KIER DISCUSSION PAPER SERIES

KYOTO INSTITUTE
OF
ECONOMIC RESEARCH

Discussion Paper No.704

"Liquidity, Interbank Market, and Capital Formation"

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July 2010

KYOTO UNIVERSITY

KYOTO, JAPAN

Liquidity, Interbank Market, and Capital Formation *

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March, 2010

Abstract

This paper presents a monetary model that links interbank markets to capital accumulation and growth. The purpose of this paper is to study how interbank markets affect real economic activities, and to find the monetary policy implications. The model shows that, in a stationary equilibrium, the economy with interbank markets attains higher capital stock than the economy without the markets, because of precautionary money savings. In addition, I find that inflationary policy is more desirable in the economy without well-functioning interbank markets.

Key words: overlapping generations, random relocation, inflation, interbank markets.

JEL Classification: E42, E51, G21

*I would like to thank Real Arai , Ryoichi Imai, Ricardo Lagos, Tomoyuki Nakajima, Akihisa Shibata, and Takashi Shimizu for their helpful comments and suggestions. Of course, all errors are mine.

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1 Introduction

A large body of empirical studies suggest that banking activities are strongly correlated with real economic activities (see Levine (1997) for a review). Interbank markets provide one of the most important functions in the financial system. These allow the reallocation of liquidity between banks. Banks with low liquidity can borrow liquidity from banks with high liquidity through the markets, and can then meet their liquidity demand. Despite their apparent importance, interbank markets have received relatively little attention in the academic literature.

The purpose of this paper is to develop a monetary model with financial intermediaries to analyze the linkage between interbank markets and real economic performance. Specifically, in order to understand how interbank markets affect real economic activities, I compare two economies under different conditions: with and without interbank markets. The economy consists of a number of regions. The number of agents who need liquidity in each region fluctuates randomly, but the aggregate demand for liquidity is constant. This setup allows for interregional transactions, as regions with liquidity surpluses provide liquidity for regions with liquidity shortages. The provision of insurance can be organized through interbank markets.

The analysis described here is based on the monetary model developed in Champ, Smith and Williamson(1996) and in Smith(2002). I employ an overlapping generations model in which spatial separation and limited communication generate a transactions role for fiat money. At the end of each period, a fraction of agents is relocated to a different location. The only asset that they can use is fiat money. This allows money to be held even when dominated in

the rate of return. Limited communication implies that relocated agents cannot transact using privately issued liabilities in the new location. Agents who are not relocated are not constrained in their transactions by the limitations on communication. They can pay for consumption goods when old with checks or other credit instruments. The other asset is a neoclasssical technology. The stochastic relocations act like shocks to agents' liquidity preferences. Idiosyncratic shocks to agents create a role for banks to provide insurance against these shocks, as in Diamond and Dybvig(1983).

The main result of the paper is to show that well-functioning interbank markets reduce banks' cash reserves and increase investments in capital. If banks cannot access the markets, banks cannot diversify liquidity risks and must hold more cash by themselves to meet liquidity demands. If banks can access the markets, they can diversify their risks and reduce their cash reserves, and make more investments in capital. In addition, I show numerically that inflationary monetary policy is more desirable in an economy without well-functioning interbank markets. Inflationary policy gives banks incentive to reduce cash reserves, and encourages them to make more investments in capital.

Several other papers have studied the functioning of interbank markets. Bhattacharya and Gale(1987) model the role of interbank markets clearly, and show that interbank markets insure banks against idiosyncratic liquidity. Aghion et al.(1999) and Allen and Gale(2000) analyze the phenomena by which banking failures are disseminated via interbank markets. Holmstrom and Tirole(1998) and Diamond and Rajan(2005) analyze the optimal liquidity provision by a central bank when interbank markets are subject to aggregate liquidity shocks and contagious failure. Allen and Gale(2009) consider incom-

plete interbank markets that result in limited hedging opportunities for banks, and they show that a central bank can implement constrained efficient allocation by using open-market operations.

The main difference between all these studies and mine is that they use static models and do not consider monetary factors explicitly. In practice, almost all interbank tradings are monetary and intertemporal activities. This paper develops a monetary model with interbank markets, and analyzes the likage between the markets and the real economy. Moreover, I study the difference in optimal money growth under different interbank regimes.

The remainder of the paper proceeds as follows. Section 2 describes the economic environment and considers the behavior of banks and government, and the nature of factor market transactions. Section 3 considers banks' problems with and without interbank markets. Section 4 discusses general equilibria, and section 5 analyzes the comparative statics consequences of monetary policy. Section 6 considers dynamic issues, while section 7 investigates welfare and optimal monetary policy issues. Section 8 concludes.

2 The Model

The model is based on Champ, Smith and Williamson(1996) and Smith(2002).

2.1 The Environment

I consider an economy consisting of an infinite sequence of two-period-lived overlapping generations. Let $t = 0, 1, 2, \ldots$ be index time. The world is divided into two spatially separated locations, and each location consists of a

number of regions of unit mass. Each region is populated by a continuum of agents of unit mass. The two locations are completely symmetric in terms of all economic activity.

All young agents are identical ex ante. They are endowed with one unit of labor when young, which they supply inelastically, and they retire when old. Young agents have no other endowments of goods or assets at any date.

All agents care only about second period consumption. Let c_t denote the second-period consumption of a representative agent born at t. Agents have the same lifetime utility, $u(c) = \ln(c)$.

