assigned to a chance of occurrence of an uncertain event. Expected utility hypothesis provides the twofold evaluation in mathematical functions. (i) The numerical assessment of preferences for the consequence of action, which is represented with the numerical utility function. (ii) The numerical assessment of the degrees of confidence for the occurrence of the uncertain event, which is represented with the probability function. Expected utility hypothesis presents a unified evaluation of both the elements in the expected utility function, where the utility and probability functions are evaluated both with the probability measure on a cardinal scale.

The mathematical expectation with the expected value $E$ is formulated as

$$E(x) = \sum_{j=1}^{n} p(\phi_j) x(\phi_j),$$ \hfill (1)

where $\phi_j$, $j = 1, \ldots, n$, denotes an occurrence of an event, $x(\phi_j)$ is a value of an outcome $x$ when an event $\phi_j$ occurs, and $p(\phi_j) = p(x(\phi_j))$.

The von Neumann-Morgenstern type expected utility function (EUF) is a generalization of the mathematical expectation for the subjective assessment of DM imposed on a decision result, which represents a conjoint measurement of both elements of (i) preference and (ii) probability in a mathematical form,

$$Eu(x) = \sum_{j=1}^{n} p(\phi_j)\, u(x(\phi_j)),$$ \hfill (2)

where $u(x(\phi_j))$ is a value of the utility function. $u(x_j), x_j \triangleq x(\phi_j)$, is assessed on a value of an outcome $x$ when an event $\phi_j$ occurs. Note that $x_j$ takes a scalar value.

The von Neumann-Morgenstern type utility function $u(x)$ is a numerical representation of human preferences, in which the numerical order of the utility function $u(x)$ corresponds to the preference order of the outcome $x$ (evaluation objects). The utility function is derived on several presumptions for the rationality of human behavior such as weak order (von Neumann-Morgenstern 1947, Savage 1954, Luce and Raiffa 1957). In this sense, the utility function $u(x)$ is a behavioral function defined in the axiomatic foundation. The concept of the von Neumann-Morgenstern type utility function makes the quantitative evaluation possible for any object in a common scale.

The von Neumann-Morgenstern expected utility theory presents the theoretical foundation of decision analysis.

**2.2 Decision analysis**

Decision processes are composed generally of two phases: Judgemental and analytical (Seo and Sakawa 1988). Decision analysis presents an art for aiding decisions with an integration of the both phases. In particular, decision analysis provides a method for heuristic construction of the mathematical forms of the utility functions and the probability functions assessed by DM, which is used with the decision tree technique. (Schlaifer 1959, Raiffa and Schlaifer 1961, Raiffa 1968, Schlaifer 1969, Pratt *et. al*. 1965 & 1995). The heuristic construction of the mathematical functions is performed step by step with judgemental processes of DM for the evaluation. In these processes, the both phases of the decision processes are combined with a proper procedure. The solution of the choice problem is found by the human judgement of DM, which is not simply generated with the mathematical calculations automatically.

**2.3 Criticism to expected utility hypothesis**

Many criticisms to the expected utility hypothesis have been raised. The criticism comes from several directions.

1. "Unrealistic" assumption of the rationality in human actions.

This criticism mainly comes from psychological experiments in laboratories. It has been asserted that actual human behaviour is often irrational and contradicting to the rationality assumptions as in Neumann-Morgenstern type expected utility hypothesis. The criticisms have mainly been directed against the normative approach or descriptive approach to decision analysis. The normative approach insists that human behaviour always must be rational. The rational behaviour with the transitivity or weak order, however, presumes the endowment of supreme wisdom with perfect knowledge; it is unrealistic in human worlds. The empirical approach asserts that the human behaviour in laboratory experiments usually reveals the contradiction or the deviation from the rationality hypothesis. The developers of decision analysis, however, have pointed out since an early stage that decision analysis intends not to be normative and nor descriptive, but only to be prescriptive. The rationality hypothesis is used as a tool for aiding decisions to avoid from falling into decision traps and from straying in decision jam. (Raiffa 1968; Bell, Raiffa and Tversky 1988). In fact, DM in the corporate management usually does not play the experimental or hobby games as in the laboratories, but the serious games in the vulnerable business environments.

2. "Single decision maker" assumption.

Decision analysis presumes the existence of DM as a single decision maker. The criticism asserts that decisions usually are not made by a single decision maker. The corporation, however, has legal personality and usually behaves as an individual in the markets. Business decisions always are presented in the markets as if the decisions have been made by the single decision maker. Internal

decision processes such as group decision making, if any, are not revealed to the public with any corporate responsibility.

3. "Expected utility function is an illusion," because nobody knows the mathematical functions empirically.

Decision analysis provides a sequential procedure for heuristic construction of the mathematical functions of DM, who does not use the mathematical functions as endowed a priori or arbitrarily.

4. "Optimality principle" is unrealistic for the use in managerial decisions with incomplete information.

Von Neumann-Morgenstern hypothesis does not presume the extremal principle, even though *Eq.*(2) can be used as an objective function in a mathematical maximization problem. DM will usually select a solution as the best-preferred solution when the EUF-value in *Eq.*(2) is in an acceptable range. The hypothesis uses EUF not in the optimality principle, but in the satisficing principle.

### 2.4 Multiobjective decision analysis

Multiobjective extensions of decision analysis have been intended since an early stage of the development.

The multiple objectives in decision problems are not simply multivariate, but also embody specific characteristics.

(1) Large-scaled objective systems. Multiobjective decision problems generally include many elements as the object of the evaluation, which constructs a large-scale decision problem.

(2) Incompatibility. The multiple objectives are usually not compatible and in conflict with each other.

(3) Incommensurability. The multiple objectives are not commensurable in a common scale, which should be scaled with different measures.

Multiobjective utility analysis has been developed for coping with these characteristics properly, which is called Multiattribute Utility Theory (MAUT) (Raiffa 1968, Keeney 1974, Keeney and Raiffa 1976). The variate that denotes a performance level of an objective in a quantitative term is called an attribute. Decision analysis assesses the attribute as the object of evaluation.

MAUT constructs the multiattribute utility function $U(x_1, x_2, \dots, x_m)$ on multiple attributes ($x_1$, $x_2, \dots, x_m$). The construction is based on the decomposition principles with the preference and utility independence assumptions, which derive the utility function in the representation form such that $U(u_1(x_1), u_2(x_2), \dots, u_m(x_m))$.

Characteristics of MAUT correspond to those of the multiobjective decision problems.

(1) Construction of a hierarchical configuration of the multiple attributes in multiple levels. In treating the multiple attributes, the multiple objectives are decomposed according to the property of

the attributes and integrated in a structured hierarchical system at multiple levels (Mesarivic, Marko and Takahara 1970).

(2) Value trade-off experiments. In reconciling the conflicting objectives among the attributes, the value trade-off experiments are executed between the attributes. Scaling constants for the attributes are assessed as the weighting parameter with this procedure. The utility function is constructed with the scaling constants in the form as $U(u_1(x_1), u_2(x_2), \ldots, u_m(x_m); k_1, k_2, \ldots, k_m, K)$.

(3) Construction of multiattribute utility function (MUF). For coping with the incommensurability for the attributes, the utility function for the attribute is assessed on a common scale between 0 to 1. The multiattribute utility function (MUF) is constructed on the singleattribute utility functions (SUF, or UNIF) and the scaling constants as its components.

The construction of MUF makes the numerical evaluation of the multiple objectives embodying the incommensurate and incompatible properties possible on a common scale. The mathematical forms of MUF are represented (Keeney 1974) as

(Additive)

$$U(x) = \sum_{i=1}^{m} k_i u_i(x_i), \quad \text{where } \sum_{i=1}^{m} k_i = 1, \ 0 < k_i \le 1. \tag{3}$$

(Multiplicative)

$$U(x) = \frac{1}{K}\left[\prod_{i=1}^{m}\{1 + Kk_i u_i(x_i)\} - 1\right], \quad \text{where } \sum_{i=1}^{m} k_i \ne 1, \ K > -1, \tag{4}$$

where $x_i$, $i = 1,\ldots, m$, is an attribute, $u_i(x_i)$ is a SUF for an attribute $x_i$, and $0 \le u_i(x_i) \le 1$, $0 \le U(x) \le 1$. The $k_i$ and $K$ are the scaling constants assessed on the attribute. $K$ is obtained as a unique solution of

$$1 + K = \prod_{i=1}^{m}(1 + Kk_i). \tag{5}$$

The uniqueness of the solution $K$ has been proved by Keeney (1974).

The numerical order of the scaling constants corresponds to the preference order of the attributes, such as

$$x_s \succ \ldots \succ x_j \succ \ldots \succ x_r \Leftrightarrow k_s > \ldots > k_j > \ldots > k_r. \tag{6}$$

Where $x_s$ denotes the attribute regarded as most important in a subsystem.

Note that $x$ in Eqs.(3) (4) is a vector. MAUT finally constructs MUF with a heuristic procedure.

MAUT proceeds in the following processes.

(1) Hierarchical structuring of multiple attributes. The multiple attributes are structured in multilevel according to their properties. The attributes are decomposed and integrated according to the objectives

hierarchy.

(2) Assessment of the singleattribute utility function. The singleattribute utility function (SUF, or uniattribute utility function UNIF) $u_i(x_i)$ for an attribute $x_i$ is assessed for each subsystem at the lowest level.

(3) The indifference experiments for the value trade-offs. The value trade-off assessment between attributes in each subsystem is executed, with which the scaling constants $k_i$ on an attribute $x_i$, $i = 1, 2, \ldots, m$, and $K$ are derived. The indifference experiments are performed between an attribute $x_s$ selected as the most important and the other attribute $x_{\bar{s}}$ in the subsystem.

(4) The calculation of MUF. MUF is calculated as in $Eq.(3)(4)$ with SUF $u_i(x_i)$ and the scaling constants $k_i$ .and $K$..

(5) Nesting of MUF. In assessing the large-scale systems including a large number of attributes, MUF is constructed with a nesting structure in multiple levels, such as, say, MUF = [MUF {MUF (MUF, MUF, SUF), MUF, MUF, SUF}, MUF, SUF]. The construction procedure of MUF is executed repetitively through the hierarchical structure in the multiple levels.

The value indifference experiments for assessment of the scaling constant $k_p$, $p = 1, \ldots, q$, in deriving MUF at the upper level are executed between the attribute $x_{sp}$, selected as most important in each subsystem $p$ at the lower level.

The construction of MUF for an overall system is make the numerical comparison possible for alternative actions of DM, which leads to choice of the most preferred solution in the complex decision environments.

(6) Sensitivity analysis. Sensitivity analysis can be performed on the attributes and SUFs, which makes the construction of alternative action plans with alternative criteria possible based on the utility analysis.

Although MAUT has provided an integrated evaluation method for the alternative action plans in complex decision environments, MAUT does not treat uncertainty in its original representation forms. Decision analysis, however, has primarily intended to cope with the uncertainty as its prominent characteristic, in particular, in the application to the business environments. We extend MAUT by introducing the probability evaluation in multiobjective decision analysis in uncertain environments.


**2.5 Expected multiobjective decision analysis**

Expected multiobjective decision analysis is an extension of the von Neumann-Morgenstern type expected utility hypothesis to the multiobjective decision environments. The method concerns the numerical construction of the expected multiattribute utility function (EMUF) embodying the probability distribution of DM. Expected multiobjective decision analysis presents a heuristic procedure for constructing EMUF, with which the numerical comparison of alternative action plans

with the multiple attributes makes possible under uncertainty.

The representation forms of EMUF are as follows.

(Additive)

$$EU(x) = \sum_{i=1}^{m} k_i \sum_{j=1}^{n} p_j(x_i(\phi_j)) u_i(x_i(\phi_j)) \quad \text{when} \sum_{i=1}^{m} k_i = 1, \ 0 \le k_i \le 1, \tag{7}$$

(Multiplicative)

$$EU(x) = 1/K \left[ \prod_{i=1}^{m} \{1 + Kk_i \sum_{j=1}^{n} p_j(x_i(\phi_j)) u_i(x_i(\phi_j)\} - 1 \right], \quad \text{when} \sum_{i=1}^{m} k_i \ne 1, \ K \ne 0, \tag{8}$$

where $x_i(\phi_j)$ is a value for an attribute $x_i$, $i = 1, \ldots, m$, to take when an event $\phi_j$, $j = 1, \ldots n$, occurs. Note that $x$ is a vector.

In *Eqs*.(7)(8) of EMUF, *Eu(x)* in *Eq*.(2) is substituted for *u(x)* in *Eqs*.(3)(4). The value of the scaling constant is assessed on an attribute $x_i$, $i = 1, \ldots, m$, and used as the weighting parameter for *Eu(x)* as the component of EMUF. The scaling constants are assessed not as the random value but as the non-random value on the attributes *x*. The method for evaluating the scaling constants in EMUF is the same as in MUF, because the probability evaluation for the uncertain events is independent of the preference evaluation for the attributes and thus introducing the probability evaluation $p_j(x_i(\phi_j))$ in EMUF as *Eqs*.(7)(8) has no effect on the evaluation of the scaling constants for the attributes. The construction of the mathematical forms of EMUF is not affected by the replacement of *Eu(x)* in the representation forms of EMUF for *u(x)* in the representation forms of MUF.


## 3. Preceding development of computer assistance

The practical use of decision analysis is generally hard work. In particular, its execution to expected multiobjective decision analysis is not easy to operate in practice. Computer assistance for the decision support is highly recommended.

Since an early stage of the development in decision analysis, computer programs have been attracted much attentions for its effective manipulation. MANECON Collection (Schlaifer 1971) is the most excellent precedent in providing the interactive processes for aiding decisions in the preference and probability evaluations in expected utility analysis. The interactive assistance in MANECON does not only mean the man-machin interaction, but alsothe interactive assistance of logical thinking processed of DM. MANECON Collection, however, only concerns the single objective decision environments under uncertainty.

MUFCAP (Multiattribute Utility Function Calculation and Assessment Program) (Sicherman 1975) has been presented as a multiobjective extension of the computer program for decision analysis.

MUFCAP supports MAUT in (i) the hierarchical configuration of the multiobjective systems structure, (ii) assessment of additional types of SUF, (iii) the value trade-off experiments between attributes, (iv) calculation of MUF, and (v) sensitivity analysis for the evaluation results. This software, however, does not concern the uncertainty in the decision environments, which is an indispensable factor in decision analysis. In addition, MUFCAP does not concern the interactive processes for the evaluations. MANECON Collection and MUFCAP are operated both in the mainframe computer systems.

We have already presented a computer programs ICOPSS (Interactive Computer Program for Subjective Systems) (Sakawa and Seo, 1980) as a successor of MUFCAP, which is written in PL/1 language the same as MUFCAP and is operated with some graphical representation on the TSS (Time Sharing Systems). ICOPSS has been applied for the regional impact analysis of construction of the Hokuriku Shinkansen in Japan, where the regional objective systems include 269 attributes decomposed to 9 levels in a hierarchical system (Seo, et al. 1986).

Recent progress of information technologies is promoting the development of computer programs to be operated in the downsizing environments with visual interface, which largely enhances the user-friendly property of computer programs. On this progress, we have presented new extensions of the existing computer programs. One is MAP (Multiattribute Utility Analysis Program), which is a visual extension of MUFCAP for multiobjective decision support. This program is written with the object-oriented language C++ and operated in the GUI environments in workstation systems. MAP has been applied to the posterior assessment of regional development in the southern part of Korea, which includes 247 attributes structured in 9 levels (Seo, Nisizaki and Park. 1999). The other is IDASS (Interactive Decision Analysis Support Systems) (Seo and Nishizaki 1997), which is a visual extension of MANECON Collection for decision support under uncertainty. IDASS is operated also in the workstation systems.

Both programs MAP and IDASS, however, still succeed the main deficiency of the precedent programs. Although IDASS supports the interactive decision processes under uncertainty and MAP executes the multiobjective evaluation both with the visual interface, these programs inherit the shortages of the preceding programs, non-multiobjective or non-interactive respectively. An integration of the both programs is expected for the multiobjective decision support under uncertainty.

In the following sections, we present a newly developed computer program MIDASS (Multiattribute Interactive Decision Analysis Support Systems) for supporting interactive multiobjective decision analysis in uncertain decision environments.


## 4. Development of MIDASS

### 4.1 General characteristics of MIDASS

MIDASS, which is a computer program for decision support in operating expected multiobjective decision analysis, is developed on the concepts of the intelligent decision support system (IDSS) (Seo and Nishizaki 1993). IDSS is a decision support system constructed on an axiomatic or disciplinary foundation with computer assistance, where the phase for human judgement is included as its prominent component.

IDSS is composed of three phases: Database Shell, Expert System Shell, and Decision Shell. Database Shell concerns to construct and manipulate a database as a data management system. Expert System Shell concerns to treat a specific knowledge base on an axiomatic or any other disciplinary foundation, where a knowledge-base management system is constructed with empirical and intuitive skills of experts as well as analytical or mathematical tools. Decision Shell concerns making decision of DM judgementally and subjectively, where a managerial decision system is constructed. These phases construct a decision loop on evolutional stages. The general conceptual framework of the IDSS is composed of the major shells and, as a particular computer-assisted system, a software program is constructed. MIDASS is developed on this concept as a computer-aided decision support system.

The general characteristics of MIDASS are as follows.

(1) MIDASS is founded on the von Neumann-Morgenstern expected utility hypoyhesis.

(2) MIDASS supports the methodological operation of decision analysis, which includes the construction of mathematical functions.

(3) MIDASS supports interactively the logical thinking processes for judgmental decisions with consistency.

(4) MIDASS is an integrated program package composed of the separate parts, which can be independently used.

(5) The programs and evaluation results in MIDASS are combined in integration on the common database that is constructed independently of the operations.

## 4.2 Program properties of MIDASS

MIDASS intends to construct EMUF (*Eqs.*(7)(8)) as a decision criterion under uncertainty.

Program characteristics of MIDASS as a computer software are as follows.

(1) Program parts decomposition.

MIDASS is composed of several parts corresponding to the components in EMUF, which are independent of each other and can be operated separately for a specific job intended.

(2) Supporting methodological transition in order.

The operation in the evaluation processes of EMUF proceeds step by step. The user starts selectively from the main menu screen. The screen transition of MIDASS is structured in order corresponding to the method of expected multiobjective decision analysis.

(3) Interactive assistance with consistency checks.

MIDASS supports interactively the evaluation processes of DM, which include the consistency checks for the assessment. Instructions for the proper selection in the evaluation are provided on the screen and, if necessarily, a message appears to suggest a possible revision. The evaluation processes proceed step by step in the screen transitions, where the user can turn back to the previous screen for the re-examination of preceding evaluations.

(4) Functional independence.

The functions of MIDASS program are operated independently of each other, which can be used separately for partial use of the MIDASS programs.

(5) Data independence.

Database of MIDASS is constructed independently of the program operations. Due to the data independence property, the availability, consistency or integrity of the database are preserved. The data independence property raises the serviceability of the database in MIDASS.   A revision of data files for a special job makes no effect on the original database. The database can be managed independently of its use.

(6) Preservation of all evaluation results.

MIDASS preserves all results of evaluation once saved as long as they are not deleted. All evaluation results are ready to use for alternative assessment in a later job.

(7) Sensitivity analysis.

MIDASS includes sensitivity analysis of the preference evaluations，which can be used for efficient construction of alternative management plans.

(8) "Fool proof" property.

MIDASS generates a message for "fool-proof." The message appears frequently in every stage of the operation for alarming and confirming user's operations, which protects the user operation from the careless misses through the evaluation works.

(9) Down-sizing with visual interface.

MIDASS is operated on the visual interface of PC systems, Windows 98, 2000 and XP at present. This property largely enhances the user-friendly property of computer utilization.

**4.3 Composition of MIDASS**

MIDASS is an integrated computer program, which is composed of the major parts of decision analysis: database construction, preference evaluation, probability evaluation and gamble evaluation.

(1) MIDASS embodies the data management system. The data management system is composed of two parts: the database management system and the job data management system. (i) The database management system constructs the data files of the attributes for general use, which is constructed

independently of a specific job. Database for MIDASS is constructed with the attributes and its prescribed values, which are the objects of the evaluation. (ii) The job data management system constructs a particular data set for a specific job, which is extracted from an indicated database file.

(2) The preference evaluation concerns the heuristic construction of the singleattribute-and multiattribute utility functions, SUF (or UNIF) and MUF, of DM.

(3) The probability evaluation concerns (i) the heuristic construction of the probability functions of DM and (ii) the calculation of posterior probability on Bayes' formula.

(4) The gamble evaluation concerns the evaluation of mathematical expectations: expected values (E), expected utility functions (EUF), and expected multiattribute utility functions (EMUF).

In the next section, the operation of MIDASS is described in some detail with illustrations, which can be used as a manual for the program operations. The "user" is used as a proxy of DM.

## 5. Operation of MIDASS

MIDASS starts from the main menu screen. Subsequent work proceeds with the screen transition.

In the main menu screen depicted in Fig.1, the major functions of MIDASS appear in the methodological order, from where the user proceeds selectively to the subsequent stages for executing the functions in the specific job. All functions of MIDASS are operated with buttons on the screen.

All functions of MIDASS, which appear on the main menu screen, become effective after the data set file is constructed for a specified job. The data input must be done in the lateral direction.

Back up of the input data and evaluation results is constructed with the <BackUp DB> button on the main menu screen. The file name is indicated with the extension ".mbs." Microsoft Access is used for making the back up file.

### 5.1 Database management

After starting MIDASS by a click on the MIDASS icon, the first screen appears as in Fig.2. In the beginning of a job operation, the user must construct a database for possible jobs in advance.

Click the <Attribute Database Management> button. The Attribute Database screen appears. MIDASS asks whether the database to be used is OLD or NEW. When the user constructs new data, the Attribute Management screen for the data input appears with a click on the <NEW> button. When the database already exists, the Select Attribute Database screen appears with the click on the <OLD> button. Then indicate the file name. Then the Attribute Management screen appears.

