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"Sustainable Public Debt, Credit Constraints,
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Sustainable Public Debt, Credit Constraints, and Social Welfare

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Abstract

Whether the sustainability of public debt is promoted or foiled by credit market imperfections depends upon the fiscal policy rules. Under the golden rule, as credit constraints dissipate, public debt is more likely sustainable, whereas under the balanced budget rule, it is less likely sustainable. We also examine the social welfare under the two different fiscal rules. The balanced budget rule is more beneficial to the super-near future generations than the golden rule, whereas the golden rule is more beneficial to the near future generations than the balanced budget rule. However, to the far future generations, the balanced budget rule once again becomes more beneficial than the golden rule.

Keywords: Fiscal sustainability; credit constraints; social welfare; heterogeneous agents; endogenous growth.

JEL Classification Numbers: E63; O40.

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1 Introduction

Recently, the accumulation of public debt has become one of the most important policy and political issues in most developed countries. The disturbances of national finance in Greece, Spain, and Portugal, which surfaced in 2010, provoked debates about the sustainability of public debt. Even other than Greece, Spain, and Portugal, many countries have built up significant amounts of public debt, often oriented to bigger governments (Neck and Sturm (2008)). For instance, the public debt in Japan has almost reached 200% of its GDP in 2009.

Whether public debt that has substantially piled up is sustainable or not is one of the most important questions in public finance. Many researchers including Chalk (2000), Rankin and Roffia (2003) and Bräuninger (2005) have provided conditions under which public debt is sustainable with specific fiscal rules in overlapping generations economies.

However, they do not take into consideration the direct relationship between economic growth and fiscal sustainability. While Yakita (2008) and Arai (2008) incorporate the government investment in the final production function so that they can study the relationship among fiscal policies, fiscal sustainability, and economic growth, no studies, to the best of our knowledge, focus on the relationship among fiscal sustainability, credit market imperfections, and economic growth.[1]

Credit market imperfections are important in understanding macroeconomic phenomena. For instance, the empirical literature on finance and growth has produced evidence for the positive effect of financial development on economic growth (e.g., King and Levine (1993), Levine, et al. (2000), and Aghion, et al. (2005)). The positive effect has been demonstrated theoretically in an extensive literature as well (e.g., Greenwood and Jovanovic (1990), Galor and Zeira (1993), Greenwood and Smith (1997), and Aghion, et al. (2005)).[2] The contri-

[1]The vital importance of the initial amount of public debt for its sustainability was discovered in Chalk's (2000) pioneering work first. By contrast, there is also an extensive literature in which the inter-temporal budget constraint of a government, rather than the initial amount of public debt, is emphasized in order to test the fiscal sustainability. See, for instance, Hamilton and Flavin (1986), Trahan and Walsh (1988), Hakkio and Rush (1991), Bohn(1998), and Chalk and Hemming(2000).

[2]Other than finance and growth, there is an extensive literature dealing with macroeconomic phenomena. For business cycles and credit market imperfections, see for instance Kiyotaki and Moore (1997). For financial

bution of this paper is to fill the gap between the literature on finance and growth and the literature on fiscal sustainability.

Following the literature pioneered by Chalk (2000), by fiscal sustainability we mean that the public-debt/GDP ratio does not diverge but converges to a certain finite level in the long run under a given fiscal policy rule. This definition implies that if the GDP growth is smaller than the public-debt growth, then the GDP cannot support public debt in the long run. Of course, in this case, public debt is not sustainable.

We address the following questions: Does the dissipation of credit market imperfections promote or foil the sustainability of public debt? Can an economy experience sustainable economic growth under a sustainable fiscal policy? If the government alters fiscal policy rules, how does the alternation affect social welfare for each generation? In order to answer these questions, we develop an overlapping generations model with public debt.

While the basic structure of our model is similar to those of Yakita (2008) and Arai(2008), heterogeneity of agents in entrepreneurial talents within a generation is assumed in our model. Agents are heterogeneous in creating capital goods, which are input for the final goods production. Due to the heterogeneity and credit constraints, less talented agents become savers and more talented agents become entrepreneurs in equilibrium. In other words, savers and entrepreneurs appear endogenously.

Credit market imperfections influence fiscal sustainability via two channels. On the one hand, if the degree of credit market imperfections is relaxed, then more resources are used by higher skilled entrepreneurs than when it is severe. Accordingly, the productivity in an economy is promoted. On the other hand, if the degree of credit market imperfections is relaxed and higher skilled entrepreneurs increase their borrowing, the demand for and supply of financial resources are changed by an increase in the interest rate.

An increase in the interest rate and productivity growth are very important for fiscal sustainability because if an interest rate goes up, then it becomes a burden of the government

market globalization and credit market imperfections, see for instance Matsuyama (2007).

and because if productivity is promoted and economic growth is boosted, then the public debt/GDP ratio decreases, other things being equal. In other words, there are two conflicting effects of the dissipation of credit market imperfections on fiscal sustainability.

In order to investigate these two conflicting effects, we examine two different fiscal policy rules. One is a so-called golden rule in which the government deficit is used for public investment which is beneficial to the future generation. The other is a balanced budget rule in which the primary balance is always zero. We will demonstrate that either one of the conflicting effects dominates the other under the two different fiscal policy rules.

These fiscal policy rules are targeted in many countries and have been used in many previous studies. The golden rule conforms to the budget rule of the Maastricht Treaty convergence criteria. Meanwhile, the level of primary balance is targeted in some other countries.[3]

While as mentioned, this paper fills the gap between the literature on finance and growth and the literature on fiscal sustainability, it can be also allocated to the literature on finance and inequality (e.g., Galor and Zeira (1993) and Banerjee and Newman (1993)). The difference between the existing literature and our paper is that our model investigates the effects of financial development on income inequality under various fiscal policies, whereas the existing literature on finance and inequality does not consider fiscal policies. In turn, while there is an extensive literature on optimal fiscal policies in growth models (e.g., Chamley (1986), Judd (1985), Barro (1990), Saint-Paul (1992), and Futagami, et al. (1993) among others), few studies consider income distribution when they study social welfare.[4] By contrast, we investigate social welfare, counting income distribution.

To measure social welfare, we do not use the concept of Pareto optimality because it does not pronounce income distribution. Since individuals in our model are heterogeneous within a generation, per capita income is not an appropriate measure for social welfare,

[3]For example, the Japanese cabinet announced that they aim to archive primary surplus by around 2020 in the Fiscal Management Strategy 2010.

[4]An exception is Fiaschi (2002) who studies social welfare with income distribution and the fiscal policy by developing an endogenous growth model.

either. Instead, we use the social welfare function that Sen (1976) derives. Sen's social welfare function is suitable for capturing income distribution as well as per capita income.

Our main findings are as follows. Under the golden rule, as credit constraints become less severe (more severe), public debt is more likely sustainable (unsustainable). Under the balanced budget rule, as credit constraints become more severe (less severe), it is more likely sustainable (unsustainable). These two completely opposite consequences are related to the two conflicting effects of credit market imperfections mentioned above. Under the golden rule, since public deficit is strictly associated with the proportion of the total output, the law of motion of public debt is not subject to the market interest rate. This implies that an efficiency-correcting effect of the dissipation of credit market imperfections is dominant. Then, with less severe (more severe) credit constraints, public debt is more likely to be sustainable (unsustainable). Meanwhile, under the balanced budget rule, an interest-rate effect is dominant, implying that the law of motion of public debt is affected by the market interest rate and it becomes a burden of the government.

Under both of the fiscal rules, even though the government debt is on a sustainable path, we obtain various growth patterns of per capita output depending upon the degree of credit market imperfections. For instance, when the degree of credit market imperfections is severe, an economy falls in a poverty trap. While this consequence is not surprising in the existing literature on finance and growth (e.g., King and Levine (1993) and Aghion, et al. (2005)), the novelty is that we clarify how growth patterns are affected by the degree of credit constraints and population growth under the two different fiscal policy rules.

The consequence on social welfare is as follows. The balanced budget rule is more beneficial to the super-near future generations than the golden rule, whereas the golden rule is more beneficial to the near future generations than the balanced budget rule. However, to the far future generations, the balanced budget rule once again becomes more beneficial than the golden rule. This alternation of dominant fiscal policy rules is caused by the differences in a tax burden effect and a crowding-out effect of each fiscal rule in each period.

This paper proceeds as follows. In the next section, we model an overlapping generations economy where individuals within a generation are heterogeneous in entrepreneurial talents. Each of them faces a credit constraint when they make a decision on investment. In section 3, we investigate the sustainability of public debt under the two different fiscal policy rules, by deriving the equilibrium dynamics for public debt and section 4 provides analyses on growth patterns when public debt is sustainable. In section 5, we study social welfare, where we take into account not only per capita consumption but also income distribution at each point in time in measuring it. We make concluding remarks in section 6.

2 Model

2.1 Individuals

An economy consists of overlapping generations, i.e., young and old agents. Time is discrete and expands from 0 to ∞. Each individual lives for two periods and exclusively obtains the utility from his second-period consumption c_{t+1}. Since he is risk-neutral, the utility function is given by $u(c_{t+1}) := c_{t+1}$. The population of young agents at time is L_t, which grows at a constant rate n, namely $L_{t+1} = (1+n)L_t$.

In the first period, each individual decides on how much he invests, borrows, and/or deposits. The budget constraints in the first and second periods are respectively given by:

$$k_t + d_t \leq w_t \tag{1}$$

and

$$c_{t+1}(1+\tilde{\tau}_{t+1}) \leq q_{t+1}\phi k_t + R_{t+1}d_t, \tag{2}$$

where k_t is an investment in a project and d_t is a deposit when positive and a debt when negative. As seen in Eq.(2), if an agent starts an investment project in the first period, then he can create capital goods ϕk_t in the second period, which are sold to the final production sector with the price q_{t+1}. R_{t+1} is the gross (real) interest rate at time $t+1$ and $\tilde{\tau}_{t+1}$ is the rate of consumption tax. If we let $1+\tilde{\tau}_{t+1} := \frac{1}{1-\tau_{t+1}}$ where $0 \leq \tau_{t+1} < 1$, then Eq.(2) is rewritten as:

$$c_{t+1} \leq (q_{t+1}\phi k_t + R_{t+1}d_t)(1-\tau_{t+1}). \tag{3}$$

We assume that there is an infinitely-lived representative financial intermediary. Following Aghion, et al. (2005), a credit constraint that the representative financial intermediary imposes on each individual is given by:

$$d_t \geq -\nu w_t, \tag{4}$$

where $\nu \in [0,\infty)$ is the measure of the degree of credit constraints. We note that individuals can borrow financial resources up to ν times their wealth, which is wages earned when they are young. In other words, we can regard w_t as down-payment for the investment. In appendix, the microfoundation for Eq.(4) is provided.[5] The non-negativity constraint for the investment is given by:

$$k_t \geq 0. \tag{5}$$

The heterogeneity of individuals in productivity of capital creation is introduced. More concretely, the productivity ϕ varies between individuals and is distributed uniformly over $[0,a]$ $(a>0)$. In other words, the distribution function of ϕ in $[0,a]$ is given by $G(\phi) := \frac{\phi}{a}$. Each individual knows his own productivity at his birth, while other individuals or even the financial intermediary does not know his productivity.[6]

Each individual maximizes c_{t+1} subject to inequalities (1), (3)-(5). The maximization problem is rewritten as:

$$\max_{d_t}(r_{t+1}-\phi q_{t+1})d_t$$

[5]This type of assumption for credit market imperfections is often imposed in the literature (e.g., Aghion, et al. (1999), Aghion and Barnergee (2005), and Aghion, et al. (2005)). The credit constraint expressed by inequality (4) can be replaced by the one associated with investment. In particular, even if we replace inequality (4) with $b_t \geq -\mu k_t$ where $\mu \in [0,1)$, this alternative credit constraint is equivalent to inequality (4) and thus the same results will be obtained.