The consumption good is produced by a representative firm, which rents capital and hires labor from young agents. A representative firm uses a constant returns to scale technology $F(K, L)$, where K and L denote capital and labor inputs, respectively. Let $f(k) \equiv F(k, 1)$ be the intensive production function, where $k = K/L$ is the capital-labor ratio. It is assumed that $f'(k) > 0 > f''(k)$ $\forall k$, that $f(0) = 0$ holds, and that f satisfies the usual Inada conditions. For simplicity, I assume that capital depreciates completely in the production process.

As in Townsend(1987), I introduce a transaction role for money by emphasizing the spatial separation and the limited communication between the two locations. Limited communication prevents privately issued liabilities from being verifiable in the other location. Money is universally recognizable and noncounterfeitable, and is therefore accepted in both locations. In particular, at each date, agents can trade and communicate only with other agents in the same location.

The timing of events within a period is as follows. First, firms rent capital

and labor, produce the final good, and pay their factors of production. Final goods are then either consumed or are invested to create next period's capital stock. Young agents in each region receive wage income and deposit it with a competitive bank. Banks use all deposits for capital investments and the purchase of money balances from the old.

After deposits have been allocated between capital investments and cash balances, a fraction π_t of young agents in each region is relocated to the other location. These agents are called "movers." The value π_t is different across regions. Relocation plays the role of a "liquidity preference shock" in the Diamond and Dybvig(1983) model, and it is natural to asume that banks arise to insure agents against these shocks. The relocation probability π_t is a random variable in each period. Because there is a continuum of young agents, it represents the fraction of all movers in each region. That is, π_t gives the size of the aggregate liquidity in a region, and higher realizations of π_t correspond to higher demand for money. This is publicly observable and is independent across regions, and identically distributed over time. Let G represent the distribution function, which is assumed to be smooth and strictly increasing on $[0, 1]$, and g the associated density function. The distribution G is common knowledge. Thus, the total of movers of each region in a location is $E(\pi) \equiv \int_0^1 \pi g(\pi) d\pi$.

To illustrate the role of interbank markets, I consider an economy where an intermediary is allowed to operate in only one region. Legal restrictions of this form were common in the United States and Japan in the past. Even in the present day, there are still many banks in both countries that operate only within a small region because of their size. As usual, I assume that there

is free entry and that competition forces banks to maximize their depositors' expected utility.

2.2 Factor Markets

The exchange of capital and labor occurs autarkically within each location. Markets in capital and labor are competitive, implying that all factors are paid their marginal product. Let R_t denote the time t rental rate for capital, and w_t denote the time t real wage rate. Then,

$$R_t = f'(k_t) \qquad \forall t \geq 0, \tag{1}$$

$$w_t = f(k_t) - k_t f'(k_t) \equiv w(k_t) \qquad \forall t \geq 0. \tag{2}$$

Note that $w'(k) > 0$ for all k.

Let the function Ω be defined as $\Omega(k) \equiv k/w(k)$. For the present, it is assumed that

$$\Omega'(k) > 0 \tag{3}$$

holds for all k. For instance, inequality (3) holds if f is any CES production function with elasticity of substitution no less than one.

2.3 Government

The government can change the money stock by making lump-sum transfers to young agents. Let M_t denote the per capita quantity of fiat money in each location at date t. Money stock grows at a constant gross rate σ, so that $M_{t+1} = \sigma M_t$. If I let τ_t denote the transfer received by a young agent at date

t, then the government budget constraint requires that

$$\tau_t = \frac{M_t - M_{t-1}}{p_t} = \frac{\sigma - 1}{\sigma} m_t \tag{4}$$

holds for all t, where m_t is the real money balance per young person at the end of date t.

2.4 Banks' behavior

As in Diamond and Dybvig(1983), the savings of all young agents will be intermediated. Banks in each region take deposits from young agents in the same region and choose how much to invest in capital i_t and money balances m_t. Let p_t denote the time t price level. The rate of return on real balances between t and $t+1$ is p_t/p_{t+1}. Banks promise a return of $d^m(\pi)$ to each mover, and $d_t(\pi)$ to each non-mover per unit on their deposits. These returns depend on π. It is assumed that there is free entry into banking and that banks are competitive, in the sense that they take the real return on assets as given. Thus, intermediaries in each region are Nash competitors on the deposit side. That is, banks announce deposit return schedules $(d_t^m(\pi),\ d_t(\pi))$, taking the announced return schedules of other banks as given.

Let $\alpha_t(\pi)$ denote the fraction of cash reserves that the bank pays out at t, and let $b_t(\pi)$ be the real balances that a bank borrows from or lends to interbank markets at the end of t. If $b_t(\pi)$ is positive, a bank borrows cash from banks in other region; while, if it is negative, a bank lends cash to them through the interbank markets. If the interbank markets are perfect, banks can use these markets to borrow or lend cash freely at the market rate. Let ϕ_t denote the gross nominal interest rate of the interbank market at date t.

After banks create their portfolio and learn the liquidity shocks of their region, the interbank markets open, and they decide whether to borrow or to lend cash at ϕ_t. If, at the end of date t, a bank in a region with high liquidity shock demands cash $b_t(\pi)$, it can borrow $p_t b_t(\pi)$ yen from other banks in regions with a low liquidity shock through the interbank markets. On the following date, the bank must pay back $\phi_{t+1} b_t(\pi) p_t$ yen to these banks. Let $r^b_{t+1} \equiv \phi_{t+1} p_t / p_{t+1}$ denote the gross real interest rate of the interbank markets. I use this rate in the following discussions instead of ϕ_{t+1}.