In the construction of the database, the worst and best values for the attributes to possibly take are assigned. Alternative value sets for the attributes are constructed in the intermediate ranges of the attributes between their worst and best values with the specification of the alternative names such as ALTx, etc. The alternative values are taken from empirical data, forecasting or reference values, etc.
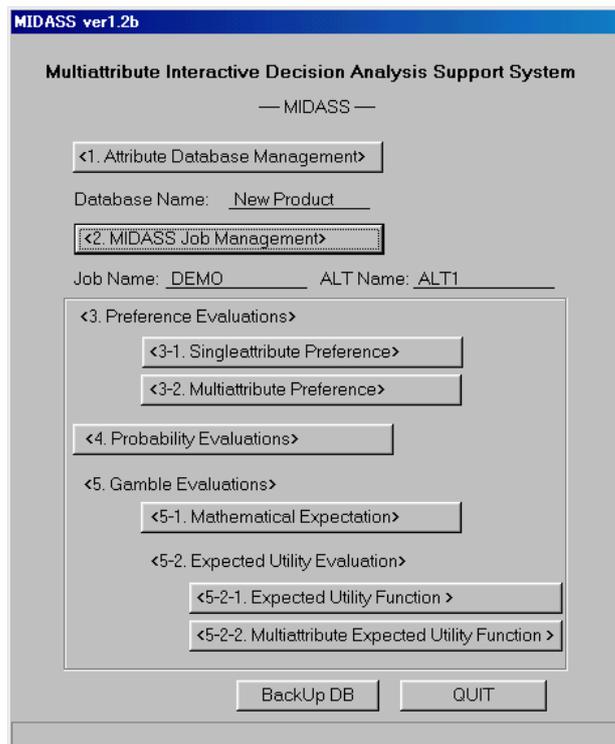
Fig. 1　Main menu screen
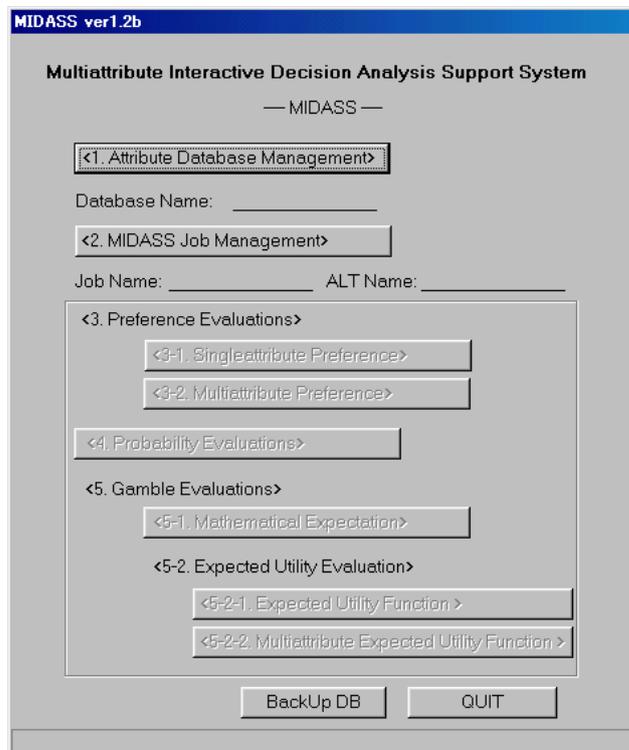


Fig. 2 First screen in operation
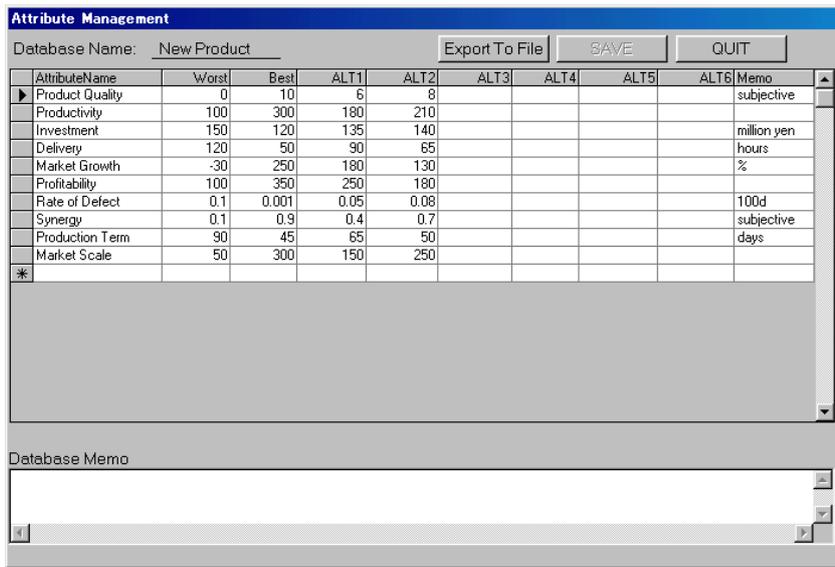
Fig.3 Database Management screen
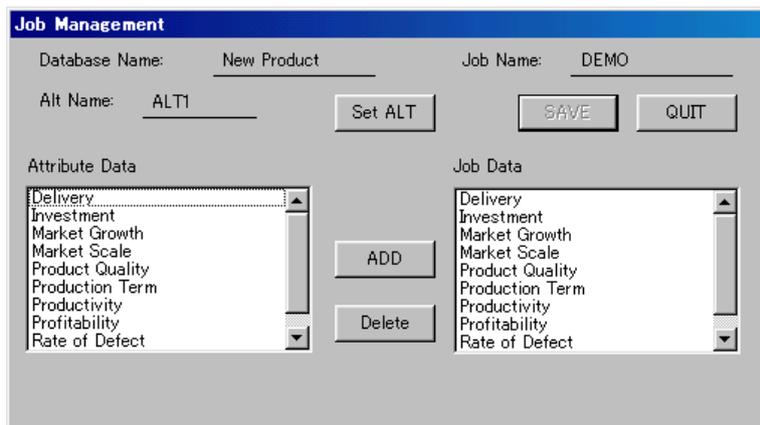


Fig.4 Job Management screen

The user proceeds to the <Job Management> screen as Fig.4 by a click of the <MIDASS Job Management> button on the main menu screen. MIDASS asks whether the job is <NEW> or <OLD>. The <Job Name> must be assigned. The job data set for a new job is constructed with the transfer of the attributes selectively from the <Attribute Data> window to <Job Data> on the <Job Management> screen. The <ADD> button is used for the transfer. The <ALT Name> is specified with the <Set ALT> button to evaluate a certain alternative in the subsequent works.

All buttons for operating all the functions of MIDASS become available on the main menu screen after the construction of the job data file.

The user can turn back to the main menu screen with the <QUIT> button on the screens.

## 5.2 Preference evaluation

## 5.2.1 Singleattribute preference evaluation

The preference evaluation of DM is executed in two ways: for the singleattribute and for the multiattribute. The utility functions of DM are constructed heuristically for the both as SUF and MUF.

The singleattribute utility function embodies DM's risk attitudes, which are represented numerically with the risk function. The risk function measures the local risk attitudes of DM's preference (Pratt 1964). The local (absolute) risk function is defined by

$$r(x) = -u''(x)/u'(x),$$ (9)

where $u(x)$ is the utility function for an attribute $x$.. When $r(x) > 0$ for all $x$, the risk attitude of DM is risk-averse and, when $r(x) < 0$, the risk attitude is risk-prone. The risk-neutral attitude is represented when $r(x) = 0$. The local proportional risk aversion is defined by

$$r*(x) = x\,r(x).$$ (10)

The risk attitude of DM is represented in assessing the certainty equivalent (CE) to the gamble evaluation for a lottery with an $n$-chance fork, which is represented as mathematical expectation.

Singleattribute preference evaluation starts with a click of the <Singleattribute Preference> button on the main menu screen (Fig.1). The Singleattribute Preference Evaluation screen appears as Fig.5.

Click first the <Select Attribute Data> button and indicate an attribute name. Then the assessment for all types of the utility functions becomes available. The input data and evaluation results are saved by assigning the file name on the evaluation screen for a type of the utility function. The user can call them in by the <LOAD> button on the menu screen. The <UNIF Set> button on the evaluation screen calculates the parameters of the mathematical form of the utility function along with its graphical representation. With the <QUIT> button, the user goes back to the preceding screen.

MIDASS supports seven types of the singleattribute utility functions, which embody the risk attitude of DM.

A. Risk-neutral preference
  1. Linear utility function
  2. Piecewise linear utility function

B. Constant risk preference
  3. Constant risk utility function
  4. Constant absolute risk-averse utility function
  5. Constant proportional risk-averse utility function

C. Decreasing risk-averse preference
  6. Sumex-type utility function
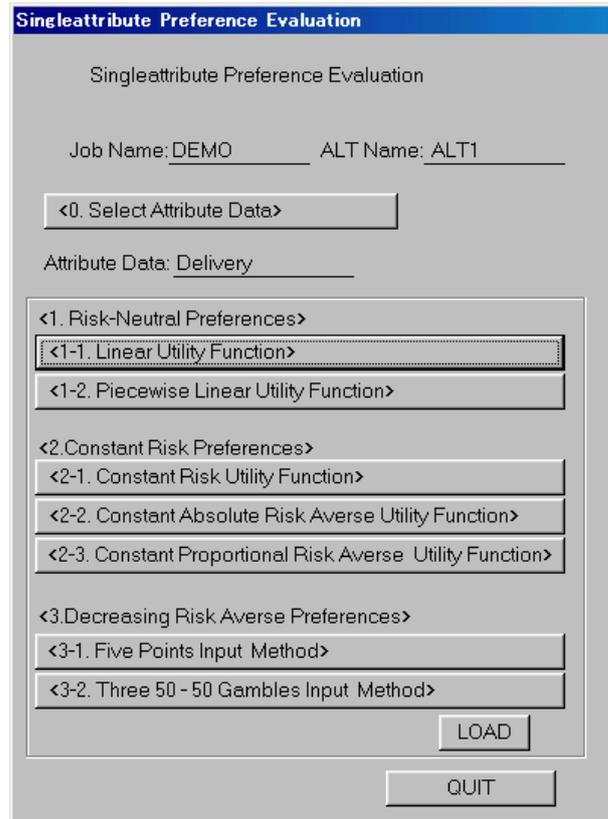  7. Piecex-average-type utility function

Fig.5 Menu screen for singleattribute preference evaluation

A. Risk-neutral preference

1. Linear utility function

The representation form of the linear utility function is

$$u(x) = ax + b,$$ (11)

where $x$ is an attribute and $u(x)$ is a utility function. The evaluation screen is illustrated for the attribute Delivery in Fig.6, which is assessed the attribute values in a descending order. After the data input, the parameters of the utility function are calculated with a click of the <UNIF Set> button, where UNIF is used for SUF (singleattribute utility function). The <Cal U(x)> button calculates the UNIF value with an input value of $x$. The <Cal inv U(x)> button calculates a value of $x$ with the input of a UNIF value. The operation is the same as in the evaluation screens for other UNIF types.

2. Piecewise linear utility function

The representation form of the piecewise linear utility function is

$$u_{n_j}(x) = a_{n_j}(x) + b_{n_j},$$ (12)

where $n_j$, $j = 1, \ldots, k$, denotes the number of a segmentation of the function. Fig.7 illustrates the evaluation of the piecewise linear utility function with five segmentations for the same attribute.

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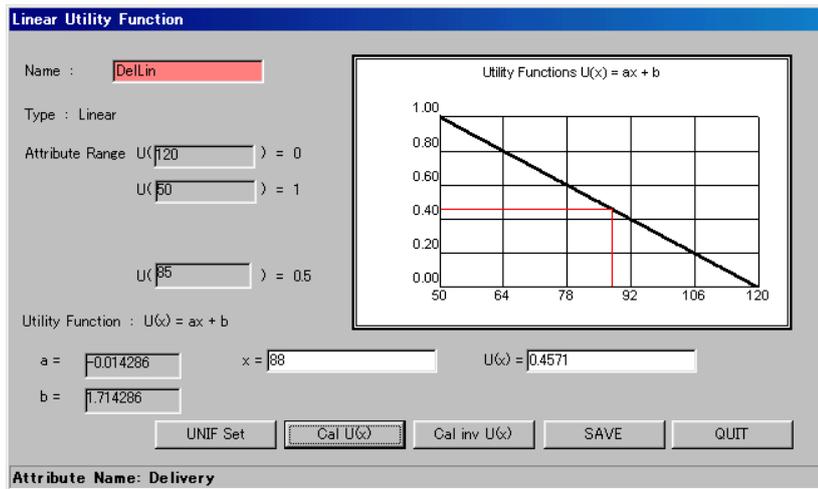
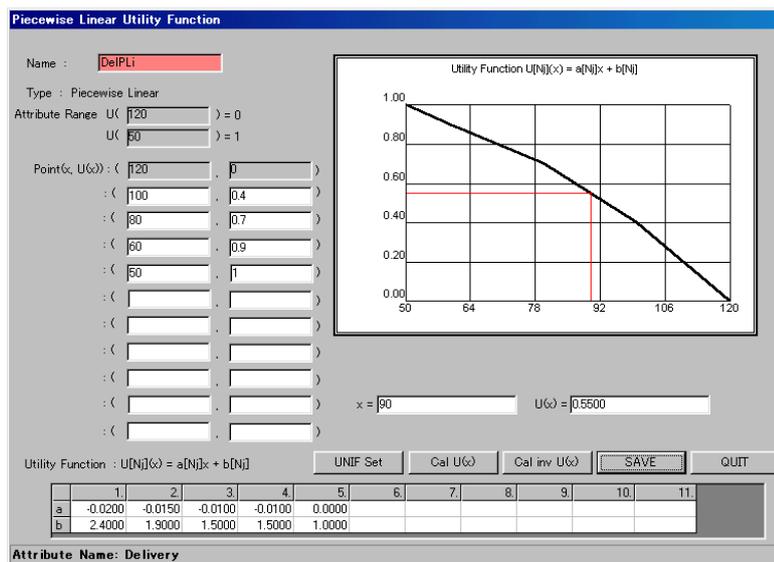Fig.6 Assessment of linear utility function (Delivery)



Fig.7. Assessment of piecewise linear utility function (Delivery)

B. Constant risk preference

3. Constant risk utility function

The constant risk utility function is represented in the mathematical form

$$u(x) = a + b \exp(-cx). \tag{13}$$

When $c > 0$, the utility function $u(x)$ represents the risk-averse attitude of DM and, when $c < 0$, the utility function $u(x)$ is risk-prone.

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The evaluation of the constant risk utility functions proceeds in three steps on the screen.

*Step 1.* Assign the file <Name> for the evaluation.

*Step 2.* As input data, assess tentatively the reference lottery with the two-chance fork and its certainty equivalent (CE) $\hat{x}$, where the reference lottery is defined to get with a 50-50 chance the best or worst value of *x*, for which the utility value 1.0 or 0.0 is assigned respectively. The expected value of the reference lottery is 0.5 and the user assesses CE with 0.5-utility value. When the CE is not satisfactory for DM, another lottery is constructed with different probability for 1.0 or 0.0 utility value. The input value of *u*(*x*) on the screen is an expected utility value for the lottery and the *x*-value is its CE, which is represented in an ordinate and in an abscissa of the graphical representation on the screen. The indifference experiment between a lottery and its CE is executed repetitively until the result is satisfactory for DM. The user can check the evaluation result for the utility function on the screen graphically.

*Step 3.* When the evaluation result is acceptable, the user preserves the mathematical form of the utility function with the <SAVE> button.

Fig.8 illustrates the assessment of the constant risk utility function for the attribute Product Quality.
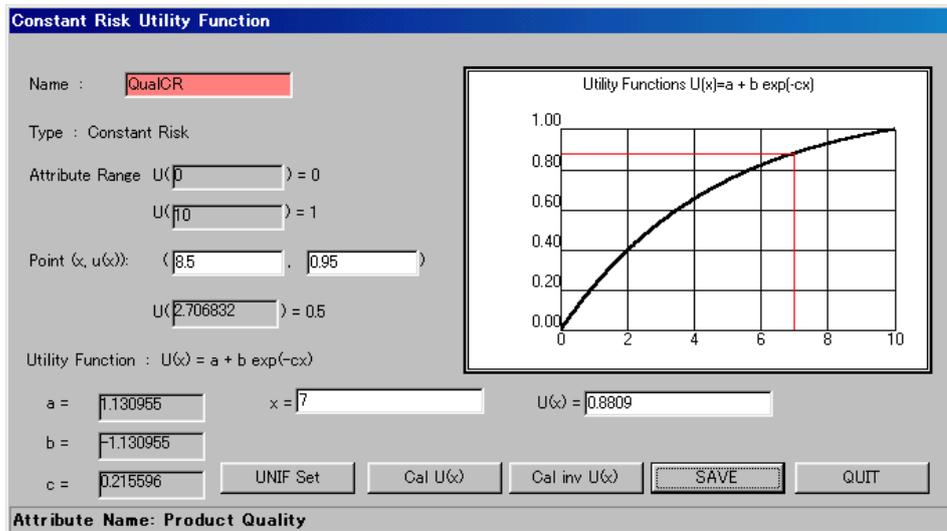


Fig.8. Assessment of constant risk utility function (Product Quality)

4. Constant absolute risk-averse utility function

The constant absolute risk-averse utility function represents the risk attitude of DM, in which a change in an attribute value *X* as a stock has no effect on the evaluation of the attribute value *x* as a flow with a chance fixed. The representation form of the utility function is

$$u(x,r) = -\frac{1}{r}(e^{-rx}-1), \qquad -\infty < x < \infty, \ -\infty < r < \infty. \tag{14}$$

The local risk aversion function is $r(x) = r$.

The utility evaluation is executed with the four points input for the worse, better and CE values of the attribute and a probability assignment for the better attribute value. MISASS calculates with the <Calculate> button the local risk aversion $r$ as the parameter of the utility function. The utility function is derived with the parameter $r$, whose values are not normalized. By assigning an attribute value, the utility value is calculated with the <CALC> button.

Fig.9 illustrates the evaluation screen for the attribute Productivity with the constant absolute risk-averse.

Once the utility function is specified, any gamble is assessed with the parameter $r$ for the risk aversion. Probability distributions for the gambles can be assigned in alternative ways: (i) gamble description by the user, (ii) call-in from a distribution file with the <Distribution in File> button, or (iii) the use of the Gaussian distribution with the assignment of mean and standard deviation SD as the input data. The gamble evaluiation is executed with the <Distribution in File> button.

MIDASS calculates CE and the risk premium (RP), $RP = x - CE$, for the described gambles or the specified probability distributions.

Fig.10 illustrates the evaluation of CE and RP for the gamble described by the user.



Fig.9 Evaluation of constant absolute risk-averse utility function (Productivity)
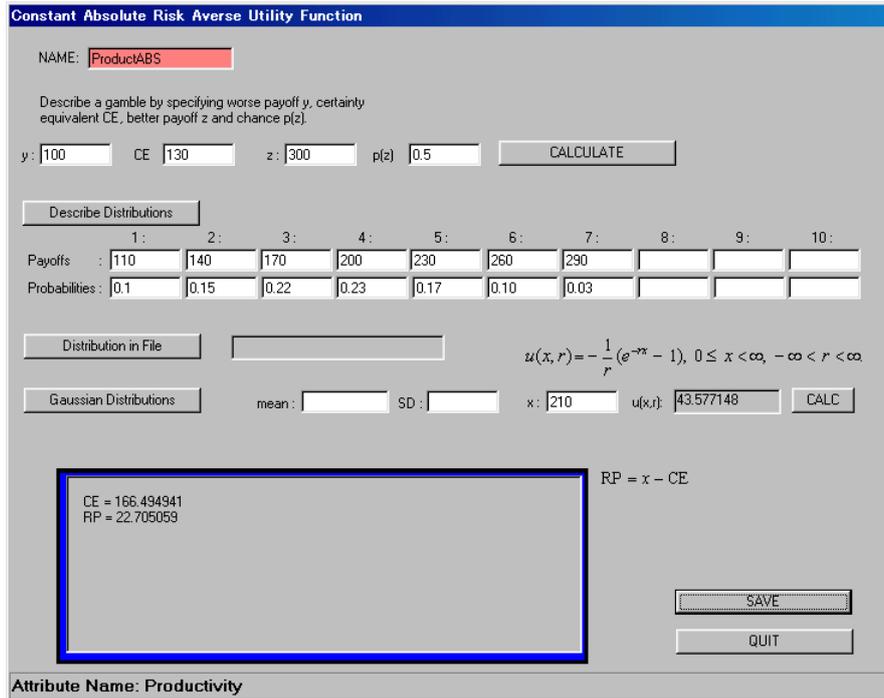
Fig.10 Illustration for constant absolute risk-averse utility function (Productivity)

5. Constant proportional risk-averse utility functions

The constant proportional risk-averse utility function represents the risk attitude of DM, in which a change in an attribute value $X$ as a stock has no effect on the evaluation for a fraction of the stock value with a chance fixed. The representation form of the utility function is

$$w\,(x,\,r) = u(\log x\,,\,r) \tag{15}$$

$$= -\frac{1}{r}(e^{-r\log x} - 1), \qquad 0 \le x < \infty, \; -\infty < r < \infty. \tag{16}$$

The local absolute risk aversion function is $r(x) = -w''(x,\,r)/w'(x,\,r) = (r+1)/x$, and the local proportional risk aversion is $r*(x) = x\,r(x) = r + 1$. The value of the local risk aversion is represented as the average value divided by the attribute $x$.

The evaluation method is the almost same as the constant absolute risk-averse case. The probability distribution for the gamble evaluation can be constructed with (i) the description of distributions by the user, or the generation by the use of (ii) the <Distribution in File> button, or (iii) the <Gamma or Lognormal Distribution> button with the assignment of mean and SD values.

In this case, the utility function values are not normalized.

Fig.11 illustrates the utility evaluation screen for the Productivity with the constant proportional risk-averse. Fig.12 illustrates the CE and RP calculations with the gamble description by the user.

Fig.11 Evaluation of constant proportional risk-averse utility function (Productivity)



Fig.12 Illustration for constant proportional risk-averse utility function (Productivity)

C. Decreasing risk-averse preference

The decreasing positive risk-averse preferences are assessed with two-types of the utility function: the sum of exponential (sumex) type and the piecewise exponential-average (piecex-average) type.

6. Sumex type utility functions (Schlaifer 1971)

The representation form of the basic (non-normalized) sumex-type utility functions is defined by

$$v(x) = -(e^{-ax} - 1) - c(e^{-bx} - 1)$$
$$= -e^{-ax} - ce^{-bx} + c + 1, \qquad a > 0, \ bc > 0. \tag{17}$$

The risk aversion function is

$$r(x) = -v''(x)/v'(x) = \frac{a^2 e^{-ax} + cb^2 e^{-bx}}{a e^{-ax} + cbe^{-bx}}, \tag{18}$$

where (i) $r(x) > 0$ everywhere, when $b, c > 0$, and (ii) $r(x) > 0$ to the left of the reflection point $x^*$ and $r(x) < 0$ to the right of the $x^*$, when $b, c < 0$. The reflection point is defined by

$$x^* = \frac{1}{a-b} \log(-a^2/(b^2 c)). \tag{19}$$

The normalized sumex utility function is defined by

$$u(x) = \frac{v(x) - v(x_0)}{v(x_1) - v(x_0)}, \tag{20}$$

where $x_0$ and $x_1$ denote respectively the worst and the best values of the attribute used as the input data.

7. Piecex-average type utility functions (Meyer and Pratt 1968)

The general representation form of the piecex-average utility functions is defined with a parameter $k$ by

$$v_k(x) = k u_1(x) + (1-k)u_2(x), \qquad 0 < k < 1, \ x_0 \le x < \infty. \tag{21}$$

The $u_1(x)$ and $u_2(x)$ are called the basis function. The local risk aversions of the basis functions $u_1(x)$ and $u_2(x)$ are nonnegative and non-increasing, but the basic functions are piecewise constant and decreasing in the steps.

The basis functions are defined by

$$u_1(x) = \begin{cases} f_2(x) & x \le y_3 \\ f_4(x) & x \ge y_3 \end{cases} \tag{22}$$

$$u_2(x) = \begin{cases} -\infty & x < x_0 \\ 0 & x = x_0 \\ f_3(x) & x_0 < x \le y_4 \\ f_5(x) & x \ge y_4 \end{cases}, \tag{24}$$

where $f_i(x)$ is the initial function defined by

$$f_i(x) = \begin{cases} a_i - b_i(e^{-r_i x} - 1) & i = 2, 3, 4, \\ a_i + b_i x & i = 5. \end{cases} \tag{25}$$

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The parameters *a, b,* and *r* in the initial functions have the values such that (i) $f_i$ passes through the input points, $i - 1$, $i$, and $i + 1$ for $i = 2, 3, 4$ and (ii) $f_i$ passes through the input points $i - 1$, and $i$ for $i = 5$. The $y_3$ and $y_4$ are the junction points, and $x_{0.25} < y_3 < x_{0.75}$, $x_{0.5} < y_4 < x_1$, $y_3 < y_4$. The subscript of *x* shows the utility value, with which the attribute value is assessed.

The piecex-average utility function is constructed when the risk aversion is decreasing and positive everywhere. The conditions for the proper fitting to the piecex-average preference function are to satisfy both (i) $r_2 > r_3 > r_4 > 0$ and (ii) $f_2(x) < f_5(x)$ for all *x* in $[x_{0.5}, x_{0.75}]$.

The evaluation method for assessing the decreasing risk-averse utility function is two-fold. One is to specify the five points for deriving the preference curve of DM; it is called the five point input method, and the other is to assess three 50-50 gambles with their certainty equivalents. This method is called the three 50-50 gamble input method and used for the sumex type utility function only. These methods are selected on the menu screen for the singleattribute preference evaluation (Fig.5).

1. Five points input method

The evaluation proceeds sequentially and interactively in the following steps.

Step 1 (Preliminary input). On the evaluation screen first appeared, the basic attribute values $V_0$ and $V_{1.0}$ assessed with the utility values 0 and 1 are assigned automatically for the selected attribute. The other three values $V_{0.25}$, $V_{0.5}$ and $V_{0.75}$ of the attribute to be assessed with the utility values 0.25, 0.5 and 0.75 respectively are indicated tentatively by DM on the screen.

Step 2 (Preliminary preference curve fitting). MIDASS depicts a preliminary preference curve of DM with a piecewise straight line by connecting the 5 input points. The user should examine this curve carefully with necessary modifications.

Fig.13 illustrates the preliminary preference curve for the attribute Market Scale on the screen.

Step 3 (Consistency check). The user proceeds to the next screen for consistency check.

