

# KIER DISCUSSION PAPER SERIES

## KYOTO INSTITUTE OF ECONOMIC RESEARCH

Discussion Paper No. 696

“Risk and Uncertainty in Health Investment”

Takao Asano and Akihisa Shibata

March 2010



KYOTO UNIVERSITY  
KYOTO, JAPAN

# Risk and Uncertainty in Health Investment <sup>\*</sup>

Takao Asano <sup>†</sup>

Akihisa Shibata <sup>‡</sup>

Faculty of Economics  
Okayama University

Institute of Economic Research  
Kyoto University

March 8, 2010

## Abstract

Extending the Grossman [12] model of health capital into a stochastic one, we analyze how the presence of Knightian uncertainty about the efficacy of health care affects the optimal health investment behavior of individuals. Using Gilboa and Schmeidler's [11] model of maxmin expected utility (MMEU) with multiple priors, we show that an agent retains the initial level of health capital if the price of health care lies within a certain range. We also show that the no-investment range expands as the degree of Knightian uncertainty rises.

*Journal of Economic Literature* Classification Numbers: D81, I11

**Key Words:** Health Investment, Risk, Uncertainty

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<sup>\*</sup>We would like to appreciate two anonymous referees for their very helpful comments and suggestions. We acknowledge financial support from the Grant-in-Aid for Scientific Research, JSPS, and the Global Center of Excellence (GCOE) program, the Ministry of Education, Culture, Sports, Science, and Technology. Needless to say, we are responsible for any remaining errors.

<sup>†</sup>Faculty of Economics, Okayama University, 3-1-1 Tsushimanaka, Kita-ku, Okayama 700-8530, Japan. e-mail: asano@e.okayama-u.ac.jp. Tel: +81-86-251-7668

<sup>‡</sup>Institute of Economic Research, Kyoto University, Yoshida, Sakyo-ku, Kyoto 606-8501, Japan. e-mail: shibata@kier.kyoto-u.ac.jp. Tel: +81-75-753-7126

# 1 Introduction

Uncertainty on the quality of products in medical care is more intense than that in standard commodities, as Arrow [1] pointed out in a seminal article. This study is an analysis of the effect of uncertainty on health care behavior. In the course of the study, a distinction is made, along the lines that of Knight [15], between “uncertainty” and “risk.” Knight argued that risk is characterized by randomness that can be measured precisely while uncertainty relates to randomness that cannot be expressed by specific probability distributions. According to this terminology, risk takes place when a unique probability measure is assigned to each event, and uncertainty takes place when only an imprecise information is available to assign a unique probability to each event. Based on Knight’s argument, Ellsberg [10] suggested the empirical importance of the distinction between risk and uncertainty.<sup>1</sup>

Extending Grossman’s [12] model of health capital, in this study, we analyze how this kind of “uncertainty” about the efficacy of health care affects health investment behavior, and show that an agent retains the initial level of health capital if the price of health care lies within a certain range. We also show that the no-investment range expands as the degree of Knightian uncertainty rises. It is natural to assume that if people remain less confident about their health condition and the efficacy of health investment, then they will hesitate to invest in their health capital because of the lack of precise information on their health condition. Let us consider the problem of being overweight. According to Puska, Nishida, and Porter [21], there exist 1 billion overweight adults, at least 300 million of them are clinically

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<sup>1</sup>Ellsberg conjectured that individuals would prefer bets with precise odds to those with imprecise odds. For example, consider a bet on drawing a yellow ball from two urns. An urn includes 50 yellow and 50 white balls, while, the other urn includes 100 yellow and white balls, but the exact numbers of the two kinds of balls are unknown. Ellsberg asserted that, in this situation, most decision-makers would prefer the first urn to the second. His conjecture was confirmed by several experimental studies, such as those by Becker and Brownson [5] and Slovic and Tversky [24]. See Oliver [19] and Wakker [25] for a comprehensive account of the Ellsberg paradox in the context of health economics.