The bank faces the following constraints on its choices i_t, m_t, $d^m_t(\pi)$, and $d_t(\pi)$. First, the bank's balance sheet requires that

$$i_t + m_t \leq w_t + \tau_t \qquad \forall t. \tag{5}$$

Second, payments to movers at t, $\pi d^m_t(\pi)(w_t + \tau_t)$, cannot exceed the date $t+1$ value of the bank's holding, and the borrowing of cash reserves. Then,

$$\pi d^m_t(\pi)(w_t + \tau_t) \leq \alpha_t(\pi) m_t \frac{p_t}{p_{t+1}} + b_t(\pi) \frac{p_t}{p_{t+1}} \qquad \forall t \tag{6}$$

must hold. Finally, real payments to non-movers cannot exceed the value of the bank's remaining reserves plus the income from capital investments minus the repayments of the interbank loan, so that

$$(1-\pi) d_t(\pi)(w_t + \tau_t) \leq [1 - \alpha_t(\pi)] m_t \frac{p_t}{p_{t+1}} + R_{t+1} i_t - b_t(\pi) r^b_{t+1}, \qquad \forall t. \tag{7}$$

Of course, $0 \leq \alpha_t \leq 1$ and $m_t \geq 0$ must hold.

Because banks behave as Nash competitors and there is free entry, banks in each region will maximize the expected utility of a representative depositor in their region,

$$\int_0^1 \Big\{ \pi \ln[d^m_t(\pi)(w_t + \tau_t)] + (1-\pi) \ln[d_t(\pi)(w_t + \tau_t)] \Big\} g(\pi) d\pi \tag{8}$$

subject to the constraints just described.

Let $\gamma_t \equiv m_t/(w_t + \tau_t)$ denote the bank's reserve-deposit ratio at date t, and let $\delta_t(\pi) \equiv b_t(\pi)/(w_t + \tau_t)$ denote the real value of the bank's borrowing or lending from the interbank markets per unit of deposits at date t.

3 An economy with and without interbank markets

3.1 An economy without interbank markets

In this section, I consider an economy in which any bank in any region cannot access other banks in other regions. That is, there are no interbank markets, and the regions are financial autarkies. Thus, all banks have to set $b_t(\pi) = 0$, or equivalently $\delta_t(\pi) = 0$, and they meet the liquidity demands of their depositors by using their own cash reserves. This environment is quite similar to Champ, Smith and Williamson(1996) and Smith(2002).

The function α_t, which is the fraction of bank reserves paid out to movers, is chosen after the realization of π, while the function γ_t, which is the fraction of reserves in the asset portfolio of the banks, is chosen before the realization of π. Hence, I solve the problem backwards, by first finding the optimal values of α_t as a function of γ_t and π. That is, I can choose α_t to solve

$$\max_{\alpha_t \in [0,1]} \pi \ln \left[\frac{\alpha_t \gamma_t}{\pi} \frac{p_t}{p_{t+1}} \right] + (1-\pi) \ln \left[\frac{(1-\alpha_t)\gamma_t}{1-\pi} \frac{p_t}{p_{t+1}} + \frac{1-\gamma_t}{1-\pi} R_{t+1} \right]. \tag{9}$$

The solution to this problem sets

$$\alpha_t(\pi) = \begin{cases} \pi(1 + \frac{1-\gamma_t}{\gamma_t} I_t) & \text{if} \quad 0 < \pi < \pi^* \\ 1 & \text{if} \quad \pi^* \leq \pi < 1 \end{cases} \tag{10}$$

where $I_t = R_{t+1}\frac{p_{t+1}}{p_t}$ is the gross nominal interest rate, and

$$\pi^* \equiv \frac{\gamma_t}{\gamma_t + (1-\gamma_t)I_t}.$$

For realizations of the liquidity shock below the critical value π^*, the bank pays out only a fraction of its reserves to movers. However, when the realization of the liquidity shock is greater than π^*, all reserves are paid out to movers, and repayments to non-movers are drawn from capital investments only. In this paper, I refer to this event as a "liquidity crisis" in the sense that there are so many movers that, even if all the bank's reserves are given to them, they will receive a lower return than the non-movers.

I now proceed to solve for the optimal value of γ_t. By substituting the optimal value of α_t into the bank's objective function, the problem can then be written as

$$\max_{\gamma_t \in [0,1]} \int_0^{\pi^*} \ln\left[\gamma_t \frac{p_t}{p_{t+1}} + (1-\gamma_t)R_{t+1}\right] g(\pi)d\pi + \int_{\pi^*}^1 \left\{\pi \ln\left[\frac{\gamma_t}{\pi}\frac{p_t}{p_{t+1}}\right] + (1-\pi)\ln\left[\frac{1-\gamma_t}{1-\pi}R_{t+1}\right]\right\} g(\pi)d\pi. \quad (11)$$

The first-order condition for this problem can be reduced to

$$\gamma_t = 1 - \int_{\pi^*}^1 G(\pi)d\pi. \quad (12)$$

This implicitly defines the solution to the bank's problem when the interbank market is unable to function are provided. If $I_t = 1$, equation (12) is satisfied only by $\gamma_t = 1$. If $I_t > 1$ holds, then equation (12) has two solutions. However, it is easy to show that the interior solution $\gamma(I_t)$ solves the optimization problem, and that $\gamma_t = 1$ is not optimal when $I_t > 1$. For future reference, I note the following properties of $\gamma(I_t)$.