MIDASS calculates with the input for the $V_{0.5}$*-value on the screen the local risk aversion $r_1$ (= R1), $r_2$ (= R2) and $r_3$ (= R3), and checks the consistency with the proper mathematical function forms by testing if the conditions (a) R1 > R2 > R3 > 0 and (b) R1 > > R2 are satisfied. The user can modifies the $V_{0.5}$*-value on the screen when the consistency check is not successful.

When the consistency is satisfied, the message appears, as depicted in Fig.14.

Step 4 (Derivation of the utility functions). Once the consistency is held, the user can proceed the derivation of the prescribed type of the utility function, either the piecex-average, or sumex type utility function selectively with the <PIECEXFIT> or <SUMEXFIT> button.

Fig.15 depicts the evaluation result of the sumex type utility function. Fig.16 shows the piecex type utility function.
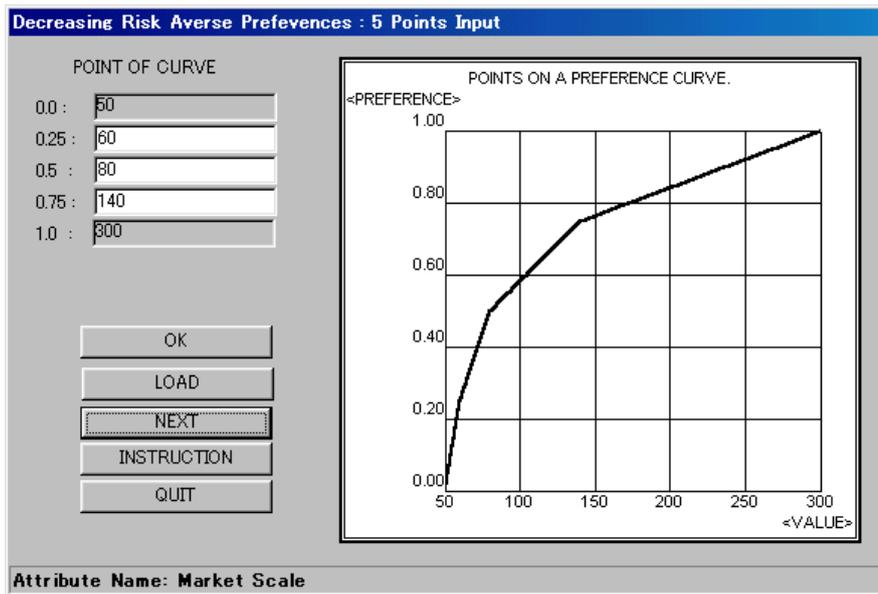
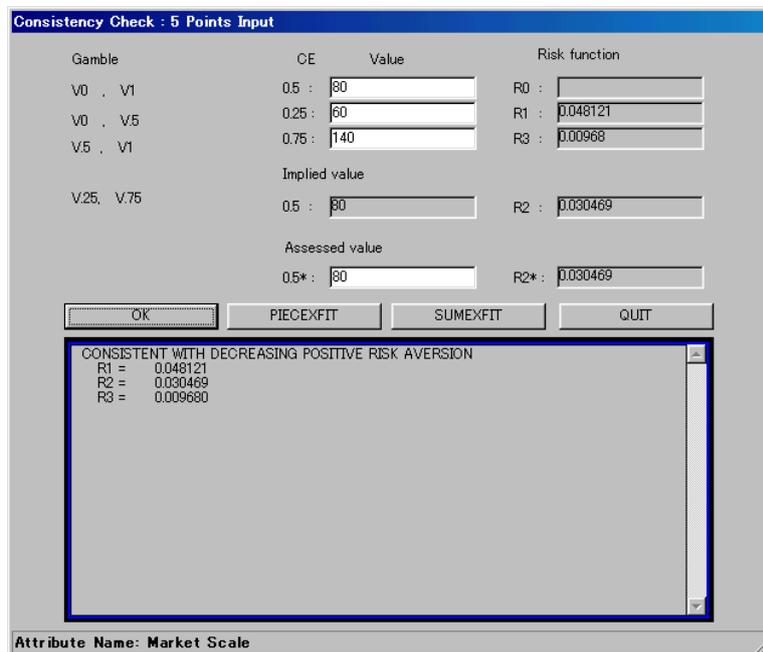Fig.13 Preliminary evaluation screen of preference (Market Scale) (I)



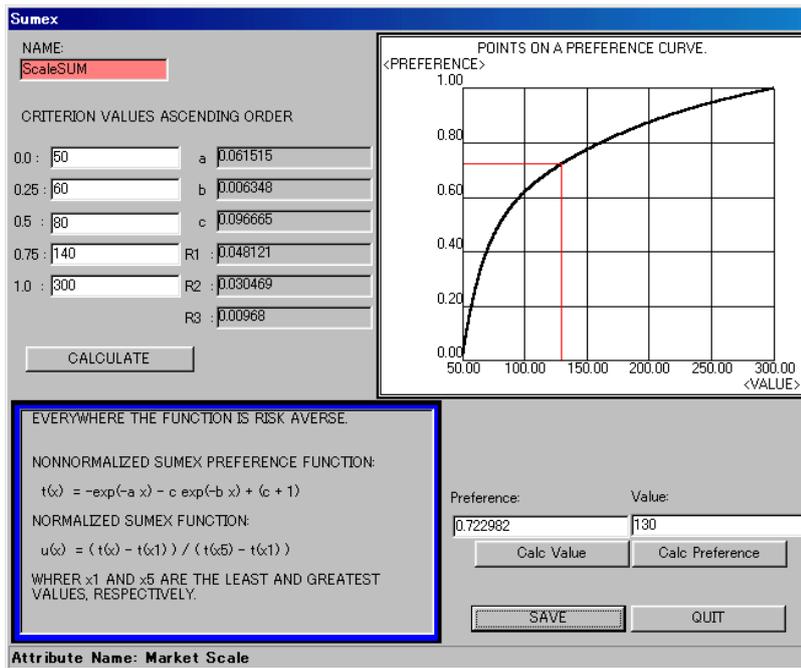Fig.14 Consistency check of the preference (Market Scale) (I)

Fig.15 Evaluation result for sumex type utility function (Market Scale)
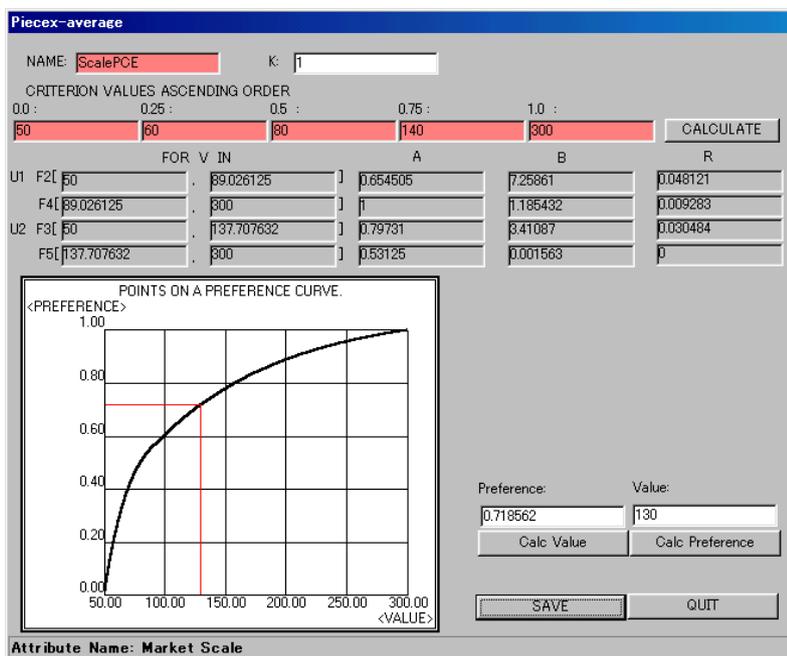


Fig.16 Evaluation result for piecex type utility function (Market Scale)

If it is necessary, the user can open the <INSTRUCTION> screen and keep it through the evaluation work on the screen as in Fig.17. The text is in the Appendix A.
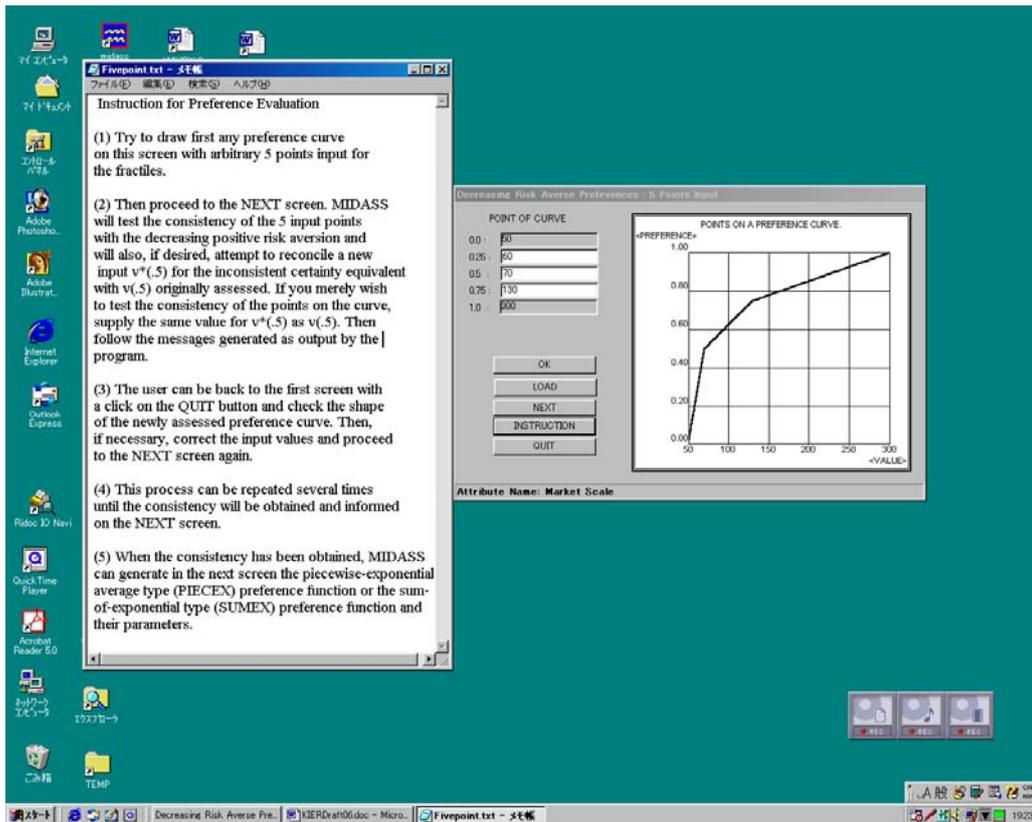
Fig.17 Assistance to user's evaluation process (Market Scale)

In the Step 4, when the user assesses the preliminary preference curve as in Fig.18 and the consistency check is not successful as shown in the <NEXT> screen as Fig.19, the user is suggested the modification of the assessment with the help messages, which appear automatically on the screen.

For the modification of the preference, (i) input the certainty equivalent (CE) $V_{0.5}*$ for an additional gamble for $V_{0.25}$ and $V_{0.75}$ obtained with the 50-50 chance. Then examine if $V_{0.5} = V_{0.5}*$. (ii) MIDASS checks the consistency again with a revised value of the risk function <R2*>, which is calculated for the modified CE $V_{0.5}*$. When the conditions for the risk functions are not satisfied, the revisions should be repeated. Schlaifer (1969, 1971) suggests that DM has the inclination to reveal the excessively large risk aversion for the narrow ranged gamble, which results in $V_{0.5} > V_{0.5}*$ and sometimes expresses a local disguised risk prone attitude of DM. For avoiding this psychological bias of the assessors, (a) $V_{0.5}$ should generally be decreased. (b) $V_{0.5}*$, however, can occasionally be increased properly.

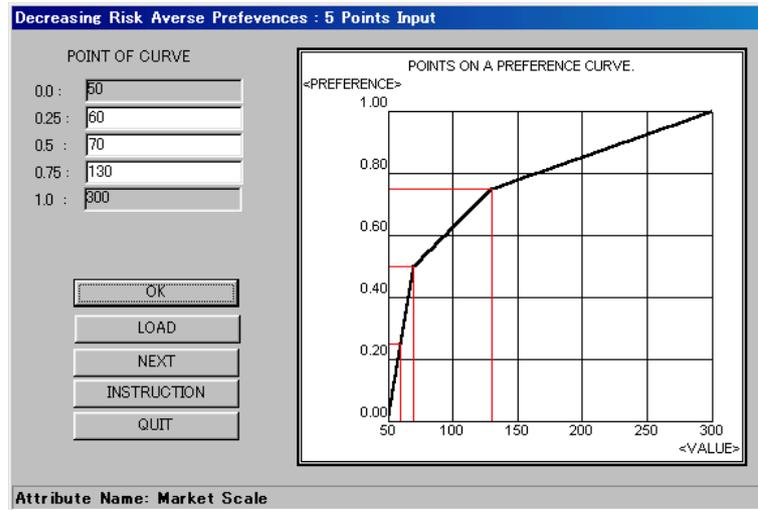The user finally obtains the proper result as in Figs.14.

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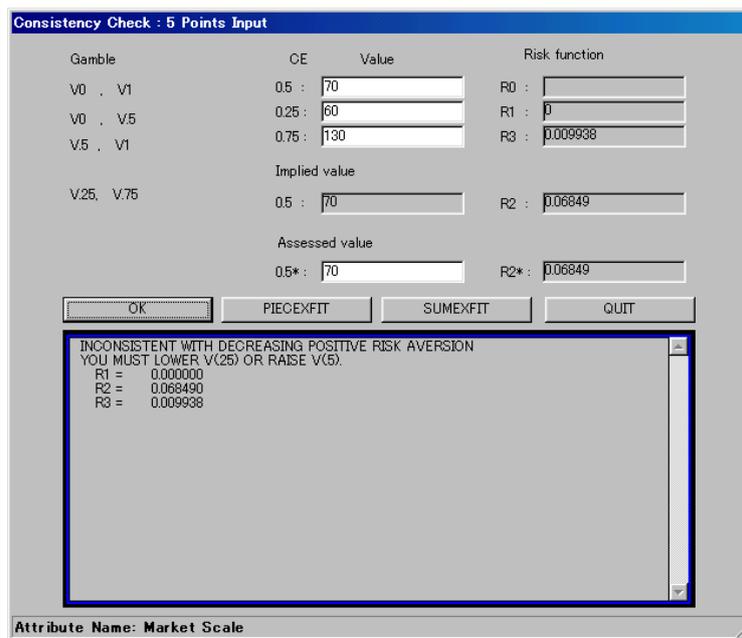Fig.18 Preliminary evaluation screen of preference (Market Scale) (II)



Fig.19 Consistency check of the preference (Market Scale) (II).

2. Three 50-50 gambles Input method

In the case of the sumex type utility function, the three 50-50 gamble input method is available.

(i) Click the <Three 50-50 Gamble Input Method> button on the menu screen. Then the valuation screen appears.

(ii) Specify the three 50-50 chance gambles and their CE on the screen. The three gambles should be constructed in an ascending order if they are not nested. If they are nested, the user can input the data

in any order. The average risk function <R> should take a positive value in a descending order.　In this method, the consistency check is not supported interactively. Fig. 20 depicts the three 50-50 gambles input method in the nested gambles case for the attribute Profitability.

　　All results of the evaluation once saved appear with the <LORD> button on the screen as in Fig.21.
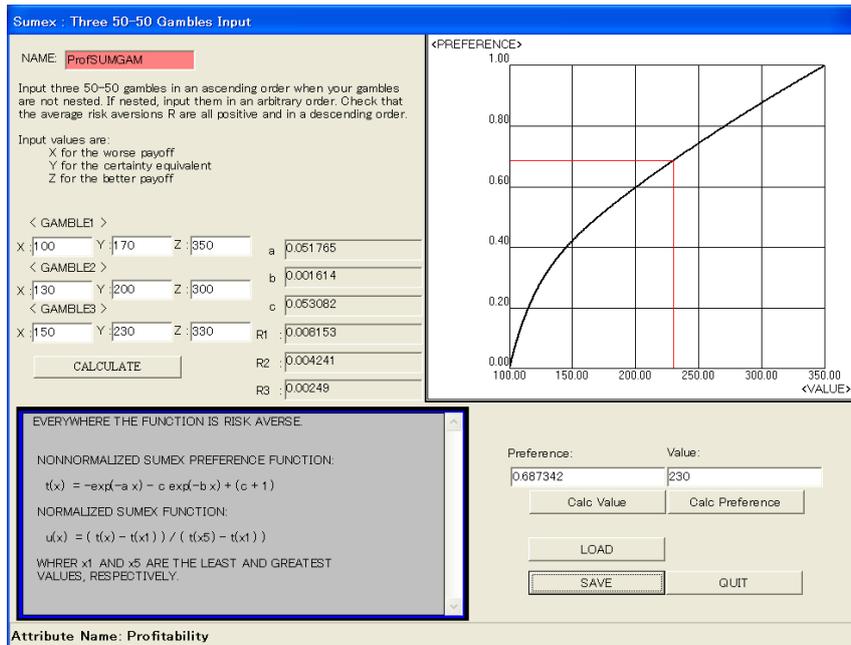


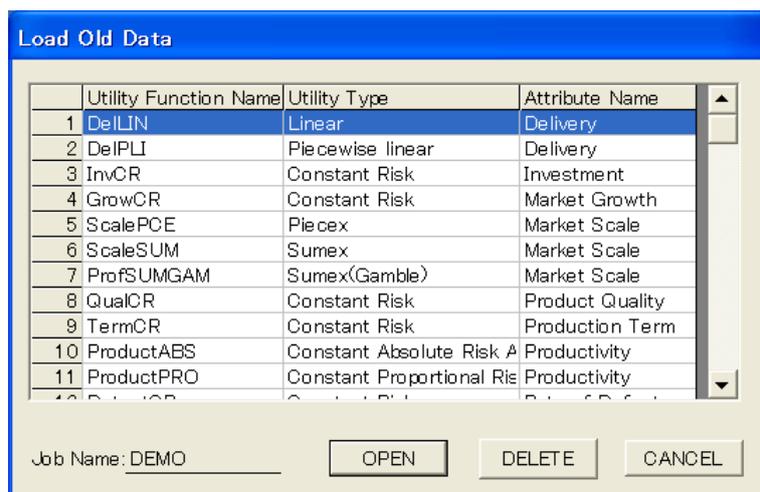Fig. 20 Demonstration of the three 50-50 gambles input method for the sumex type (Profitability)



Fig.21 Load screen for saved evaluation data

**5.2.2 Multiattribute preference evaluation**

Multiattribute preference evaluation starts from the main menu screen as depicted in Fig.1. The screen for multiattribute preference evaluation is arranged according to the method of multiattribute utility analysis, which is shown in Fig.22.

Multiattribute preference evaluation finally aims the evaluation of an over all multiattribute utility function for the prescribed job.
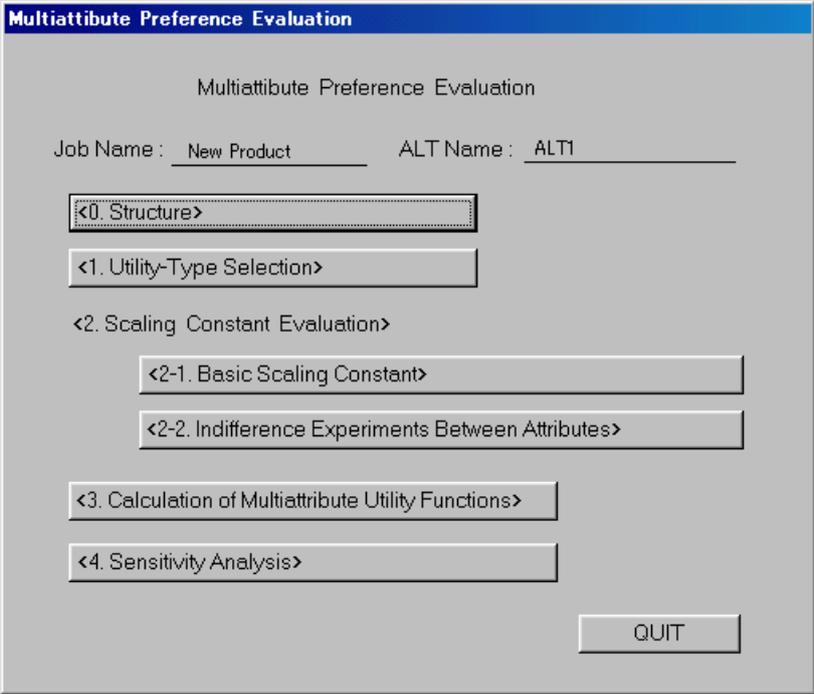


Fig.22 Menu screen for multiattribute preference evaluation

The multiattribute utility analysis proceeds in order.

Step 1. Structuring of the attributes.

First, all the attributes selected for the present job are structured in a hierarchical system on multilevel, which represents an objectives hierarchy of DM.

The <Structure> button is used on the menu screen. The <structure> screen appears with the job name at the top level as Fig.23, which is tentatively assigned for the MUF name. With a click of the right button of the mouse on the name, the pop-up menu appears with the <Add MUF>, <Add UNIF>, <Rename>, and <Delete> buttons. UNIF denotes the uni(=single)attribute utility function. The UNIF name to be included in MUF is selected from the list of Attribute Name as in Fig.24, which appears with a click on the <Add UNIF> button in the pop-up menu. Assign a UNIF name first and select an attribute name for it from the list of the attributes appearing on the screen. The Flag shows with a circle that the attribute has been selected for MUF. The MUF or UNIF name can be changed or deleted

with the < Rename> or <Delete> button respectively on the pop-up menu.

The hierarchical configuration of the objective systems is constructed in order by the assignment of MUF and UNIF names with the <Add MUF> and <Add UNIF> buttons in the pop-up menu.

Fig.25 illustrates the <Structure > screen for the hierarchical configuration. The large-scaled structure can be folded up in a compact diagram.
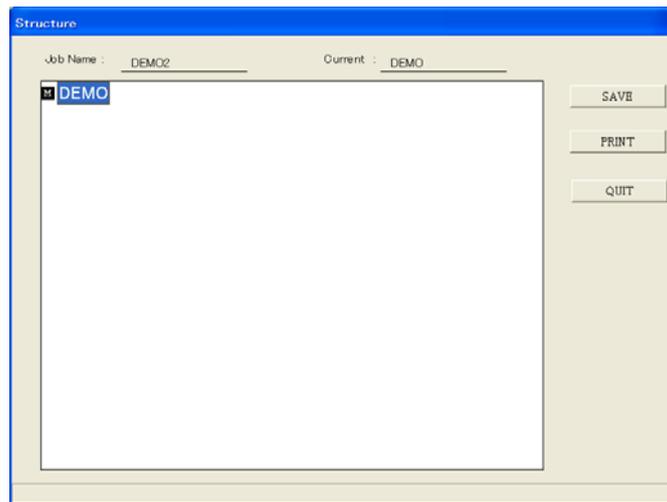


Fig.23 Structure screen first appeared
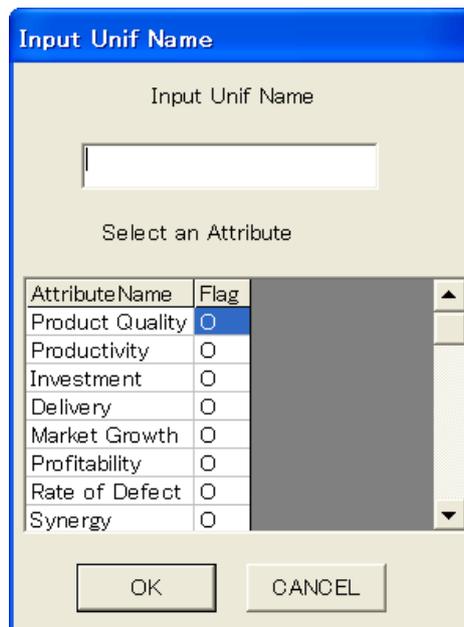


Fig.24 Unif Name input screen
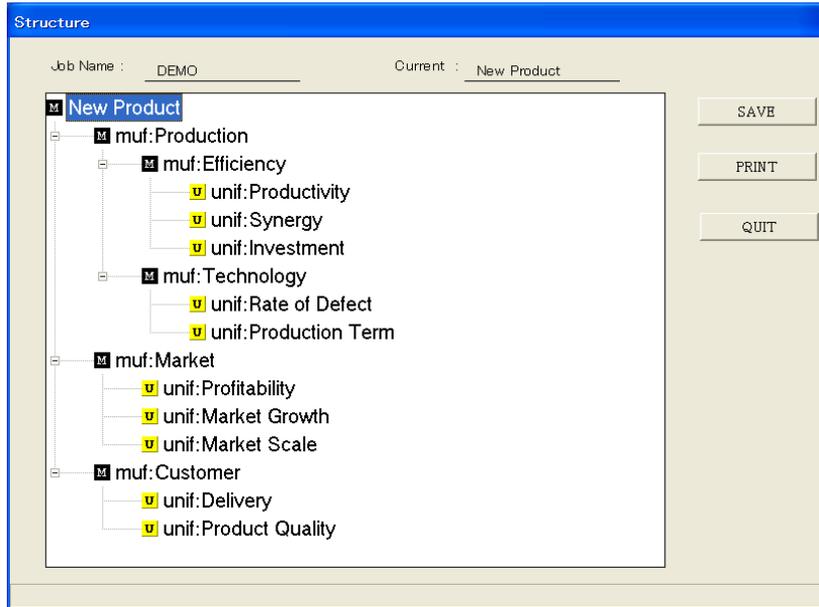
Fig.25 Screen for MUF structure

Step 2. Selection of UNIF type.