[6]As discussed in Microfoundation I in appendix, only when an individual chooses not to repay his obligations, the financial intermediary monitors the individual. This assumption is along the same line as the costly state verification approach (e.g., Townsend (1979)). In this paper, we abstract from our model a case in which officials in the financial intermediary monitor individuals' talents. Instead, they check individuals' down-payments (or equivalently investments) when they loan to the individuals.

subject to

$$-\frac{\mu}{1-\mu}w_t \leq d_t \leq w_t,$$

where $\mu := \frac{\nu}{1+\nu}$. When $r_{t+1} - \phi q_{t+1} > 0$, it is optimal for an individual to choose $d_t = w_t$ and $k_t = 0$, whereas when $r_{t+1} - \phi q_{t+1} < 0$, then it is optimal to choose $d_t = -\frac{\mu w_t}{1-\mu}$ and $k_t = \frac{w_t}{1-\mu}$. Formally, we obtain:

Lemma 1 *Let $\phi_t := \frac{R_{t+1}}{q_{t+1}}$. The following hold.*

- *If $\phi_t > \phi$, then $k_t = 0$ and $d_t = w_t$.*
- *If $\phi_t < \phi$, then $k_t = \frac{w_t}{1-\mu}$ and $d_t = -\frac{\mu w_t}{1-\mu}$.*

2.2 Financial Intermediary

The financial sector is competitive and thus the representative financial intermediary cannot acquire profits from its business.[7]

In addition to imposing credit constraints, the financial intermediary accepts deposits from individuals and loans resources to them. The financial intermediary buys government bonds with an excess of saving. Let B_{t+1} be government bonds issued at time t and redeemed at time $t+1$. The balance sheet of the financial intermediary is given by:

$$F_t + B_{t+1} = D_t,$$

where F_t and D_t are respectively the aggregate loan and deposit. We assume that B_1 is greater than zero, namely, at time zero the public debt piles up. In order to finance the public debt, the government has to issue the government bonds. In other words, B_1 is a predetermined variable.[8]

[7]This type of financial intermediary is often assumed in the literature. See for instance Grandmont (1983) and Rochon and Polemarchakis (2006).

[8]Implicitly, we assume that all the transactions including the government sector and the private sector are done in real terms, implying that there are no nominal contracts nor nominal money. Therefore, B_1 cannot jump at time zero.

2.3 Production Sector

A representative firm produces final goods from capital goods and labor. We assume a Cobb-Douglas production function as follows:

$$Y_t = AZ_t^{\alpha}(g_t L_t)^{1-\alpha},$$

where Y_t is the output, L_t is the aggregate young labor, and Z_t is the aggregate capital goods. Following Baro (1990), we incorporate per capita capital stock of the government g_t into the production function. From a view point of the private agents, g_t is exogenously given. We assume that both private and public capital goods depreciate entirely in one period.

The production sector is perfectly competitive so that the production factors are paid their marginal products:

$$q_t = \alpha Y_t / Z_t$$

$$w_t = (1-\alpha) Y_t / L_t,$$

where q_t is the price of the capital goods and w_t is the wage rate at time t.

2.4 Government

The budget constraint of the government at time $t+1$ is given by:

$$B_{t+2} + T_{t+1} = R_{t+1} B_{t+1} + I_{t+1}^g, \tag{6}$$

where T_{t+1} is a tax collected at time $t+1$ and I_{t+1}^g is the investment executed by the government.

The tax collected at time $t+1$ is given by:

$$T_{t+1} = \tau_{t+1}\left[\int_0^a (q_{t+1}\phi k_t + R_{t+1} d_t) L_t dG(\phi)\right],$$

which is reritten by using lemma 1 as follows:

$$T_{t+1} = \tau_{t+1}\left[\frac{q_{t+1} w_t L_t}{1-\mu} F(\phi_t) + R_{t+1} B_{t+1}\right]. \tag{7}$$

We will compare the outcomes of two different fiscal policy rules. The first one is a so-called golden rule, where the government can borrow only to invest in public capital goods. It holds that $B_{t+2} - B_{t+1} = \lambda I^g_{t+1}$, where $\lambda \in [0, 1)$. In addition, the government invests a certain proportion of output of a corresponding year in public capital goods, i.e., it holds that $I^g_{t+1} = \theta Y_{t+1}$ where $\theta \in (0, 1)$. Thus, it follows that $B_{t+2} - B_{t+1} = \lambda I^g_{t+1} = \lambda\theta Y_{t+1}$. We call this fiscal rule the "golden rule," following the literature on public finance. While this fiscal rule seems ad hoc, in reality by the Stability and Growth Pact in the European Union, the member countries must respect that public debt should be less than 60% of the GDP.[9]

The second one is a balanced-budget fiscal policy where the primary balance is equal to zero, namely, $T_{t+1} - I^g_{t+1} = 0$.[10] In addition to this fiscal policy rule, the government invests a certain proportion of output in public capital goods: $I^g_{t+1} = \theta Y_{t+1}$ as in the golden rule case. We call this fiscal policy the "balanced budget rule."

Under both of the fiscal policy rules, we assume that $\theta \leq \alpha$ so as for the tax rate τ_{t+1} to be less than one for all $t \geq 0$.

2.5 Market equilibrium

From $I^g_{t+1} = g_{t+1}L_{t+1} = \theta Y_{t+1}$, the first-order conditions of the production sector becomes:

$$q_t = \alpha A^{1/\alpha}\theta^{(1-\alpha)/\alpha} := \bar{q} \tag{8}$$

$$w_t = (1-\alpha)A^{1/\alpha}\theta^{(1-\alpha)/\alpha} z_t, \tag{9}$$

where $z_t := \frac{Z_t}{L_t}$.

From lemma 1, we obtain the total supply of capital goods at time $t+1$ as follows:

$$Z_{t+1} = \int_{\phi_t}^{a} \frac{w_t}{1-\mu}\phi L_t dG(\phi),$$

or equivalently,

$$z_{t+1} = \frac{(1-\alpha)\bar{q}F(\phi_t)}{\alpha(1-\mu)(1+n)} z_t, \tag{10}$$

[9]Yakita (2008) investigates the sustainability of public debt with this type of fiscal policy setting in an overlapping generations economy with a perfect credit market.

[10]Alternatively, we could assume that the primary balance is not equal to zero such that $T_{t+1} - I^g_{t+1} = \eta Y_{t+1}$. See our companion paper, Arai(2008) and Kunieda (2010) for the investigation into this fiscal policy.

where $F(\phi_t) = \int_{\phi_t}^{a} \phi dG(\phi) = \frac{a}{2}(1 - (\frac{\phi_t}{a})^2)$.

The total loan at time t is given by $F_t = \frac{\mu w_t L_t}{1-\mu}(1 - G(\phi_t))$ and the total deposit is given by $D_t = w_t L_t G(\phi_t)$. Thus, from the balance sheet of the financial intermediary, $B_{t+1} = D_t - F_t$ is given by:

$$B_{t+1} = w_t L_t \frac{G(\phi_t) - \mu}{1 - \mu}. \tag{11}$$

As mentioned, at time zero public debt piles up and thus B_1 is predetermined. Since the government debt is financed by excess saving in the private sector, ϕ_0 is subject to the initial government debt so that the government bond market clears. We should note that even though q_{t+1} and R_{t+1} are forward-looking variables, their expectations are not independently established. Accordingly, ϕ_0 is a predetermined variable as well.

3 Sustainability

In the literature, public debt is said to be sustainable if the public-debt/GDP ratio converges to a certain finite level in the long run. We follow this definition for fiscal sustainability in discussing the sustainability of public debt in what follows.

3.1 Under the golden rule

Under the golden rule, it follows that $B_{t+2} - B_{t+1} = \lambda\theta Y_{t+1}$. The government imposes taxes on private agents so that the government budget constraint (6) holds. The tax rate τ_{t+1} is computed from the fiscal policy rule, and from Eqs.(6)-(11) we obtain a tax rate τ_{t+1} as follows:

$$\tau_{t+1} = \frac{\phi_t(G(\phi_t) - \mu) + \theta F(\phi_t)/\alpha - (1-\alpha)(G(\phi_{t+1}) - \mu)F(\phi_t)/(\alpha(1-\mu))}{F(\phi_t) + \phi_t(G(\phi_t) - \mu)}.$$

We note that under the assumption $\theta \leq \alpha$, τ_{t+1} is always no greater than one.

From $B_{t+2} - B_{t+1} = \lambda\theta Y_{t+1}$, we have:

$$\frac{G(\phi_{t+1}) - \mu}{1 - \mu} = \frac{\alpha(1-\mu)}{(1-\alpha)\bar{q}F(\phi_t)}\left(\frac{G(\phi_t) - \mu}{1 - \mu}\right) + \frac{\lambda\theta}{1-\alpha}. \tag{12}$$

Since $B_1 \geq 0$, we have $G(\phi_0) - \mu \geq 0$. By applying a mathematical induction to Eq.(12), we note that for all $t \geq 0$, $G(\phi_t) - \mu \geq 0$. Accordingly, we only investigate the case in which $\phi_t \geq a\mu$ and thus $B_{t+1} \geq 0$ for the dynamical system (12).

We transform the dynamical system (12) into a system with respect to the public debt/GDP ratio. Denoting $\frac{\tilde{B}_{t+1}}{1-\alpha} := \frac{B_{t+1}}{(1-\alpha)Y_t} = \frac{G(\phi_t)-\mu}{1-\mu}$, we rewrite Eq.(12) for $t \geq 1$ as:

$$\begin{aligned}\tilde{B}_{t+1} &= \frac{\alpha(1-\mu)}{(1-\alpha)\bar{q}} \frac{\tilde{B}_t}{F[a((1-\mu)\frac{\tilde{B}_t}{(1-\alpha)} + \mu)]} + \lambda\theta \\ &= \frac{2\alpha}{(1-\alpha)a\bar{q}} \frac{\tilde{B}_t}{\left(1 - \frac{\tilde{B}_t}{1-\alpha}\right)\left(1 + \mu + (1-\mu)\frac{\tilde{B}_t}{1-\alpha}\right)} + \lambda\theta := \Psi_x(\tilde{B}_t; \lambda, \mu, \theta), \qquad (13)\end{aligned}$$

where we should note that $\tilde{B}_t \in [0, 1-\alpha)$, i.e., B_t should be supported by the savings of young agents at time t.

Lemma 2 $\Psi_x(\tilde{B}_t; A, \lambda, \mu, \theta)$ *is strictly increasing and strictly convex with respect to* $\tilde{B}_t$.

Proof: Let us define a function as $\tilde{\Psi}(\tilde{B}_t) := \frac{\tilde{B}_t}{F[a((1-\mu)\frac{\tilde{B}_t}{1-\alpha}+\mu)]}$. It suffices to demonstrate that $\tilde{\Psi}(\tilde{B}_t)$ is strictly increasing and convex with respect to $\tilde{B}_t$. $\frac{\partial\tilde{\Psi}(\tilde{B}_t)}{\partial\tilde{B}_t} > 0$ is obvious because $F(.)$ is decreasing with respect $\tilde{B}_t$. By taking a derivative of $\tilde{\Psi}(\tilde{B}_t)$, we obtain $\frac{\partial\tilde{\Psi}(\tilde{B}_t)}{\partial\tilde{B}_t} = \frac{1}{F(.)} + \frac{\tilde{B}_t((1-\mu)\frac{\tilde{B}_t}{1-\alpha}+\mu)a\frac{1-\mu}{1-\alpha}}{(F(.))^2}$. From the last, $\frac{\partial\tilde{\Psi}(\tilde{B}_t)}{\partial\tilde{B}_t}$ is increasing with respect to $\tilde{B}_t$ and thus $\tilde{\Psi}(\tilde{B}_t)$ is strictly convex again because $F(.)$ is decreasing with respect $\tilde{B}_t$. $\square$

The most important parameters for the sustainability of public debt under the golden rule are λ and μ, which are associated with the government policy and the degree of credit market imperfections, respectively. The zero deficit case in which $\lambda = 0$ is a suitable benchmark to discuss the sustainability of public debt. Therefore, we start our study with this case.