Lemma 1

$$(i) \quad \gamma(1) = 1 \tag{13}$$

$$(ii) \quad \lim_{I_t \to \infty} \gamma(I_t) = E(\pi) \tag{14}$$

$$(iii) \quad \gamma'(I_t) < 0 \quad for \ all \ I_t. \tag{15}$$

Note that inequality $\gamma(I_t) > E(\pi)$ holds for all I_t.

3.2 An economy with interbank markets

Next I analyze an economy in which interbank markets open and create a balance between the supply and demand of banks' cash reserves. The essential purpose of the markets is to allow banks with high liquidity needs to borrow cash from banks with low liquidity. After the realization of π, a bank determines the real amount b that it would like to borrow(lend). On the following date, the bank returns (obtains) $r^b b$, where r^b is the real interest rate of the market.

Both the fraction of bank reserves paid out to movers α_t, and the real value of the liquidity needs δ_t, are chosen after the realization of π, while $\hat{\gamma}_t$, the reserve deposit ratio of a bank in the economy with a perfect interbank market, is chosen before the realization of π. Hence, as in the previous section, I first solve for the optimal values of α_t and δ_t, as functions of $\hat{\gamma}_t$ and π. That is, I can choose α_t and δ_t to solve

$$\max_{\alpha_t \in [0,1], \delta_t} \pi \ln \left[\frac{\alpha_t \hat{\gamma}_t}{\pi} \frac{p_t}{p_{t+1}} + \frac{\delta_t}{\pi} \frac{p_t}{p_{t+1}} \right] + (1-\pi) \ln \left[\frac{(1-\alpha_t)\hat{\gamma}_t}{1-\pi} \frac{p_t}{p_{t+1}} + \frac{1-\hat{\gamma}_t}{1-\pi} R_{t+1} - \frac{r^b_{t+1} \delta_t}{1-\pi} \right] \tag{16}$$

The solution to this problem sets

$$\alpha_t(\pi) = 1, \tag{17}$$

$$\delta_t(\pi) = -(1-\pi)\hat{\gamma}_t + \frac{\pi R_{t+1}}{r^b_{t+1}}(1-\hat{\gamma}_t). \tag{18}$$

Equation (17) says that it is optimal for banks not to pay cash reserves to non-movers at all, without depending on the realizations of π. When demand for liquidity is fairly low, i.e., equation (18) is negative, the bank is able to meet the demand by using its own cash reserves, and it is optimal to lend the remaining reserves to other banks with high liquidity shocks, through the interbank market. On the other hand, when demand for liquidity is high enough, i.e., when equation (18) is positive, the bank pays out all its reserves to movers and, in addition, borrows cash from other banks with low liquidity shocks, through the interbank markets.

Next, I determine the optimal value of $\hat{\gamma}_t$. To do so, I substitute the optimal values of α_t and δ_t into the bank's objective function, so that the only remaining variable to be determined is $\hat{\gamma}_t$. The problem can then be written as

$$\max_{\hat{\gamma}_t \in [0,1]} \int_0^1 \left\{ \pi \ln \left[\frac{p_t}{p_{t+1}} \frac{1}{r^b_{t+1}} \right] + (1-\pi) \ln[\hat{\gamma}_t r^b_{t+1} + (1-\hat{\gamma}_t) R_{t+1}] \right\} g(\pi) d\pi. \tag{19}$$

The optimal choice of reserve-deposit ratio $\hat{\gamma}_t$ must be given by

$$\hat{\gamma}_t = \begin{cases} 0 & \text{if} \quad R_{t+1} > r^b_{t+1} \\ \in [0,1] & \text{if} \quad R_{t+1} = r^b_{t+1} \\ 1 & \text{if} \quad R_{t+1} < r^b_{t+1}. \end{cases} \tag{20}$$

If $R_{t+1} > r^b_{t+1}$, banks will be willing to invest all their deposits in capital, and to meet the liquidity demand of depositors by using cash borrowed from other banks through the interbank market. In this case, no one holds cash, and,

consequently, the real value of money is zero. If $R_{t+1} < r^b_{t+1}$, banks will be willing to hold their all deposit in cash, and to lend their remaining reserves to other banks through the interbank market. In this case, capital investments do not occur, and outputs and wage income will be zero. In an equilibrium, it is necessary that the no-arbitrage condition, $R_{t+1} = r^b_{t+1}$, holds.