obese. Puska, Nishida, and Porter [21] state that “obesity and overweight pose a major risk for serious diet-related chronic diseases, including type 2 diabetes, cardiovascular disease, hypertension and stroke, and certain forms of cancer.” Even though we recognize that losing our weight is necessary for our health, in general, quite a few of us do not diet in order to lose their weight. This paper shows that we tend to behave in this way when we are cautious about the effects of efforts such as cutting the amount of fatty and sugary foods and engaging in daily physical exercise. The reason that people have difficulty in controlling their diet can be also explained from the point of view of time-inconsistency caused by hyperbolic discounting as mentioned in Laibson [14]. Or it may be because people cannot resist the temptation of good food and have a self control problem as analyzed in Gul and Pesendorfer [13]. This paper provides another angle to understand the overweight problem from the Knightian uncertainty viewpoint.

While this paper studies the optimal health investment under Knightian uncertainty and shows that the degree of Knightian uncertainty can reduce the amount of health investment, in the previous literature on health investment, there are several related studies which also analyze the optimal health investment under stochastic environments. Using a stochastic version of the Grossman model, Dardanoni and Wagstaff [8] showed that, under mild conditions, a greater risk stimulates health care (health investment). Selden [23] showed, however, that the result by Dardanoni and Wagstaff can be reversed according to how the risk is introduced. Applying Arrow’s [2] model of portfolio selection to health care management, Chang [7] constructed a unified model including Dardanoni and Wagstaff’s and Selden’s models as its special cases. Moreover, Picone, Uribe and Wilson [20] constructed a dynamic version of the Grossman model and examined the effects of uncertainty on health investment. Simulating the model, they showed that either a first-order stochastic dominant shift or a second-order stochastic dominant shift in risk will stimulate health capital investment, especially in the early

stage of life.

All of the aforementioned studies, however, included only risk factors and, therefore, were limited to the effects of risk on health investment behavior, while the role of uncertainty was entirely ignored. Because most people have only imprecise information about the efficacy of health care behavior, it is quite natural to formulate decision-making problems by introducing Knightian uncertainty. Thus, in this paper, using a version of the maxmin expected utility (MMEU) model of Gilboa and Schmeidler [11], we construct a simplified version of the Grossman model with Knightian uncertainty and analyze how the presence of Knightian uncertainty affects the health investment behavior of individuals.

In the MMEU theory, an agent's beliefs are captured by a set of probability measures and his preferences are represented by the minimum of expected utilities over the set of probability measures. In a closely related paper, Schmeidler [22] axiomatized Choquet expected utility (CEU) theory and showed that an agent's beliefs are captured by a *non-additive measure* and his preferences are represented by the *Choquet integral*. Moreover, Schmeidler [22] proved that, when an agent's beliefs are captured by a *convex* non-additive measure, CEU is equivalent to a special case of MMEU, which endows CEU with the meaning of *uncertainty aversion* and makes it possible to analyze his behavior under Knightian uncertainty through the convexity of non-additive measures.<sup>2</sup> Because we would like to consider a

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<sup>2</sup>Let  $\mu$  be a convex non-additive measure,  $\mathcal{M}$  be the set of probability measures on  $(S, 2^S)$ , and let  $B(S, \mathbb{R})$  denote the space of bounded functions from  $S$  into  $\mathbb{R}$ . Define the *core* of  $\mu$  by  $\text{core}(\mu) = \{Q \in \mathcal{M} | (\forall E \in 2^S) \mu'(E) \geq Q(E) \geq \mu(E)\}$ , where  $\mu'$  is the conjugate of  $\mu$ . Then

$$(\forall X \in B(S, \mathbb{R})) \int X(s)\mu(ds) = \min \left\{ \int X(s)Q(ds) \mid Q \in \text{core}(\mu) \right\},$$

where the left-hand side of the equality is in the sense of the Choquet integral. This relation implies that it is the convexity of non-additive measures that serves the bridge between the MMEU theory and the CEU theory, and makes it possible to analyze the decision-makers' behavior under Knightian uncertainty through the Choquet integral. For definitions of non-additive measures, the convexity, and the Choquet integral, see Appendix.

situation in which an agent is uncertainty-averse, we assume that his beliefs are described by a convex non-additive measure and analyze our MMEU maximizer's problem using the CEU theory.<sup>3</sup>

The organization of this paper is as follows. Section 2 is an illustrative example based on the main result of this study. Section 3 is the main result of this study presented in a formal framework of the CEU theory. Section 4 contains further analyses of the main result. Section 5 is the conclusion. The definitions and some mathematical results regarding the Choquet integrals and proofs of theorems in this study are in the Appendix.