Before proceeding to the MUF evaluation, the UNIF types for all the attributes to be used in the MUF must be selected. The screen for the selection appears with a click of the <Utility-Type Selection> button on the menu screen. Fig.26 illustrates this screen.

Note that the UNIF types of Constant Absolute/Proportional Risk Averse are not available for MUF evaluation.
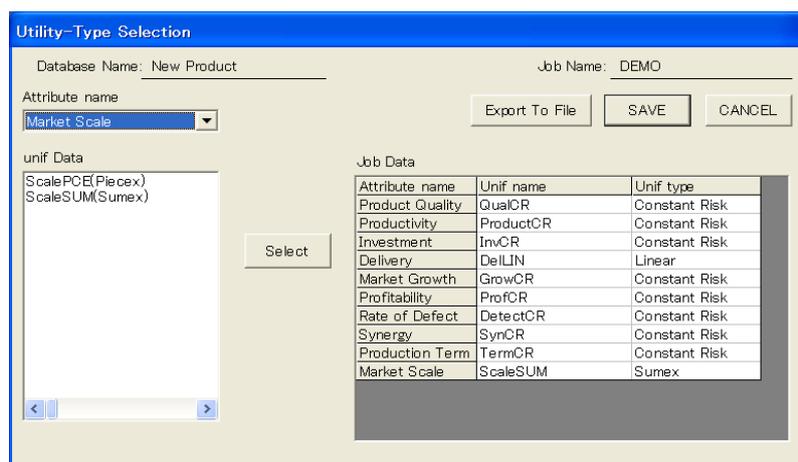


Fig.26 Screen for selection of UNIF type

Step 3. Assignment of scaling constants.

In MUF construction, the scaling constants are assessed for UNIF and MUF at the lower levels in the hierarchical structure. The evaluation of the scaling constants is executed as a result of the assessment of the value trade-off between attributes in each subsystem. The indifference experiment between the attributes is used for assessing the value trade-off numerically. MIDASS supports this experiment with drawing the indifference map on the screen, where the user can modify the input data for the assessment visually.

The assignment of the scaling constants proceeds in the following steps.

Step 3-1. Construction of the preference order among the attributes in each subsystem.

Numerical order of the scaling constants corresponds to the preference order of the attributes. The construction of the preference order is used for the relative scaling in the assignment of the value indifference points between attributes, which should be consistent with the preference order of DM..

Step 3-2. Assignment of the value of the basic scaling constant.

The value of the scaling constant is assessed first for the most preferable, or regarded as most important, attribute in each subsystem, which is called the basic scaling constant. The assessment is executed with an indifference experiment between a lottery and its certainty equivalent. The lottery is constructed with two chance forks. One has all best values of the attributes with a probability $p$ and the other has all worst values of the attributes with a probability $1 - p$. The certainty equivalent (CE) is set such that the most preferable attribute takes its best value and all other attributes take their worst values. The user assesses the probability $p$ for deriving the indifference between the lottery and CE.

The assignment starts with the <Basic Scaling Constant> button on the menu screen. Then the same screen as the Structure screen appears with the different screen name. Click a MUF name at a lower level with the right button. The Input Base No. screen appears as in Fig.27. The user specifies the number of the basic attribute on the screen, which is assessed with the basic scaling constant. With the <OK> button, proceed to the Set Basic k screen as in Fig.28, on which the indifference experiment for assessing the basic scaling constant is executed. The probability for deriving the indifference between the lottery and CE is assessed. Fig.29 shows the output screen for the evaluation, where the value of the basic scaling constant appears and from where the evaluation for other scaling constants proceeds with the <Indifference Map> button as explained in Step 3-3.

Alternatively the user can go to the next screen with the <CANCEL> button in Fig.27. The same screen appears. The number of the basic attribute can be assigned with the <Set Base> button. Proceed to the next screen with the <Set k> button. Then the input screen for the assignment of the basic scaling constant appears as in Fig.28.

All the results of the evaluation for the scaling constants in the subsystem Market are depicted in Fig.30.
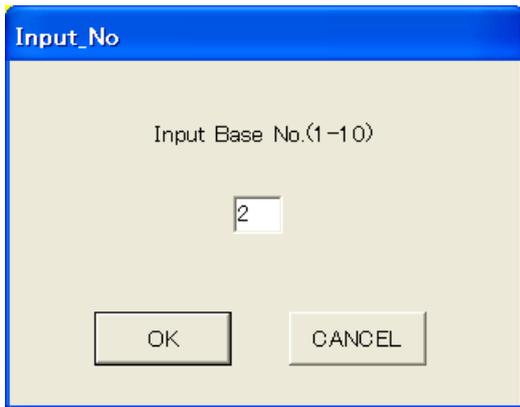
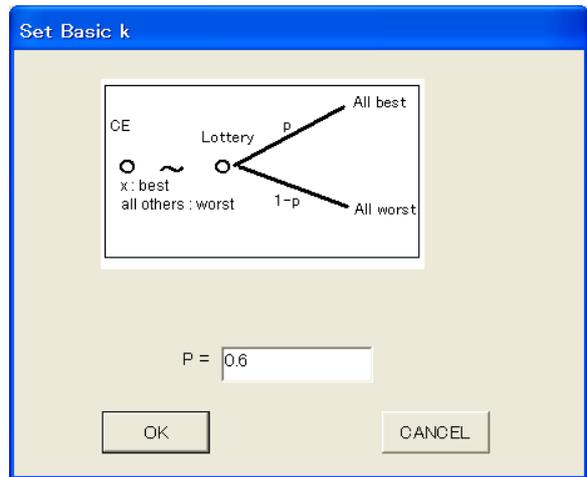Fig.27 Screen for basic attribute number input



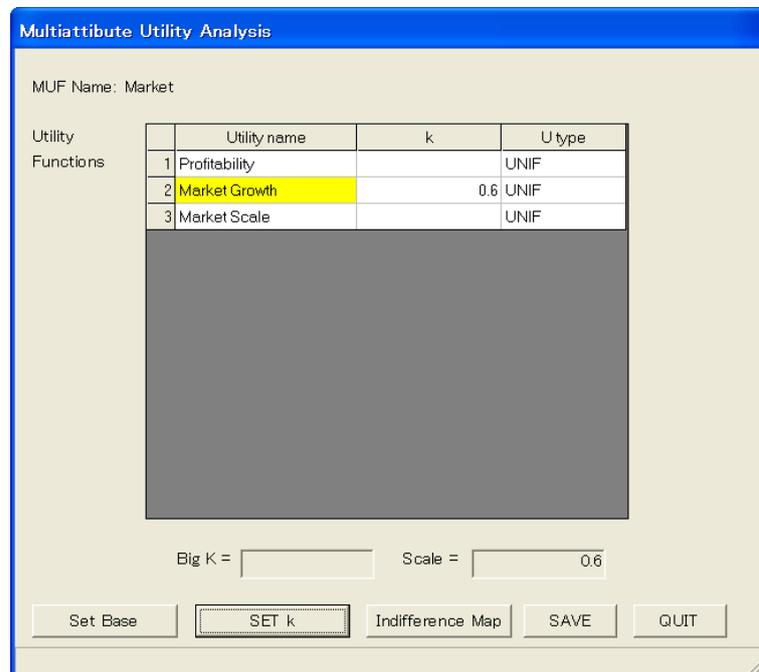Fig.28 Indifference experiment screen for basic scaling constant



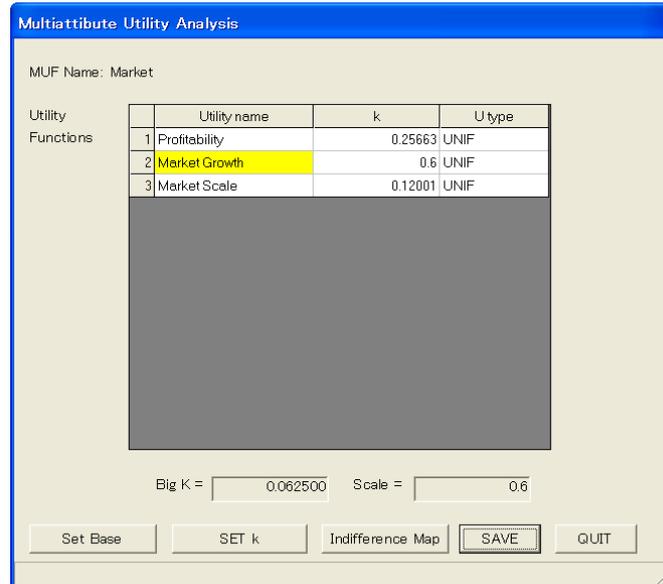Fig.29 Screen for the scaling constant evaluation

Fig.30 Evaluation results of the scaling constants

Step 3-3 Assessment of all other scaling constants.

Assessment of the scaling constants for all other attributes is executed with the <Indifference Map> button on the evaluation screen as Fig.29. The same screen as Fig.27 appears. The number of an attribute is assigned as the opponent in the indifference evaluation. Then, with the <OK> button, the Indifference Map Between Two Attributes screen appears for the value indifference experiment between the two attributes. The most preferable attribute name appears always on the horizontal axis.

Assess the value indifference points for the two attributes as the input data. Then an indifference map is generated automatically with the <Indifference Eval> button. When the user specifies a value of one attribute, the indifferent point for the other attribute is calculated with the <Point Calc> button and drawn on the screen.

Fig.31 and 32 depict the evaluation results for three attributes in the MUF named as Market. Note that the numerical order of the scaling constants, $k_{\text{Market Growth}} > k_{\text{Profitability}} > k_{\text{Market Scale}}$, corresponds to the preference order of the attributes, $x_{\text{Market Growth}} \succ x_{\text{Profitability}} \succ x_{\text{Market Scale}}$. The relative scale of the basic attribute to the opponent should be carefully checked in finding the indifferent points on the screen, which should correspond to the preference order for the attributes.

All the evaluation results of the scaling constants appear as in Fig.30. Check again that the numerical order of the scaling constants corresponds to the preference order of the attributes.

The <Indifference Experiments between Attributes> button on the menu screen also executes the same evaluation works thorough the similar screen transitions.
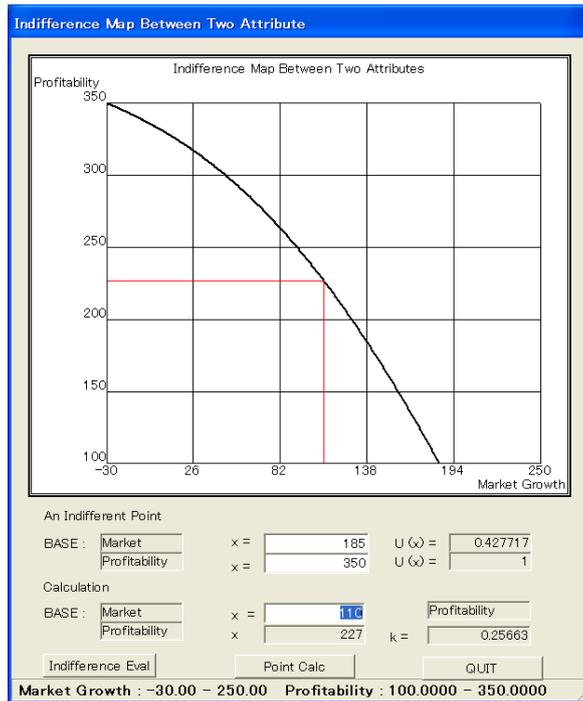
34

Fig. 31 Indifference experiment between Market Growth and Profitability
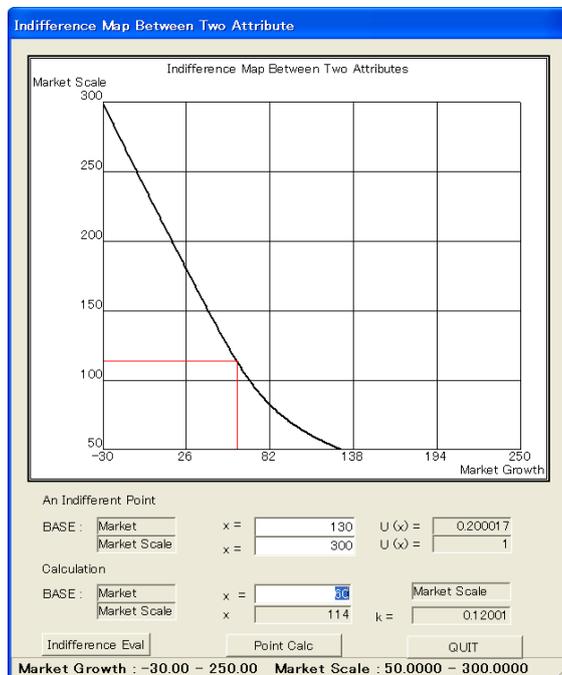


Fig.32 Indifference experiment between Market Growth and Market Scale

The hierarchical configuration of MUF structure includes the nesting of MUF at the lower level as shown in Fig.25. The assessment of the scaling constants for MUF at the upper level is executed in order between the basic attributes selected as most important in the subsystems at the lower level.

Fig.33 illustrates the evaluation of the scaling constants at the highest level, where the MUF named New Product is constructed on three MUFs named Production, Market, and Customer. Fig.34 and Fig.35 depict the indifference experiment between Customer and Market and between Customer and Production respectively, where $x_{Customer} \succ x_{Market} \succ x_{Production}$.

Step 4. Calculation of MUF values

Calculation of MUF is executed with the <Calculation of Multiattribute Utility Function> button on the menu screen. All the results of the evaluation appear on the screen with the related information.

Fig.36 depicts the final evaluation results for the MUF of the system named New Product. The output can be exported in the MS Excel form with the <Export to File> button.

Step 5 Sensitivity analysis.

MIDASS executes the sensitivity analysis for MUF and UNIF values, which calculates the differential values for the UNIF and attribute values.

Sensitivity analysis is useful for the construction of alternative management plans according to alternative criteria, which is based on the utility assessment in the objectives configuration for the existing system New Product.

Fig.37 shows the results of sensitivity analysis for the objective system New Product.



Fig.33 Evaluation of scaling constants for New Product

Fig.34 Indifference experiment between Customer and Market



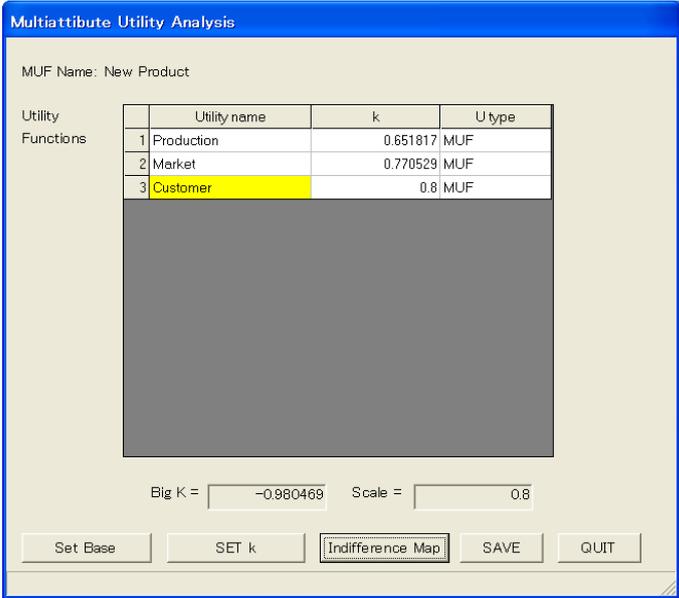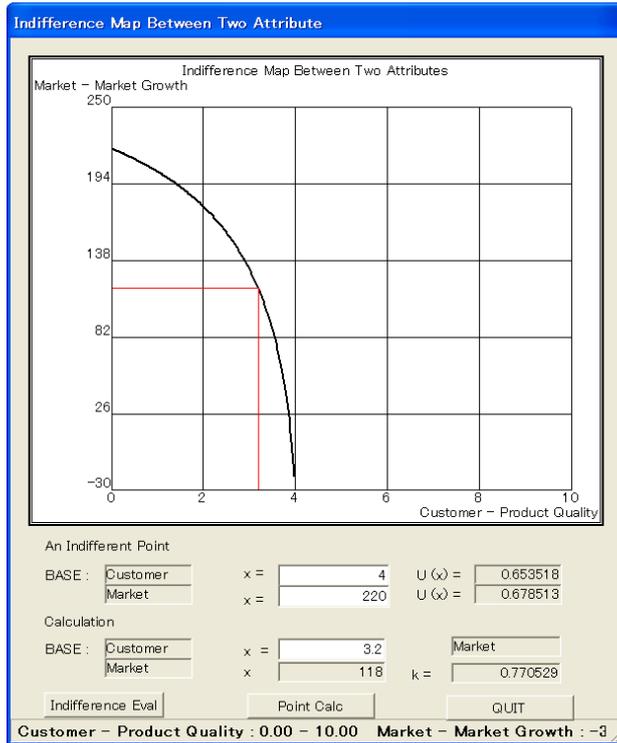Fig.35 Indifference experiment between Customer and Production

Fig.36 Evaluation results for MUF: New Product



Fig.37 Sensitivity analysis

**5.3 Probability valuation**

**5.3.1 Evaluation of subjective distributions**

MIDASS supports the evaluation of the probability distribution for the attribute. MIDASS can construct the statistical probability distributions with the data input that is subjectively assessed by DM. Input data for assessing the distributions are different according to the distribution types. MIDASS generates as output the evaluation results along with the graphical representation.

MIDASS supports selectively alternative evaluations for the 11 types of distribution, which correspond to the numerical data characteristics of the attributes, which are treated as the uncertain quantities. Bayesian statistics are derived with sample data.

1. Continuous distributions
A. 0-1 attribute values
 (1) Beta distribution
 (2) Bounded lognormal distribution
B. Nonnegative attribute values
 (3) Lognormal distribution
 (4) Logstudent distribution
 (5) Gamma-$q$ distribution
 (6) Weibull distribution
C. Unrestricted attribute values
 (7) Gaussian (or normal) distribution
 (8) Arcsinh-normal distribution
 (9) Logistic distribution
 (10) Student distribution
 2. Piecewise quadratic distribution
 3. Discrete distribution
 4. Bayesian statistics
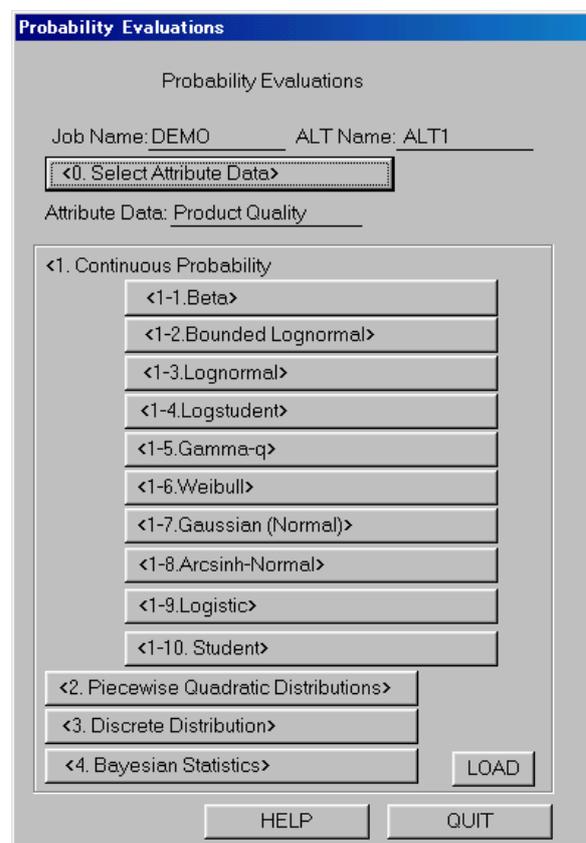


Fig.38 Menu screen for probability evaluation

The user starts with the <Probability Evaluation> button on the main menu screen in Fig.1. The menu screen for the probability evaluation appears as in Fig.38. First an attribute name should be specified on the Select Attribute screen, which appears with the <Select Attribute Data> button as in Fig.39. Then all the functions for the evaluation become available on the menu screen. The user selects a proper distribution type in order according to the numerical characteristics of the attribute.
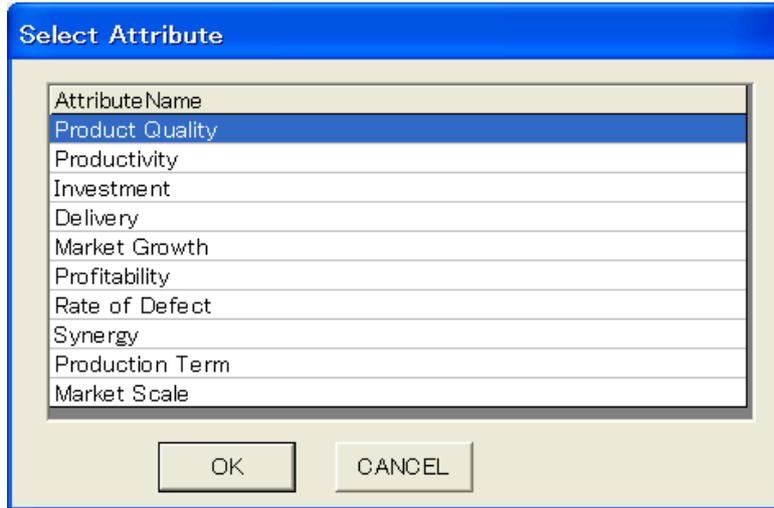
Fig.39 Screen for selecting an attribute name

The evaluation screen for the distribution appears with a click on a proper distribution button on the menu screen. Evaluation results are saved with the grouped approximation. The way of grouping the data should be indicated on every evaluation screen assigning (i) whether by equal probability or by equal width and (ii) the number of brackets in the grouping. The graphical representation can be enlarged by a click on the screen. The user can turns back with the <QUIT> button on the screens.

In the following, <x x x> indicates the related button.

**5.3.2. Statistical Probability Distributions**

I. Continuous probability distributions

A.  Distribution families with [0, 1]- attribute values

Input data are different for the distribution types.

1. Beta distribution (<Beta>)

The Beta cumulative function is represented with parameters *B* and *C* in the following form

$$k(p;\, B,\, C) \;=\; \int_0^p \frac{\Gamma(C)}{\Gamma(B)\Gamma(C-B)}\, t^{B-1}(1-t)^{C-B-1}\, dt\,, \tag{25}$$

where $t \le p$ is the uncertain quantity, and $0 \le t \le 1,\ B > 0,\ C > B.$ $\Gamma(\bullet)$ is the complete gamma function and defines the generalized factorial, $\Gamma(r) = (r-1)!$ .

The input values are assessed on the screen for (i) the 0.25 and 0.75-fractiles on the cumulative distribution, or alternatively for (ii) moments as (a) mean $m = [0, 1]$ and (b) the standard deviation $s \;=\; [0, \sqrt{m(1-m)}\,]$ . (iii) Direct input of parameters *B* and *C* is also possible.