Lemma 3 *Suppose that* $\lambda = 0$. *Then, the following hold:*

- *If* $1 + \mu \leq \frac{2\alpha}{a(1-\alpha)\bar{q}}$, *then there is no non-trivial steady state for the dynamical system (13).*

- *If* $1 + \mu > \frac{2\alpha}{a(1-\alpha)\bar{q}}$, *then there is a non-trivial steady state for the dynamical system (13).*

Proof: Since $\Psi_x(\tilde{B}_t)$ is strictly increasing and convex and $\Psi_x(0) = 0$, it follows that if $\Psi'_x(0) \geq 1$, then there is no non-trivial steady state, whereas if $\Psi'_x(0) < 1$, then there is a non-trivial steady state. Therefore, the proof follows from the fact that $\Psi'_x(0) = \frac{\alpha(1-\mu)}{(1-\alpha)\bar{q}} \frac{1}{F(a\mu)} \geq (<)1$ is rewritten as $F(a\mu) \leq (>) \frac{\alpha(1-\mu)}{(1-\alpha)\bar{q}} \Longleftrightarrow 1 + \mu \leq (>) \frac{2\alpha}{a(1-\alpha)\bar{q}}$. $\square$

When the average productivity of agents is small or when the public investments are few (i.e., a or θ is small), the first case in proposition 1 is more likely obtained. In this case, the public debt is unsustainable if $\tilde{B}_1$ is greater than zero. Even though the economy exhibits endogenous growth initially, the government debt crowds out the private investments and the number of investors decreases. Consequently, the growth rates are reduced as time goes by and then the total saving cannot support the built-up public debt.

Henceforth, we focus on the second case of lemma 3, i.e., the case in which there exists a non-trivial steady state if the government conducts the zero-deficit fiscal policy. Figure 1 provides a phase diagram for this case. Let B^x denote the non-trivial steady state of the dynamical system. We note that if $\tilde{B}_1$ is greater than B^x, the public debt is not sustainable.

[Figure 1 around here.]

While this result is not surprising in the existing literature, our interest is in what would happen if credit market imperfections become less severe. Since it is easily shown that with $\tilde{B}_t$ given, $\frac{F[a((1-\mu)\frac{\tilde{B}_t}{1-\alpha}+\mu)]}{1-\mu}$ is increasing with respect to μ, $\Psi_x(.)$ is decreasing with respect to μ. Then, it follows from lemma 2 that the non-trivial steady state is increasing with μ. This means that as credit market imperfections become less severe, the sustainable public debt is more likely achievable in the golden rule. This happens because as credit market imperfections are resolved, resource allocation is corrected and then productivity of the whole economy is improved. As a consequence, the rate of economic growth goes up, which implies that the economy can support more public debt than when credit market imperfections are severe. Meanwhile, under the golden rule, the dynamical system (13) is independent of the interest rate. Therefore, the burden of the government does not increase even though the interest rate goes up.

Next, we will see the positive deficit case, which is a simple extension of the zero deficit case. As seen in Figure 2, if λ gradually goes up from zero to one with other things being equal, another non-trivial steady state appears because the increase in λ shifts up the dynamical system (13). We call the high steady state B_H, which was actually B^x when $\lambda = 0$, and the low steady state B_L, which was the trivial steady state. From the dynamical system (13), the locus for $\Delta\tilde{B} := \tilde{B}_{t+1} - \tilde{B}_t = 0$ (henceforth, we call this the $\Delta\tilde{B} = 0$ locus) is given by $\tilde{B}_t = B_L$ and $\tilde{B}_t = B_H$. From Figure 2, we note that if $\tilde{B}_t < B_L$ or $\tilde{B}_t > B_H$, then $\Delta\tilde{B} > 0$, which implies that $\{\tilde{B}_t\}$ is increasing. If $B_L < \tilde{B}_t < B_H$, then $\Delta\tilde{B} < 0$ and thus $\{\tilde{B}_t\}$ is decreasing. If the public debt is sustainable, $\{\tilde{B}_t\}$ eventually converges to B_L.

[Figure 2 around here.]

Since $\Psi_x(\tilde{B}_t)$ is convex and increasing with respect to $\tilde{B}_t$, if λ increases, then B^H decreases. This implies that as the government deficit goes up, the region where the initial public debt provides the feasible path of the future public debt narrows. In particular, both λ and θ are close to one, the public debt is unsustainable no matter what initial value it takes because $\tilde{B}_t$ should be less than one.

Again, these consequences are not surprising in the existing literature (e.g., Yakita (2008) and Arai (2008)). However, there is novelty in that as μ goes up, $\Psi_x(\tilde{B}_t)$ shifts down and the public debt becomes more likely sustainable as in the case in which the government conducts the zero-deficit fiscal policy. Proposition 1 below provides a summary of this section.

Proposition 1 *Suppose the government follows the golden rule. Then, as credit market imperfections become less severe, public debt becomes more likely sustainable and the rate of growth of public debt goes down.*

3.2 Under the balanced budget rule

Under the balanced budget rule, the tax rate for $t \geq 1$ is obtained from Eqs.(6), (7), and (10) as follows:

$$\tau_t = \frac{\theta}{\alpha + \tilde{B}_{t+1}}.$$

Again, we can verify that under the assumption $\theta \leq \alpha$, τ_t is always no greater than one because $\tilde{B}_t \geq 0$ for all $t \geq 1$.

Under the balanced budget rule, the dynamical system for $\{\tilde{B}_t\}$ for $t \geq 1$ is given by $\tilde{B}_{t+1} = R_t \tilde{B}_t$, or equivalently,

$$\begin{aligned} \tilde{B}_{t+1} &= \frac{\alpha(1-\mu)}{1-\alpha} \frac{a\big(\mu + (1-\mu)\frac{\tilde{B}_t}{1-\alpha}\big)\tilde{B}_t}{F(a(\mu + (1-\mu)\frac{\tilde{B}_t}{1-\alpha}))} \\ &= \frac{2\alpha}{1-\alpha} \frac{\tilde{B}_t\big(\mu + (1-\mu)\frac{\tilde{B}_t}{1-\alpha}\big)}{\big(1 - \frac{\tilde{B}_t}{1-\alpha}\big)\big(1 + \mu + (1-\mu)\frac{\tilde{B}_t}{1-\alpha}\big)} := \Psi_p(\tilde{B}_t;\mu). \end{aligned} \tag{14}$$

Lemma 4 *$\Psi_p(\tilde{B}_t;\mu)$ is strictly increasing and strictly convex with respect to $\tilde{B}_t$.*

Proof: Since $\frac{\tilde{B}_t}{F(a(\mu+(1-\mu)\frac{\tilde{B}_t}{1-\alpha}))}$ is strictly increasing and strictly convex with respect to $\tilde{B}_t$ from the proof of lemma 2 and $\mu + (1-\mu)\frac{\tilde{B}_t}{1-\alpha}$ is increasing and convex with respect to $\tilde{B}_t$, the claim is proven. □

lemma 5 below provides us with the information about a phase diagram for the dynamical system (14).

Lemma 5 *The following hold:*

- *If $1 + \frac{1}{\mu} \leq \frac{2\alpha}{1-\alpha}$, then there is no non-trivial steady state for the dynamical system (14).*
- *If $1 + \frac{1}{\mu} > \frac{2\alpha}{1-\alpha}$, then there is a non-trivial steady state for the dynamical system (14).*

Proof: Since $\Psi_p(\tilde{B}_t)$ is strictly increasing and convex and $\Psi_p(0) = 0$, it follows that if $\Psi_p'(0) \geq 1$, then there is no non-trivial steady state, whereas if $\Psi_p'(0) < 1$, then there is a non-trivial steady state. The proof follows from the fact that $\Psi_p'(0) = \frac{a\alpha\mu(1-\mu)}{1-\alpha}\frac{1}{F(a\mu)} \geq (<) 1$ is rewritten as $F(a\mu) \leq (>) \frac{a\alpha\mu(1-\mu)}{1-\alpha} \iff 1 + \frac{1}{\mu} \leq (>) \frac{2\alpha}{1-\alpha}$. □

In lemma 5, the greater is μ, the more likely we have the first case. However, if α is around one third, as assumed in the literature, the first case is implausible. We obtain a phase diagram for the second case of lemma 5 in Figure 3. The non-trivial steady state is denoted by B^p. As in the case of the golden rule, only when $\tilde{B}_1 \in [0, B^p]$, the public debt is sustainable.

[Figure 3 around here.]

It is easily shown that $\partial\Psi_p(\tilde{B}_t)/\partial\mu > 0$ with $\tilde{B}_t$ given. This implies that as a credit constraint is relaxed, public debt is less likely sustainable. This is because, contrasting with the golden rule, the balanced budget rule is subject to the interest rate. As credit market imperfections are resolved, the interest rate goes up with other things being equal because financial resources are intensively used by capable agents. The higher interest rate becomes a burden of the government. As in the previous section, a summary is given in proposition 2 below.

Proposition 2 *Suppose the government follows the balanced budget rule in the sense that $T_{t+1} - I^g_{t+1} = 0$ for all $t \geq 0$. Then, as credit market imperfections become more severe, public debt becomes more likely sustainable and the rate of growth of public debt goes down.*

3.3 Discussion

We have clarified how credit market imperfections affect the sustainability of public debt, depending upon the fiscal rules. This discovery has an important policy implication.

The parameter μ is thought of as a measure of the degree of financial development. Now suppose that the government is concerned about the sustainability of the public debt. As demonstrated in our model, if an economy has a poorly developed financial sector, the government should conduct the balanced budget rule because the smaller is μ, the more likely is the public debt sustainable. On the other hand, if an economy has a fully developed financial sector, the government should execute the golden rule because as μ becomes big, the public debt is more likely sustainable.

Probably, the governments in the real world more or less follow an intermediate case between the golden rule and the balanced budget rule such as a convex combination of them unless a fiscal regulation is imposed as in the EU countries. The policy implication of the current model tells us in which direction a government should shift its fiscal policy depending upon the country's financial development so that it can attain the sustainable public debt.

4 Economic Growth

In this section, we investigate economic growth in terms of per capita output when public debt is sustainable. Our study is focused on the balanced growth paths in which the growth rates of per capita output and capital goods are the same and constant. In both cases of the golden rule and the balanced budget rule, as the parameter of credit market imperfections μ goes up, the growth rate of an economy goes up. While this consequence is not surprising in the existing literature (e.g., King and Levine (1993) and Aghion, et al. (2005)), the novelty is that we clarify a relationship between credit market imperfections and economic growth when an economy incurs public debt.

4.1 Under the golden rule

From $w_t = (1-\alpha)Y_t/L_t$ and Eq.(10), we obtain:

$$y_t = \frac{(1-\alpha)\bar{q}F(\phi_{t-1})}{\alpha(1-\mu)(1+n)} y_{t-1}, \tag{15}$$

where $y_t := Y_t/L_t$. The dynamical system of this economy for $t \geq 1$ consists of Eqs.(15) and Eq.(13) that is inserted again below:

$$\tilde{B}_{t+1} = \frac{\alpha(1-\mu)}{(1-\alpha)\bar{q}} \frac{\tilde{B}_t}{F(\phi_{t-1})} + \lambda\theta, \tag{16}$$

where $\phi_{t-1} = a((1-\mu)\frac{\tilde{B}_t}{(1-\alpha)} + \mu)$.