## 2 An Illustrative Example

Incorporating Knightian uncertainty into a simple two-period version of Grossman's [12] model of health capital, we provide an illustrative example that shows that an agent keeps the initial level of health capital if the price of health care lies within a certain range.<sup>4</sup> Suppose that there is an agent who invests in his health capital, either in a positive or a negative amount.<sup>5</sup> Furthermore, we assume that he is an MMEU maximizer and that he determines his health investment on the basis of the worst-case scenario, that is, his beliefs are captured by a set of probability measures  $\mathcal{M}$ , and his preferences are represented by the minimum of expected utilities over  $\mathcal{M}$ .

Let  $p$  be the price of health care at  $t = 1$ , and let  $X(s)$  be the health condition at  $t = 2$  in state  $s$ . We assume here that there are two states, *good*

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<sup>3</sup>The models by Gilboa and Schmeidler [11] and Schmeidler [22] can explain the Ellsberg paradox fairly well and have been widely employed by applied studies. See, for example, Dow and Werlang [9] and Berliant and Konishi [6].

<sup>4</sup>This example is based on Nishimura and Ozaki [18].

<sup>5</sup>Typical examples of negative investment are to smoke heavily and to drink heavily. It seems natural to consider only non-negative health investment. Even if we restrict our analysis to the case of non-negative health investment, we can easily obtain essentially the same results as those derived in this paper, that is, we can show that there exists a lower bound beyond which he never invests in his health capital and the lower bound becomes smaller as the degree of Knightian uncertainty increases.

(g) or *bad* (b), and that the good and bad health conditions are  $X(g) = G$  with probability  $q_g$  and  $X(b) = B$  with probability  $1 - q_g$ , respectively.<sup>6</sup> Moreover, we assume that he is risk-neutral and an MMEU maximizer and that  $\mathcal{M}$  is the set of probability measures  $q_g$ .<sup>7</sup>

The expected return from a positive health investment is represented as

$$-p + \min \{q_g G + (1 - q_g)B \mid q_g \in \mathcal{M}\}. \quad (1)$$

Similarly, the expected return from a negative health investment is given by

$$p + \min \{-(q_g G + (1 - q_g)B) \mid q_g \in \mathcal{M}\} = p - \max \{q_g G + (1 - q_g)B \mid q_g \in \mathcal{M}\}, \quad (2)$$

where "negative health investment" stands for the agent's behavior that causes damage on individuals' health. As easily understood, the agent makes a positive health investment if (1) is positive, and he makes a negative health investment if (2) is positive. If both (1) and (2) are negative, he never makes this health investment, that is, he does nothing for his health capital and keeps his initial health capital intact if

$$\min \{q_g G + (1 - q_g)B \mid q_g \in \mathcal{M}\} < p < \max \{q_g G + (1 - q_g)B \mid q_g \in \mathcal{M}\}. \quad (3)$$

For simplicity, we assume that the true probability of his good health condition,  $q_g$ , is  $1/2$ , but because of the presence of Knightian uncertainty, he is not confident about  $q_g$ . In order to capture this situation, we assume here that his beliefs are represented by a set of probability measures  $\mathcal{M} = [1/2 - \epsilon, 1/2 + \epsilon]$  for any  $\epsilon \in (0, 1/2)$ ,<sup>8</sup> where we can interpret  $\epsilon$  as the degree

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<sup>6</sup>We assume that  $G > B$ .

<sup>7</sup>Since there exists Knightian uncertainty about his health condition at  $t = 2$ , his beliefs cannot be captured by a unique probability measure.

