Discussion Paper No.716

“Endogenous Voter Turnout and Income Redistribution”

Go Kotera

August 2010
Endogenous Voter Turnout and Income Redistribution*

Go Kotera†
Graduate School of Economics, Kyoto University‡
August, 2010

Abstract

In this paper, a simple model is proposed to endogenize voting behavior that incorporates a sense of duty to vote. We assume that a sense of duty to vote is an increasing function of a person’s human capital and the public faith in politics, and those with a higher sense of duty often vote. Then, we examine the relationship between income redistribution policy and human capital accumulation. From our assumption, the voter turnout is expected to gradually increase as human capital accumulates. However, we show that, in some cases, the positive relationship between voter turnout and human capital accumulation is not necessarily hold. In addition, the effect of growing inequality on the redistribution policy is investigated.

Keywords: Voter turnout, Human capital, Income distribution, Redistribution

JEL Classifications: D31, D72

*I am grateful to Akihisa Shibata, Sovannroeun Samreth, Daishin Yasui, an anonymous referee and editors for their useful comments and suggestions.
†E-mail: koteragou@livedoor.com
‡Yoshida Honmachi, Sakyo-ku, Kyoto 6068501, Japan
1 Introduction

Many researchers have recently examined income distribution. Among them, Meltzer and Richard (1981), Persson and Tabellini (1994), and Benabou (1996, 2000) have analyzed models of income inequality with redistribution policies. In these models, the income redistribution policies are related to the median voter theorem or similar ideas. Thus, a decisive agent is the median voter or that at a specific percentile point, which is determined exogenously, of income distribution. When the median voter theorem does hold, the problem of who is the pivotal voter can be replaced by that of who is the median of the distribution that consists of the voting participants. Such a distribution is obtained by endogenizing the voting behavior of agents. In this paper, we will endogenize the agents’ choice of whether or not to vote because doing so seems to have some significance. First, according to earlier studies, the influence of the taxation-redistribution scheme on economic performance seems to be broadly divided into two types: (i) a negative effect of reducing labor supply and investment; and (ii) a positive effect, such as insurance against uncertainty and the imperfection of (human) capital market. As described above, endogenizing voting behavior entails deciding who is the pivotal voter and thus directly affects the (post-tax) income distribution and economic performance. Second, voter turnout can be regarded as a measure of democracy. In the field of political science, what kind of political regime, including dictatorship, oligarchy, and democracy, is desirable is a fundamental issue. Especially, democracy has been historically looked with doubtful eyes at least until the middle of 20th century because it may fall into the populism. As shown later in this paper, the more educated and wealthier citizens tend to vote with greater regularity and number, which increases their influences at the polls. Hence, it is believed that a low turnout is more meritocratic and, conversely, a high turnout is more
populistic in a democratic economy. Lijphart (1997) argues that turnout should be higher because an unequal turnout is biased against less well-to-do citizens and, even if the abstention is the expression of their affirmative willingness, their will is never counted, i.e., "voice and exit are often alternative ways of exerting influence but with regard to voting the exit option spells no influence; only voice can have an effect". Lijphart discusses the low voter turnout in the U.S., which is particularly lower than that in other developed countries, and making voting compulsory might be an alternative for this. Third, in association with the first and the second, voter turnout can affect the economic performance through income redistribution. To the best of our knowledge, few studies have analyzed the relationship between voting behavior and economic growth. One exception is the work of Mueller and Stratmann (2003), who presented cross-sectional evidence showing that high levels of democratic participation are associated with more equal income distribution and high participation rates are related to larger government sectors that lead to slower economic growth. In a theoretical study, Arawatari (2008) endogenizes voting behavior, although differently from the way this is done in our study, and investigates its effect on growth and income distribution.

The rest of the paper is organized as follows. Section 2 is a survey of the relationship between voting behavior and earlier research, in particular, that concerning rational choice theory serving as a base for the construction of a voting model; in addition, the objectives of the study are stated. Section 3 is the introduction of a simple endogenous voting model. Section 4 contains a proposal for a standard income redistribution model and a derivation of a political equilibrium. Section 5 contains the results of an analysis of the model and an examination of the relationship among voter turnout, income redistribution, and income inequality. Section 6 concludes.
2 Related Literature

In the literature of rational choice theory, the determinants of political participation and the reason that people vote have been an important topic for a long time. The rational voter hypothesis was first developed by Downs(1957). He observed that the rational voters vote when a payoff is anticipated: \( PB - C > 0 \), where \( P \) is the probability that a person’s vote is the decisive one and \( B \) is the benefit that a person would gain from having his/her preferred candidate win rather than lose. \( C \) represents the cost of voting, such as the opportunity cost. However, probability(or expected benefit) is practically bound to be minuscule in an election with a large number of electors and therefore, a person’s expected payoff is approximately \(-C\). The reason that people have for voting(i.e., the paradox of voting) need to be examined.\(^1\) To explain this paradox, Riker and Ordeshook(1968) propose a solution by modifying the inequality \( PB + D - C > 0 \), in which \( D \) is a sense of duty to vote or the utility from voting behavior itself. This modification suggests that voting is not undertaken as an instrumental act to determine a winning candidate but, rather, as a private or symbolic act from which satisfaction is derived independently of the outcome of the election. Furthermore, many empirical studies are consistent with the fact that this term is very significant for explaining voting behavior. Schram(1992), for instance, shows that party choice(the choice of preferred policy) and turnout decisions are made sequentially and independently and that "The role of a sense of 'Civic Duty' in the decision to vote or abstain and the origin of this sense are examples of processes that are related to the act of casting of a vote per se but do not play a role in the individual party choice." Thus, a sense of duty affects not the decision-making that occurs in the ballot booth but the deci-