Fig.40 illustrates the evaluation of the beta distribution on the screen.
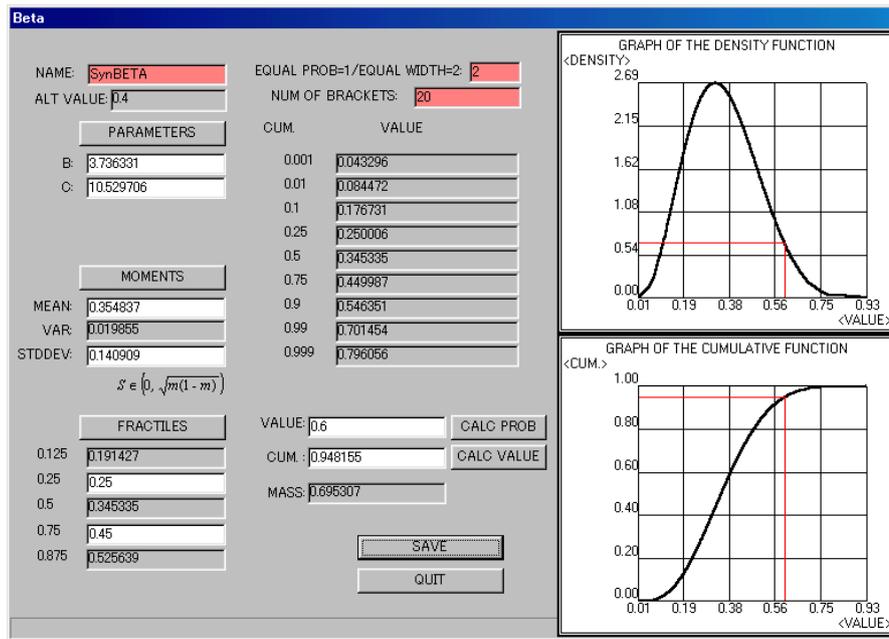
Fig.40 Evaluation screen of Beta distribution (Synergy)

2. Bounded lognormal distribution (<Bounded Lognormal>)

The bounded lognormal distribution is the distribution of the uncertain quantity $t$ having parameters $\mu$ and $\sigma$, if $x = \log[t / (1 - t)]$ has a normal distribution with the parameters, the mean $\mu$ and the standard deviation $\sigma$, and $0 \le t \le 1$, $-\infty < \mu < \infty$, $\sigma > 0$.

The input data are (i) the parameters $\mu$ (MU) and $o$ (SIGMA), or (ii) the attribute values for 0.25 and 0.75-fractiles.

The Beta and bounded lognormal distributions are both flexible. They have wide applicability.

Fig.41 depicts the evaluation screen for the bounded lognormal distribution.

B. Distribution families with nonnegative attribute values

3. Lognormal distribution (<Lognormal>)

The lognormal distribution is the distribution of the uncertainty quantity $t$ having parameters $\mu$ and $\sigma$, for which $x = \log t$ has a normal distribution with the parameters $\mu$ and $o$, where $x \ge 0$, $-\infty < \mu < \infty$, $\sigma > 0$.

The distribution is skewed with a long tail to the right.

The input is (i) the 0.25 and 0.75-fractiles, or (ii) the parameters $\mu$ and $\sigma$.

Fig.42 illustrates the evaluation of the lognormal distribution and Fig.43 depicts the lognormal distribution for the same attribute Synergy as in Fig.40 (Beta) and Fig.41 (Bounded lognormal).
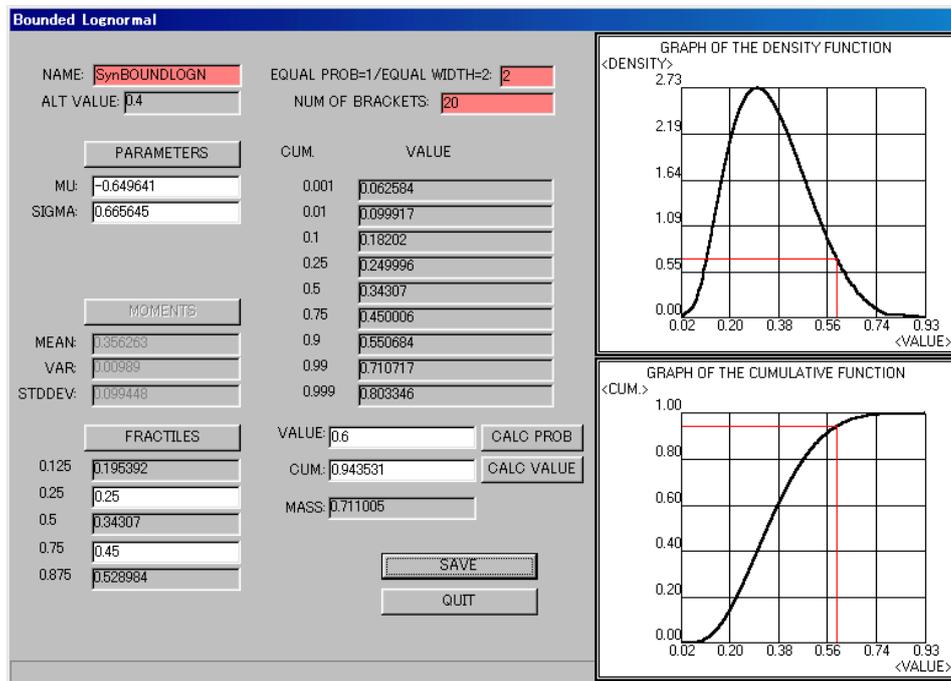
Fig.41 Evaluation screen for bounded lognormal distribution (Synergy)
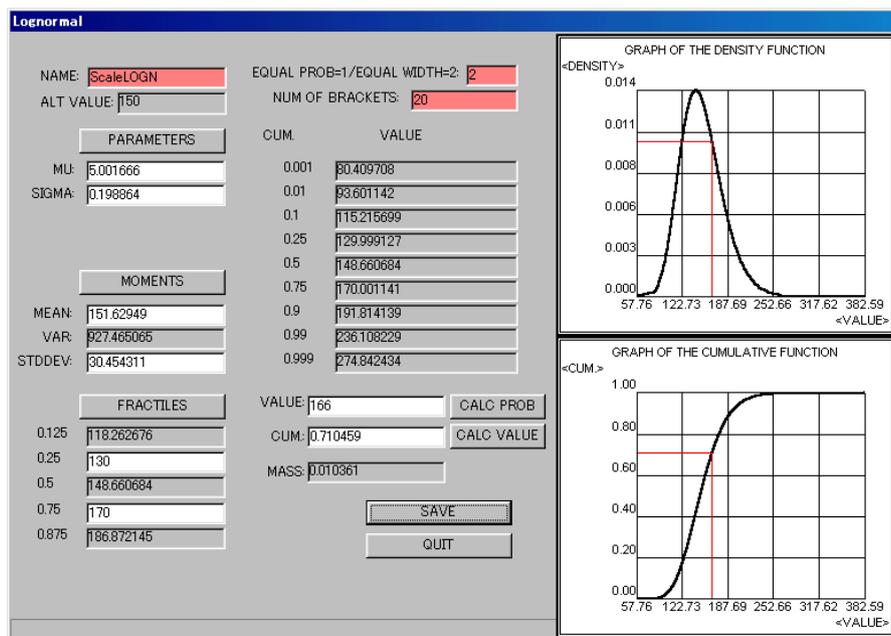


Fig.42 Evaluation of lognormal distribution (Market Scale)
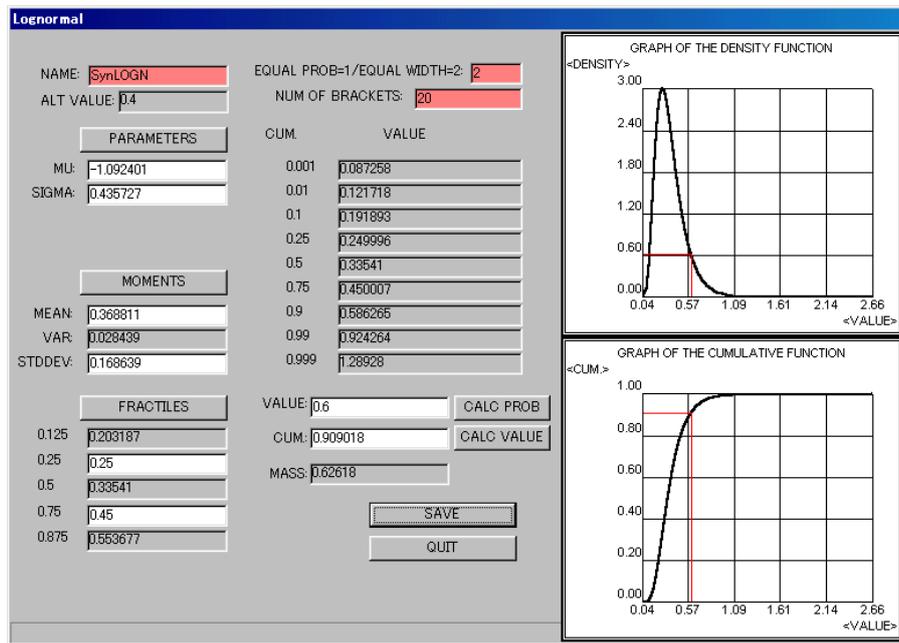
Fig.43 Evaluation of lognormal distribution (Synergy)

4. Logstudent distribution (<Logstudent>)

The logstudent distribution is the distribution of the uncertain quantity *t* having the parameters, $\mu$, and $\sigma$ and $\nu$ (degree of freedom), for which $x = \log t$ has a student distribution with the parameters $\mu$, $\sigma$ and $\nu$, where $t \geq 0, \ -\infty < \mu < \infty, \ \sigma > 0, \ \nu > o$.

The standard student density function is defined by

$$f(x, \nu) = \frac{\nu^{\frac{1}{2}\nu}}{B\left(\frac{1}{2}, \frac{1}{2}\nu\right)} (\nu + x^2)^{-\frac{1}{2}(\nu+1)} \tag{26}$$

where $B(p, q)$ is the complete beta function, and $-\infty < x < \infty, \ \nu > 0$. The general student distribution of an uncertain quantity *y* is defined when $x = (y - \mu)/\sigma$ has a standard student distribution with the argument $-\infty < y < \infty$, and parameters, $-\infty < \mu < \infty, \ \sigma > 0, \ \nu > 0$.

The length of the tail of the logstudent distributions is controllable in only one direction with the 0.25 and 0.75-fractiles fixed, which can be longer, but not shorter, than in the lognormal distributions.

The input data is (i) the 0.25, 0.75, and 0.875-fractiles, or (ii) the parameters $\mu, \sigma,$ and $\nu$.

Fig.44 illustrates the evaluation of the logstudent distribution.
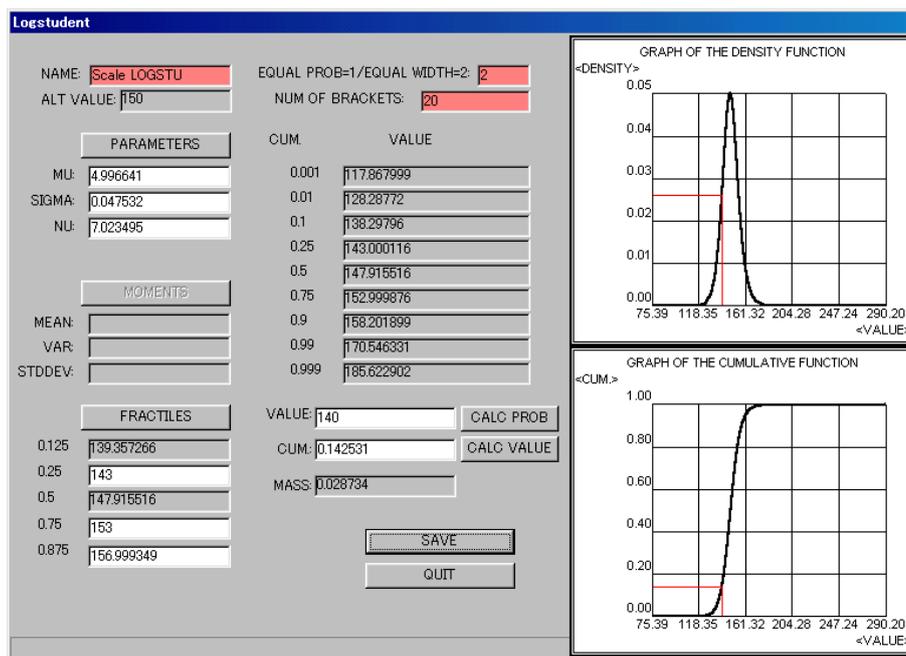
Fig.44 Evaluation of logstudent distribution (Market Scale)

5. Gamma-*q* distribution (<Gamma-q>) (Raiffa and Schlaifer 1961, Schlaifer 1971)

The gamma-*q* distribution is the distribution of the uncertain quantity *t* having the parameters *q, r,* and *s,* for which $x = (t/s)^q$ has the standard gamma distribution with a parameter *q,* and $t \geq 0, \ q \neq 0, \ r > 0, \ \text{and} \ s > 0$.

The standardized gamma normalized density function is defined with the parameter *r* by

$$f(x \, ; \, r) \ = \ \frac{e^{-x} \, x^{r-1}}{(r-1)!} \, , \quad x \geq 0, \ r > 0 , \qquad (27)$$

where $(r-1)! \ \underset{=}{\Delta} \ \Gamma(r)$ is the complete gamma function or generalized factorial. When $x = yt$ has a standard gamma distribution with the parameter *r*, the uncertain quantity *t* is said to have the gamma-1 distribution with parameters *r* and *y*.

The gamma-1 normalized density function is defined by

$$f(x \, ; \, r, \, y) = \frac{e^{-yx} \, (yx)^{r-1}}{(r-1)!} \, y, \quad x \geq 0, \ r, \, y > 0. \qquad (28)$$

The characteristics of the gamma-*q* distribution are that (i) one side of the tail is longer, and the other side is shorter, than in the lognormal distributions, (ii) when $q > 0$, the tail is longer to the left, and when $q < 0$, the reverse, and (iii) when *q* is approaching 0, the distribution form is approaching the lognormal type.

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The input data are (i) a direct assignment of the parameter $q$, and (ii) the specification of (a) the 0.25 and 0.75-fractiles, or (b) input of the other parameters $r$ and $s$.

Fig.45 depicts the screen for the assessment of the gamma-$q$ distribution for $q = 1$. Fig.46 illustrates the gamma-1.5 distribution for the same data. Fig.47 shows the case of $q = -1$.
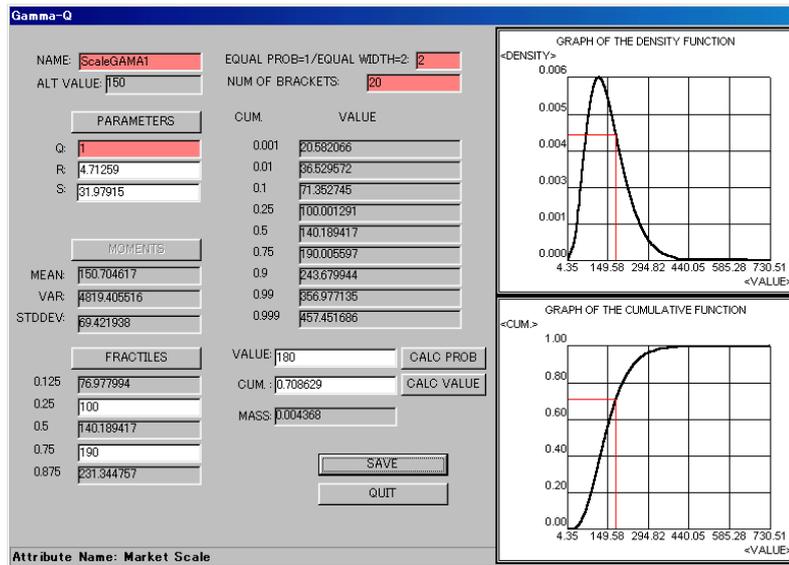


Fig.45 Evaluation of Gamma-1 distribution (Market Scale)
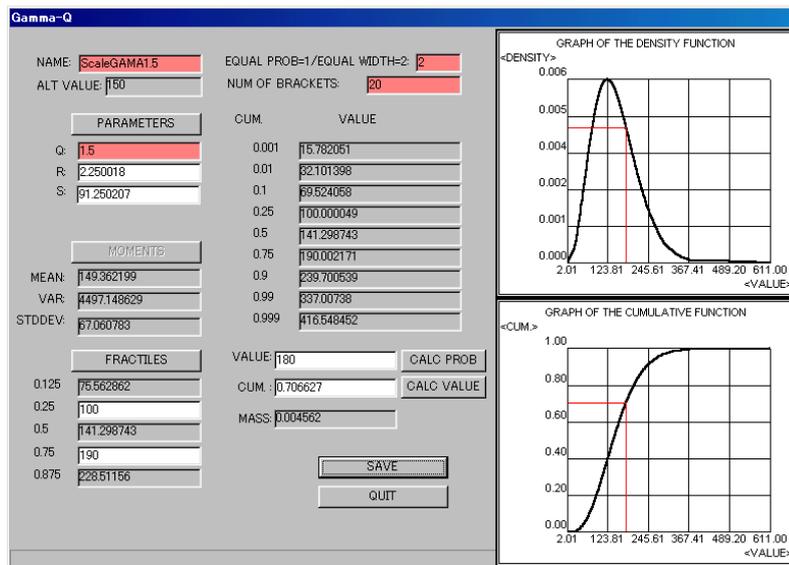


Fig.46 Evaluation of Gamma-1.5 distribution (Market Scale)
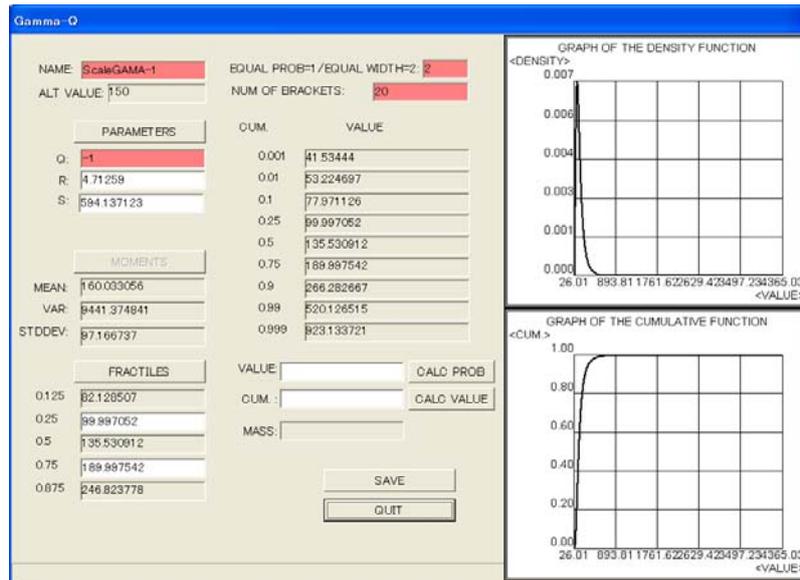
Fig.47 Evaluation of Gamma-$-1$ Distribution (Market Scale)

The selection method among the three types of distribution families suitable to the [0, 1]-uncertain quantities has been presented (Schlaifer 1971).

Step 1. Try to fit preliminarily a lognormal distribution by assessing the 0.25 and 0.75-fractiles.

Step 2. Then examine the tails of the distribution for its modifications.

(1) When the both tails are desired to be extended, the logstudent distribution is tried by assessing the 0.875-fractile.

(2) When the user wishes to extend one tail and to shorten the other, then use the gamma-q distribution on the same 0.25 and 0.75-fractiles.

The $q$ values are changed in the following rules.

(i) Set $q =1$, when the user wants the tail to be extended to the left.

(ii) Set $q = -1$, when the user wants the tail to be extended to the right.

(iii) Adjust the $q$ values to DM's opinion by doubling or halving it.

According to the increase (decrease) of the $q$ value, the tail extends more to the left (right)-side.

Step 3. If it is OK, then save the result.

6. Weibull distribution (<Weibull>)

MIDASS supports the evaluation of Weibull distributions, which is included independently as a service program for users' convenience and used for the reliability studies in special engineering fields in the processes of making decisions.

The density function is represented as

$$f(t) = \alpha\beta^{-\alpha}t^{\alpha-1}\exp(-(\frac{t}{\beta})^{\alpha}). \qquad (29)$$

The cumulative distribution function is

$$F(x) = 1 - \exp(-(\frac{t}{\beta})^{\alpha}) \qquad (30)$$

where $\alpha$ and $\beta$ are parameters and $t$ is a variable.

MIDASS calculates the distribution in the case $\beta = 1$. The user can assign any new file name for a special job on the screen. Input data are the parameter A ( $= a$ ) or the 0.25-fractile value.

Fig.48 illustrates the evaluation screen for the Weibull distribution.
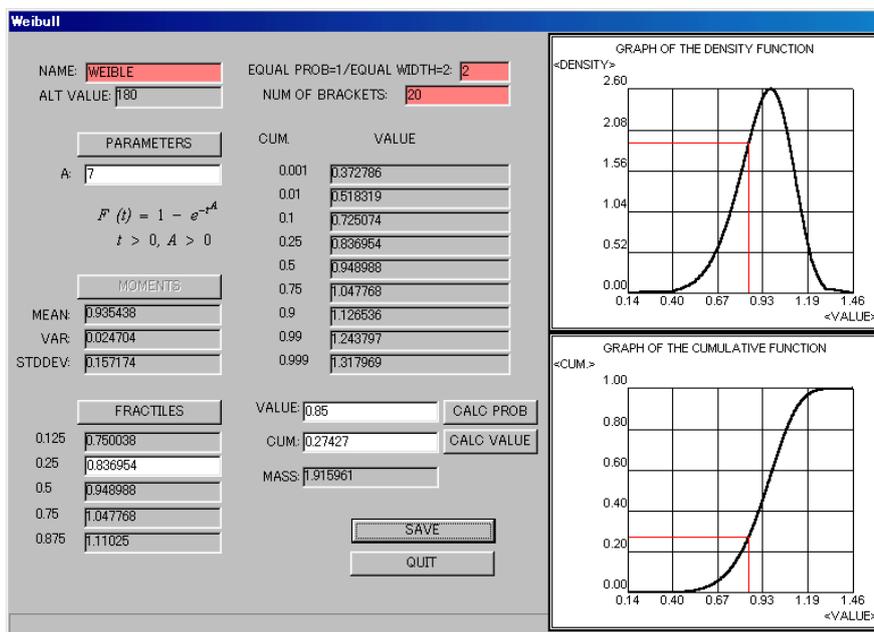


Fig.48 Evaluation of Weibull distribution

C. Distribution families with unrestricted attribute values

7. Gaussian (or normal) distribution (<Gaussian (Normal)>)

The Gaussian, or normal, distribution is the distribution of the uncertain quantity $t$ having parameters $\mu$ and $\sigma$, and $-\infty < t < \infty$, $-\infty < \mu < \infty$, $\sigma > 0$, for which $x = (t-\mu)/\sigma$ has a standard normal distribution with parameters $\mu = 0$, $\sigma = 1$.

The standard normal density function $f(x)$ is defined as

$$f(x) = (2\pi)^{-1/2} e^{-x^2/2}, \quad -\infty < x < \infty. \qquad (31)$$

The Gaussian (Normal) density function for the uncertainty quantity $t$ is represented by

$$f(t) = \frac{1}{\sqrt{2\pi o^2}} \exp\left(-\frac{(t-\mu)^2}{2o^2}\right),$$

(32)

where $\sigma > 0$, and $\sigma^2$ is a variance.

The Gaussian distribution is a symmetric distribution, whose shape is inflexible. Although the normal distribution is not necessarily suitable for the heuristic derivation of DM's distribution, it sometimes can be used as a good approximation to the unknown distributions with a wide variety.

Input data are (i) the 0.25 and 0.75-fractiles, or (ii) the parameters $m$ (mean) and $s$ (standard deviation).