<sup>8</sup>This class of multiple priors is referred to as  $\epsilon$ -contaminations. See Nishimura and Ozaki [17] for an axiomatic foundation of the  $\epsilon$ -contamination. In Section 4, we provide the definition of  $\epsilon$ -contaminations, and analyze the effect of increases in Knightian uncertainty on an interval in which the agent never invests in her health capital.

of his error about the true probability. Under this setting, we have

$$\min \{q_g G + (1 - q_g)B \mid q_g \in \mathcal{M}\} = \frac{1}{2}G + \frac{1}{2}B - \varepsilon(G - B) \quad (4)$$

$$\text{and} \quad \max \{q_g G + (1 - q_g)B \mid q_g \in \mathcal{M}\} = \frac{1}{2}G + \frac{1}{2}B + \varepsilon(G - B). \quad (5)$$

Under our assumption that  $G > B$  and  $\varepsilon > 0$ , it is apparent from (4) and (5) that we can have a situation in which (3) holds, that is, there is an interval in which he never invests in his health capital. Moreover, an increase in  $\varepsilon$  leads to a decrease in the right-hand side of (4), and an increase in  $\varepsilon$  leads to an increase in the left-hand side of (5), which implies that the interval in which he never invests in his health capital *expands* as  $\varepsilon$  increases. Since an increase in  $\varepsilon$  implies an increase in Knightian uncertainty, this result implies that a lesser confidence in the probability of his good health condition discourages his health investment more severely. Finally, we consider a situation in which  $\varepsilon = 0$ , that is, there is no Knightian uncertainty in the probability of his good health condition. In this situation, (4) is equal to (5), which implies that there is no interval in which he never invests in his health capital.

### 3 Health Investment under Knightian Uncertainty

In this section, introducing Knightian uncertainty into a simple two-period version of the Grossman [12] model, we provide a formal model of health investment. In order to analyze how the presence of Knightian uncertainty about the efficacy of health care affects individual health investment behavior, we assume that an agent is a CEU maximizer, that is, his beliefs are captured by a convex non-additive measure, and his preferences are represented by the Choquet expected utility. Under this assumption, we account for the existence of an interval in which he never invests in his health capital. As we explain in Section 2, the existence of the interval is not accounted for if an agent maximizes the standard expected utility, that is, if his beliefs are captured by a unique probability measure and his preferences are rep-



resented by the expected utility. Before we present the main result of this study, we will briefly explain the basic setup.

Let  $S$  be the space of the states of the world, let  $2^S$  be the power set of  $S$ , and let  $(S, 2^S)$  be a measurable space. Let  $B(S, \mathbb{R})$  denote the space of bounded functions from  $S$  into  $\mathbb{R}$ . Let  $X(s)$  denote the health condition in state  $s$  at time  $t = 2$ , where  $X \in B(S, \mathbb{R})$ . Let  $H_1$  and  $H_2$  be health capitals at  $t = 1$  and  $t = 2$ , respectively, and let  $H_2 : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$  be defined by

$$H_2(X(s), M) \equiv (1 - \delta)H_1 + MX(s),$$

where  $\delta$  denotes the depreciation rate and  $M \in \mathbb{R}$  denotes the amount of health investment.<sup>9</sup> We assume that an agent's income depends on the level of his health capital and that the income-generating function  $\Psi : \mathbb{R} \rightarrow \mathbb{R}$  is a function of the health capital at  $t = 2$ ,  $H_2$ , where  $\Psi$  is concave and continuously differentiable with  $\Psi'(\cdot) > 0$  and  $\Psi''(\cdot) \leq 0$ .<sup>10</sup>

Let  $A$ ,  $Y_1$  and  $p$  denote the initial wealth, the income at  $t = 1$ , and the price of health capital, respectively. We assume that  $u : \mathbb{R} \rightarrow \mathbb{R}$  is a concave and twice-continuously differentiable function with  $u'(\cdot) > 0$  and  $u''(\cdot) \leq 0$ . Moreover, we assume that an agent is *uncertainty averse* and that he has to decide whether or not to make a health investment at  $t = 1$ .<sup>11</sup> The objective is to choose the amount of the health investment  $M$  in order to

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<sup>9</sup>Even if we consider an additive case such as

$$H_2(X(s), M) = (1 - \delta)H_1 + I(M) + X(s),$$

the result is not essentially affected. That is, we can derive the existence of the no-investment interval. Note that in this additive case, the interval obtained is not the same as the one in Theorem 1.