\(^1\)From the point of view of rational choice theory, a detailed and inclusive discussion about the paradox of voting can be found in Feddersen(2004).
Blais (2000) examines in detail which variables, $P$, $B$, $C$, or $D$, affects the voting behavior and how. According to his study, $P$, the probability that their vote decides who wins, matters in such a way that the closeness of the election fosters voter turnout but its impact is small and marginal. As for $B$, the benefit that he/she would gain from having his/her preferred candidate win, the fact that more important elections, such as legislative elections, attract more voters than local elections, in which the stakes are perceived to be smaller, is consistent with rational choice theory; however, turnout in presidential elections in countries where the president has little power, such as Austria, Iceland, and Ireland, is remarkably high and has an ambiguous impact. With regard to $C$, the cost of voting, such as the opportunity cost and bad weather, is strikingly small, and the influence on turnout is only marginal. Then, he concludes that terms $P$, $B$, and $C$ lack validity for explaining voting behavior and $D$ is the most significant variable. He uses a questionnaire to screen individuals and measure their sense of duty and clarifies that those with a strong duty for voting genuinely believe that it would be wrong not to vote. He concludes that "it is difficult to argue that sense of duty is not powerful motivation for voting." 

In this study, we discuss not only what the motivation for voting is but also what important attributes voters have. Knack (1992) found that in the U.S., individuals are significantly more likely to vote if they are married, reside in the jurisdiction where they vote and attend church regularly. Among the characteristics associated with high-voter turnout, education is the most influential. 

2 For example, Ashenfelter and Kelly (1975): "The theory of voting that is best supported by our results is that which posits a sense of duty or obligation as the primary motivation for voting." Furthermore, Bartels and Brady (2003) survey the outcome of political participation in political science for a few decades and conclude that the assumption of narrow self-interest and rational agents is insufficient to explain voting behavior.

3 He also shows that rational choice theory has a little validity for those with a weak sense of duty to vote.

4 See also Verba et al. (1978), Mueller (2003), and their references.
is noteworthy that voter turnout is declining in developed countries despite the
growing numbers receiving higher education.\textsuperscript{5} Arawatari(2008) refers to this
phenomenon as the "New paradox of voting." He focuses on the cost of voting
and emphasizes that the diffusion of mass media first lowers the cost of accessing
information about the candidates and their policies but later increases the
cost of filtering information. He states that this U-shaped relationship of cost
with information introduces an inverted U-shaped relationship of turnout with
information (proportional to human capital in economy).\textsuperscript{6}

Finally, we discuss the Meltzer-Richard hypothesis\textsuperscript{7}; higher inequality will
create a majority for more redistribution. This negative relation between equality and redistribution is derived by Meltzer and Richard(1981) and is based on the idea that, with a right-skewed income distribution, any voting rule that reinforces the role of individuals below the mean income provides an incentive for redistribution from rich to poor citizens. The Meltzer-Richard model has been interpreted as a theory of government size, and, although it has been tested with cross-country data, it has received little support. For example, research indicates that the degree of inequality in the Scandinavian countries is low, while their governments are large. Their model is static, but Persson and Tabellini(1994) and Alesina and Rodrick(1994) show that the disincentives introduced by redistribution last over time and lower the growth rate theoretically and empirically. These studies extend the empirical cross-country analysis to economic growth. However, the validity of this hypothesis is mixed, e.g., Saint-Paul and Verdier(1996). Then, the lack of evidence in favor of the Meltzer-

\textsuperscript{5}The decline of voter turnout is accurately reported by Lijphart(1997) and references therein. According to his survey, participation in the U.S. presidential elections declined from 60-65\% in the 1950s and 1960s to 50-53\% in the 1980s and 1990s.

\textsuperscript{6}Unlike Arawatari(2008), Larcinese(2007a) shows a monotonic relationship between information and turnout. Thus, political knowledge has a sizeable influence on the probability of voting, and socio-demographic variables can have a vast impact on turnout through their effect on political knowledge. Therefore, mass media significantly influences political participation.

\textsuperscript{7}The survey about the Meltzer-Richard hypothesis can be found in Benabou(1996) as well as in Borge and Rattso(2004) and their references.
Richard hypothesis motivates the modification of theory both theoretically and empirically.

In this paper, we will focus on a sense of duty to vote, $D$, because, from earlier studies, $D$ seems to be the most important term, and propose a simple model that describes the relation between human capital accumulation and the voter turnout in an economy. Furthermore, as is later shown in the paper, it is also possible to explain the declining voter turnout in several developed countries since approximately 1970 and the relationship between redistribution and income inequality.

3 Voting Model

In this section, we develop a voting model. In an economy, agents are distributed on a continuum $[0, 1]$, and an individual, $i$, is initially endowed with human capital, $h^i$. As there are a large member of voters, all individuals know that nobody can directly affect the outcome through a person’s own vote; in other words, they calculate their expected payoff under $P = 0$, when they decide whether or not to vote.