Figure 49 depicts the evaluation screen for the Gaussian distribution of the attribute Market growth.
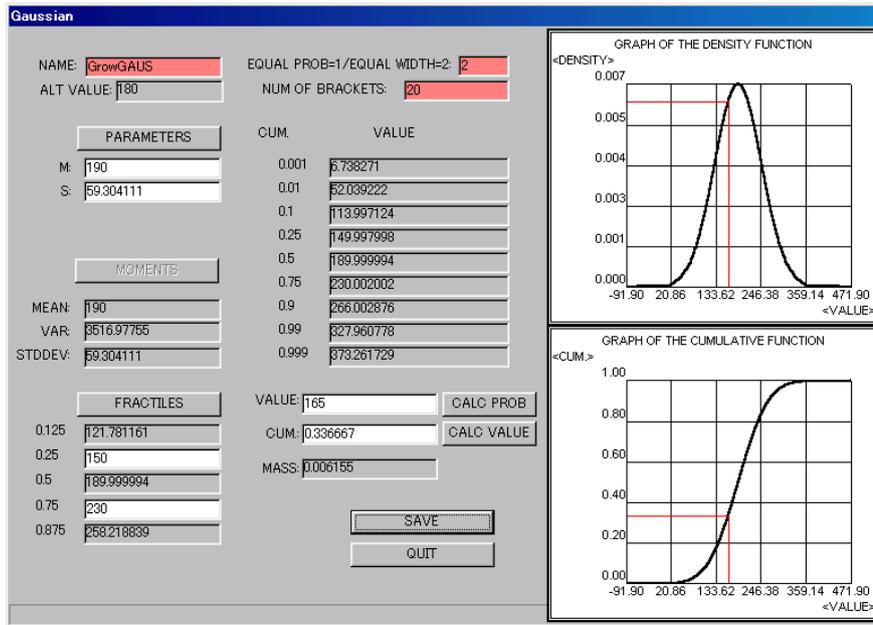


Fig.49 Evaluation of Gaussian distribution (Market Growth)

8. Arcsinh-normal distribution (<Arcsinh-Normal>) (Schlaifer 1971)

The arcsinh-normal distribution is a distribution of the uncertain quantity $t$ having parameters $\mu$, $\sigma$, $m$ and $s$, and $-\infty < t < \infty$, $-\infty < \mu < \infty$, $\sigma > 0$, $-\infty < m < \infty$, $s > 0$, for which $x = \text{arcsinh}\left[(t - \mu)/\sigma\right]$ has a normal distribution with parameters $\mu$ and $\sigma$.

The arcsinh-normal distribution can be symmetric, or is skew with the very long tail either to the left or the right.

The input data is (a) the parameters $\mu$, $\sigma$, $m$, and $s$, or (b) three quartiles including the median (i.e., the 0.25, 0.5 and 0.75-fractiles), and (ii) the octile for the longer tail (i.e., the 0.125 or 0.875-fractile). If the distribution is symmetric, specify the 0.875-fractile.

The distribution is difficult to evaluate directly. Try this family when the user is not satisfied with the evaluation for Gaussian distribution.

The evaluation procedure proceeds sequentially in the following order.

Step 1. Start first with a Gaussian distribution. If the distribution is not satisfied by the user, then try the arcsinh-normal distribution with reference of the original fractile values.

Step 2. Fix a value of a fractile first such as the median, for example, generated in the Gaussian distribution. Then narrow the substantial range of the distribution. (i) Try to increase the values for the 0.125 and 0.25-fractiles and decrease for the 0.75 and 0.875-fractiles. As the upper fractiles (0.75 and 0.875) values decrease and/or the lower fractiles (0.125 and 0.25) values increase, the statistical range of the distribution is narrowed (i.e. the width of distribution shape is widened). (ii) Or alternatively, try the reverse. Then the substantial range is widened (i.e. the width of distribution shape is narrowed). (iii) When the distribution is symmetric, the tails must be at least as long as the tails of a Gaussian distribution with the same interquartile range.

Step 3. If the distribution is still not appropriate for user's opinion, then change the $\mu$ and/or $\sigma$ value.

Note that, in the parameter input, (i) the $\sigma$-value should be set as a relatively very small and (ii) the octile values should not be too close to the adjacent quartile. This means that the tail should have appropriate length to this distribution type.

Fig.50 demonstrates the evaluation screen for the attribute Market growth.
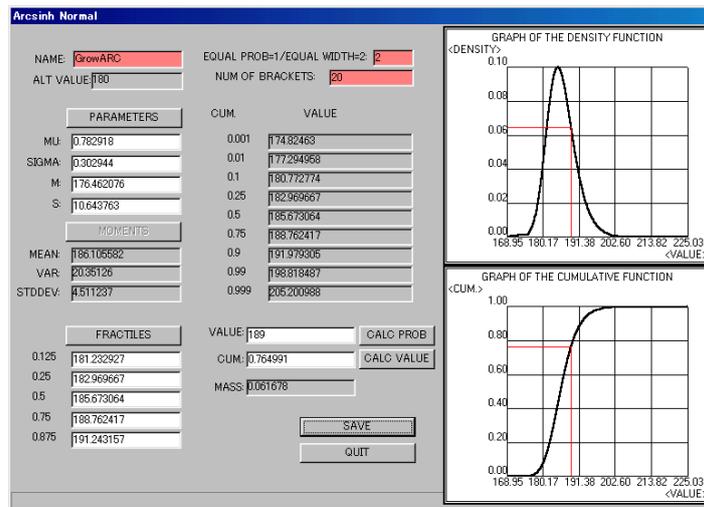


Fig.50 Evaluation of arcsinh-normal distribution (Market Growth)

9. Logistic distribution (<Logistic>)

The logistic distribution is known as "law of growth." The distribution function form is represented as

$$F(x) = \frac{1}{1 + e^{-\alpha x - \beta}} \quad , \tag{33}$$

where $\alpha > 0$.

Input data are parameters (ALPHA, BETA), moments (mean, standard deviation), or the 0.25 and 0.75 fractiles.

Fig.51 illustrates the evaluation of the logistic distribution for the attribute Market Growth.
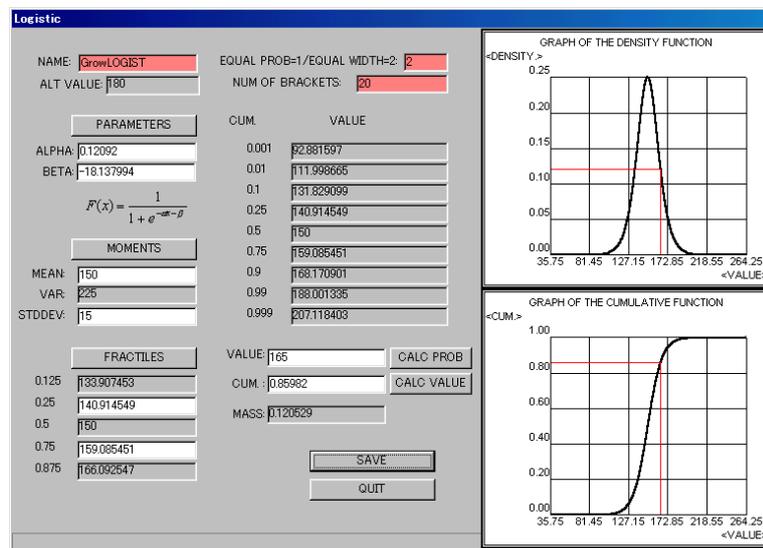


Fig.51 Evaluation of logistic distribution (Market Growth)

10. Student *t*-distribution (<Student>)

The Student *t*-distribution is used in user' probability evaluation as an alternative to the normal distribution.

So far, however, the Student *t*-distribution has been used for determining the statistical confidence interval of the population mean when the population distribution is assumed to be Gaussian but the population variance is unknown. This property is also useful for the pretreatment of data bases.

The student *t*-distributions $f(t)$ is a distribution defined with the Student *t*-statistics,

$$t(\mu) = \frac{(\bar{x} - \mu)}{s / \sqrt{n}} \quad , \qquad\qquad -\infty < t < \infty , \tag{34}$$

and the density function is represented as

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$$f(t) = \frac{\Gamma\,[\,(\nu+1)\,/\,2\,]}{\sqrt{\pi\,\nu}\;\,\Gamma\,(\nu/2)}\,(1+\frac{t^2}{\nu})^{-(\nu+1)/2} \tag{35}$$

where $\bar{x}$ is the sample mean, $s$ is the sample standard variation and $\mu$ is the population mean. The distribution has the parameter $\nu > 0$, which is the degree of freedom, $\nu = n - 1$, and $n$ is the number of a sample. The shape of the distribution is symmetric and the dispersion becomes smaller as the $\nu$-value is larger and approach the normal distribution.

Input data are $\mu$, SIGMA $(= s\,)$, $\nu$.

Fig.52 illustrates the evaluation of the Student $t$- distribution.
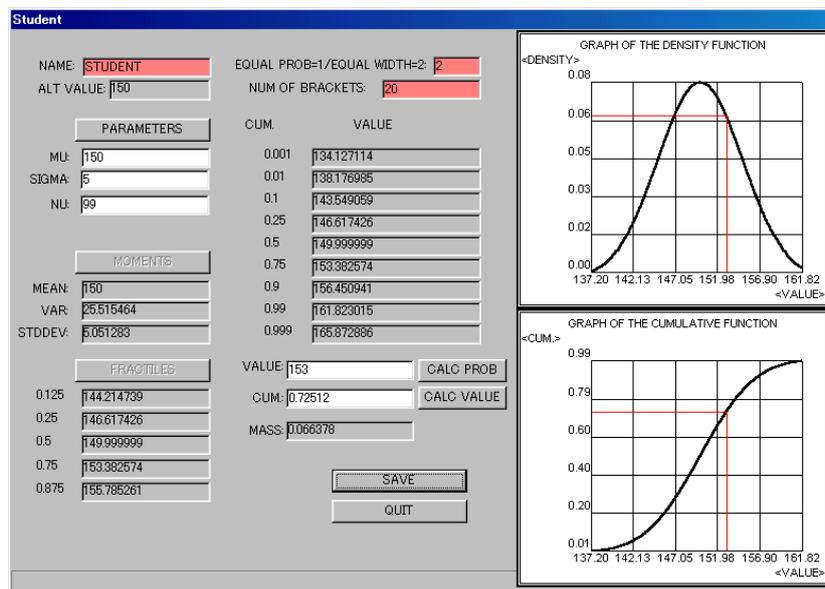


Fig.52 Evaluation of Student $t$-distribution (Market Scale)

The user proceeds in order with the evaluation of the distributions in each distribution family according to the numerical characteristics of the attribute values or the variates, until the proper distribution family has been found to coincide to user's opinion. The instruction screen for the selection can be used with the <HELP> button on the menu screen for the probability evaluation. Keep it on the screen through the evaluation work as depicted in Fig.53. The text is put in Appendix B.

All the evaluation results specified with a file NAME "xxxx" are preserved with the <save> button on the screen and call them in for later use. Tentative evaluation results for the attribute can be saved with the different file names as, say, "xxx1", "xxx2", etc. These preliminary results can be used as a reference for the modification of the evaluation or as an alternative for later use.
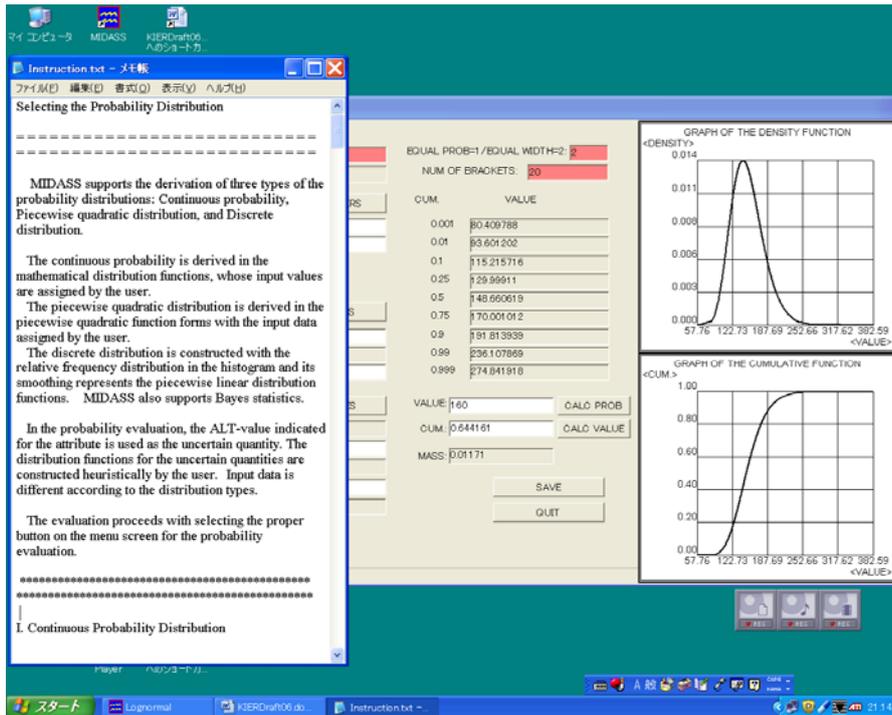
Fig.53 Evaluation screen with instruction

As seen above, some distributions can be used independently in the processes of statistical analysis for the pretreatment of the original data as a primary material. In such a case, the programs can be initiated with a proper button on the menu screen by assigning a proper attribute name from the Select Attribute screen, for which the database can be constructed separately for this purpose.

II. Piecewise quadratic distributions

MIDASS constructs the piecewise quadratic distribution heuristically with the <Piecewise Quadratic Distribution> button. Specify the 3-25 input points, or preferably 10-12 points, with equal width for cumulative distribution. The program checks if these input values are consistent with the piecewise quadratic functions and, if not, a message is generated for the corrections. Note that the assessor has an inclination to assess an excessively sharp mode and an excessively plain tail.

Input data are the cumulative distribution values in the specified points and the corresponding attribute values. Calculation is executed with the <OK> button. The mean (MEAN), variance (VAR) and standard deviation (STDEV) of the assessed distribution are calculated automatically and appear on the screen.

Fig.54 depicts the evaluation screen of the piecewise quadratic distribution for the attribute Delivery.
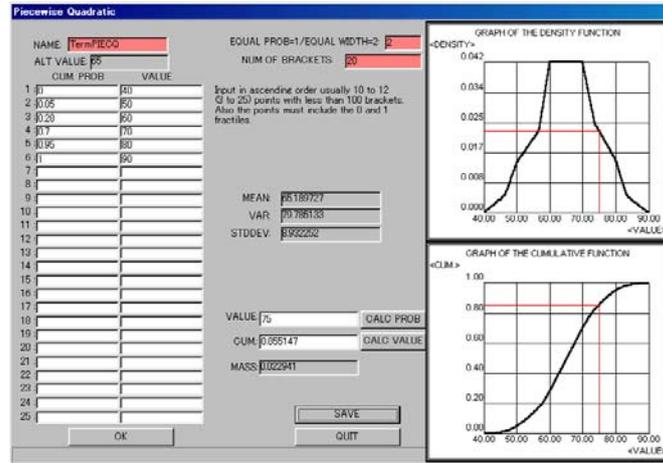
Fig.54 Evaluation of piecewise quadratic distribution (Delivery)

III Discrete distributions

The user can construct with the <Discrete Distribution> button the discrete distribution for an empirical data. This distribution is used mainly for the elementary statistical processing in various fields. MIDASS includes this type of evaluation as a service program for pretreatment of database. The relative frequency distributions in the cumulative form are used widely, say, in industrial management fields, such as for constructing the Pareto map used in ABC analysis for the inventory control. This distribution, however, also can be used in the direct assessment of the distribution in decision analysis.

Fig.55 illustrates the discrete distribution.
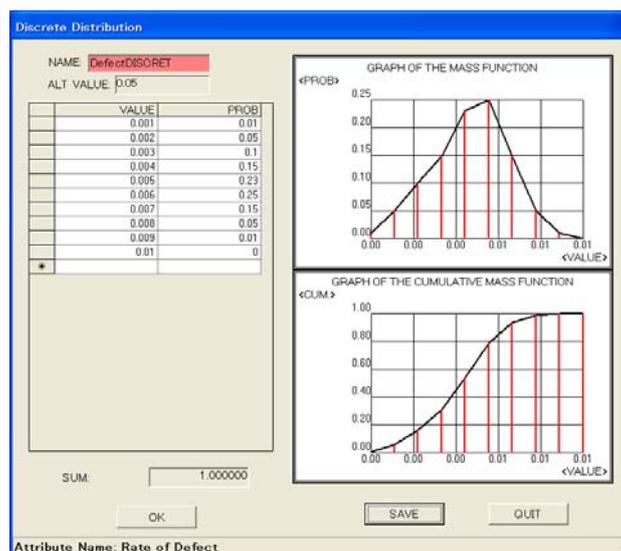


Fig.55 Screen for discrete distribution (Rate of Defect)

A list of all the evaluation results for the distributions once saved appear with the <LOAD> button on the menu screen, as shown in Fig.56. With a click on the file name (Probability Function Name), the evaluation screen is called out. The modification of the evaluation can be performed on the screen.



Fig.56 Load screen for the evaluation results

### 5.3.3 Posterior Probability Distributions

1. Bayes' theorem

DM's subjective probability can be modified as the posterior probability with sample information. Bayesian statistics support this modification with Bayes' theorem.

Denote $E_s$, $s = 1, 2, \ldots, p$, as the sample event that is known with the sample information and $\theta_j$, $j = 1, 2, \ldots, n$, as the population event that is unknown in the natural world. The posterior probability of $p(\theta_j | E_s)$ is the probability of DM for possible occurrence of population event $\theta_j$ after the sample information $E_s$ has been obtained. The Bayes formula constructs DM's posterior probability with two components: (1) DM's prior probability $p(\theta_j)$ for the natural event and (2) the conditional probability $p(E_s | \theta_j)$ of the sample event $E_s$ that is conditioned to possible occurrence of the natural event $\theta_j$. The Bayes formula is represented as

$$p(\theta_j | E_s) = \frac{p(\theta_j)\, p(E_s | \theta_j)}{\sum_{j=1}^{n} p(\theta_j)\, p(E_s | \theta_j)} \quad, \quad j = 1, 2, \ldots, n, \quad s = 1, 2, \ldots, p, \quad (36)$$

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where the numerator represents the joint, or path, probability in the reversed probability diagram and the denominator defines the unconditional probability $p(E_s)$ for a sample event $E_s$. When the conditional probability $p(E_s | \theta_j)$ is assigned to the possible value $E_s$ taken as given, the sample likelihood function for the possible $\theta_j$-values is defined with $L(\theta_j | E_s{}^*) \underline{\Delta}\, p(E_s{}^* | \theta_j)$, $j = 1, 2,$ … , $n$. Then the Bayes formula is expressed by $p(\theta_j | E_s{}^*) \underline{\underline{\Delta}}\, C(E_s{}^*) L(\theta_j | E_s{}^*) p(\theta_j)$, where

$C(E_s{}^*) \underline{\underline{\Delta}}\, \dfrac{1}{p(E_s{}^*)}$ is a normalizing factor and does not depend on the occurrence of the population event.

Although the Bayes formula is based on the subjective probability of DM $p(\theta_j)$, which is assessed prior to the acquisition of the sample information, the sample likelihood function $f_L(E^* | \theta)$, where $E^*$ denotes a value of $E$ fixed, becomes more important as the number of the sample information (observation) is increasing. In result, the sample likelihood functions in the Bayes formula can be replaced with the mathematical functions under many realistic conditions; it leads to the use of statistical cumulative functions in the assessment of DM's posterior distribution.

2. Evaluation methods of the posterior distributions (Schlaifer 1969)

A. Case of the two-valued populations

Denote $s(\theta_j)$ be an uncertainty quantity for the possible consequence as a population event $\theta_j$. The two-valued population is defined when the consequence of an event $j$ takes one of two alternatives. For example, $s(\theta_j)$ denotes a possible value of an alternative consequence, either "success" or "failure." The two-valued population usually is defined for qualitative data and represented as a fraction, i.e. [0-1]-value. The population data for the uncertain quantity is $s$ and the sample statistics are the number of the sample data $n$ and the number of the sample information (or sample observation), say, "success" $r$.

Denote a sample event obtained with the sample information by $n \to r$, which indicates that $n$ sample data yields $r$ successes. The evaluation of the Bayes formula proceeds in the following steps.

Step 1. Assess DM's prior probability $p(s)$ for the possible value $s$ of an uncertain natural event $\theta_j$.

Step 2. Assess the conditional probability $p(n \to r | s)$ for the sample event $n \to r$ when $s$ is given.

Step 3. Calculate the joint probability $p\{s \cap (n \to r | s)\} \underline{\Delta}\, p(s) p(n \to r | s)$.

Step 4. Calculate the unconditional probability $p(n \to r)$ for the sample event $n \to r$.

Step 5. Calculate the posterior probability of DM by the Bayes formula.

$$p(s | n \to r) = \frac{p(s)\, p(n \to r | s)}{p(n \to r)} \tag{37}$$

The posterior probability $p(s\,|\,n \to r)$ in Eq.(37) can be used hereafter as the unconditional probability of DM when the sample event $n \to r$ is obtained.

Although the Bayesian formula uses the subjective probability of DM as its primal factor, it is known that the evaluation results are consistent with the use of the non-Bayesian statistical distribution under some conditions.

Assume the equiprobable sampling with replacements be executed. Then the following propositions has been presented.

Proposition 1. (Use of the binomial mass function for the conditional probability)

When the sampling is equiprobable with replacement, DM's conditional probability $p(n \to r\,|\,s)$ is required by the logical consistency to be equal to the value of the binomial mass function $b^*(n,\,r,\,s)$, which is defined by

$$b^*(n,\,r,\,s) = \binom{n}{r} s^r(1-s)^{n-r}$$

$$= \frac{n!}{r!\,(n-r)!}\,s^r(1-s)^{n-r}. \tag{38}$$

More important result is the use of the beta cumulative functions for the evaluation of DM's posterior distributions.

Proposition 2 (Use of the beta cumulative function for DM's posterior distribution).

Assume that, under the equiprobable sampling with replacement, (i) assignment of DM's prior probability is uniform, and (ii) the number of the population members $N$ is reasonably larger relative to the number of the sample information $n$, i.e. $N >> n$. Then the limiting form of the cumulative distribution function for DM's posterior probability is identical to the beta cumulative distribution

$$k(s;\,B,\,C) = \int_0^s \frac{\Gamma(C)}{\Gamma(B)\Gamma(C-B)}\,t^{B-1}(1-t)^{C-B-1}\,dt \qquad t \le s, \tag{39}$$

with parameters, $B = r+1$ and $C = n+2$. In addition, the mean is nearly equal to $B/C$, and the mode is approximately expressed by $(B-1)/(C-2) = r/n$.

The uniform distributions are defined when (i) DM assigns the equal probabilities to logically possible values of the uncertain quantity, and (ii) these values are equally spaced. Both the assumptions are not unrealistic in the incomplete knowledge environments.

The nearly uniform distributions are also defined. Define the interval $I_{n,r}$ of the sample likelihood function for the conditional probability $p(n \to r\,|\,s)$, where the values of a cumulative distribution take between 0.001 and 0.999. Then the nearly uniform distribution is defined when the following conditions are satisfied: (i) DM does not assign extremely high values of the prior probability to any value $s$ outside the interval $I_{n,r}$, and (ii) DM assigns roughly equal prior probabilities to all values of $s$ inside $I_{n,r}$, where the roughly equal means the greatest one of the probabilities is no more than four

times the least.

Proposition 3.

When DM's prior distributions are nearly uniform, the cumulative distributions of DM's posterior probability are approximated very closely by the beta cumulative distributions.

The similar device is used also in the many-valued population case.

B. Case of the many-valued population

The many-valued population is defined when each member of the population takes one of many possible values, which is applied to any quantitative data. In the present discussion, the average $A$ of possible values of the population data is used as the uncertain quantity. The sample statistics are the number of the sample data $n$, the average $a$ of the sample values, and the sample variance $v$.

The Bayesian evaluation proceeds in the following steps.

Step 1. Assess DM's prior probability $p(A)$ for the possible value of the population average $A$.

Step 2. Assess DM's conditional probability $p(n \rightarrow a, v \mid A)$ of the sample values $(a, v)$ obtained for the $n$ sample data, where the possible value $A$ for the population event is taken as given.

Step 3. Calculate the joint probability of the sample event $p(A) p(n \rightarrow a, v \mid A)$.