<sup>10</sup>Chang [7] defines  $H_2 : \mathbb{R} \rightarrow \mathbb{R}$  by  $H_2(M) = (1 - \delta)H_1 + I(M)$ , where  $I : \mathbb{R} \rightarrow \mathbb{R}$  denotes some production function satisfying some regularity conditions. Moreover, he assumes that the income-generating function  $\Psi : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$  is a real-valued function of the amount of the health investment  $M$  and some random variable  $\varepsilon$ , satisfying some regularity conditions, denoted by  $\Psi(H_2, \varepsilon)$ . If we assume that  $\lim_{M \rightarrow 0} I'(M)$  exists and is finite, then we can also derive our main result within his framework.

<sup>11</sup>As we explain in Introduction, we assume that agent's beliefs are captured by a convex non-additive measure, and her preferences are represented by the Choquet integral.

maximize

$$\int u((A + Y_1 - pM) + \Psi(H_2(X(s), M))) \mu(ds),$$

where  $\mu$  is a convex non-additive measure and the integral is in the sense of the Choquet integral. This agent is assumed to maximize his Choquet expected utility of his terminal wealth with a convex non-additive measure. In this setting, we have the following theorem.

**Theorem 1.** *Suppose that the price of health capital  $p$  satisfies the following:*

$$\int_S \frac{d\Psi}{dH_2} X(s) \mu(ds) < p < \int_S \frac{d\Psi}{dH_2} X(s) \mu'(ds),$$

where  $\mu'$  is the conjugate of  $\mu$ . Then, the agent never invests in this health capital.

The integrand  $d\Psi/dH_2 X$  denotes the marginal product of the health investment evaluated at  $M = 0$ . If this agent's beliefs are captured by a probability measure, then the interval does not exist. This theorem implies that the decreased confidence in his health condition at  $t = 2$  makes him hesitate about the health investment in either a positive or a negative direction.

It is worth mentioning the economic mechanism that leads to our main result. In the two-period model of our paper, the reason that individuals invest in health is because it will bring a better health status tomorrow, which in turn will increase the income tomorrow. The presence of Knightian uncertainty makes the health condition tomorrow very uncertain. It discounts the expected return of the health investment and makes individuals more likely not engaged in the health investment.

## 4 An Increase in Knightian Uncertainty

In this section, we analyze the effect of increases in the degree of Knightian uncertainty on the interval in which the agent does not invest in his health

capital and show that an increase in the degree of the Knightian uncertainty expands the length of the interval. For that purpose, we assume that his beliefs are captured by the  $\varepsilon$ -contamination of the true probability measure  $P_0$ .<sup>12</sup>

First, a formal definition of  $\varepsilon$ -contaminations is presented. Let  $P_0$  be the true probability measure about his health condition at  $t = 2$ , and let  $\varepsilon \in (0, 1)$ . The  $\varepsilon$ -contamination of  $P_0$  is defined by

$$\mathcal{P}_0 \equiv \{(1 - \varepsilon)P_0 + \varepsilon Q \mid Q \in \mathcal{M}\}.$$

If his beliefs are captured by the  $\varepsilon$ -contamination of  $P_0$ , then, with  $(1 - \varepsilon) \times 100\%$ , he is certain about the true probability measure of his future health condition, but, with  $\varepsilon \times 100\%$ , he fears that his beliefs in his future health condition are completely wrong. If  $\varepsilon = 0$ , then the set of probability measures  $\mathcal{P}_0$  is reduced to the singleton,  $P_0$ . An increase in  $\varepsilon$  implies that an agent is more uncertain about the true probability measure  $P_0$ . Thus,  $\varepsilon$  can be considered to be a parameter that captures the degree of Knightian uncertainty.<sup>13</sup>