As described in the previous sections, we regard a sense of duty as the main determinant of voting and formulate $D$ more specifically. If whether or not to vote largely depends on the moral obligation to vote, recent decline in voter turnout, especially, that in the U.S. is due to a decline in the sense of duty. The basic idea is borrowed from Abramson and Aldrich (1982); the dramatic decline of voter turnout in the U.S. since the early 1960s can be largely explained by two factors, the weakening voter identification with the political parties and declining beliefs in the responsiveness of government. Hence, in the current paper, we regard weakening party identification and disappointment with government as responsible for the decrease in the sense of duty to vote. From the
1960s to the 1970s, many nations around the world faced various difficulties. In the U.S., the civil rights movement and the antiwar campaign against Vietnam War had a growing influence; furthermore, the Watergate Scandal took place in 1972. Other developed countries also experienced two oil crises, stagflation, and financial crises. It would appear that these incidents made people lose confidence in (large) governments and that such criticism of the welfare state later made several countries switch political regimes from large governments to small governments. Examples of such regimes are those of Ronald Reagan in the U.S. (Reaganomics) and Margaret Thatcher in the U.K. (Thatcherism). On the basis of these historical events, it seems that the moral obligation to vote increases with confidence in government, including individual party identification, and the decreases with the government scandals. Furthermore, we regard the scandals as a kind of rent-seeking, such as peculation of tax revenue or inefficient supply of public goods by the government, and define that the peculation rate of tax revenue is \( \theta \in [0, 1] \) and the confidence in the government, denoted as \( R(\cdot) \), is a strictly decreasing and concave function of \( \theta \), \( R(0) = \bar{R} > 0 \), \( R(1) = 0 \).

Keeping in mind that more educated people tend to vote, we here take the following position about a sense of duty to vote and the choice of voting behavior:

**Assumption 1** Individuals decide their voting behavior on the basis of the term \( D \), a sense of duty to vote or the utility by the voting behavior itself. It is supposed that \( D \) of individual \( i \) is an increasing function with respect to his/her own human capital, \( h^i \), and the confidence in the government shared commonly by all individuals, \( R(\cdot) \), and \( C > 0 \), the cost of voting. Hence, the behavior of

---

8Branton(2003) shows that individual-level party identification is consistently associated with voting behavior across each of the various types of ballot propositions.

9Of course voting behavior can be analyzed in different ways. For instance, in Funk(2007), voting is the social norm, and a person’s voting behavior is interpreted as the signal that he/she performs his/her social duty. Furthermore, Blais(2000) indicates that delegitimation of authority, such as secularization, makes it more difficult to ascertain whether voting is good and not voting is bad.
an individual \( i \) is

\[
\text{given } \theta, \quad D \left( h^i, R(\theta) \right) - C = \begin{cases} 
> 0 & \rightarrow \text{ vote} \\
= 0 & \rightarrow \text{ indifference} \\
< 0 & \rightarrow \text{ abstention}
\end{cases}
\]

For simplicity, let us suppose that \( D \) is a linear function, \( D \left( h^i, R(\theta) \right) = \phi h^i + \psi R(\theta) \), \( \phi > 0 \), \( \psi > 0 \) and the threshold of human capital can then be shown below:

\[
D(\hat{h}, R(\theta)) - C = 0 \iff \hat{h} \equiv \hat{h}(\theta) = \frac{C - \psi R(\theta)}{\phi}
\]

(1)

The derogation of the credibility of the government by an increase of the rate of peculation disappoints eligible voters, and, therefore, only the more educated people vote. It follows that \( \hat{h} \) increases with \( \theta \). \( H(\cdot) \) is defined as the cumulative distribution function of human capital, and then voter turnout \( \pi \) is given by

\[
\pi = 1 - H(\hat{h})
\]

As \( \hat{h} \) is an increasing function of \( \theta \), turnout \( \pi \), ceteris paribus, decreases with the peculation rate \( \theta \). Moreover the pivotal voter is the one who is the median of the distribution that consists of the voting participants, and his/her percentile of the original distribution function \( \hat{p} \) is thus derived by

\[
\frac{\hat{p} - H(\hat{h})}{1 - H(\hat{h})} = \frac{1}{2} \iff \hat{p} = \frac{1}{2} + \frac{1}{2} H(\hat{h}) \geq \frac{1}{2}
\]
Namely, the percentile of the decisive agent is higher than the median of the cumulative distribution function of human capital, and this is straightforward because the rich tend to vote more than the poor.\textsuperscript{10}

Now, let us consider that the economy consists of two groups of individuals: the rich, denoted as \( r \), and the poor, denoted as \( p \), and it is assumed that the rich are a fraction, \( \delta < 1/2 \), of all individuals. Thus, the poor are a majority in society, and their favorite policy is adopted if the policy follows a simple majority voting. Then the distribution function of human capital is

\[
H(h) = \begin{cases} 
0 & \text{if } h \in [0, h^p) \\
1 - \delta & \text{if } h \in [h^p, h^r) \\
1 & \text{if } h \in [h^r, \infty) 
\end{cases}
\]

4 The Behavior of Agents and Political Equilibrium

The time structure of actions of individuals (potential voters) and the government is as follows.

1. Individuals are "leaders." Thus, they decide whether or not to vote and their preferred tax rate, expecting accurately how much the government speculate the tax revenue. Tax policy \( \tau \) is determined through their voting behavior and majority voting.