Step 4. Calculate the unconditional probability $p(n \rightarrow a, v)$ for the sample event.

Step 5. Divide each joint probability with the unconditional probability. DM's posterior probability $p(A \mid n \rightarrow a, v)$ is obtained with the Bayes formula.

The posterior probability can be used as DM's unconditional probability hereafter when the sample event $(n \rightarrow a, v)$ is obtained.

In the equiprobable sampling with replacement for the many-valued population data, large-sample theory in the non-Bayesian statistics is applied.

Proposition 4. (Applicability of large-sample theory)

Assume the following conditions be satisfied in the equiprobable sampling with replacement: (i) The sample size $n$ is enough large, at least more than 25 and preferably about 100, and (ii) DM's prior information for the spread of the population values is negligible relative to the sample information for the spread of the sample values. Then large-sample theory is applicable and the values of DM's conditional probability $p(n \rightarrow a, v \mid A)$ for the sample event, which are represented with the sample likelihood function $f_L(A)$ of the uncertainty quantity $A$ for the possible sample values of he sample event $(n \rightarrow a, v)$ taken as given, can be evaluated by $k g*(u)$,

$$p(n \rightarrow a, v \mid A) \approx k g*(u), \qquad (40)$$

where $g*(u)$ is the standard Gaussian density function,

$$g*(u) = (2\pi)^{-1/2} e^{-u^2/2}, \qquad (41)$$

$$u = ( a - A )\sqrt{ n / v } . \tag{42}$$

The $k$ is the constant, whose value does not depend on $A$ and is canceled out in the process of the calculation of the Bayes formula.

The use of large-sample theory leads to the Gaussian approximation to DM's posterior probability.

Proposition 5. (Gaussian approximation to the posterior distribution)

Assume the sample statistics $n$, $a$, and $v$ obtained from the equiprobable sampling with replacement. Assume the following conditions be satisfied; (i) DM's prior distribution for $A$ is uniform, (ii) possible values of the uncertain quantity $A$ are very close together, (iii) the sample likelihood function for DM's conditional probability can be calculated with the use of large-sample theory. Then the cumulative distribution for DM's posterior probability is almost identical to the standard Gaussian cumulative distribution,

$$k (u) = \int_{-\infty}^{u} (2\pi)^{-1/2} e^{-t^2/2} \, dt, \qquad t \leq u, \tag{43}$$

with parameters $M$ (= mean) and $S$ (= standard deviation), where

$$M = a, \quad S = \sqrt{v / n}, \tag{44}$$

and $t \leq u$, and $t$ is the uncertain quantity.

The Gaussian approximation can be applied also to the case with the nearly uniform prior probability for $A$.

Proposition 6.

Assume that (i) DM does not assign the extremely high values of the probability to the uncertainty quantity $A$ outside the interval extending from $A = a - 3\sqrt{v / n}$ to $A = a + 3\sqrt{v / n}$, and (ii) DM assigns roughly equal probabilities to all values of $A$ inside this interval. Then DM's posterior distribution is almost the same as that to be assigned in the case of the strictly uniform prior probability. DM's posterior distribution can be very closely approximated by the standard Gaussian cumulative distribution with the parameters already defined.

These results suggest the appropriateness of the use of the sample means as in the point estimates.

The Gaussian approximation also is applicable to the case of the equiprobable sampling without replacement.

Proposition 7.

When large-sample theory is applicable, DM's distribution posterior to equally probable sampling without replacement can be identical with the distribution for the equiprobable sampling with replacement, namely

$$p ( n \to a, v \mid A ) \approx k \, g^* (u). \tag{45}$$

The argument $u$, however, should be changed to

$$u = (a - A)\sqrt{n/v} \ \sqrt{(N-1)/(N-n)}, \tag{46}$$

where $N$ denotes the size of the population. Consider that the sample size $n$ is usually small relative to the population size. Then the factor $\sqrt{(N-1)/(N-n)}$ in Eq.(46) can be presumed to be equal to 1, which can be negligible.

3. Evaluation of the Bayes formula with MIDASS

MIDASS estimates the Bayes formula independently with a specified file name saved in advance in the following steps.

Step 1. The operation starts with the <Bayesian Statistics> button on the menu screen for the Probability Evaluation in Fig.38.

Step 2. Load Old Data screen appears. The list of the old data saved in the probability evaluations appears, say, as shown in Fig.57. The user specifies the distribution file name to be used for the Bayes estimation, which is usually constructed independently on the proper data base.
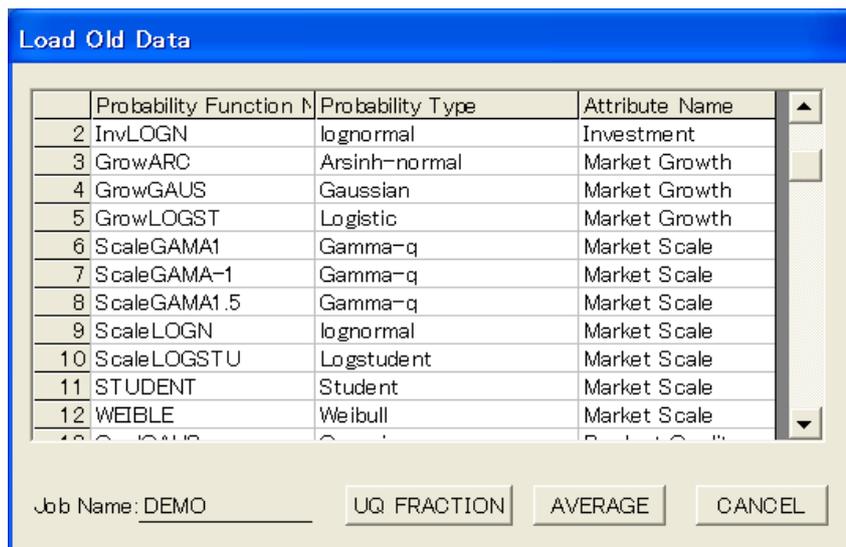


Fig.57 Load screen for Bayesian estimation

Step 3. The type of the population data, either the two-valued population or the many-valued population, is specified on the screen. When the uncertainty quantity takes a fraction, the two-valued population is used with the ＜UQ FRACTION＞ button. When the uncertain quantity takes any numerical value, the many-valued population is used with the <AVERAGE> button. The many-valued population data is constructed as an average of the original population data.

Step 4. The evaluation screen appears.

The input data for the Bayes estimation is processed with the grouped approximation. The input data is different according to the property of the population data.

A. Case of two-valued population.

The sample statistics, *n* and *r,* are specified as the input data.

All the evaluation results appear with a click of the <OK> button. All the related values are automatically calculated and appear on the screen. Fig.58 illustrates an example for the Bayes estimation. The existing file name is used here for the provisional experiment.
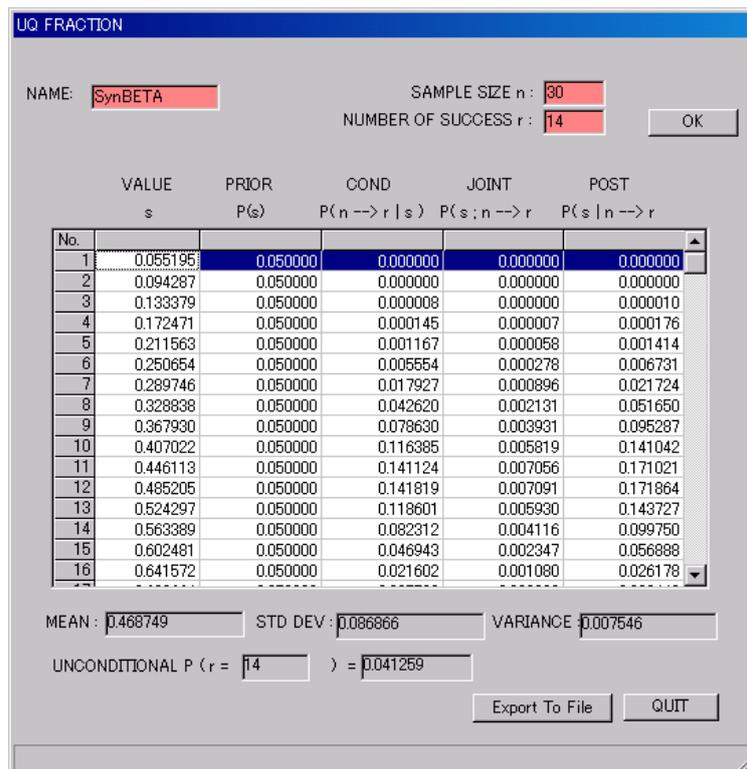


Fig.58 Bayesian estimation for two-valued population

B. Case of many-valued population

The evaluation method is in two ways on the AVERAGE screen.

(i) The sample statistics *n, a,* and *ν* are specified as input data. Click the <SUMMARY STAT> button. All the evaluation results appear as in Fig. 59.

(ii) Alternatively the user can construct the input data for the Bayes evaluation. Input the sample size *n* and click the <SAMPLE VALUE> button. The input screen appears. The user constructs the input data with the <SAMMPLE VALUES＞button as shown in Fig.60.

The evaluation is executed automatically and the results are shown as in Fig.61.

The evaluation of DM's prior probability, which is assessed with the 0.5-0.5 uncertain value, is improved largely in the posterior probability after the Bayesian estimation.
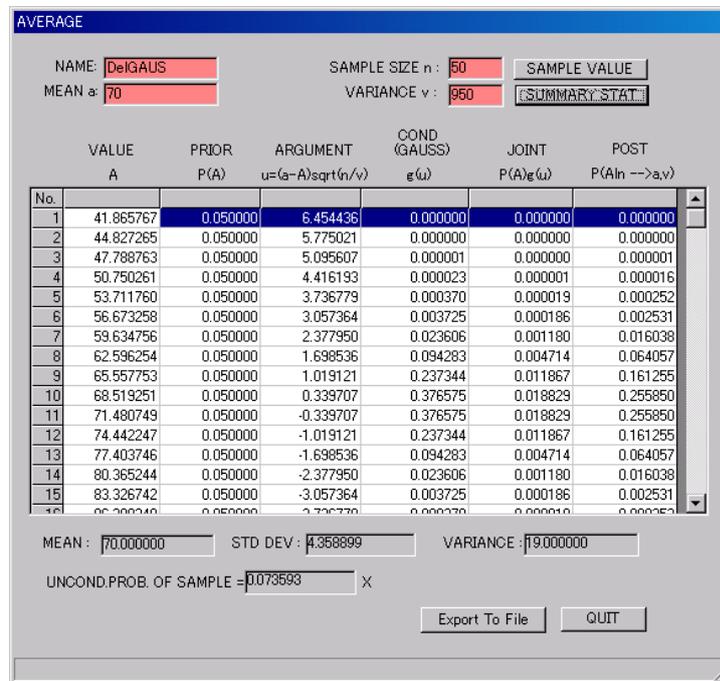


Fig.59 Bayesian estimation for many-valued population
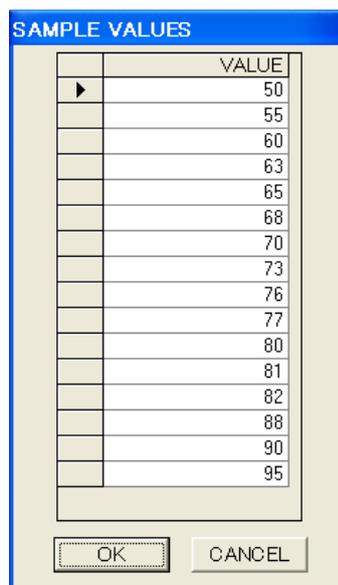


Fig.60 Input screen for sample data of many-valued population

AVERAGE

NAME: DelGAUS   SAMPLE SIZE n : 16   SAMPLE VALUE
MEAN a: 73.3125   VARIANCE v : 152.214   SUMMARY STAT

| No. | VALUE A | PRIOR P(A) | ARGUMENT u=(a-A)sqrt(n/v) | COND (GAUSS) g(u) | JOINT P(A)g(u) | POST P(A|n -->a,v) |
|---|---|---|---|---|---|---|
| 1 | 41.865767 | 0.050000 | 10.195465 | 0.000000 | 0.000000 | 0.000000 |
| 2 | 44.827265 | 0.050000 | 9.235306 | 0.000000 | 0.000000 | 0.000000 |
| 3 | 47.788763 | 0.050000 | 8.275148 | 0.000000 | 0.000000 | 0.000000 |
| 4 | 50.750261 | 0.050000 | 7.314989 | 0.000000 | 0.000000 | 0.000000 |
| 5 | 53.711760 | 0.050000 | 6.354830 | 0.000000 | 0.000000 | 0.000000 |
| 6 | 56.673258 | 0.050000 | 5.394672 | 0.000000 | 0.000000 | 0.000000 |
| 7 | 59.634756 | 0.050000 | 4.434513 | 0.000021 | 0.000001 | 0.000021 |
| 8 | 62.596254 | 0.050000 | 3.474355 | 0.000954 | 0.000048 | 0.000916 |
| 9 | 65.557753 | 0.050000 | 2.514196 | 0.016915 | 0.000846 | 0.016241 |
| 10 | 68.519251 | 0.050000 | 1.554037 | 0.119259 | 0.005963 | 0.114508 |
| 11 | 71.480749 | 0.050000 | 0.593879 | 0.334444 | 0.016722 | 0.321120 |
| 12 | 74.442247 | 0.050000 | -0.366280 | 0.373059 | 0.018653 | 0.358196 |
| 13 | 77.403746 | 0.050000 | -1.326438 | 0.165521 | 0.008276 | 0.158926 |
| 14 | 80.365244 | 0.050000 | -2.286597 | 0.029211 | 0.001461 | 0.028047 |
| 15 | 83.326742 | 0.050000 | -3.246755 | 0.002051 | 0.000103 | 0.001969 |
| 16 | 86.288240 | 0.050000 | -4.206914 | 0.000057 | 0.000003 | 0.000055 |

MEAN : 73.312500   STD DEV : 3.084385   VARIANCE : 9.513428

UNCOND.PROB. OF SAMPLE = 0.052075   X

Export To File   QUIT

Fig.61 Bayesian estimation for many-valued population (II)

## 5.4 Gamble evaluation

The evaluation of the mathematical expectations as the numerical results of decision analysis is presented as the gamble evaluation by DM. MIDASS executes the gamble evaluation on three ways: (i) the expected values for an attribute, (ii) the expected utility values for the single attribute utility function (EUF), and (iii) the expected multiattribute utility functions (EMUF). The evaluation starts on the main menu screen of MIDASS in Fig.1.

### 5.4.1 Expected value evaluation

The evaluation of the mathematical expectation for an attribute starts with the <Mathematical Expectation> button. The evaluation screen appears. The user should indicate first an attribute name. The Select Attribute screen appears with the <Attribute Name> button. Then user selects a distribution for the attribute with the <Prob Data> button on the evaluation screen. The Select Prob Name screen appears. Return to the Mathematical Expectation screen with the <OK> button. The evaluation results for the mathematical expectation appear with the <CALC> button on the screen. The expected value $E(x)$ is calculated along with the pair $(x, p(x))$.

Fig.62 depicts an example for the attribute Profitability.
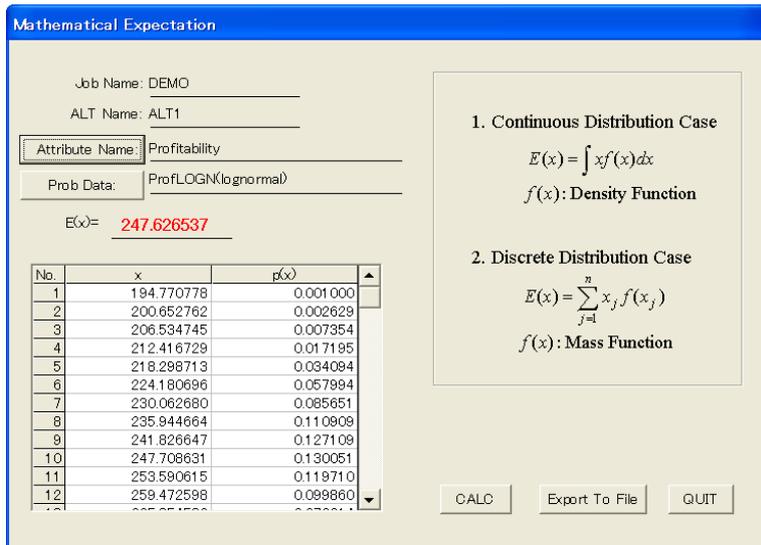
Fig.62 Evaluation of mathematical expectation (Profitability)

### 5.4.2 Expected utility evaluation

The evaluation starts with the <Expected Utility Function> button on the main menu screen in Fig.1. The evaluation screen for Expected Utility Function appears. The Select Attribute screen appears with a Click of the <Attribute Name> button as shown in Fig.63. Select an attribute on the screen. Similarly the Select Unif Name and the Select Prob Name screens appear with the click of the <Unif Data> and the <Prob Data> buttons respectively as in Figs.64 and 65. Select the UNIF type and the distribution type for the attribute on these screens.
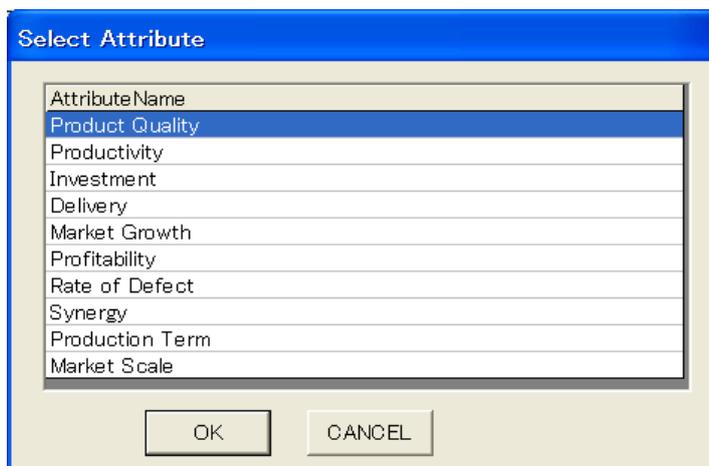


Fig.63 Selection of an Attribute
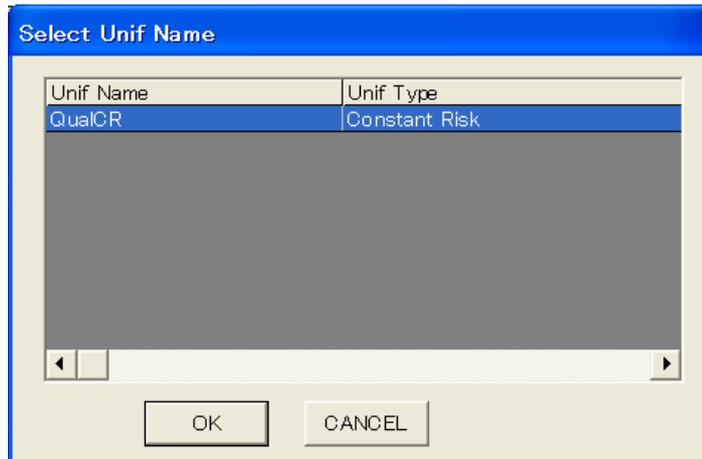
Fig.64 Selection of UNIF Type
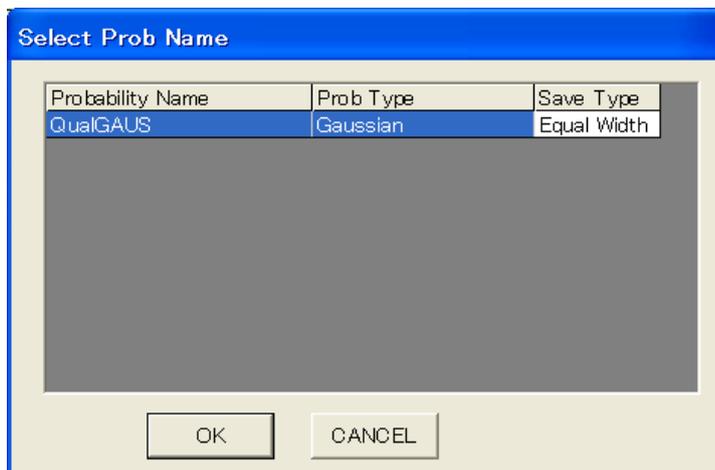


Fig.65 Selection of Probability type

The values for the components of the expected utility function assessed for an attribute $x$ appear with a click on the <CALC> button, which are shown as the ($x$-$u(x)$-$p(x)$ ) tripple. The value of the expected utility function $EU(x)$ assessed with the specified utility function and distribution for the attribute $x$ is calculated and appears along with the mathematical formula.

Fig.66 illustrates the evaluation results for the expected utility function of the attribute Product Quality.
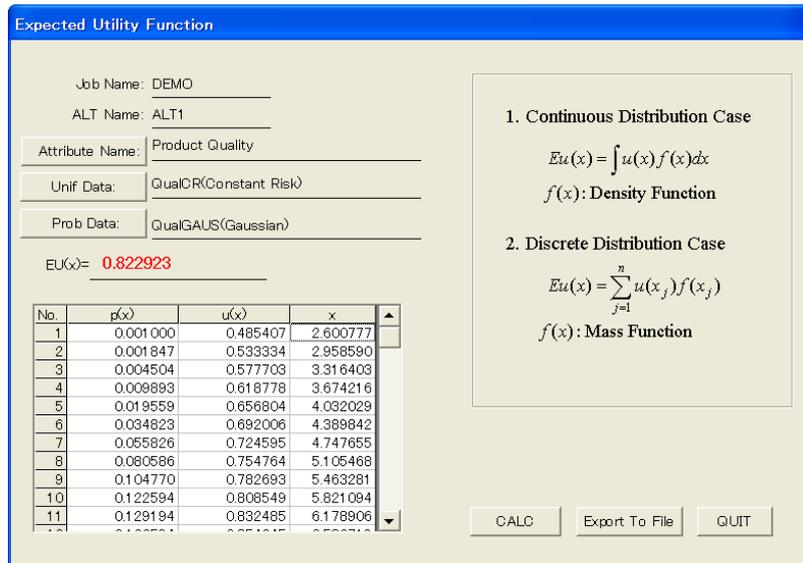
Fig.66 Evaluation Screen for expected utility value (Product Quality)

### 5.4.3 Expected multiattribute utility evaluation

The evaluation of the expected multiattribute utility function starts with the <Multiattribute Expected Utility Function> button on the main menu screen of MIDASS in Fig.1.

With a click, the Probability-Type Selection screen appears. Note that the UNIF types to be selected for the attributes have already been specified and saved in the process of the MUF evaluation. The distribution types for all the attributes are selected on the screen as shown in Fig.67. The Probability-Type Selection screen generates a list of the set of Unif type, Prob name, and Prob type selected for the Attribute name shown in the screen, which is used in the evaluation of EMUF. The hidden item appears with a click on the arrow in the screen.

The evaluation of EMUF is executed after the selection of the distribution types for all the attributes is completed. The evaluation results for EMUF appear with a click of the <save> button on the Probability-Type Selection screen. The Multiattribute Expected Utility Function screen appears. MIDASS generates all of the EMUF values at each level in the hierarchical structure along with the evaluation results for their components.

Note that the evaluation results are based on the alternative values of the data set (say, "ALT x"), which has been selected for the specified job in advance with the <Set ALT> button on the Job Management screen as in Fig.4.