Let  $\varepsilon'$  and  $\varepsilon$  be in  $(0, 1)$ , let  $\varepsilon' > \varepsilon$ , and define  $\theta_1$  and  $\theta_2$  by

$$(\forall E) \quad \theta_1(E) = \begin{cases} (1 - \varepsilon')P_0(E) & \text{if } E \neq S \\ 1 & \text{if } E = S \end{cases} \quad (6)$$

$$(\forall E) \quad \theta_2(E) = \begin{cases} (1 - \varepsilon)P_0(E) & \text{if } E \neq S \\ 1 & \text{if } E = S, \end{cases} \quad (7)$$

where  $E$  is an event and  $P_0$  is a probability measure on  $(S, 2^S)$ . It can be shown that  $\theta_1$  and  $\theta_2$  are convex non-additive measures and that  $\text{core}(\theta_1) = \mathcal{P}_1$  and  $\text{core}(\theta_2) = \mathcal{P}_2$ , where  $\mathcal{P}_1 \equiv \{(1 - \varepsilon')P_0 + \varepsilon'Q \mid Q \in \mathcal{M}\}$  and  $\mathcal{P}_2 \equiv \{(1 - \varepsilon)P_0 + \varepsilon Q \mid Q \in \mathcal{M}\}$ . Therefore, the intervals of an agent whose beliefs are represented by the  $\varepsilon'$ -contamination of  $P_0$  and the  $\varepsilon$ -contamination of

<sup>12</sup>As we explain in this section, the  $\varepsilon$ -contamination of  $P_0$  can be represented by core of some convex non-additive measure. Therefore, an agent can be considered to be a CEU maximizer as in Section 3. For the relation between MMEU and CEU, see footnote 2.

<sup>13</sup>For applications of  $\varepsilon$ -contaminations to economics, see Nishimura and Ozaki [16] and Asano [3].

$P_0$  are

$$\int_S \frac{d\Psi}{dH_2} X(s) \theta_1(ds) < p < \int_S \frac{d\Psi}{dH_2} X(s) \theta'_1(ds), \text{ and}$$

$$\int_S \frac{d\Psi}{dH_2} X(s) \theta_2(ds) < p < \int_S \frac{d\Psi}{dH_2} X(s) \theta'_2(ds),$$

respectively.

Our main theorem on the effect of Knightian uncertainty is as follows:

**Theorem 2.** *If the degree of Knightian uncertainty increases, then the interval in which the agent does not invest in his health capital expands.*

*Proof.* See Appendix. □

This theorem states that the more uncertainty averse he is about his future health condition, the more he hesitates about his health investment in either a positive or a negative direction. As mentioned in Introduction, even though we recognize that losing our weight is necessary for our health, in general, quite a few of us do not diet in order to lose their weight. This kind of behavior can be explained by Theorem 2. Moreover, this theorem implies that the more cautious we get about effects of diet, the less we diet.

## 5 Conclusion

Introducing Knightian uncertainty into the Grossman [12] model of health capital, we analyzed the effect of Knightian uncertainty on the health investment behavior of individuals. We showed that, in the presence of Knightian uncertainty, when the price of health care lies within a certain interval, an agent does nothing, that is, he keeps the initial level of his health capital. We also showed that the length of the no-investment interval expands as the degree of uncertainty increases.

## Appendix

Let  $(S, 2^S)$  be a measurable space. A set function  $\mu : 2^S \rightarrow [0, 1]$  is a *non-additive measure (or normalized capacity)* if (a)  $\mu(\emptyset) = 0$ , (b)  $\mu(S) = 1$ , and (c)  $E, F \in 2^S$  and  $E \subset F$  imply  $\mu(E) \leq \mu(F)$ , where  $\emptyset$  denotes the empty set. A non-additive measure  $\mu$  is *convex* if  $\mu(E \cup F) + \mu(E \cap F) \geq \mu(E) + \mu(F)$  for all  $E, F \in 2^S$ . Let  $\mu$  be a non-additive measure on  $(S, 2^S)$ . Then, the *conjugate*  $\mu'$  of  $\mu$  is defined by  $\mu'(E) = 1 - \mu(E^c)$  for all  $E \in 2^S$ , where  $E^c$  denotes the complement of  $E$ .