2. The government is "follower." The government determines the peculation

\textsuperscript{10}The models that endogenize voting behavior in a democratic society resemble those that analyze the process of democratization or the extension of suffrage, such as that of Acemoglu and Robinson(2000), in that the distribution of voter and pivotal voter changes, although they differ regarding whether or not the poor are allowed to participate in politics. In fact, Bourguignon and Verdier(2000), who study the dynamics of inequality, democratization, and economic development in a political economy model of growth in which education is both the engine of growth and a determinant of political participation, point out that their model would be applicable to the case in which the citizens do not use their voting rights.
rate $\theta$, given $\tau$, and facing the trade-off between the benefit of expropria-
tion and the cost of derogation of the credibility.

So, first, we solve the optimization problem of the "follower", government.

4.1 The behavior of the government

Although the government cannot affect the policy, tax rate $\tau$, which is decided
by majority voting, through the process of taxation and redistribution, it is
assumed that they can peculate a part of the tax revenue $\theta \tau Y$, where $\theta \in [0,1]$
represents peculation rate, and $Y$ is the aggregate output. Let us also assume
that the peculated tax disappears and does not affect the social welfare.$^{11}$ Given
the tax revenue $\tau Y$, the objective function of the government consists of the
amount of peculation from tax revenue and credibility by individuals, $R(\cdot)$; thus,
there exists a trade-off between payoff by peculation and the loss of confidence
in the government.$^{12}$ Then, the optimization problem of the government is

$$\text{given } \tau \max_\theta \theta \tau Y + R(\theta)$$

For simplicity, let us assume that $R(\theta) = R(1 - \theta^2)$ and $R$ is a parameter that
expresses the extent of credibility of the government. The solution is as follows

$$\theta = \min \left[ \frac{\tau Y}{2R}, 1 \right] \equiv \theta(\tau)$$

$$\Leftrightarrow \theta(\tau) = \begin{cases} \frac{\tau Y}{2R} & \text{if } \frac{Y}{2R} > 1 \text{ and } 0 \leq \tau < \frac{2R}{Y}, \text{ or } \frac{Y}{2R} \leq 1 \\ 1 & \text{if } \frac{Y}{2R} > 1 \text{ and } \frac{2R}{Y} \leq \tau \leq 1 \end{cases}$$

\( \text{(2)} \)

$^{11}$These assumptions can be interpreted to represent the inefficiency of the government, e.g., the useless policy of parceling out public works projects that do not increase economic welfare.

$^{12}$Generally, confidence or reputation should be analysed by dynamic models such as repeated games.
The peculation rate $\theta$ increases as the tax revenue $\tau Y$ increases.\textsuperscript{13}

### 4.2 The behavior of individuals

The income of an individual $i$ is $y_i'$, $(y_i' > y^p)$, the average income is $\delta y^r + (1 - \delta) y^p = \bar{y} (= Y)$ and $y^r > y^p$ is equivalent to $\xi - \delta > 0$. Now, the Gini coefficient can be obtained as $\xi - \delta$ from a simple calculation. The fraction of the total income possessed by the rich is $\delta y^r / \bar{y} = \xi$. From the definition of $\xi$, $y^r = \xi \bar{y} / \delta$, and $y^p = (1 - \xi) \bar{y} / (1 - \delta)$. Let us also assume that the utility of an agent equals his/her disposable income\textsuperscript{14} and that redistribution policies are linear, with a flat marginal tax rate $\tau$ and a lump-sum uniform transfer equal to $T = \bar{y} \left( \tau (1 - \theta) - \tau^2 / 2 \right)$. Thus, redistribution is costly, and the loss is normalized by average income or aggregate output. This implies that the equilibrium tax rate does not depend in an arbitrary way on the scale of the economy.\textsuperscript{15}

Individuals choose their preferred policy while expecting the peculation by the government.\textsuperscript{16} Thus, the optimization problem of an individual $i$ about his/her preferred policy is

$$
\max_{\tau} \left( 1 - \tau \right) y_i' + \bar{y} \left( \tau (1 - \theta (\tau)) - \frac{\tau^2}{2} \right)
$$

\textsuperscript{13}If the utility function is not linear in $\theta \tau Y$, the optimal rate of peculation might not be an increasing function of $Y$. When F.O.C. is satisfied, in general, a marginal increase of $Y$ generates two effects: first, the marginal utility increases because of an increment of the peculation; second, the marginal utility itself diminishes. Herein, the second effect does not exist because of the linearity of the utility function in $\theta \tau Y$. Therefore, the larger the economy scale is, the more inefficient the government is.

\textsuperscript{14}Assuming the "warm-glow" utility function and the same linear taxation and lump-sum transfer redistribution with cost, the indirect utility function can be expressed by disposal income.

\textsuperscript{15}Although it’s possible to interpret that the cost of expropriation by the government is included in the second term, $-\bar{y} \tau^2 / 2$, herein I distinguish the first term from the second term as follows: the former is the intentional cost by the government and the latter is the otherwise, cost e.g., creating bureaucracy and distorting the investment and labor supply incentives.