Fig.68 illustrates the Multiattribute Expected Utility Function screen with the output of all the evaluation results for EMUF.
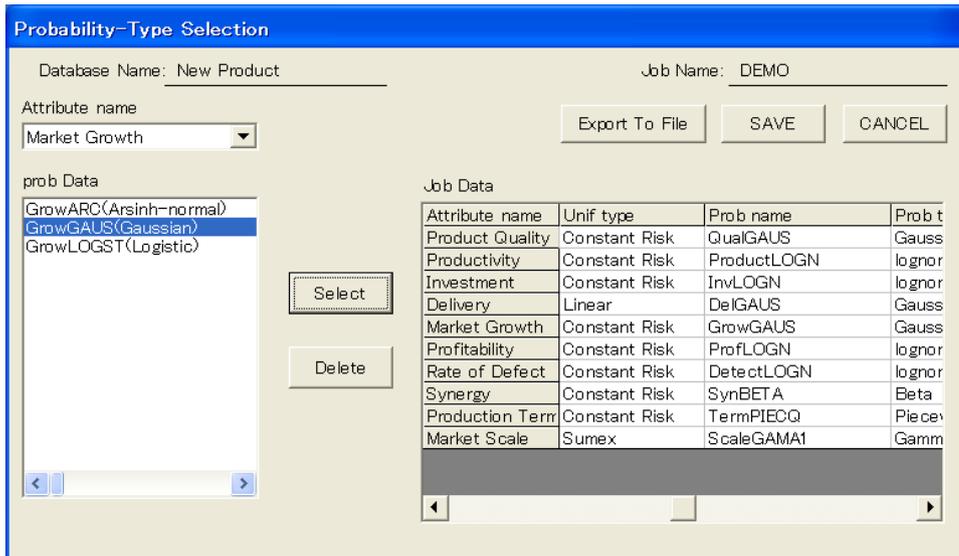
Fig.67 Screen for selection of the distribution types for attributes
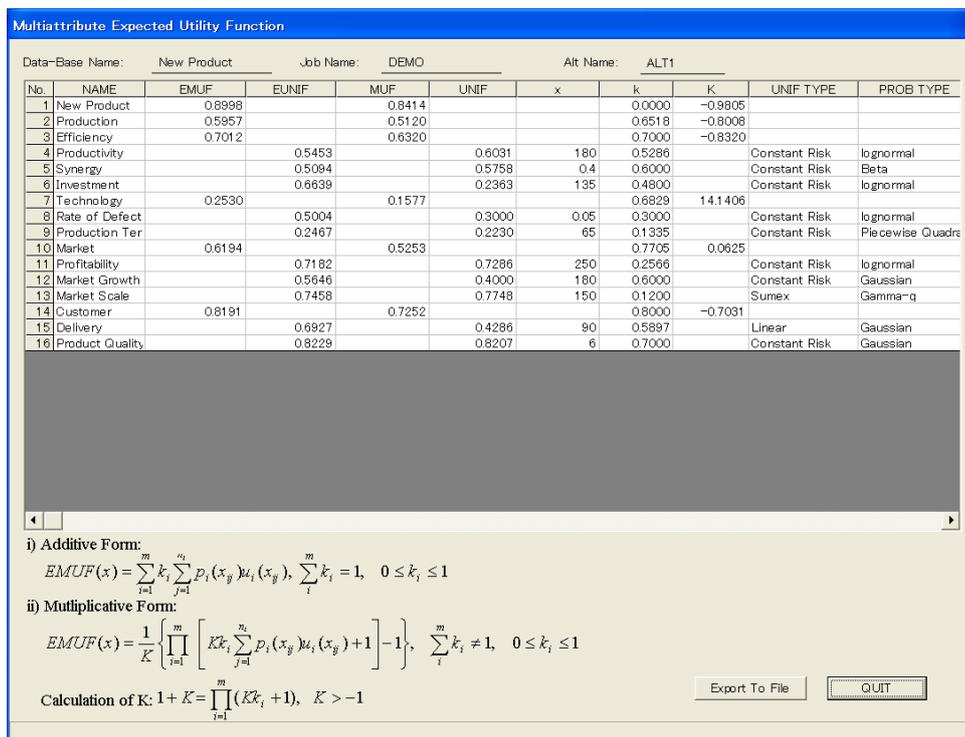


Fig.68 Demonstration of Evaluation Results for EMUF (New Product)

**6. Concluding Remarks**

Decision analysis is a classical method, which has been well-established over a long period and still is attracting much attention to its importance for coping with the complex decision problems. In particular, the multiobjective extension of this method should be seen in a new light. In a new era of the management, for example, the concept of the value based management (VBM), or the enterprise value creative management, is pervading in the business worlds, which requires to manage the multiobjective point of view properly. The concept of the customer satisfaction (CS) management is also inspired along with the increasing recognition of the customer-perceived value concept (Kotler and Armstrong 2005). The corporate social responsibility (CSR) is increasing its importance in the making-money structure of enterprises. For example, ISO 9000:2000 presents the eight principles for the quality management system, in which the customer focus is regarded as the most basic one. ISO9001:2000 demands to be compatible with the requirements of ISO14001:2004 environmental management systems (EMS). The balanced score card (BSC) also is well known as one of the efficient tools for executing these principles in practice (Kaplan and Norton 1992, 2001; Tanner 2002). In SSME (Service Sciences, Management, and Engineering), which is also recently inspired, a cross disciplinary approach as "research at a crossroads" is stressed and "services is a people business," and *its* "profitability" is raised as its characteristics (e.g *Technology Review,* special issue on Research in Development, may 2005, MIT).

In addition, under the increasing versatility in the business environments, the valuation of corporate management is much concerned with the treatment of the uncertainty. Decision analysis has been presented an effective method in using decision tree analysis (Copeland et al. 1995, Copeland and Antikarov 2001, Goodwin and Wright 2004, Koller et. al. 2005). In this context, expected utility theory should be recaptured and its multiobjective extensions are expected. Although the foundations for those developments have already been presented, the operational use of the methods in an integrated form has not yet been developed. For effective operation of this approach, further development in the conceptual construction of an integrated methodology is required, which is an intelligent decision support system with the computer assistance.

This paper has discussed the background of decision analysis and presents an integrated method for its strategic use of multiobjective decision analysis. A new computer program MIDASS has been presented with the method for heuristic construction of the expected multiattribute utility function (EMUF), which can be used as a unified criterion in the corporate management under the uncertainty. As an example, a decision problem for new product development has been presented with illustrations.

There exist still some rooms to be considered in MIDASS.

(1) Decision analysis usually starts with the construction of the decision tree diagram.

The expected multiobjective decision analysis presumes the construction of the decision tree with the $n$-chance forks, where consequences with the multiple attributes occur at each chance fork. MIDASS is used for evaluating EMUF at the nodes of the decision tree structure. EMUF must be embedded in the decision tree structure. Decision analysis, however, usually constructs the decision tree in multiple stages. MIDASS must be used repeatedly at every node in the multi-staged decision tree structure. In this situation, a device for more effective operation of MIDASS will be required, which needs further evolution of the MIDASS program.

(2) In MIDASS, the assignment of DM's evaluation is made with deterministic values. The evaluation in decision making, however, is ambiguous in twofold: One is the cognitive ambiguousness both in the utility assessment and the probability assessment by DM. Two is the incompleteness of information to be obtained by DM. The assessments for the ambiguity usually are performed with the ambiguous quantities such as fuzzy numbers (Seo 1992, 1997; Nishizaki and Seo 1994) or with the random values such as in the probabilistic utility functions (Luce and Suppes 1965; Seo 2000). The extension of MIDASS to these directions has much academic interest. Even so, however, the effort for these directions will introduce much complexity in the decision support systems, which may sacrifice seriously the user-friendly property in the decision support systems.

MIDASS supports at present the decision ambiguity in three ways.

(i) The thinking processes of DM for the evaluations are constructed sequentially with the interactive assistance not only by the computer, but also by the internal self-examination of DM, in which the evaluation can go back to the preceding steps and revise it repetitively.

(ii) MIDASS includes the Bayes theorem for coping with the ambiguity in the probability evaluation of DM. The use of sample information is included in the decision tree structure (Raiffa 1968, etc.). The acquisition of the sample information, however, is sometimes not realistic for the business decisions environments, in particular, in such the case as the repeated experiments.

(iii) For coping with the decision ambiguity, the construction of decision tree structure at multiple stages is recommended, where the phases for revising the evaluation results can be included.

In further evolution of MIDASS, the development of the method to embed it more effectively and repeatedly in the decision tree structure should be expected. MIDASS presents a first but primal step for this work with the selective and manual operations.

MIDASS software disk is available from the first author on your reruest.
Mail, with your affiliation, to the e-mail address: fseo@kier.kyoto-u.ac.jp

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## Appendix

### A. Instruction for Preference Evaluation (<INSTRUCTION>)

Evaluation for the decreasing risk-averse preferences with the 5 points input method proceeds in following order.

(1) On the evaluation screen first appeared, assign tentatively the input values for three fractiles, 0.25, 0.5, and 0.75. The values for 0.0 and 1.0 points are automatically indicated. MIDASS draws a preference curve for the input data with a click on the <OK> button.

(2) Proceed to the Consistency Check screen with the <NEXT> button. MIDASS checks the consistency of the 5 input points with the decreasing positive risk aversion functions.

(i) Confirm the 50-50 gambles and their certainty equivalent (CE) values for the user sequentially.

The 50-50 chance lottery technique is used for the construction or confirmation of the preference curve for the user. The lottery technique assesses a lottery and its certainty equivalent (CE). Three lotteries with the 50-50 chance forks are constructed, whose consequences for the attribute are $v(0.0)$ or $v(1.0)$, $v(0.0)$ or $v(0.5)$, and $v(0.5)$ or $v(0.0)$ and their certainty equivalents (CE) are assessed with 0.5, 0.25, and 0.75-values of the preferences respectively. The three lotteries are constructed with the substitutions of the assessed attribute values in order. Then assign the CE values as input data on the screen.

The $v(x.x)$ denotes the value of an attribute, for which user's preference is assessed with the x.x-value. Note that the worst value of the attribute is assessed with the 0.0-value of the preference, which is represented as $v(0.0)$, and the best value is assessed with 1.0, which is represented as $v(1.0)$. On the screen, the parenthesis of $v(x.x)$ is omitted.

(ii) Check the input values for CE carefully. Then supply presumably the Assessed value $v^*(.5)$ as the same as the Implied value $v(.5)$. (iii) Follow the messages generated by MIDASS. The user can revise the Assessed value $v^*(.5)$ on the screen deliberately according to the suggestions.

(3) The user can be back to the first input screen, if necessarily, with a click on the <QUIT> button. Check the shape of the preference curve on the screen, which is newly generated with the revised input values. Then proceed to the Consistency Check screen again with the NEXT button.

(4) This process can be repeated many times until the user is satisfied with the consistency, which is informed by a message on the screen.

(5) When the consistency has been obtained for the decreasing risk aversion, choose the utility function type to be assessed, the piecewise-exponential average type (PIECEX) or the sum-of-exponential type (SUMEX) with the <PIECEXFIT> button or the <SUMEXFIT> button on the screen respectively.

(6) The valuation results for user' utility functions appear on the next screen.

B. Instruction for Selecting the Probability Distribution (<HELP>)

MIDASS supports the derivation of three types of the probability distributions: Continuous probability, Piecewise quadratic distribution, and Discrete distribution. The continuous probability is derived in the mathematical distribution functions, whose input values are assigned by the user. The piecewise quadratic distribution is derived in the piecewise quadratic function forms with the input data assigned by the user. The discrete distribution is constructed with the relative frequency distribution in the histogram and its smoothing represents the piecewise linear distribution functions. MIDASS also supports Bayes statistics.

In the probability evaluation, the ALT-value indicated for the attribute is used as the uncertain quantity. The distribution functions for the uncertain quantities are constructed heuristically by the user. Input data is different according to the distribution types.

The evaluation proceeds with selecting the proper button on the menu screen for the probability evaluation.

I. Continuous Probability Distribution

The evaluation of the continuous distributions are not so much "flexible" due to the use of the mathematical distribution forms, but more "reasonable" and easier to generate and manipulate in the subsequent works.

MIDASS supports three groups of the mathematical distributions. The selection of the distribution type depends on the quantitative characteristics of the data (i.e. the attribute value as an uncertain quantity).

A. Family Group 1, for which the attribute data takes the values between 0 and 1.

B. Family Group 2, for which the attribute data takes the nonnegative values (i.e. larger than 0).

C. Family Group 3, for which the attribute data takes the unrestricted values (from minus infinity to plus infinity).

Input data for the assessment are usually parameters, moments, and/or fractiles of the distribution.
< General attention! >

(1) Note that the assessor usually has an inclination to assess the 0.25 and 0.75-fractiles with substantially shorter tails than they should be.

(2) The graph on the screen can be enlarged with a click when the numerical values on the horizontal axis are shown with congestion.

A. Family Group 1. Attribute data takes 0 to 1 values.

The qualitative or fractional data are suitable to this Group.

1. Beta distribution: <Beta>

This type may have a wide variety of shapes and easy to assess by the user.

The distribution may be either symmetric or skew depending on the parameters.

-------- Suggestion --------------------------

The evaluation is recommended in the following process.

(1) Try this distribution type first.

(i) Assess the values for the 0.25 and 0.75-fractiles.

(ii) Alternatively, if it is available, input the mean (MEAN), $m$, and the standard deviation (STDDEV) values as the moments. The MEAN-value must be in between 0 and 1. The STDDEV-value must be in between 0 and Sqrt($m(1 - m)$).

(2) When the user is not satisfied with the skewness of the assessed distribution, the parameter adjustment will be helpful for the correction of the skewness.

(i) When the parameter $B$ increases, the distribution skews to the right side having a longer tail to the left side. When the parameter $B$ decreases, the distribution skews to the left-side having a longer tail to the right side.

(ii) When the parameter $C$ increases with the fixed $B$-value, the variance (dispersion) will become smaller, and vice versa.

<Attention!>

The parameter $C$ should be larger than the parameter $B > 0$.


2. Bounded lognormal distributions: <Bounded Lognormal>

This family closely resembles the beta distributions, except in the cases when the beta distribution is nearly uniform and when the substantial probability is assigned to the data values very close to 0 or 1.

-------- Suggestion --------------------------

(1) When the user is not satisfactory with the beta distribution, try this distribution type.

(2) Assess the distribution first with the same fractile values as the beta distribution.

(3) When the distribution shape is not satisfactory to the user, try to change the values of the two fractiles to the same direction, either increase or decrease.

(4) Try, for example, to decrease the values a bit both for the 0.25 and 0.75-fractiles. Then the distribution will skew to the left side. When the fractiles increase, the distribution skews to the right side.

(5) If desired, the parameter values can be corrected on the reference of the values generated by the program.

<Attention!>

The parameter MU takes an unrestricted value, but the SIGMA should be positive.

B. Family Group 2: Attribute data takes nonnegative values.

This family is applied to most popular data and is used widely. Economic and managerial data have generally nonnegative characteristics.

The user is recommended to proceed in order with the following distribution types.

1. Lognormal distributions: <Lognormal>

The shape of this family is always skew with a long tail to the right side.

--------- Suggestion ------------------

(1) Try this distribution type first in this Group. Input the values for the fractiles as indicated.

(2) If the spread from the 0.25 to the 0.75-fractile is small relative to the distance between the origin and the median, then the shape of the distribution will be nearly symmetric.

As the spread increases, the distribution becomes more and more skew.

(3) The parameter MU is an unrestricted value and the SIGMA is a positive value.

2. Logstudent distributions: <Logstudent>

The tails of the logstudent may be longer than the tails of the lognormal, but they cannot be shorter. The control over the tail length works only in one direction.

--------- Suggestion ------------------

(1) When the user is not satisfactory with the lognormal distribution, try this distribution type.

(2) When you wish to extend both tails longer than in the lognormal distributions, use this type of distribution with the values for the 0.25 and 0.75-fractiles fixed and increase a bit for the 0.875-fractile. The value of the 0.875-fractile for the logstudent may be greater than that of the lognormal distribution, but cannot be smaller.

(3) For the correction of the skewness, the user also can change the parameters, SIGMA for example. When the value increases, the shape skews to left side. When the value decreases, the shape moves to the center position and becomes more close to the symmetric type.

(4) The parameter MU is unrestricted, but SIGMA and NU should take positive values.

3. Gamma-$q$ distributions: <Gamma-q>

When the user wishes to extend one tail longer and shorten the other in the lognormal distribution, use this distribution type with the values for the 0.25 and 0.75-fractiles fixed.

The parameter $q$ controls the length of the tails of the distribution and can be any positive or negative value other than 0.

The program requests the user to specify the parameter $q$ directly as input data. Two additional assignments are either other two parameters or the values for the 0.25 and 0.75-fractiles of the distribution.

------- Suggestion -----------------------------

(1) Try first the same values for the 0.25 and 0.75-fractiles as the preceding distribution.

(2) Follow the subsequent steps.

Step 1. Start with $q = 1$ when you wish to extend the left tail, or start with $q = -1$ when you wish to extend the right tail.

Step 2. If this shape is not satisfactory to the user, then double or halve the $q$-values sequentially. As the $q$-value increases, the shape of the distribution skews to the right side having a longer tail to the left side. As the $q$-value decreases, the distribution skews to the left side having a longer tail to the right side.

(3) If the parameter $q$ is positive, the left tail is longer and the right tail is shorter than in the lognormal distribution. When the $q$ is negative, the reverse is true.

(4) When the parameter $q$ approaches 0 from either side, the shape of the distribution approaches the lognormal distribution.

(5) The two parameter values, R and S, should be positive.

＜Attention!＞

(1) The $q$-value should not be set very close to 0.

(2) The values of the fractiles are fixed in the Step 1 and the Step 2.


4. Weibull distributions: <Weibull>

Weibull distribution is used independently to assess the extreme distributions, which is applied to the data having a very low probability for the extreme but critical values in either side of the distribution. This distribution type is used in the extreme statistics such as in reliability or risk analysis.

The Weibull distribution has a widely flexible property.

The attribute value $x$ is used as $t = x / b$ in the calculation, where $b > 0$ is the scale parameter.

----- Suggestion --------------------------

(1) To use the shape parameter A as input data is recommended.

(2) The parameter A should be a positive integer.

(3) When the value of the parameter A increases, the distribution moves to the right side with a longer tail to the left side and a smaller variance. When the A decreases, the distribution skews to the left side with a longer tail to the right side and a larger variance.


C. Family Group 3: Attribute data takes unrestricted values.

This distribution family so far has been known in the use for the statistical estimation of the population parameters and the testing for the statistical hypothesis. In decision analysis, this family can be used for preliminary statistical treatment in the database construction. Direct assessment with

this family, however, is also useful due to the many variations of the parameters to be assigned in the probability evaluation.

1. Gaussian (Normal) distributions: <Gaussian (Normal)>

The Gaussian distribution is the most well-known symmetric distribution. The shape of the distribution is inflexible and thus may not necessarily suitable for user's direct evaluation. In the cases, however, where the law of large numbers is applied, this type of the distribution plays an important role in the evaluation of unknown distribution, whose specification is difficult. The distribution is also used often in the statistical experiments, such as the parameter estimation and statistical testing.

---------Suggestion--------------------------

(1) Although the normal distribution is not suitable for the heuristic derivation of DM's distribution, it sometimes can be used as a good approximation to unknown real distributions with wide variations.

(2) The parameter input for M and S is recommended, although the values for the 0.25 and 0.75-fractiles can be also assigned.

(3) The parameter M takes an unrestricted value, but the parameter S should be a positive value.

2. Arcsinh-Normal distributions: <Arcsinh-Normal>

The arcsinh-normal distribution is used as an alternative to the Gaussian distribution. If the user is not satisfied with the Gaussian distribution, then try this family.

The shape of this distribution can be either symmetric or skew, with the very long tail to either the left side or the right side.

----- Suggestion ------------------------------

The valuation of the distribution proceeds in the following steps.

Step 1. Start with a Gaussian distribution. If the distribution is not satisfied by the user, then try the arcsinh-normal distribution with reference of the original fractile values.

Step 2. Fix a value of a fractile such as the median, for example, generated in the Gaussian distribution. Then try to narrow the substantial range of the distribution.

(i) Increase the values for the 0.125 and 0.25-fractiles and decrease the values for the 0.75 and 0.875-fractile with the 0.5-fractile value fixed. As the upper fractile (0.75 and 0.875)-values decrease and/or the lower fractile (0.125 and 0.25)-values decrease, the statistical range of the distribution becomes narrower (i.e. the width of the distribution shape is widened).

(ii) Alternatively, try the reverse. Then the substantial range of the distribution is widened (i.e. the width of distribution shape is narrowed).

(iii) When the distribution is symmetric, the tails must be long at least as the same as the tails of the Gaussian distribution with the same interquartile range.

Step 3. If the distribution is still not appropriate for user's opinion, then change the $\mu$ and/or $\sigma$

value.

< Attention! >

(1) In the parameter input, the SIGMA should be set as a relatively very small value.

(2) The value of the octile should not be too close to the adjacent quartile. This means that the tail should have an appropriate length for this distribution type.


3. Logistic distributions: <Logistic>

The logistic distribution is known with the "law of growth" and used in the cumulative forms for evaluating the growth curves in various fields, such as biological, chemical, demographical, social, etc.

------ Suggestion ------------------

(1) The parameter often is assessed by regression analysis using the time-series data.

(2) The parameter ALPHA should take a positive value.


4. Student Distributions: <Student>

The Student distribution is an alternative symmetric distribution, whose shape is resemble to the Gaussian distribution, but has a relatively sharp and lower median with the "heavy" tails in the both side. This distribution is used when the user is not satisfactory with other distribution types.

The distribution is also known for the use in the statistical experiments, such as the parameter estimation and the testing of the statistical hypothesis, for example, in quality control, etc.

------ Suggestion ------------------

(1) The general Student distribution is assessed with the parameter MU (= M), whose value is unrestricted, and the parameters SIGMA (= S) and NU (degree of freedom), whose values take the positive values. The MU is not defined when NU is less than or equal to1, and the SIGMA is infinite when Nu is less than or equal to 2.

(2) The standard or unit Student distribution (called the t distribution) is assessed with the parameters, MU = 0 and SIGMA = 1. The parameter NU and SIGMA should take positive values although MU is unrestricted. .

(3) The Cauchy distribution is the t distribution with the parameter NU = 1. The MU value is not defined in this distribution.

(4) The moments is not used the same as the other cases in this family, Gaussian and Arcsinh-normal distributions.


II. <Piecewise Quadratic Distribution>

When the user intends to derive a quadratic distribution heuristically, the distribution can be constructed with the piecewise quadratic curve. The assessment is executed with the specification of

several input points on the screen. Then MIDASS derives the piecewise quadratic distribution.

This process, however, often may confront with a difficulty in assessing the distribution consistent with the piecewise quadratic type. The assessment should proceed carefully and patiently.

------- Suggestion -----------

1. As input values, specify 3-25 points, or preferably 10-12 points with equal width, which must include the 0 and 1-fractiles. Then the program checks if these input values are consistent with the piecewise quadratic forms and, if not, a message is generated to prompt the correction of the input values.

2. Input data should take a finite range.

3. All cumulative distributions in the discrete type have the general shape as follows.

(1) The steepest, or modal, portion of the graph is a linear segment.

(2) If the linear segment is not at the end of the left-side, the graph has a convex shape to the left-side when viewed from below and is tangent to the $F = 0$ axis (usually the lower horizontal axis).

(3) If the linear segment is not at the end of the right-side, the graph is concave to the right side when viewed from below and is tangent to the $F = 1$ axis (usually the upper horizontal line).

<Attention!>

(1) Note that the assessor usually has an inclination to assess an excessively sharp mode, the steepest point, and an excessively plain tail.

(2) When the graph has the S shape, the user should not select the mode of the distribution as the input value. Instead, the user should select two points marking the ends of the linear segment.

III. <Discrete Distribution>

The construction of the database for the attributes often requires the pretreatment and checks of the original data obtained from the production sites. The elementary statistical treatment of data base is usually to make the relative frequency distribution, which is derived in the form of the histogram in the descriptive statistics. The smoothing of a shape of the histogram derives the discrete distribution. When the user prefers to assess the discrete distribution, MIDASS constructs the discrete distributions.

The relative frequency distributions are used widely in various fields of the operational management, for example, in the field of the industrial management such as in derivation of the Paretian map, which is used in ABC analysis for the inventory control and in "seven tools" for the quality control. The "control map" in the "seven tools" is also constructed with the discrete distribution.

The application fields of the discrete distributions are extended further to various fields.

IV. <Bayesian Statistics>

MIDASS supports the Bayesian statistics for the two-valued population and the many-valued

population. The data as an uncertain quantity for the two-valued population takes a fraction value and the data for the many-valued population takes any value. Note that the input data for the many-valued population is the average of the original population data. Specify a file name for the probability distribution, whose values are already preserved. Use the button for one of the population types, <UQ FRACTION> or <AVERAGE>. Then MIDASS calculates the Bayes formula with the input of the sample statistics as indicated.

The posterior distribution of DM is generated with the Bayes formula, which revises the prior distribution of DM.

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