In the balanced growth path, it follows that $\tilde{B}_t = B_L$. To make the dynamical system simple and to clarify the effects of the degree of credit constraints and population growth, we let $\tilde{y}_t := (\tilde{B}_{t+1} - \lambda\theta)y_t$. Then, the dynamical system consisting of Eqs.(15) and (16) is redefined as the one consisting of Eq.(16) and:

$$\tilde{y}_t = \frac{1}{1+n}\big(\tilde{y}_{t-1} + \frac{\lambda\theta\tilde{y}_{t-1}}{\tilde{B}_t - \lambda\theta}\big). \tag{17}$$

Since Eqs.(16) and (17) are respectively independent of population growth and the degree of credit constraints, we can easily investigate the effects of them. We note that since $\{\tilde{B}_t\}$ is bounded and $B_L > \lambda\theta$, if $\{\tilde{y}_t\}$ grows unboundedly, then the economy exhibits endogenous growth, whereas if $\{\tilde{y}_t\}$ converges to zero, then the economy shrinks and per capita output converge to zero.

We denote the growth rate of per capita output in the balanced growth path as $\gamma_y := \frac{\tilde{y}_t}{\tilde{y}_{t-1}}$. From Eq.(17), γ_y is given by $\frac{1}{1+n}(\frac{B_L}{B_L-\lambda\theta})$. Henceforth, we assume $n \geq 0$.[11] Then, we note that if $\gamma_y > 1 \Longleftrightarrow B_L < (\frac{1}{n}+1)\lambda\theta := B_z$, then the economy exhibits endogenous growth and if $\gamma_y < 1 \Longleftrightarrow B_L > B_z$, then the economy shrinks as time goes by and falls in a poverty trap.

Since B_L is independent of population growth, if the rate of population growth is zero, then the economy endogenously grows without fail. This is because if the rate of population growth is equal to zero, the condition for the sustainability of public debt guarantees large productivity enough for the economy to exhibit endogenous growth. By contrast, if population growth is so big that $B_L > B_z$, it falls in a poverty trap. Meanwhile, we note that if the government conducts a zero deficit fiscal policy, namely, $\lambda = 0$, then the economy experiences endogenous growth because $0 = B_L < B_z$ always holds.

While B_L is not affected by population growth, it is subject to the degree of credit market imperfections. From the discussion in section 2, it is easily noted that $\partial B_L/\partial\mu < 0$. Since $\lim_{n\to 0} B_z = \infty$ and $\lim_{n\to\infty} B_z = \lambda\theta$, the patterns of economic growth depend upon the degree of credit market imperfections, i.e., μ under a relatively large rate of population growth.

[11]When $-1 < n < 0$, the economy always attains endogenous growth.

To see this, the transition paths to the balanced growth paths of the two countries, whose degrees of credit constraints are different, are illustrated in Figure 4. Let us suppose that there are two countries, country 1 and country 2 and that country i with $\mu = \mu_i$ has B_L^i. We assume that the parameters other than μ are the same between the two countries. However, due to the difference in the degree of credit constraints, it follows that $B_L^1 \in (B_z - \epsilon, B_z)$, and $B_L^2 \in (B_z, B_z + \epsilon)$ where ϵ is very small. In this case, we have $\mu_2 < \mu_1$. The difference between μ_1 and μ_2 is very small or even infinitesimal. Although the fundamentals of the two economies are almost the same and they start with the same amount of per capita capital, the fates of the two countries are completely different.

At first, both of them follow very similar transition paths as seen in Figure 4. Policymakers in each country must be happy because public debt is small and they seemingly experience economic growth. However, when country 2 reaches B_z, the economy starts shrinking. While the policymaker in country 2 sees endogenous growth of country 1, they probably do not understand what is going on in their own country. This example demonstrates that due to an infinitesimal difference in the degrees of credit constraints, the fates of countries are quite different.

[Figure 4 around here.]

4.2 Under the balanced budget rule

Since the characteristics of the balanced growth path under the balanced budget rule are simple relative to the case of the golden rule, we provide a brief sketch of them. In the balanced gowth path, it holds that $\tilde{B}_t = 0$ and thus $\gamma_z = \frac{a(1-\alpha)\bar{q}(1+\mu)}{2\alpha(1+n)}$.

In contrast with the case of the golden rule, even though population growth is zero, the economy might not be able to exhibit endogenous growth if the productivity parameters a and A or the degree of credit market imperfections μ is so small that $\frac{a(1-\alpha)\bar{q}(1+\mu)}{2\alpha} < 1$. This is because the condition for the sustainability of public debt is independent of the productivity parameters a and A. In other words, the sustainable condition for public debt does not require for the productivity parameters to be high.

This result implies that under the balanced budget, while as in the case of the golden rule, an infinitesimal difference in the degree of credit constraint can give quite different economic growth patterns, growth-enhancing factors, such as financial sector development, do not necessarily lead to sustainable public debt.

5 Social Welfare

5.1 Gini coefficient and Sen's social welfare function

In this section, we investigate social welfare when an economy incurs public debt and faces credit market imperfections. In our model, since individuals are heterogeneous, it is not enough to measure social welfare simply by aggregating consumption in the second period. Then, we take into account income distribution between agents.

On the basis of our consideration, Sen's (1976) social welfare function is used, which is given by:

$$SW_t := \bar{c}_t(1 - G_t) \tag{18}$$

from $t = 1$ onwards, where $\bar{c}_t$ is per capita disposable income and G_t is a Gini coefficient at time t. For simplicity, we assume that population is normalized to one, implying that there is no population growth ($n = 0$), and ϕ has a uniform distribution in $[0, 1]$ ($a = 1$) henceforth.[12]

From a national income identity, we have per capita income (or equivalently per capita consumption) $\bar{c}_t$ as follows:

$$\bar{c}_t = Y_t - K_t - I_t^g, \tag{19}$$

where K_t is the aggregate investment at time t. Since we have $K_t = \frac{(1-\alpha)Y_t(1-G(\phi_t))}{1-\mu}$ and

[12]Even though we assume $n \neq 0$ and $a \neq 1$, the results obtained below will not change.

$I_t^g = \theta Y_t$, Eq.(19) is rewritten as:

$$\bar{c}_t = (\alpha - \theta + \tilde{B}_{t+1})Y_t. \tag{20}$$

It is surprising that given Y_t, if $\tilde{B}_{t+1}$ increases, then $\bar{c}_t$ goes up. This happens because of a general equilibrium effect of a fiscal policy, namely, if $\tilde{B}_{t+1}$ goes up, the tax rate goes down because I_t^g is determined by a fiscal policy rule. Eq.(20) means that since we employ an overlapping generations model, Ricardian equivalence does not hold. There is a possibility that an increase in public debt raises social welfare in an economy.[13]

We have derived $\bar{c}_t$ from a macroeconomic perspective. In turn, in order to obtain the Lorenz curve, we will compute $\bar{c}_t$ from a microeconomic perspective. Consumption is given by $c_t = \phi_{t-1}\bar{q}w_{t-1}(1-\tau_t)$ for individuals with $\phi < \phi_{t-1}$ and $c_t = \frac{\phi - \mu\phi_{t-1}}{1-\mu}\bar{q}w_{t-1}(1-\tau_t)$ for individuals with $\phi > \phi_{t-1}$, respectively. Therefore, we obtain:

$$\begin{aligned} \frac{\bar{c}_t}{\bar{q}w_{t-1}(1-\tau_t)} &= \phi_{t-1}\int_0^{\phi_{t-1}} d\phi + \frac{1}{1-\mu}\int_{\phi_{t-1}}^{a}(\phi - \mu\phi_{t-1})d\phi, \\ &= \frac{1}{2(1-\mu)}(\phi_{t-1}^2 - 2\mu\phi_{t-1} + 1). \end{aligned} \tag{21}$$

From Eq.(21), the Lorenz curve, $L(x;t)$, is given by:

$$L(x;t) = \int_0^x \frac{c_t}{\bar{c}_t}d\phi = \begin{cases} \frac{2(1-\mu)\phi_{t-1}x}{\phi_{t-1}^2 - 2\mu\phi_{t-1}+1} & \text{if } 0 \leq x < \phi_{t-1} \\ \\ \frac{\phi_{t-1}^2 - 2\mu\phi_{t-1}x + x^2}{\phi_{t-1}^2 - 2\mu\phi_{t-1}+1} & \text{if } \phi_{t-1} \leq x \leq 1. \end{cases} \tag{22}$$

Since the Gini coefficient is formulated by $G_t = 1 - 2\int_0^1 L(x;t)dx$, we have:

$$1 - G_t = -\frac{2(\phi_{t-1}^3 - 3\phi_{t-1}^2 + 3\mu\phi_{t-1} - 1)}{3(\phi_{t-1}^2 - 2\mu\phi_{t-1} + 1)}. \tag{23}$$

From Eqs.(20) and (23), the log of Sen's social welfare function is given by:

$$\log[SW_t] = \log\left[-\frac{2(\phi_{t-1}^3 - 3\phi_{t-1}^2 + 3\mu\phi_{t-1} - 1)}{3(\phi_{t-1}^2 - 2\mu\phi_{t-1} + 1)}\right] + \log[(\alpha - \theta + \tilde{B}_{t+1})] + \log[Y_t], \tag{24}$$

where $\phi_{t-1} = (1-\mu)\frac{\tilde{B}_t}{1-\alpha} + \mu$. Sen's social welfare consists of the three parts. The first term of the right-hand side is an effect of equity and the second term is an effect of allocation within

[13]At a glance, this phenomenon seems related to dynamic efficiency. In fact, it is not related to it because an increase in $\bar{c}_t$ associated with $\tilde{B}_{t+1}$ is an issue of resource allocation within a period.

a period. The third term comes from per capita output. Since the first and second terms converge to the steady-state values, the social welfare is dominated by per capita output in the long run and thus an economy with the higher rate of growth can eventually attain the higher social welfare. However, on the transitional path, the first and second terms are important, implying that some generations may gain the higher welfare, whereas the other generations incur the lower welfare under a specific fiscal rule.

This discussion is politically very important because the government in power in a period tends to increase the social welfare of the current generation at the sacrifice of the social welfare of the future generations who are not born yet. Therefore, it is significant to do numerical experiments of the evolution of social welfare.

5.2 Numerical analysis

In order to compute the social welfare, we have to pin down five parameters: α, A, λ, θ, and μ. Following the macroeconomics literature, let α take one third, $\alpha = 1/3$. θ is the government expenditure share of the gross domestic product. From the International Financial Statistics created by the International Monetary Fund, we compute the average of the government expenditure share in the United States from 2000 to 2007 and obtain $\theta = 0.154$. We can also observe the total central government debt of the United States in the OECD database. We regard an increase in it as the budget deficit, $B_{t+1} - B_t$, and compute $\lambda = 0.086$, which is the average value from 2000 to 2007.[14] As with μ, we note from lemma 1 that μ is the ratio of the newly issued aggregate private credit to the investments. By using the database for financial structure created by Levine, et al. (2000) and updated by themselves in 2010, we compute $\mu = 0.636$, which is the average value from 2001 to 2005. The parameter A is determined so that for any value $\mu \in [0, 1)$ and $\theta = 0.154$, the second case of lemma 3 may hold and we set $A = 6$. While this value is ad hoc, A is eventually cancelled out when we compare the two fiscal policy rules. Then, a parameter set is given

[14] Of course, if we include the budget deficits of the local governments in our computation, then λ should be greater than 0.086.

as follows:

$$(\alpha, A, \mu, \lambda, \theta) = (1/3, 6, 0.636, 0.086, 0.154).$$

From Eqs.(9) and (10), the law of motion of output for $t \geq 1$ is given by:

$$Y_t = \frac{\bar{q}(1-\alpha)}{2\alpha}\left(1 - \frac{\tilde{B}_t}{1-\alpha}\right)\left(1 + \mu + (1-\mu)\frac{\tilde{B}_t}{1-\alpha}\right)Y_{t-1}. \tag{25}$$

Meanwhile, under the golden rule, the law of motion of the public debt/GDP ratio for $t \geq 1$ is given by:

$$\tilde{B}_{t+1} = \frac{2\alpha}{(1-\alpha)\bar{q}}\frac{\tilde{B}_t}{\left(1 - \frac{\tilde{B}_t}{1-\alpha}\right)\left(1 + \mu + (1-\mu)\frac{\tilde{B}_t}{1-\alpha}\right)} + \lambda\theta, \tag{26}$$

and under the balanced budget rule, it is given by:

$$\tilde{B}_{t+1} = \frac{2\alpha}{1-\alpha}\frac{\tilde{B}_t\left(\mu + (1-\mu)\frac{\tilde{B}_t}{1-\alpha}\right)}{\left(1 - \frac{\tilde{B}_t}{1-\alpha}\right)\left(1 + \mu + (1-\mu)\frac{\tilde{B}_t}{1-\alpha}\right)}. \tag{27}$$

The cutoff ϕ_{t-1} is determined by:

$$\phi_{t-1} = \frac{1-\mu}{1-\alpha}\tilde{B}_t + \mu. \tag{28}$$

The social welfare is computed recursively by using Eqs.(24), (25), (26) (under the golden rule), and (28) or Eqs.(24), (25), (27) (under the balanced budget rule), and (28).