\textsuperscript{16}Even if the government and individuals behave independently from each other, the following result does not change.
Substituting Equation (2) into the objective function, the solution for the rich is

$$\tau^r = 0,$$  \hspace{1cm} (3)

and the solution for the poor is

$$\tau^p = \frac{\bar{R}(\xi - \delta)}{(\bar{R} + \bar{y})(1 - \delta)}$$  \hspace{1cm} (4)

Since the policy variable is unidimensional and this objective function is single-peaked with respect to $\tau$, the median voter theorem is applicable. Here, for simplicity, let us assume that $y^i \equiv f(h^i) \equiv h^i$, $\bar{y} = \delta y^r + (1 - \delta) y^p = \delta h^r + (1 - \delta) h^p \equiv \bar{h}$.

### 4.3 Political equilibrium

In this subsection, combining the voting behavior and policy decision as discussed above, we derive a political equilibrium or taxation-redistribution scheme. In this model, the equilibrium of an economy is divided into three states as shown below.

When the pivotal voters are the rich, i.e., $h^p \leq \hat{h}(\theta^r) \leq h^r$, denoted as $\theta(\tau^i) = \theta^i$, from Equations (2) and (3),

$$\left(\theta^r, \tau^r, \pi^r\right) = (0, 0, \delta)$$  \hspace{1cm} (5)

This is the case in which only the rich vote. Then, taxation-redistribution is not conducted, and, therefore, peculation does not take place.\(^\dagger\)

On the other hand, in the case that the pivotal voters are the poor, i.e.,

---

\(^\dagger\)In this state, the tax revenue that is used for redistribution is zero, and this does not necessarily mean that there is not any taxation-redistribution scheme and public service, such as a police service and minimum bureaucracy.
\( \hat{h}(\theta^p) \leq h^p \), from Equations (2) and (4),

\[
(\theta^p (\hat{h}), \tau^p (\hat{h}), \pi^p) = \left( \frac{\hat{h}\left(\xi - \delta\right)}{2 \left(\bar{R} + \hat{h}\right) \left(1 - \delta\right)}, \frac{\bar{R}\left(\xi - \delta\right)}{(\bar{R} + \hat{h}) \left(1 - \delta\right)} , 1 \right)
\] (6)

In this case, even if the government peculates a part of the tax revenue, the human capital of the poor is sufficiently large to cast ballots, and all individuals vote.

Moreover, there is another case such that \( \hat{h}(\theta^r) < h^p < \hat{h}(\theta^p) \). In this case, clearly the state is neither (5) nor (6); namely, the human capital of the poor is higher than the threshold in which the rich are the pivotal voters and lower than the threshold in which the poor are the pivotal voters. Then, \( h^p \in (\hat{h}(\theta^r), \hat{h}(\theta^p)) \) must be the threshold for the existence of an equilibrium because only the state that can be an equilibrium is that the winner in an election is neither the poor nor the rich. Then, the peculation rate of this case is as follows:

\[
h^p = \frac{C - \bar{R} \left(1 - \theta^2\right)}{\phi}
\]

\[
\Rightarrow \theta = \sqrt{\frac{\phi}{\psi R} \left(\frac{(1 - \xi) \hat{h}}{1 - \delta} - \frac{C - \psi R}{\psi} \right)} \equiv \theta^m(\hat{h})
\]

Furthermore, the fact that \( h^p \) is the threshold means that the poor are indifferent regarding whether or not to vote. Therefore, the number of the poor who participate in voting can be from 0 to \( 1 - \delta \), but, if the number of poor who vote exceeds the number of rich \( \delta \), the poor become the pivotal voters. The correspondent peculation rate is not \( \theta^m \) but \( \theta^p \), and this is not consistent with the condition that the poor are indifferent about voting behavior. The case in which the number of the poor who vote falls below \( \delta \) is similar; the correspondent rate of peculation is not \( \theta^m \) but \( \theta^r \). Then, the number of the poor that is consistent with \( \theta^m \) is only \( \delta \), and voter turnout is \( 2\delta \). This is also consistent
with the condition of an equilibrium in which the winner in an election is neither
the poor nor the rich. The tax rate $\tau^m$, which corresponds to $\theta^m$, is derived
from Equation (2), and, therefore, this state is defined as follows\(^{18}\)

$$
(\theta^m (\bar{h}), \tau^m (\bar{h}), \pi^m) = \left( \sqrt{\frac{\phi}{\psi R (1 - \xi) \bar{h} - \frac{C - \psi R}{\phi}}}, \frac{2R \theta^m (\bar{h})}{\bar{h}}, 2\delta \right)
$$

(7)

Definition 1 A political equilibrium is composed of a trio of factors. This trio
could be three types, $(\theta^i, \tau^i, \pi^i)$, $i = r, p, m$, expressed by (5), (6), and (7).\(^{19}\)
Henceforth, we call equilibria defined by (5), (6) and (7) as R state, P state,
and M state respectively.

5 Analysis

5.1 The effect of human capital accumulation

Assumption 2

$$
\frac{C - \psi R}{\phi} < \frac{\xi h}{\delta} \iff \bar{h} > \frac{\delta (C - \psi R)}{\xi \phi} \equiv h
$$

Assumption 2 assures that the rich, at least, cast ballots. It follows that,
as described above, we assume a developed and democratized society in which
some people vote. Thus, the early stage of the process is the $R$ state. In the
$R$ state, the tax rate determined by majority voting is 0. Thus, peculation
does not arise, and the threshold of human capital regarding whether or not
vote is $\bar{h} (0) = (C - \psi \bar{R}) / \phi$. After that, when the human capital of the poor

\(^{18}\)This state is unstable in the sense that the equilibrium cannot be sustained even if the
number of poor participants changes only slightly. However, I do not refer to the stability
of this state because it is the switch of the redistribution scheme that is the foucas of this paper.
\(^{19}\)In fact, another equilibrium $(\theta, \tau, \pi) = (0, 0, 0)$ exists. However, as described below, we
focus on the established democracy and thus so exclude the case in which any no individual
go to the polls.
exceeds $\hat{h}(0)$, the peculation and the redistribution by the government accrue, and the state becomes $M$. In addition, when human capital accumulates, the state attains $P$, and the poor become the majority force.