Let  $B(S, \mathbb{R})$  denote the space of bounded functions from  $S$  into  $\mathbb{R}$  and let  $X \in B(S, \mathbb{R})$ . The integral of  $X$  with respect to a non-additive measure  $\mu$  is called *the Choquet integral*, and is defined by

$$\begin{aligned} & \int X(s) \mu(ds) \\ &= \int_0^\infty \mu(\{s \in S \mid X(s) \geq \alpha\}) d\alpha + \int_{-\infty}^0 [\mu(\{s \in S \mid X(s) \geq \alpha\}) - 1] d\alpha, \end{aligned}$$

where integrals on the right hand side are in the sense of Riemann integrals.

**Fact 1.**

$$(\forall X, Y \in B(S, \mathbb{R})) \quad X \geq Y \Rightarrow \int X(s) \mu(ds) \geq \int Y(s) \mu(ds).$$

**Fact 2.**

$$(\forall X \in B(S, \mathbb{R})) (\forall a \in \mathbb{R}) (\forall b \in \mathbb{R}_+) \quad \int (a+bX(s)) \mu(ds) = a+b \int X(s) \mu(ds).$$

**Fact 3.**

$$(\forall X \in B(S, \mathbb{R})) \quad \int X(s) \mu(ds) = - \int (-X(s)) \mu'(ds),$$

where  $\mu'$  is the conjugate of  $\mu$ .

**Fact 4** (Schmeidler [22]). *Let  $\mu$  be a convex non-additive measure, and let  $X \in B(S, \mathbb{R})$ . Then*

$$\int_S X(s) \mu(ds) = \min \left\{ \int_S X(s) Q(ds) \mid Q \in \text{core}(\mu) \right\}, \text{ and}$$

$$\int_S X(s)\mu'(ds) = - \int_S -X(s)\mu(ds) = \max \left\{ \int_S X(s)Q(ds) \mid Q \in \text{core}(\mu) \right\}.$$

**Fact 5.** Let  $X, Y \in B(S, \mathbb{R})$ , and let  $\mu$  be a convex non-additive measure. Then,

$$\int_S (X(s) + Y(s))\mu(ds) \geq \int_S X(s)\mu(ds) + \int_S Y(s)\mu(ds).$$

Let  $\mu$  be a convex non-additive measure, let  $X \in B(S, \mathbb{R})$ , and define  $\mathcal{P}(\mu, X)$  by

$$\mathcal{P}(\mu, X) \equiv \operatorname{argmin} \left\{ \int X dQ \mid Q \in \text{core}(\mu) \right\}.$$

Since  $\mu$  is convex, the set  $\mathcal{P}(\mu, X)$  is non-empty.

Let  $h : S \times \mathbb{R} \rightarrow \mathbb{R}$  satisfy the following:

$$(\forall z \in \mathbb{R}) \quad h(\cdot, z) \text{ is measurable} \tag{8}$$

$$(\forall z \in \mathbb{R}) \quad h(s, \cdot) \text{ is differentiable and concave} \tag{9}$$

Then,  $\int_S h(s, z)\mu(ds)$  is concave with respect to  $z$  if  $\mu$  is convex.

**Proposition 1.** Let  $\mu$  be a convex non-additive measure, and let  $h : S \times \mathbb{R} \rightarrow \mathbb{R}$  satisfy the conditions (8) and (9). Then

$$\begin{aligned} (\forall z \in \mathbb{R}) \quad \frac{d}{dz_-} \int h(s, z)\mu(ds) &= \max \left\{ \int \frac{\partial h}{\partial z}(s, z)Q(ds) \mid Q \in \mathcal{P}(\mu, h(\cdot, z)) \right\} \\ (\forall z \in \mathbb{R}) \quad \frac{d}{dz_+} \int h(s, z)\mu(ds) &= \min \left\{ \int \frac{\partial h}{\partial z}(s, z)Q(ds) \mid Q \in \mathcal{P}(\mu, h(\cdot, z)) \right\}, \end{aligned}$$

where  $d/dz_-$  and  $d/dz_+$  denote the right derivative and the left derivative, respectively.