We set the initial level of public debt and output as in table 1.

Case 1	B_1(initial public debt) $= 1$	Y_0(initial output) $= 3$
Case 2	B_1(initial public debt) $= 1$	Y_0(initial output) $= 5$
Case 3	B_1(initial public debt) $= 1$	Y_0(initial output) $= 10$

Table 1: The setting on initial level of public debt and output in the three cases.

We examine the social welfare for the six cases, i.e., three cases [Case1, case2, case3] $\times$ two cases [GR(the golden rule), BBR(the balanced budget rule)]. We compute the social welfare, SW_t, in each period for $t \geq 1$ until the economy converges to the balanced growth path.

For each case, we derive a ratio of social welfare of the golden rule to that of the balanced budget rule, which is standardized as follows:

$$SWratio_t := \frac{SW_t^{GR} - SW_t^{BBR}}{SW_t^{BBR}} \times 100(\%),$$

where SW_t^i is defined as social welfare at time t under the i fiscal rule ($i = GR, BBR$).

[Figures 5 , 6 , and 7 around here]

Figures 5, 6, and 7 provide the results. We note that relative to the golden rule (the balanced budget rule), the balanced budget rule (the golden rule) provides the higher (lower) social welfare for the far-future generations. This consequence is intuitive. Since the public-debt/GDP ratio, $\tilde{B}_t$, converges to zero under the balanced budget rule, the private investment is not crowed out by public debt in the long run. Meanwhile, under the golden rule, the public-debt/GDP ratio converges to a certain positive level. This implies that the private investment is retarded by public debt even in the long run. Therefore, the growth rate in the golden rule is less than in the balanced budget rule in the far future. Nevertheless, as seen in Figure 5, it takes very long time for the balanced budget rule to overtake the golden rule when the economy starts with very low per capita output.

The balanced budget rule provides the higher social welfare for the super-near future generations than the golden rule, whereas it provides the lower social welfare for the near future generations.

We can understand this alternation of the dominant fiscal rules by looking at Eq.(20). As seen in the later, the social welfare under both rules is mostly affected by per capita consumption and the effect of the Gini coefficient is limited. Under the balanced budget rule, even more government bonds are issued in the super-near future than under the golden rule, implying that the balanced budget rule imposes the lower rate of tax than the golden rule. The crowding-out effect of public debt on per capita output is limited in the super-short run. Therefore, the balanced budget rule dominates the golden rule in the super-short run.

Meanwhile, in the near future, more government bonds are issued under the golden rule than the balanced budget rule and the golden rule imposes the lower rate of tax than the balanced budget rule in the short run. The crowding-out effect of public debt on per capita output is still limited in the short run. Therefore, the golden rule dominates the balanced budget rule. However, since in the long run, the crowding-out effects of public debt on per capita output accumulate, the level of per capita output in the balanced budget rule becomes far more than that in the golden rule. Thus, the balanced budget rule becomes a dominant fiscal rule once again.

We compare the effects of $1 - G_t$ and $\bar{c}_t$ on social welfare. Since the social welfare function consists of these two components, it is important to investigate which factor more significantly affects social welfare. We define $Gratio_t$ as a ratio of $1 - G_t$ of the two fiscal rules, which is standardized as:

$$Gratio_t = \frac{(1 - G_t^{GR}) - (1 - G_t^{BBR})}{1 - G_t^{BBR}} \times 100(\%).$$

Likewise, $\bar{c}ratio_t$ is a ratio of per capita consumption under the two rules:

$$\bar{c}ratio_t = \frac{\bar{c}_t^{GR} - \bar{c}_t^{BBR}}{\bar{c}_t^{BBR}} \times 100(\%).$$

By the numerical calculation, we find that the effect of per capita consumption on social welfare is much greater than the effect of inequality in all cases. As seen in Figures 8-13, the scale of the consumption ratio is about ten times as large as that of $(1 - G_t)$. It is not surprising that the Gini coefficient has a small effect on social welfare in the long-run because an economy endogenously grows. In the short run, however, the effect of the Gini coefficient on social welfare is very small relative to the effect of per capita consumption as well. In the super-near future generations, the balanced budget rule leads to more equity than the golden rule. However, in the near and far future generations, the golden rule provides more equity than the balanced budget rule.

[Figures 8-13 around here.]

6 Concluding Remarks

Does the dissipation of credit market imperfections promote or foil the sustainability of public debt? In order to answer this question, we have developed an overlapping generations model with public debt. Our findings are as follows. Whether the sustainability of public debt is promoted or not when credit market imperfections are relaxed depends upon the fiscal policy rules. Under the golden rule, as credit constraints become less severe (more severe), public debt is more likely sustainable (unsustainable). On the contrary, under the balanced budget rule, as credit constraints become more severe (less severe), it is more likely sustainable (unsustainable).

We have also found that a fiscal policy has significant effects on social welfare via economic growth, resource allocation, and income distribution. For the social welfare, we have compared the outcomes of the golden rule and the balanced budget rule. The dominant fiscal policies alternates depending upon generations. The balanced budget rule is more beneficial than the golden rule for the super-near future generations and the golden rule is more beneficial than the balanced budget rule for the near future generations. However, for the far future generations, the balanced budget rule becomes more beneficial than the golden rule once again.

Appendix

Microfoundation for credit constraints

In this appendix, we provide two kinds of microfoundations for credit constraints. One is based on the idea of Aghion and Banerjee (2005) and the other is based on that of Antras and Caballero(2009).

Microfoundation I

Following Aghion and Banerjee (2005), we provide a microfoundation for a credit constraint facing each individual.[15] The credit market imperfections simply come from the possibility that borrowers may not repay the financial intermediary their obligation.

Each agent prepares his own wealth w_t to invest, which is wages earned when he is young. If he is a borrower, his total resources are $k_t = w_t - d_t$. From Eq.(2), the return on one unit of investments is $q_{t+1}\phi$. If a borrower earnestly repays the financial intermediary his obligation, then he gets a net income, $q_{t+1}\phi k_t + R_{t+1}d_t$. In turn, if he chooses not to repay his obligations, then he incurs a cost, δk_t to hide his revenues. When he does not repay his obligations, the financial intermediary monitors the borrower and it is able to capture the borrower with probability p_{t+1}. In this case, the expected income of the borrower is given by $q_{t+1}\phi k_t - \delta k_t + p_{t+1}R_{t+1}d_t$

Under this loan contract, the incentive compatibility constraint so as for a borrower not to default is given by:

$$q_{t+1}\phi k_t + R_{t+1}d_t \geq [q_{t+1}\phi - \delta]k_t + p_{t+1}R_{t+1}d_t, \tag{29}$$

which is rewritten as:

$$d_t \geq -\frac{\delta}{R_{t+1}(1-p_{t+1})}k_t, \tag{30}$$

The left-hand side of Eq.(29) is the revenues when the borrower invests in a project and repay the financial intermediary, whereas the right-hand side of Eq.(29) is the gain when the borrower defaults. Eq.(30) is independent of the return on one unit of investments $q_{t+1}\phi$.

In order to attain the probability p_{t+1} to detect the borrower's stalling, the financial intermediary incurs an effort cost, $d_t C(p_{t+1})$, which is increasing and convex with respect to p_{t+1}. As in Aghion and Banerjee (2005), we assume $C(p_{t+1}) = \kappa \log(1-p_{t+1})$, where κ is strictly greater than δ so that all borrowers face severer credit constraints than their

[15] Aghion, et al. (1999) and Aghion, et al. (2005) provide a microfoundation for a credit constraint in the same line as Aghion and Banerjee (2005).

natural debt limits. The financial intermediary can choose an optimal probability to solve a maximization problem such that:

$$\max_{p_{t+1}} \; -p_{t+1}R_{t+1}d_t - \kappa \log(1-p_{t+1})d_t.$$

Since $-d_t > 0$, this maximization problem is rewritten as:

$$\max_{p_{t+1}} \; p_{t+1}R_{t+1} + \kappa \log(1-p_{t+1}).$$

From the first-order condition, we have:

$$R_{t+1} = \frac{\kappa}{1-p_{t+1}}. \tag{31}$$

As the interest rate R_{t+1} increases, the financial intermediary chooses the high probability to detect the borrower's hiding his revenues. This is because if R_{t+1} goes up, borrowers have higher incentives to default and thus the financial intermediary makes more efforts to detect their stalling even though it incurs more costs. From Eqs.(30) and (31), we obtain:

$$d_t \geq -\frac{\delta}{\kappa}k_t,$$

or equivalently,

$$d_t \geq -\frac{\delta}{\kappa-\delta}w_t. \tag{32}$$

Since the individual's productivity ϕ is not observable, the financial intermediary does not impose individual-specific credit constraints. However, it has to know how much wealth, w_t, individuals have. As long as it imposes a credit constraint given by inequality (32) on individuals, no one will default in equilibrium. Since $\delta < \kappa$, we can let $\nu := \frac{\delta}{\kappa-\delta} \in [0,\infty)$ and thus:

$$d_t \geq -\nu w_t,$$

which is a credit constraint in the main text.

Microfoundation II

We extend a microfoundation for a credit constraint developed by Antras and Caballero (2009) to the one suitable for our model. In particular, we consider a participation constraint of the financial intermediary and an incentive compatibility condition of borrowers so as for them not to default.

We impose an assumption on the behavior of borrowers. It is assumed that at the end of the first period of their lifetime and after investment has occurred, any borrower can escape with no cost from engaging his investment project, taking some of fractions of his investment, $(1-\mu)(w_t - d_t)$, where $0 < \mu < 1$ and not repaying the financial intermediary his obligations. In this case, he will be engaged in capital production somewhere and sell capital goods in a market.

If a borrower walks away at the end of the first period, then the financial intermediary can take back the amount of the remainder of investment, $\mu(w_t - d_t)$. We assume that the financial intermediary can lend the remainder of investment in the financial market again. Therefore, when the financial intermediary makes a financial contract with a borrower, it faces a participation constraint such that:

$$R_{t+1}\mu(w_t - d_t) \geq -R_{t+1}d_t,$$

or equivalently

$$d_t \geq -\frac{\mu}{1-\mu}w_t. \tag{33}$$

On the other hand, the incentive compatibility constraint for a borrower not to escape from engaging his project at the end of the first period is given by:

$$\phi q_{t+1}(w_t - d_t) + R_{t+1}d_t \geq \phi q_{t+1}(1-\mu)(w_t - d_t). \tag{34}$$

For individuals with ϕ such that $R_{t+1} - \mu\phi q_{t+1} \leq 0$, Eq.(34) always holds. Therefore, we focus on individuals with ϕ such that $R_{t+1} - \mu\phi q_{t+1} > 0$. Then, Eq.(34) is rewritten as:

$$d_t \geq -\frac{\mu}{(\phi_t/\phi) - \mu}w. \tag{35}$$

Since $\phi_t/\phi \leq 1$ in equilibrium, it follows that $-\frac{\mu}{(\phi_t/\phi)-\mu} \leq -\frac{\mu}{1-\mu}$, implying that Eq.(35) is redundant.