Substituting (6) into (1), the threshold whether the poor are pivotal voters or not is defined as

$$\hat{h}(\theta^p(\hat{h})) = \frac{C - \psi \hat{R}}{\phi} \left( 1 - (\theta^p(\hat{h}))^2 \right)$$

$$= \frac{C - \psi \hat{R}}{\phi} + \frac{\psi \hat{R}}{\phi} \frac{\hat{h}(\xi - \delta)}{2(\hat{R} + \hat{h})(1 - \delta)}^2 \equiv \hat{h}^p(\hat{h})$$

$$\hat{h}^p(0) = \frac{(C - \psi \hat{R})}{\phi}, \quad \lim_{\hat{h} \to \infty} \hat{h}^p(\hat{h}) = \frac{C - \psi \hat{R}}{\phi} + \frac{\psi \hat{R}}{4\phi} \left( \frac{\xi - \delta}{1 - \delta} \right)^2$$

$$\hat{h}^{p''}(\hat{h}) > 0, \quad \hat{h}^{p''}(0) = 0, \quad \lim_{\hat{h} \to \infty} \hat{h}^{p''}(\hat{h}) = 0$$

$$\hat{h}^{p''}(\hat{h}) \geq 0 \iff \hat{h} \leq \frac{\hat{R}}{2}$$

It follows that the shape of threshold $\hat{h}^p(\hat{h})$ is an increasing cubic curve with the inflection point $\hat{h} = \hat{R}/2$. Then, the threshold of human capital regarding whether or not to vote is defined as follows

$$\hat{h}(\hat{h}) \equiv \begin{cases} \frac{C - \psi \hat{R}}{\phi} & \text{if } \frac{C - \psi \hat{R}}{\phi} \leq \hat{h} \leq \frac{C - \psi \hat{R}}{\phi} < \frac{1 - \delta}{1 - \xi} \frac{C - \psi \hat{R}}{\phi} \\ \min[\hat{h}^m(\hat{h}), \hat{h}^p(\hat{h})] & \text{if } \frac{1 - \delta}{1 - \xi} \frac{C - \psi \hat{R}}{\phi} \leq \hat{h} \end{cases}$$

In the current paper, $\bar{h}$ is parameter but human capital of the rich and the poor, and thresholds can be viewed as functions of $\bar{h}$. Then, as shown later, we can produce a figure that expresses the relationship between the transition of the redistribution scheme and human capital accumulation under constant inequality. Subsequently, I can state the following proposition.
Proposition 1  Under Assumptions 1 and 2, in equilibrium, 20

1. An economy in which sufficient human capital has been accumulated is necessary in P state with $\tau^p = 0$.

2. The transition of the states with human capital accumulation is as follows:

(a) When $\hat{h}'(\bar{R}/2) \leq (1 - \xi) / (1 - \delta)$, the state is $R \rightarrow M \rightarrow P$, and voter turnout is $\delta \rightarrow 2\delta (\rightarrow 1)$.

(b) When $(1 - \xi) / (1 - \delta) < \hat{h}'(\bar{R}/2)$, under the condition that $(C - \psi \bar{R}) / \phi$ is within a certain range 21, the state transits in the direction of $R \rightarrow M \rightarrow P \rightarrow M \rightarrow P$, as human capital accumulates. The transition of voter turnout is $\delta \rightarrow 2\delta \rightarrow 1 \rightarrow 2\delta (\rightarrow 1)$; thus, the relation between human capital accumulation and the transition of voter turnout is an inverted U-shape. Otherwise the state is $R \rightarrow M (\rightarrow P)$ and voter turnout is $\delta \rightarrow 2\delta (\rightarrow 1)$.

Proposition 1-1 is proved straightforwardly from $0 = \lim_{\bar{h} \to \infty} \hat{h}'(\bar{h}) < (1 - \delta) / (1 - \xi)$ and $\lim_{\bar{h} \to \infty} \tau^p (\bar{h}) = 0$, which suggest that, when human capital accumulates significantly, all individuals who have attained a high level of education vote, and voter turnout becomes 1. Then, we regard this situation as a kind of extreme. When $\hat{h}'(\bar{R}/2) \leq (1 - \xi) / (1 - \delta)$ (proposition 1-2(a)), once $\hat{h}'(\bar{h})$ crosses the straight line $(1 - \xi) \bar{h}/ (1 - \delta)$, regardless of how $\bar{h}$ increases, $\hat{h}'(\bar{h})$ and $(1 - \xi) \bar{h}/ (1 - \delta)$ never cross because the maximum slope of $\hat{h}'(\bar{h})$ is less than that of $(1 - \xi) \bar{h}/ (1 - \delta)$. Therefore, once the $R$ state transits to the $M$ state and the $M$ state transits to the $P$ state, the reverse never occurs and voter turnout monotonically increases.