4 General Equilibrium

An equilibrium in this economy is characterized by the market clearing conditions for real balances, capital, the interbank market, and the government budget constraint. Because the supply of real balances is equal to $m_t = M_t/p_t$ and the demand for real balances is given $\gamma(I_t)(w_t + \tau_t)$, market clearing for real balances is

$$m_t = \gamma(I_t)(w_t + \tau_t). \tag{21}$$

Next, the time $t + 1$ capital stock must equal the level of investments at t. From the bank's balance sheet constraint (5), this requires that

$$k_{t+1} = i_t = [1 - \gamma(I_t)](w_t + \tau_t). \tag{22}$$

Therefore, the government budget constraint (4) must hold. By using the government budget constraint $\tau_t = \frac{\sigma-1}{\sigma} m_t$ in equation (21), we have

$$w_t + \tau_t = \frac{w(k_t)}{1 - \frac{\sigma-1}{\sigma}\gamma(I_t)}. \tag{23}$$

By substituting this equation into (21) and (22), the expressions for m_t and

k_{t+1} are rewritten as

$$m_t = \frac{\gamma(I_t)w(k_t)}{1-\frac{\sigma-1}{\sigma}\gamma(I_t)}, \tag{24}$$

$$k_{t+1} = \frac{[1-\gamma(I_t)]w(k_t)}{1-\frac{\sigma-1}{\sigma}\gamma(I_t)}. \tag{25}$$

Finally, the interbank market clearing condition is

$$\int_0^1 \delta_t(\pi)g(\pi)d\pi = 0. \tag{26}$$

These three conditions, together with the given initial values of k_0 and M_0, describe the equilibrium path of the economy for a fixed money growth rate σ.

4.1 Equilibrium without interbank markets

We first start by deriving the equilibrium in which the interbank markets are non-operating. In this case, the optimal reserve-deposit ratio is $\gamma(I_t)$ which satisfies equation (12). By substituting equation (24) into the identity $I_t = R_{t+1}p_{t+1}/p_t = R_{t+1}\sigma m_t/m_{t+1}$, we can obtain the dynamic system that defines the evolution of equilibrium sequences $\{k_t,\, I_t\}$:

$$I_t = R_{t+1}\sigma\frac{\gamma(I_t)w(k_t)}{\gamma(I_{t+1})w(k_{t+1})}\frac{1-\frac{\sigma-1}{\sigma}\gamma(I_{t+1})}{1-\frac{\sigma-1}{\sigma}\gamma(I_t)}, \tag{27}$$

$$k_{t+1} = \frac{[1-\gamma(I_t)]w(k_t)}{1-\frac{\sigma-1}{\sigma}\gamma(I_t)} \tag{28}$$

In order to study the steady state equilibrium without the interbank market, I impose $k_{t+1} = k_t = k$ and $I_{t+1} = I_t = I$ in equations (27) and (28). It is easy to verify that the steady state equilibrium must satisfy the following conditions:

$$I = \sigma f'(k) \tag{29}$$

$$\Omega(k) = \frac{1-\gamma(I)}{1-\frac{\sigma-1}{\sigma}\gamma(I)} \tag{30}$$

4.2 Equilibrium with interbank markets

Next, I proceed to describe the equilibrium in which the interbank market is fully functioning. In this case, the optimal reserve-deposit ratio is $\hat{\gamma}(I_t)$, which satisfies equation (20). In addition, in equilibrium, the no-arbitrage condition

$$R_{t+1} = r^b_{t+1} \tag{31}$$

must hold for all t. In equilibrium, we cannot have $\gamma_t = 0$, since movers would then have zero consumption. In addition, we cannot have $\gamma_t = 1$ either, since there is then too much liquidity in the interbank markets, and $R < r^b$ will no longer hold. By using this condition and the interbank market clearing condition (26), we obtain

$$\hat{\gamma}_t = E(\pi). \tag{32}$$

Figure 1 illustrates the determination of the reserve-deposit ratio and the real interest rate of the interbank markets in equilibrium. It is easy to show that $\delta(\pi) = \pi - E(\pi)$. This solution means that a bank in each region will hold money, i.e., a share of deposits equal to the share of movers in the regions as a whole, $E(\pi)$. For the liquidity shock of a region below the value $E(\pi)$, the bank will borrow cash from other banks at r^b_t and pay out all its reserves, plus the liquidity it obtains from the loan, to movers in the region. When the liquidity shock is larger than $E(\pi)$, the bank will pay out only a fraction of its reserves to movers, and lend the remainder to other banks through the interbank markets.[1]

Now, we have the following lemma.

[1]Under logarithmic utility, the optimal reserve-deposit ratio does not depend on the opportunity cost of holding reserves I_t.

Lemma 2 *For all σ, we have*

$$\hat{\gamma}_t < \gamma_t. \tag{33}$$

Proof of the lemma is straightforward from Lemma 1. Lemma 2 states that the cash reserves of banks that are able to access to the interbank markets are less than the reserves of banks that are unable to access these markets. In other words, banks in an economy with interbank markets make more capital investments than banks in an economy without such markets. Interbank markets play an important role in risk sharing between banks. If the interbank markets are unable to function, banks have to hold precautionary cash reserves against liquidity shocks. Figure 2 illustrates the reserve-deposit ratios of the two types of economies, as a function of the nominal interest rate.

By substituting equation (32) into equations (25) and (27), we can obtain the dynamic system that defines the evolution of equilibrium sequences $\{k_t, I_t\}$:

$$I_t = \sigma f'(k_{t+1}), \tag{34}$$

$$k_{t+1} = \frac{[1 - E(\pi)]w(k_t)}{1 - \frac{\sigma-1}{\sigma}E(\pi)}. \tag{35}$$

The dynamic properties of equation (34) are the same as the properties of the standard Diamond(1965) model. Then the steady state values I and k may be obtained from equations (33) and (34), as solutions to

$$I = \sigma f'(k),$$

$$\Omega(k) = \frac{1 - E(\pi)}{1 - \frac{\sigma-1}{\sigma}E(\pi)}. \tag{36}$$

Figure 3 depicts the determination of the steady state equilibria. Clearly, there is a unique steady state equilibrium in both banking economies. $E_1(E_2)$ is the steady state equilibrium without(with) interbank markets. It is easy to see that the economy with interbank markets attains higher capital accumulation than the economy without interbank markets, i.e., $\hat{k}^* > k^*$ for a given money growth rate, σ. As a result, interbank risk sharing leads to high outputs.