To be summarized, if the financial intermediary imposes a credit constraint $d_t \geq -\frac{\mu}{1-\mu}w_t$, which is the participation constraint of the financial intermediary, borrowers never default. By letting $\frac{\mu}{1-\mu} := \eta$, we have a credit constraint $d_t \geq -\eta w_t$ in the main text.

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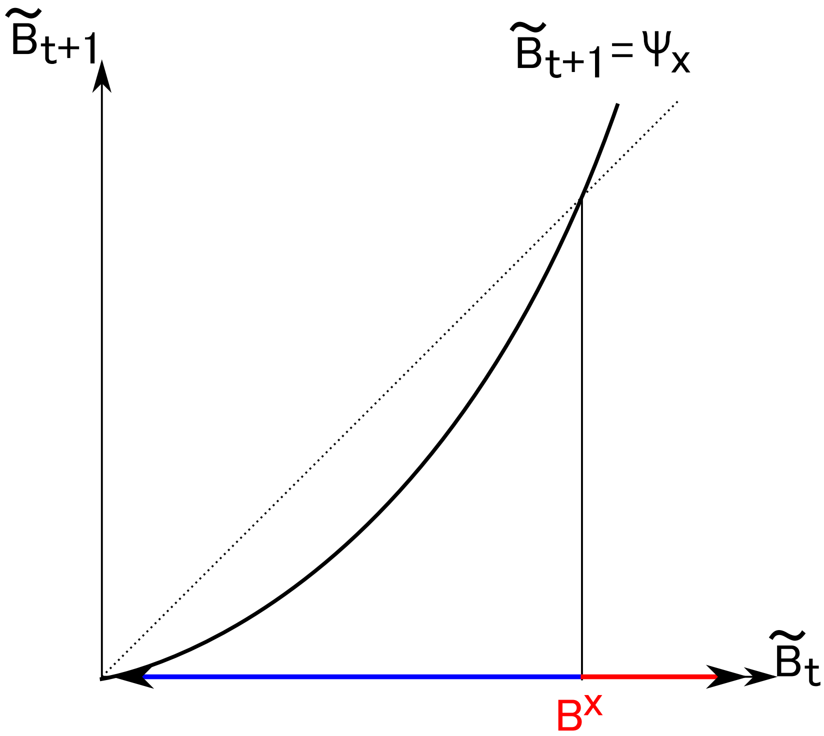

Figure 1: The phase diagram under the golden rule when deficit is zero.

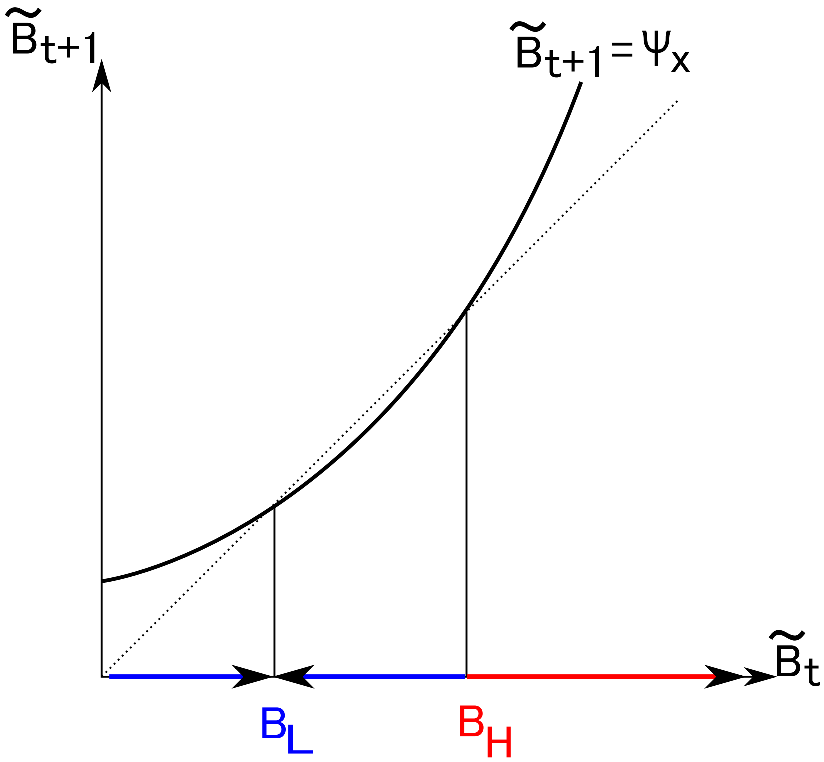

Figure 2: The phase diagram under the golden rule when deficit is non-zero.

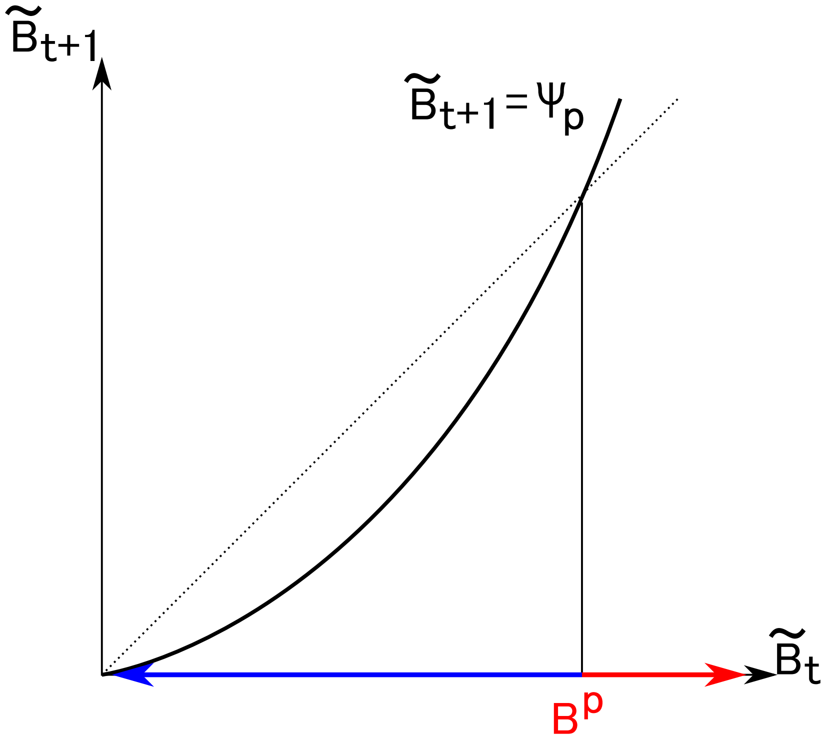

Figure 3: A phase diagram under the balanced budget rule.

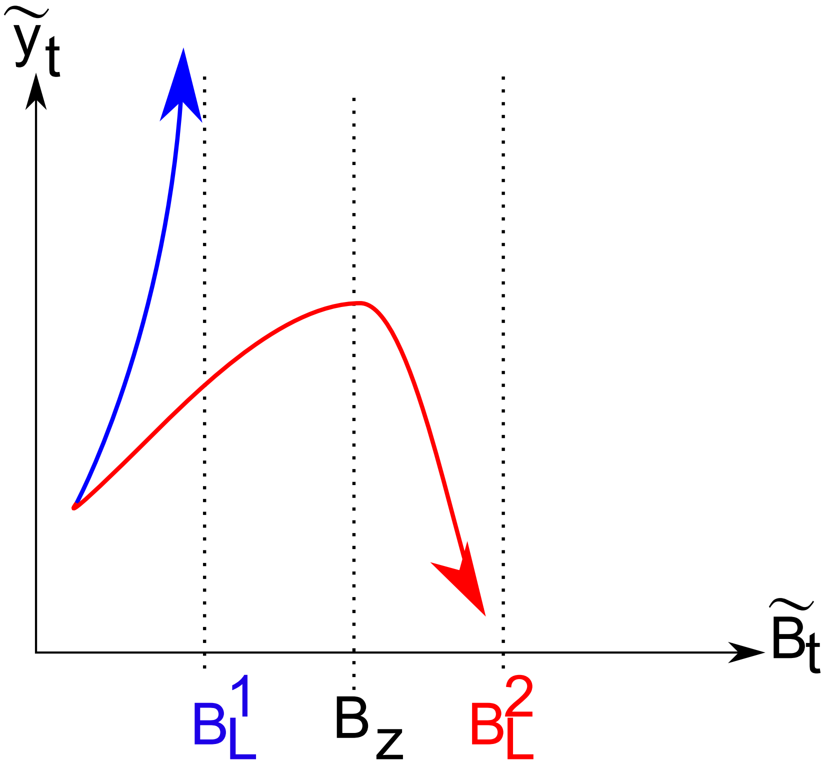

Figure 4: A Different Growth Pattern.

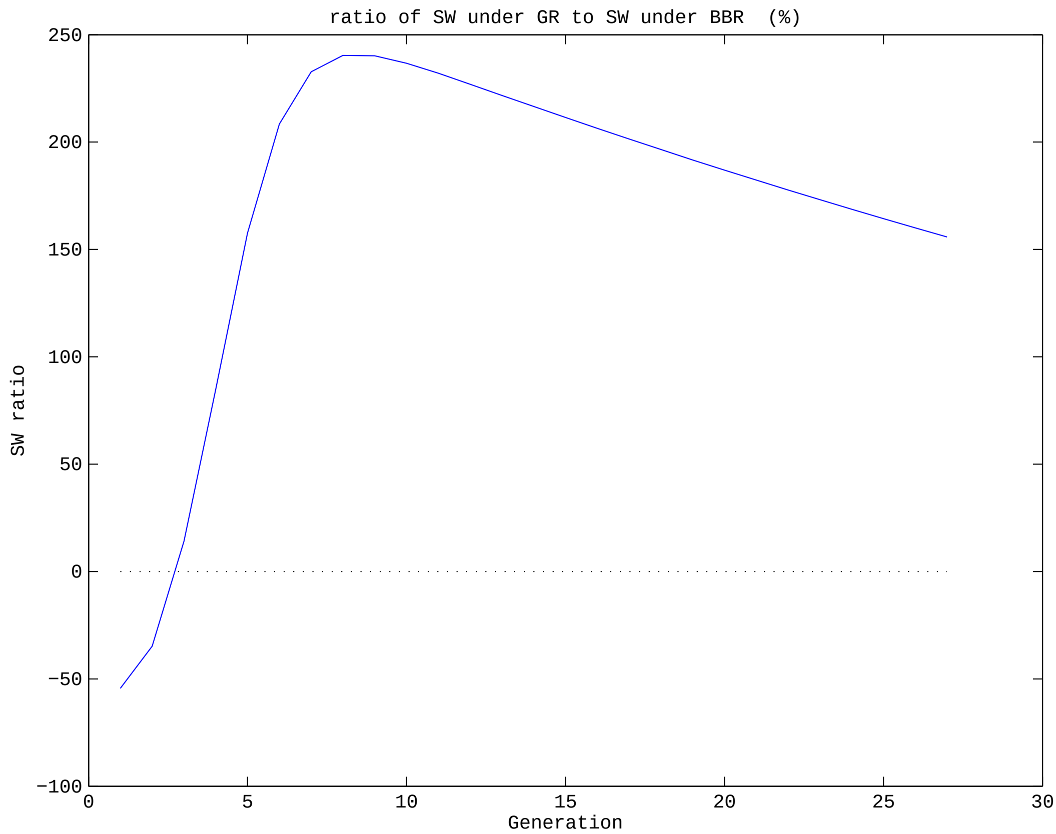

Figure 5: The social welfare ratio under the case 1 ($B_1 = 1$ and $Y_0 = 3$). [Horizontial axis: the number of generations, Vertical axis: the social welfare ratio.]