20Although, by definition, $\xi = \delta h' / \bar{h}$ is in conjunction with $\bar{h}$, herein it is assumed that any point on the horizontal axis is positioned in such a way that $\xi$ is constant.

21By using the discriminant of the third-degree equation, the rigorous condition under which the states transit in this way is derived. However, the calculation is complicated and the implication is difficult to interpret, so I omit it. See also footnote 22.
Proposition 1-2(b) is one of the main results of this paper, with constant \( \xi \), voter turnout can be an inverted U-shape as \( \bar{h} \) increases. This implies that, even if \( \bar{h} \) increases or in spite of proceeding the popularization of higher education, voter turnout may decrease. This situation takes place when a cubic curve crosses straight line \((1 - \xi) \bar{h}/(1 - \delta)\) three times, as shown in Fig.1. (The thick line expresses threshold \( \hat{h}(\bar{h}) \)). The inequality \((1 - \xi)/(1 - \delta) < \hat{h}^{ss}(\bar{R}/2) = 2\psi(\xi - \delta)^2/27\phi(1 - \delta)^2\) is the necessary condition that \( \hat{h}^{ss}(\bar{h}) \) and \((1 - \xi) \bar{h}/(1 - \delta)\) cross at three times. This condition is met more easily when \( \xi \) and \( \psi \) are large and \( \delta \) and \( \phi \) are small, i.e., income inequality is high and a sense of duty to vote depends more on the confidence in the government than on an individual’s own human capital. This means that, in this economy, the redistribution is highly needed because income inequality is high and individuals tend to trust the government even if it appropriates a part of tax revenue. Therefore, \( \bar{h} \) is small, and marginal peculation is large; nonetheless, the poor require more redistribution, and the state transits from \( M \) to \( P \). As \( \bar{h} \) increases more, however, the demand for redistribution becomes small, and larger peculation in the \( P \) state than in the \( M \) state, exceeds the effect that marginal peculation diminishes; as a result, the state reverses the \( M \) state and voter turnout decreases from 1 to 2\( \delta \). Under constant inequality, this case expresses the mechanism, as described in Section 3, whereby individuals are disappointed with the the government scandals and the inefficient policies and develop a distrust of the government. As a consequence, the sense of duty to vote declines, and some of the poor stop going to vote. This process is similar to the fact that, in the 1970s,

\[22\] The parameters in which this inequality holds certainly exist. For instance, when \( \xi \to 1 \), inequality holds; \( 0 \approx (1 - \xi)(1 - \delta) < \hat{h}^{ss}(\bar{R}/2) = 2\psi(\xi - \delta)^2/27\phi(1 - \delta)^2 \approx 2\psi/27\phi \). Then, fixed \( \psi \), \( \phi \), the cubic curve crosses a straight line \( \hat{h}^{ss}(\bar{h}) \) three times by adjusting an intercept of the cubic curve \((C - \psi \bar{R})/\phi \).

\[23\] Alesina and Angeletos(2005) point out that bigger government raises the possibilities for corruption. Then, those who are especially productive in rent seeking may prefer the high corruption regime. On the other hand, the poor may prefer a high level of redistribution even at a cost of high corruption. This indicates the possibility that a large corrupt government may draw support from an unlikely coalition of the very poor and the rich insiders.
the developed countries faced the difficulties, e.g., two oil crises, an increase of budget deficit from inefficient and excessive social service, and stagflation, and as a result, several countries, such as U.S. and U.K., cut back the government size and, coincidentally, voter turnout gradually began to decline.

Fig.1 Human capital accumulation and the transition of the redistribution schema

5.2 The effect of income inequality

As already indicated, the Gini coefficient in this model is expressed by $\xi - \delta$. By definition, $\xi = \delta \hat{h} / \hat{h}$ changes in conjunction with $\hat{h}$, but, in the previous subsection, we assumed that $\xi$ was constant. Thus, alternatively in this subsection, we analyze the effect of an increase of $\xi$, the fraction of the total income possessed by the rich.

An increase of $\xi$ affects the equilibrium through two channels. First, an increase of $\xi$ directly increases the income of the rich and decreases the income of the poor. In the figures, the angle between two straight lines widens. Then, the
range in which the redistribution scheme is \( R \) becomes large, and the possibility that an economy is in the \( R \) state becomes high. Second, \( \partial \hat{h}^P (\bar{h}) / \partial \xi > 0 \) means that when income inequality grows, the poor require a higher tax rate, and the government appropriates more. Consequently, the confidence in the government decreases, and the threshold at which all the poor vote or not becomes high and in figures, the cubic curve remains the same intercept \( (C - \psi \bar{R}) / \phi \) and moves up. When \( \xi = \delta \), thus, the individuals are completely equal, the equilibrium is always in the \( P \) state with \( (\theta^p, \tau^p, \pi^p) = (0, 0, 1) \), and the threshold is horizontal; for any \( \bar{h} \), \( \hat{h}^{\theta^p} (\bar{h}) = 0 \). Now, if \( \xi \) increases slightly, \( h^p = (1 - \xi) \bar{h} / (1 - \delta) \) decreases, and the threshold \( \hat{h}^P (\bar{h}) \) becomes a moderate cubic curve; then, the distinction of the states arises as drawn in Fig.2(a). In this case, the boundary between the states \( M \) and \( P \) is relatively small, and an economy reaches the \( P \) state with less human capital accumulation. Moreover, as \( \xi \) increases, the slope of \( h^p (\bar{h}) \) decreases, and that of the threshold \( \hat{h}^{\theta^p} (\bar{h}) \) increases more. Then, two lines can cross three times, as shown in Fig.1, if inequality holds \( (1 - \xi) / (1 - \delta) < \hat{h}^{\theta^p} (\bar{R} / 2) \). When \( \xi \) increases further, the boundary between the states \( M \) and \( P \) becomes larger. Thus, to reach the \( P \) state, more human capital accumulation is needed.(Fig.2 (b))