5 Comparative Statics

An increase in the money growth rate shifts the locus defined by equation (27) upwards and that defined by equations (28) and (32) to the right, in Figure 4. As a result, an increase in the rate of money creation leads to a higher steady state nominal rate of interest and capital stock in both economies. That is, the Tobin effect prevails in both economies. The main reason for the presence of the effect is that any seigniorage collected is rebated to the young, which increases deposits, and also imcreases the investment in capital. In addition to the wealth effect of inflation, there is the portfolio effect in an economy without interbank markets. In this economy, inflation raises the cost of holding money and increases investments, since the reserve-deposit ratio is decreasing in the nominal interest rate. In an economy with interbank markets, however, money demand is interest-invariant for the case of logarithmic utility. More generally, money demand will respond to the interest rate, and there exists the portfolio effect in an economy with markets.

6 Dynamics

In this section, I investigate dynamical equilibria. To simplify the discussion, I consider only the situation of Cobb-Douglas production, $f(k) = Ak^{\alpha}$, with $\alpha \in (0, 1)$. First, I begin by studying the dynamic equilibrium with interbank markets. From equation (35), we can obtain the law of motion for the per capita capital stock, which is given by

$$k_{t+1} = \frac{(1-\alpha)[1-E(\pi)]Ak_t^{\alpha}}{1-\frac{1-\sigma}{\sigma}E(\pi)}. \tag{37}$$

Define $\psi \equiv \frac{(1-\alpha)[1-E(\pi)]A}{1-\frac{1-\sigma}{\sigma}E(\pi)}$. For a given value of σ, ψ is a positive constant. Hence the dynamic properties of equation (37) are the same as the properties of the standard Diamond(1965) model. Clearly, the dynamic equation has a unique positive steady state, $\hat{k}^* = \psi^{\frac{1}{1-\alpha}}$, and the steady state is globally stable.

Next, we consider the dynamic equilibrium without interbank markets. With the Cobb-Douglas production, the dynamic system consisting of equations (27) and (28) is rewritten as

$$I_t = \frac{\alpha\sigma\gamma(I_t)\left[1-\frac{\sigma-1}{\sigma}\gamma(I_{t+1})\right]}{(1-\alpha)[1-\gamma(I_t)]\gamma(I_{t+1})}, \tag{38}$$

$$k_{t+1} = \frac{(1-\alpha)[1-\gamma(I_t)]Ak_t^{\alpha}}{1-\frac{\sigma-1}{\sigma}\gamma(I_t)}. \tag{39}$$

It will be useful to analyze the local dynamics in the neighborhood of the steady state. I linearize equations (38) and (39) in the neighborhood of the steady state, so that

$$(k_{t+1}-k^*, I_{t+1}-I^*)' = J(k_t-k^*, I_t-I^*)',$$

where J is the Jacobian matrix,

$$J = \begin{pmatrix} \frac{\partial k_{t+1}}{\partial k_t} & \frac{\partial k_{t+1}}{\partial I_t} \\ \frac{\partial I_{t+1}}{\partial k_t} & \frac{\partial I_{t+1}}{\partial I_t} \end{pmatrix}.$$

Let D and T denote the determinant and the trace, respectively, of J. Straightforward algebraic manipulation establishes that

$$D = -\frac{1-\alpha}{\sigma}\frac{\gamma(I)[1-\gamma(I)] - I\gamma'(I)}{\gamma'(I)} > 0, \tag{40}$$

$$T = \alpha - \frac{1-\alpha}{\alpha\sigma}\frac{\gamma(I)[1-\gamma(I)] - I\gamma'(I)}{\gamma'(I)} > 0. \tag{41}$$

Since D and T are both positive, it is clear that $D > -T-1$ holds. In addition, from (40) and (41), we have $T = \alpha + \frac{D}{\alpha}$. Thus, $D < T - 1$ iff $D > \alpha$. This inequality holds for any I if α is sufficiently small. This implies that the steady state equilibrium without interbank markets is a saddle. Since the eigenvalues of J are positive, the paths approaching the steady state equilibrium display monotonic increases or decreases in k_t and I_t.

7 Welfare and Optimal Monetary Policy

We now compare welfare with or without the interbank market. Let W denote the steady-state expected utility of a representative depositor. We view the government as choosing the rate of money growth, σ, to maximize

$$W = \int_0^1 \Big\{\pi \ln[d_t^m(\pi)(w_t + \tau_t)] + (1-\pi)\ln[d_t(\pi)(w_t+\tau_t)]\Big\} g(\pi)d\pi, \tag{42}$$

so that W is the government's objective function.