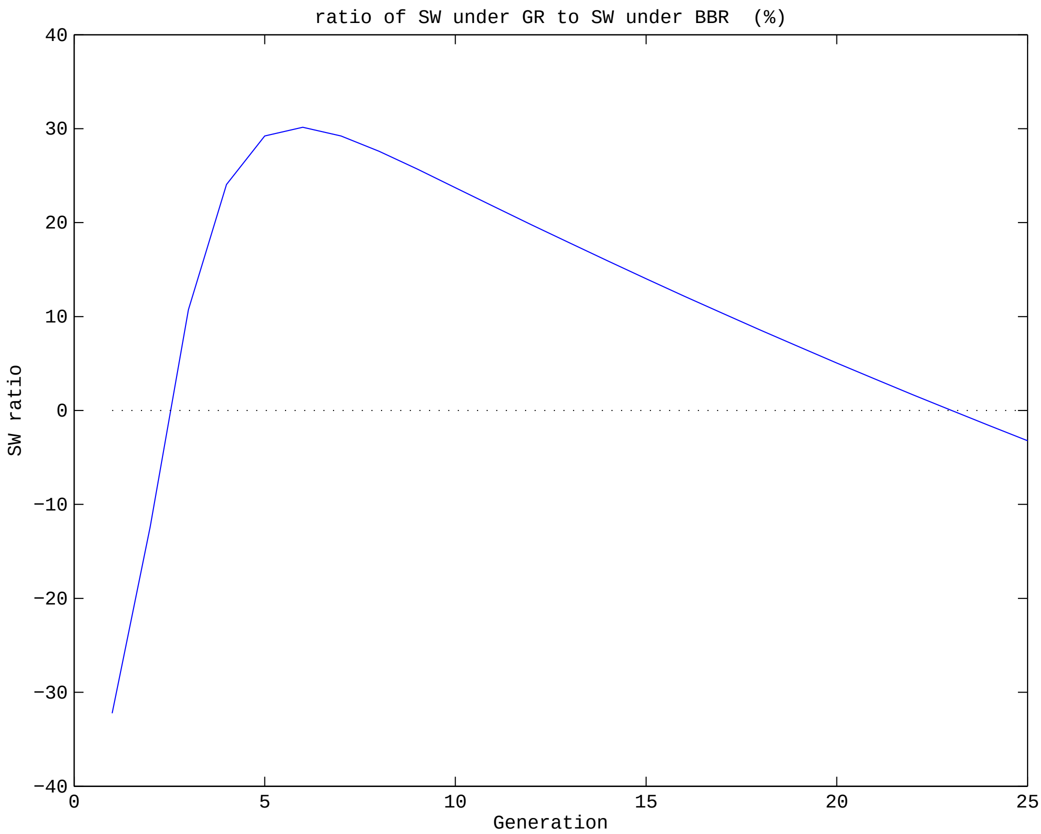

Figure 6: The social welfare ratio under the case 2 ($B_1 = 1$ and $Y_0 = 5$). [Horizontial axis: the number of generations, Vertical axis: the social welfare ratio.]

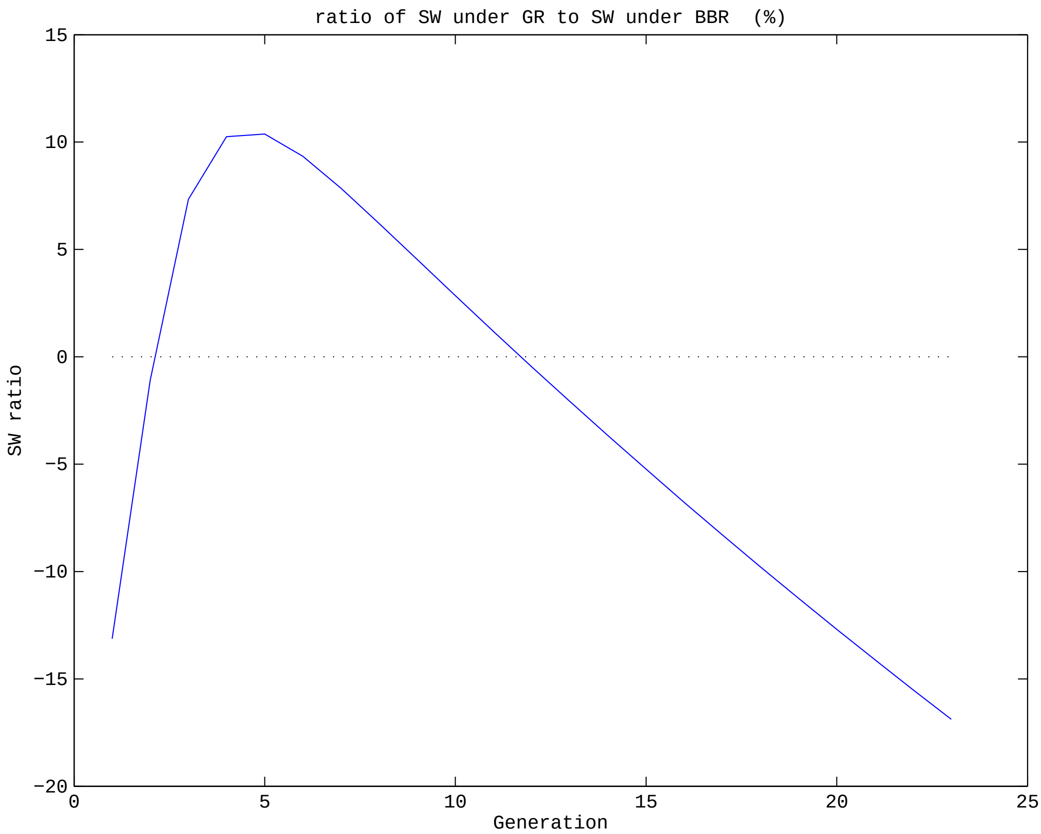

Figure 7: The social welfare ratio under the case 3 ($B_1 = 1$ and $Y_0 = 10$). [Horizontial axis: the number of generations, Vertical axis: the social welfare ratio.]

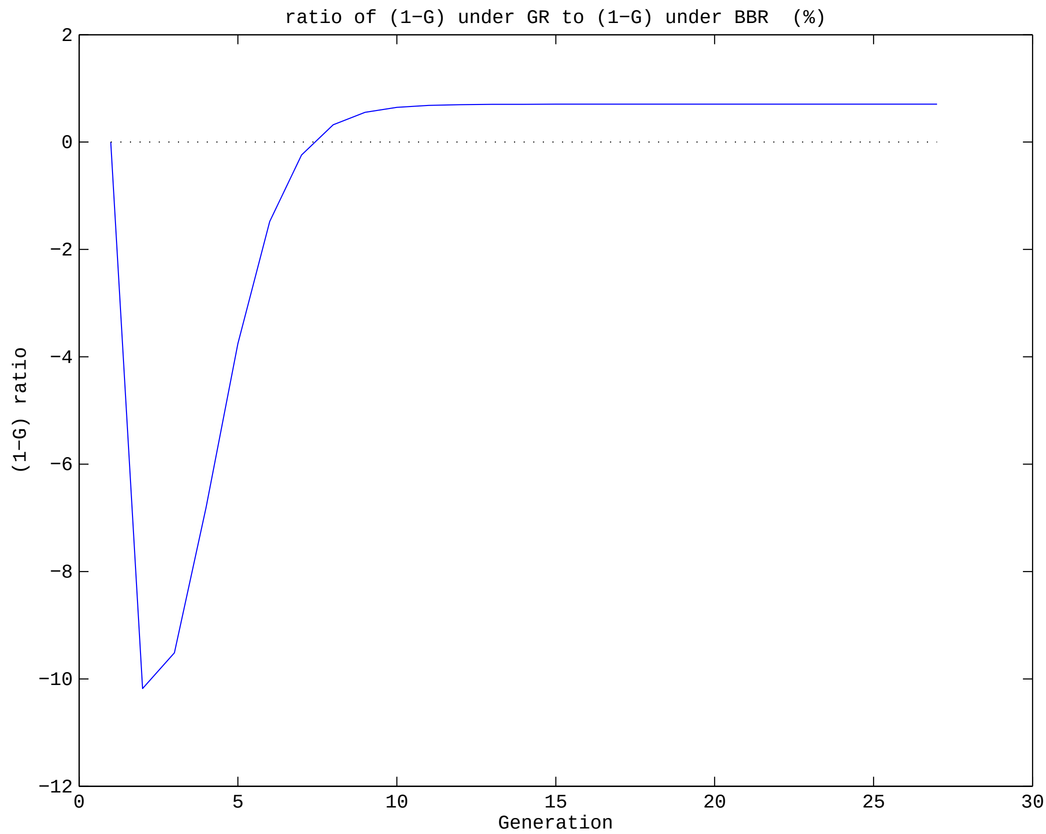

Figure 8: *Gratio* under the case 1 ($B_1 = 1$ and $Y_0 = 3$). [Horizontial axis: the number of generations, Vertical axis: *Gratio*.]

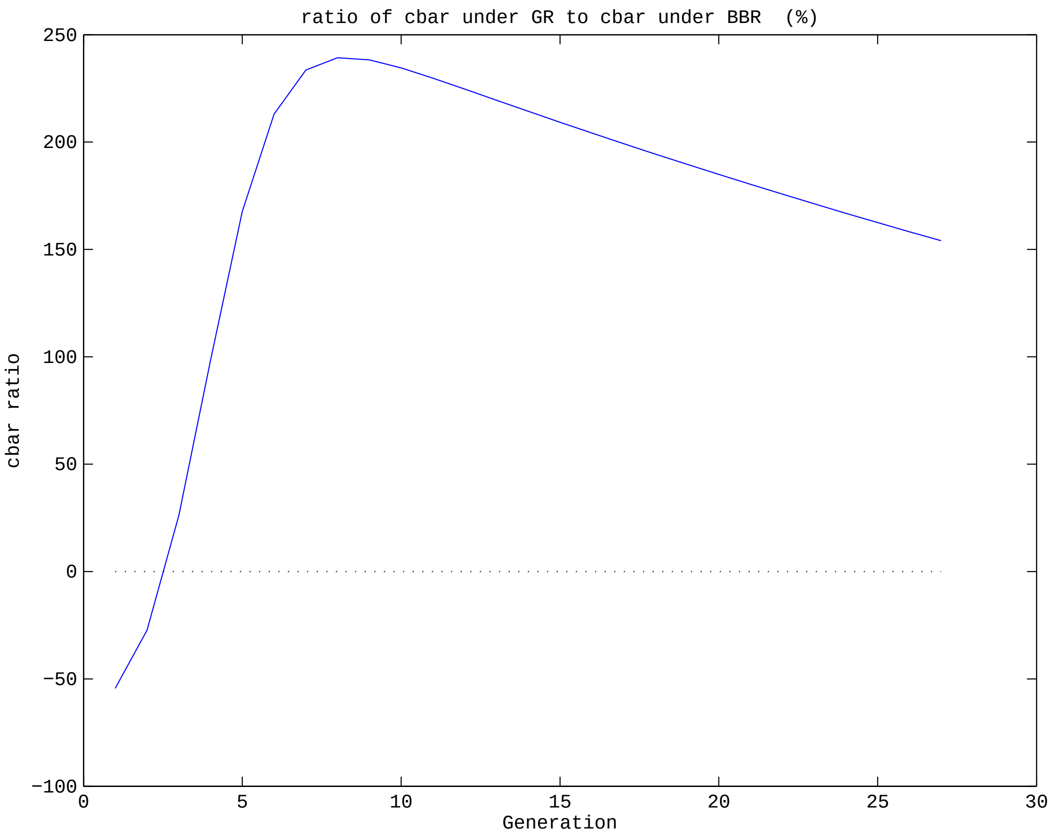

Figure 9: $\bar{c}ratio$ under the case 1 ($B_1 = 1$ and $Y_0 = 3$). [Horizontial axis: the number of generations, Vertical axis: $\bar{c}ratio$.]