Fig.2(a) Inequality is low

Fig.2(b) Inequality is high
At this point, there is an interesting relationship between inequality and redistribution in this model. In the standard by Meltzer ana Richard (1981), more unequal income distribution requires more redistribution, but this relationship is only slightly supported empirically or is at least controversial. Theoretically, Benabou(2000) emphasizes that, when a decisive agent is richer than the median and the insurance and credit markets are imperfect, redistribution is U-shaped with respect to inequality. This takes place due to the following mechanism. Under low inequality, redistributive policy generates \textit{ex ante} gains as insurance against uncertainty. As inequality rises, the proportion of those who stand to lose from redistribution increase and the demand for distribution decreases. At high enough levels of inequality, however, redistribution increases with inequality by the standard effect of the Meltzer-Richard hypothesis. However, endogenizing voting behavior as in this model, the validity of the Meltzer-Richard hypothesis is also ambiguous from a theoretical standpoint but in a different way from Benabou(2000), because growing inequality lowers the human capital of the poor who are participating less and less in elections. For instance, inequality becomes high and the figure transits from 2(a) to 2(b), the boundary between the states $M$ and $P$ shifts to the right, a part of $P$ state in the Fig.2(a) becomes the $M$ state in the Fig.2(b). Then, the tax rate of the state-switching part decreases from $\tau^p$ to $\tau^m$. As for the boundary between the states $R$ and $M$ and the case that the state-transition goes through Fig.1, the tax rate of the state-switching part also decreases.

**Proposition 2** Under Assumptions 1 and 2, even if inequality $\xi$ grows, the redistribution does not increase in most cases.

Empirically, Larcinese(2007a) shows that the relationship of inequality with redistribution and social spending is far from clear; in particular, the inclusion of country-specific features can alter the results. On the contrary, turnout vari-
able is robust to the introduction of country-specific effects in the regression to public spending on social services and then, he concludes that an increase in turnout generates more social spending. This robust positive relationship between turnout and redistribution is consistent with the model of this paper because, in this model, an increase of turnout means that many poor who require redistribution vote.

6 Concluding Remarks

Voting frequency and a good education are correlated positively, as many empirical studies show. Then, under the same inequality, voter turnout could be high or low when human capital accumulation influences the voting behavior of individuals. Thus, in an economy in which human capital does not accumulate, only the more educated, namely, the richer people with a high sense of duty to vote cast ballots, and voter turnout is low; on the other hand, when the sufficient human capital accumulates, the poor also have a strong moral obligation to vote, and, consequently, voter turnout becomes high. However, an exception could be considered. When income inequality is high and individuals are responsive to government scandals, a situation in which all individuals go to the polls could arise, despite the fact that human capital does not accumulate so much. Then, the peculation rate becomes so large that individuals, especially the poor, distrust the government, and a part of the poor stops voting. Hence, turnout decreases even if human capital accumulates. This mechanism could be an explanation for the decline of voter turnout in some developed countries since approximately the 1970s and the various degrees of voter turnout in several countries with similar inequality.

Increasing inequality often discourages those who have less education from casting ballots because their sense of duty to vote is not strong. This means that,
even if inequality increases, only the well-educated, and rich people vote, and
income redistribution is not conducted. This result is different from that of the
traditional economic theory as in Meltzer and Richard (1981); large inequality
requires large redistribution. Then, this model could be one of the answers to
the fact that the traditional theory is little supported empirically.

Finally, as an extension of this model, it is essential to consider that income
and income distribution are endogenized, and, thus, an economic model should
be constructed. This seems to be significant because such a model would be use-
ful to analyze the relationship and interaction between economy and politics or
economic growth and democracy, e.g., a taxation-redistribution scheme, which
is determined by voting behavior and income distribution, affects economic per-
formance and, conversely, determines voting behavior and income distribution.

References

Abramson, P. R. and Aldrich, J. H. (1982), "The Decline of Electoral Participa-
tion in America", American Political Science Review 76, 502-521

Acemoglu, D. and Robinson, J. A. (2000), "Why Did the West Extend the
Franchise? Growth, Inequality, and Democracy in Historical Perspective", Qua-
terly Journal of Economics, CXV, 1167-1199


Alesina, A. and Angeletos G.-M. (2005), "Corruption, Inequality, and Fair-
ness", Journal of Monetary Economics 52, 1227-1244

Arawatari, R. (2008), "Informatization, Voter Turnout and Income Inequal-
ity", Journal of Economic Inequality forthcoming

Context", American Economic Review 93(2), 156-161


Blais, A.(2000), To Vote or Not to Vote. University Pittsburgh Express


Government”, *Journal of Political Economics* 89(5), 914-927