Using the steady-state equilibrium conditions, we can rewrite indirect utility with and without the interbank market as

$$\begin{aligned} W(\sigma) = \ln\left[\frac{w[k(\sigma)]}{1 - \frac{\sigma-1}{\sigma}\gamma[I(\sigma)]}\right] + G(\pi^*)\ln\frac{1}{\sigma}\big[\gamma[I(\sigma)] - \{1-\gamma[I(\sigma)]\}I(\sigma)\}\big] \\ + \int_{\pi^*}^1 \left[\pi \ln\left(\frac{\gamma[I(\sigma)]}{\pi\sigma}\right) + (1-\pi)\ln\left(\frac{I(1-\gamma[I(\sigma)])}{\sigma(1-\pi)}\right)\right] g(\pi)d\pi, \end{aligned} \tag{43}$$

and

$$\hat{W}(\sigma) = \ln\left[\frac{w[k(\sigma)]}{1-\frac{\sigma-1}{\sigma}E(\pi)}\right] + \int_0^1 \left[\pi \ln\left(\frac{1}{\sigma}\right) + (1-\pi)\ln\left(\frac{I(\sigma)}{\sigma}\right)\right] g(\pi)d\pi. \tag{44}$$

In this section, I assume that $f(k) = Ak^{\alpha}$, with $\alpha \in (0,1)$. From this assumption, we can obtain the optimal monetary policy analytically in the economy with interbank markets as follows.

Lemma 3 *Welfare with interbank markets* $\hat{W}(\sigma)$ *is maximized at*

$$\hat{\sigma}^* = \frac{\alpha}{(1-\alpha)[1-E(\pi)]}.$$

To characterize the solutions to equation (12), I make the additional assumption that π is uniformly distributed, so that $g(\pi) = 1$ holds for all $\pi \in [0,1]$.

Since it is not possible to pursue the welfare function without interbank markets analytically, I will resort to numerical analysis. Figure 5 presents the welfare of both as a function of the rate of money growth.[2] The solid line represents welfare where interbank markets are functioning well, while the dashed line represents welfare without interbank markets.

From Figure 5, the following findings are obtained. First, welfare with interbank markets is higher than welfare without interbank markets, for any $\sigma > 0$. The intuition is simple. Banks that cannot undertake bank-to-bank transactions are forced to hold precautionary cash holdings. This type of cash holding is an unproductive activity. When the interbank markets are functioning well,

[2]I set $\alpha = 0.35$ and $A = 5$. Because I assume that money is dominated in the rate of return, $\hat{\sigma}^*$ must satisfy $f'(\hat{k}^* \mid_{\sigma=\hat{\sigma}^*}) \geq \frac{1}{\hat{\sigma}^*}$. This condition is reduced to $E(\pi) \geq \frac{1-2\alpha}{1-\alpha}$. $\alpha = 0.35$ and $E(\pi) = \frac{1}{2}$ satisfy this condition.

however, this type of cash holdings will turn into investments in capital, and the consumption of non-movers will increase.

Next, optimal money growth differs between the two economies. To be more precise, inflationary policy is more desirable in the economy with interbank markets than in the economy without them. In fact, numerical calculations show that the optimal money growth with the markets is $\hat{\sigma}^* \approx 1.07$, while the optimal money growth without the markets is $\sigma^* \approx 1.43$. Banks in the economy without markets tend to have more cash reserves than the liquidity demand of the overall economy. The extra cash reserves are unproductive, and inflationary policy reduces these reserves. As a result, the consumption of non-movers increases, and welfare improves.

8 Conclusion

This paper has developed a monetary model of interbank markets. I show that interbank markets have a function that allows banks to diversify their liquidity risks, and, as a result, this reduces banks' cash reserves and increases investments in capital. In sum, the markets can affect real economic activities and lead to higher income per capita. The results obtained here are consistent with the empirical observations that countries with developed financial systems are wealthy. In addition, I demonstrate that, if an economy does not have well-functioning interbank markets, then an inflationary policy will be desirable. An inflationary policy gives banks incentive to reduce cash reserves, and encourages them to make more investments in capital. The results suggests that the optimal monetary policy depends on the level of financial development. In this paper, I assume logarithmic utility for reasons of analytical convenience.

It would be interesting to see how the results obtained here change under the more general CRRA utility form. I leave this question for future research.

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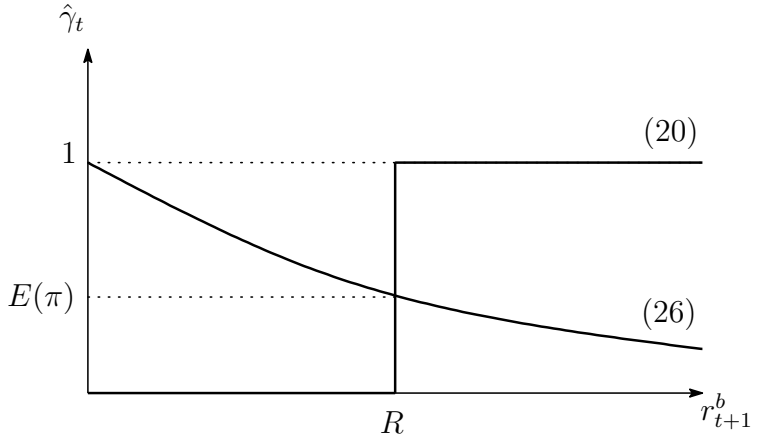