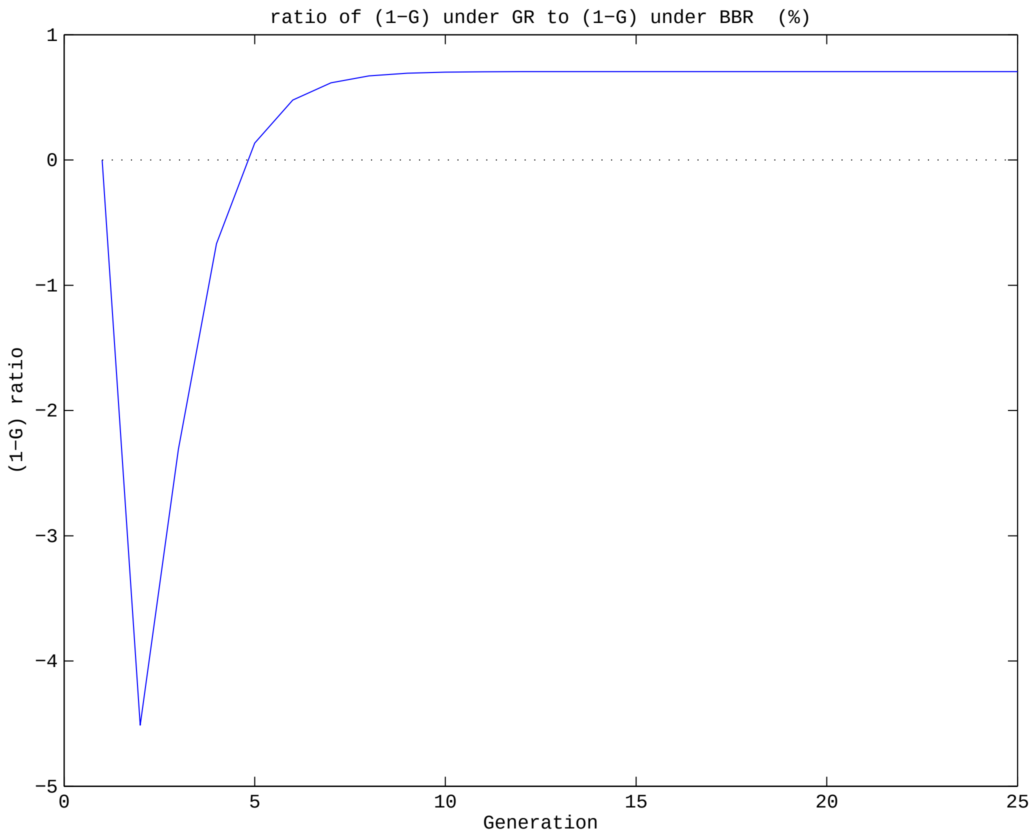

Figure 10: *Gratio* under the case 2 ($B_1 = 1$ and $Y_0 = 5$). [Horizontial axis: the number of generations, Vertical axis: *Gratio*.]

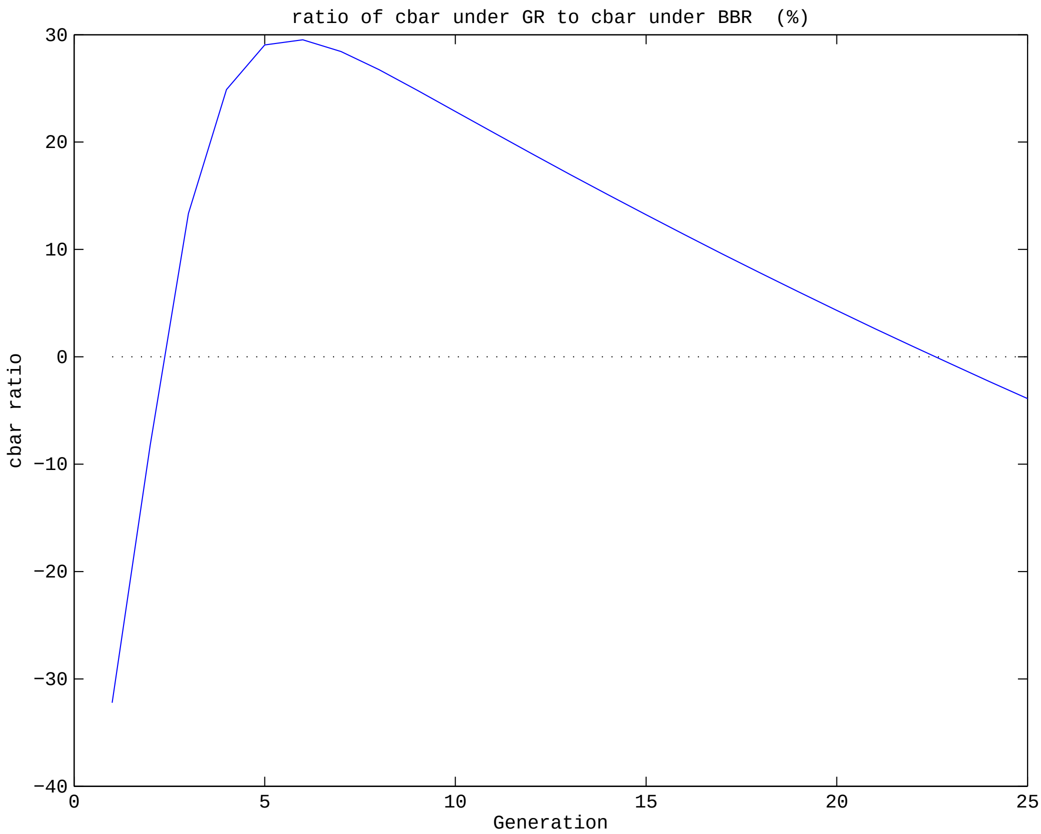

Figure 11: $\bar{c}ratio$ under the case 2 ($B_1 = 1$ and $Y_0 = 5$). [Horizontial axis: the number of generations, Vertical axis: $\bar{c}ratio$.]

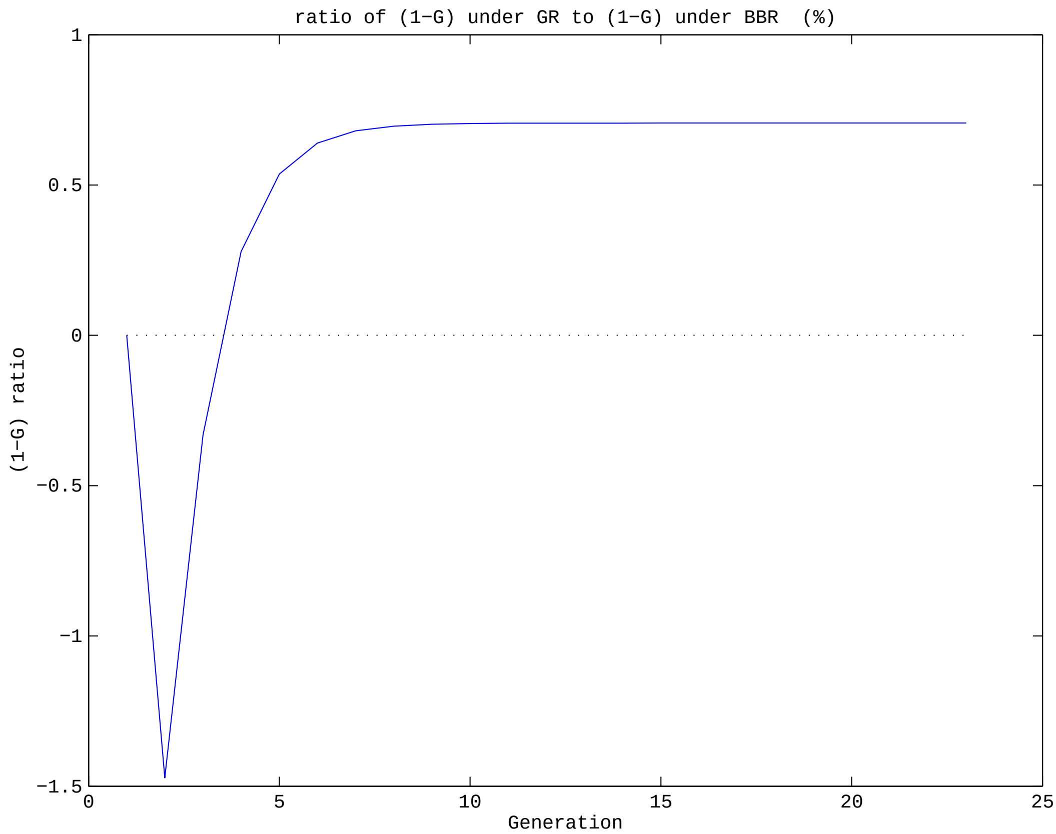

Figure 12: *Gratio* under the case 3 ($B_1 = 1$ and $Y_0 = 10$). [Horizontial axis: the number of generations, Vertical axis: *Gratio*.]

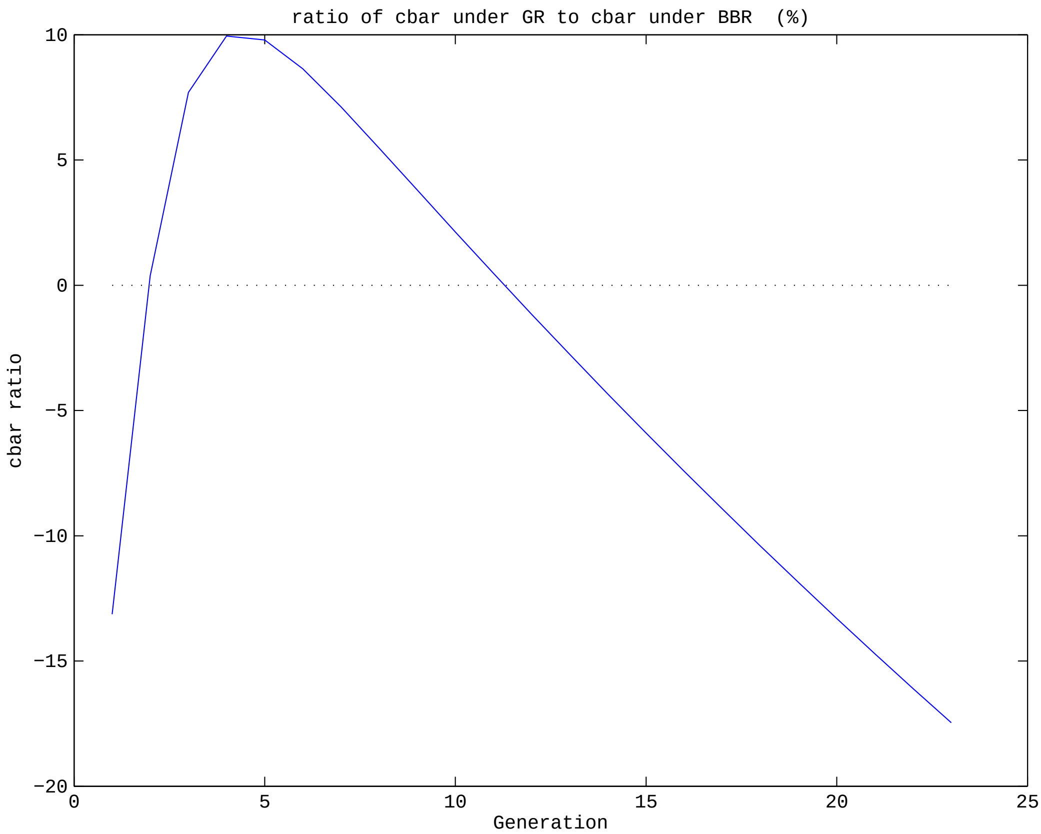

Figure 13: $\bar{c}ratio$ under the case 3 ($B_1 = 1$ and $Y_0 = 10$). [Horizontial axis: the number of generations, Vertical axis: $\bar{c}ratio$.]