*Proof.* See Aubin [4]. □

*Proof of Theorem 1.* Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be defined by  $f(M) = \int u((A + Y_1 - pM) + \Psi(H_2(X(s), M))) \mu(ds)$ . From Fact 5 together with

the assumptions that  $\Psi$  and  $u$  are concave, it follows that  $f$  is a concave function with respect to  $M$ . Therefore,  $M = 0$  is optimal if

$$\begin{aligned} f'_+(0) &\equiv \frac{d}{dM_+} \int_S u((A + Y_1 - pM) + \Psi(H_2(X(s), M))) \mu(ds) \Big|_{M=0} \\ &< 0 < \frac{d}{dM_-} \int_S u((A + Y_1 - pM) + \Psi(H_2(X(s), M))) \mu(ds) \Big|_{M=0} \equiv f'_-(0). \end{aligned}$$

From Proposition 1, it follows that

$$\begin{aligned} &\min \left\{ \int_S u'(A + Y_1 + \Psi(H_2(X(s), 0))) \left( -p + \frac{d\Psi}{dH_2} X(s) \right) Q(ds) \mid Q \in \mathcal{P}(\mu, u(W)) \right\} \\ &< 0 < \max \left\{ \int_S u'(A + Y_1 + \Psi(H_2(X(s), 0))) \left( -p + \frac{d\Psi}{dH_2} X(s) \right) Q(ds) \mid Q \in \mathcal{P}(\mu, u(W)) \right\} \\ \Leftrightarrow &\min \left\{ \int_S \frac{d\Psi}{dH_2} X(s) Q(ds) \mid Q \in \text{core}(\mu) \right\} < p < \max \left\{ \int_S \frac{d\Psi}{dH_2} X(s) Q(ds) \mid Q \in \text{core}(\mu) \right\} \\ \Leftrightarrow &\int_S \frac{d\Psi}{dH_2} X(s) \mu(ds) < p < \int_S \frac{d\Psi}{dH_2} X(s) \mu'(ds), \end{aligned}$$

where the first equivalence follows from  $u'(\cdot) > 0$  and  $\mathcal{P}(\mu, u(W)) = \text{core}(\mu)$ , and the second equivalence follows from Fact 4.  $\square$

*Proof of Theorem 2.* In order to prove Theorem 2, we provide the concept of *being more uncertainty averse* proposed by Dow and Werlang [9] and present mathematical results related to the concept. Let  $\mu$ ,  $\mu_1$ , and  $\mu_2$  be non-additive measures on  $(S, 2^S)$ , and let  $c(\mu, A) \equiv 1 - \mu(A) - \mu(A^c)$ . Dow and Werlang [9] state that  $\mu_1$  is *more uncertainty averse than*  $\mu_2$  if  $c(\mu_1, A) \geq c(\mu_2, A)$  for all  $A$ .

**Theorem 3** (Dow and Werlang [9]). *Let  $\mu_1$  and  $\mu_2$  be non-additive measures on  $(S, 2^S)$ . Then, the following statements are equivalent.*

- (i)  $\mu_1$  is more uncertainty averse than  $\mu_2$ .
- (ii) For any  $X \in B(S, \mathbb{R})$ ,

$$\int X(s) \mu'_1(ds) - \int X(s) \mu_1(ds) \geq \int X(s) \mu'_2(ds) - \int X(s) \mu_2(ds),$$

where  $\mu'_1$  and  $\mu'_2$  are conjugates of  $\mu_1$  and  $\mu_2$ , respectively.

**Lemma 1.** *Let  $\varepsilon'$  and  $\varepsilon$  be in  $(0, 1)$ . Let  $\theta_1$  and  $\theta_2$  be defined by (6) and (7), respectively. If  $\varepsilon' > \varepsilon$ , then  $c(\theta_1, E) \geq c(\theta_2, E)$  for all  $E$ .*

*Proof.* The proof is immediate. □

Combining Theorem 3 and Lemma 1 proves Theorem 2. □



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