Figure 1. Equilibrium with interbank markets

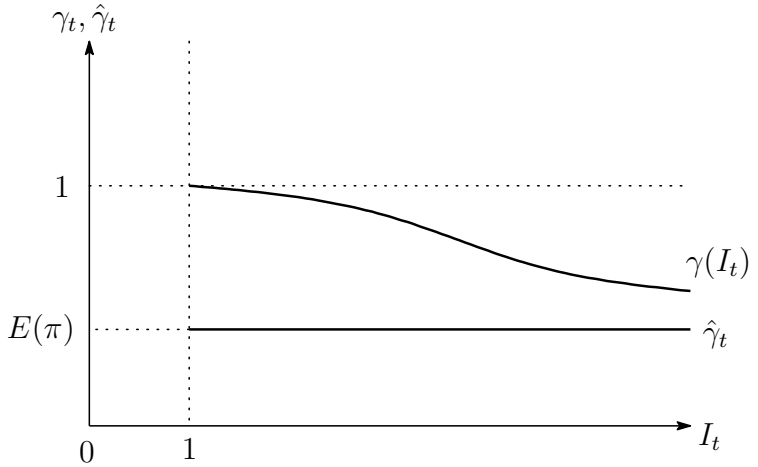

Figure 2. The reserve-to-deposit-ratio

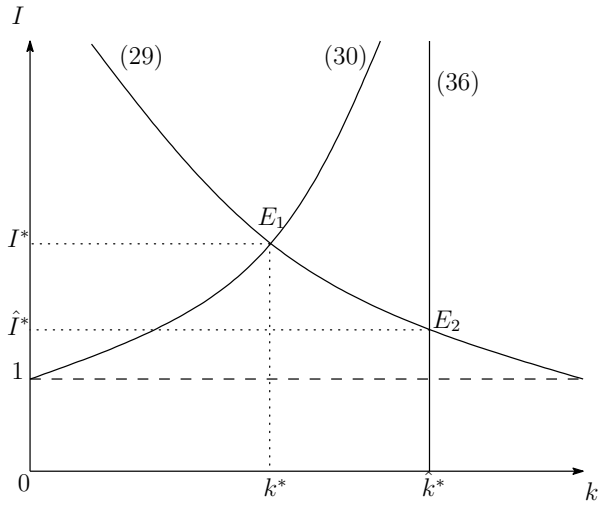

Figure 3. Steady state equilibria with and without interbank markets

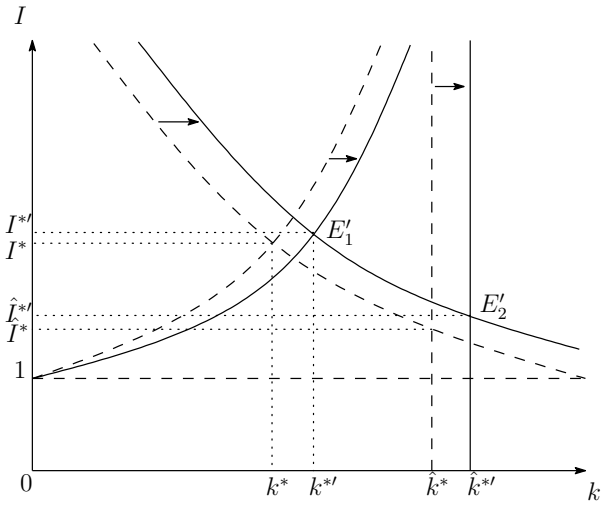

Figure 4. An increase in the rate of money growth

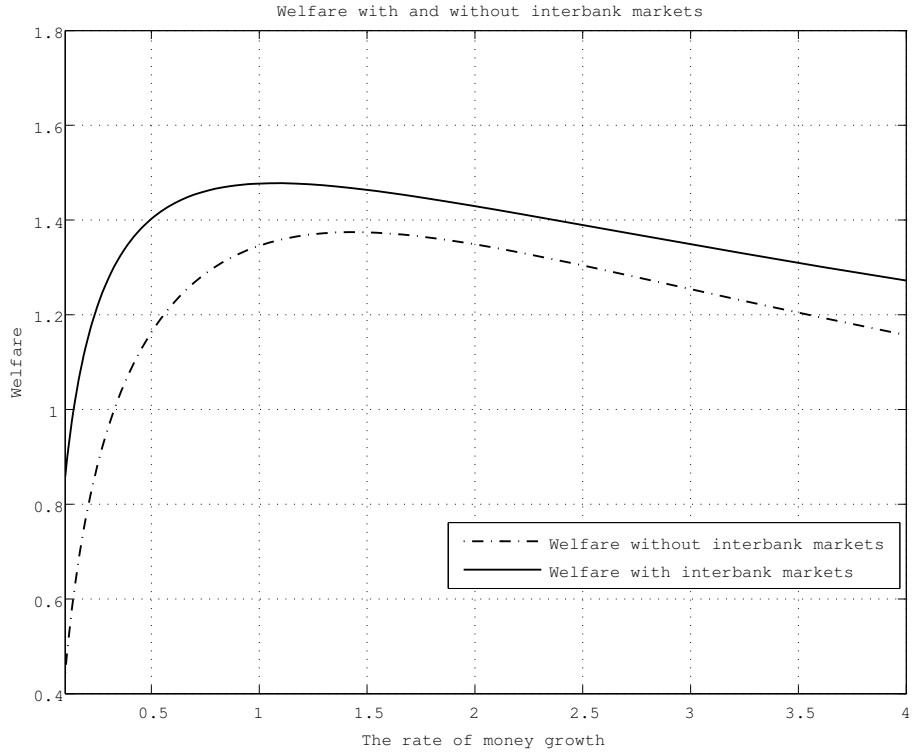

Figure 5. Welfare with and without interbank markets.