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Simulation for prediction of distribution of critical current among specimens under low applied bending strains near the average irreversible strain in Bi2223 composite tape

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Abstract

When many multi-filamentary Bi2223 composite specimens with different irreversible bending strain to each other are tested, number of damaged specimens among all increases with bending strain in the low bending strain region near the average irreversible strain. The present work attempted to predict the distribution of critical current among many specimens in such a low bending strain region with a proposed simulation method that uses the damage evolution model, Monte Carlo method and the experimental data measured at limited strains. With the proposed simulation method, statistical features in variation of distribution of critical current values among specimens near the average irreversible bending strain were elucidated.

Keywords: Superconductors (A); critical current density (C), Stress effects (C)

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1. Introduction

Under tensile/bending applied strain, critical current of Bi2223 composite tape is reversible up to the irreversible strain at which the damage of the Bi2223 filaments takes place first, but beyond the irreversible strain, the critical current decreases irreversibly with increasing applied strain due to the damage evolution[1-13]. The irreversible strain and the relation of critical current to applied strain are different among specimens due to the difference in damage evolution [1-9]. For reliability and safe design, it is needed to develop a description method of the critical current distribution among specimens near the average irreversible strain.

In bent samples, filaments are damaged when the tensile strain along a sample length direction reaches the tensile fracture strain of the filaments embedded in the composite tape, given by $\varepsilon_f - \varepsilon_r$ where $\varepsilon_f$ and $\varepsilon_r$ are the tensile fracture- and residual- strains of the filaments, respectively [7-10, 15]. Recently, the authors have proposed a damage evolution model for analysis of distribution of critical current [7-9], in which the difference in the damage evolution among the specimens was expressed by the distributed $\varepsilon_f - \varepsilon_r$ values in combination with the shape of the core (the region in which Bi2223 filaments are bundled into Ag). With this model, the measured distribution of the normalized critical current ($I_c/I_{c0}$ where $I_c$ and $I_{c0}$ are the critical current values at arbitrary and zero bending strains, respectively) was described well. However, the distribution of $I_c$-values could not be described in that form.

The aim of the present work is to develop a simulation method for prediction of the distributions both of $I_c$ and $I_c/I_{c0}$ values among many specimens in bent Bi2223 multifilamentary composite tape by extending the approach in our preceding work [7-9] and to elucidate the change in distributions of $I_c$ and $I_c/I_{c0}$ values in low bending strain region where the number of damaged specimens increases with increasing strain due to the distributed irreversible strain.

2. Experimental procedure

The multifilamentary Bi2223/Ag/Ag alloy composite tape, fabricated by the powder-in-tube method at Korea Electrotechnology Research Institute (KERI), was used as the experimental sample. It contained 55 Bi2223 filaments. The thickness $t$ and width $W$ of
the sample were 0.23 and 4.1 mm, respectively.

Figure 1(a) shows the as-observed transverse cross-section of the sample [8]. When the thickness direction is enlarged by a factor of 3, the shape of the core can be more clearly observed, as shown with a broken curve in Fig.1(b). As the damage of the Bi2223 filaments existing in the core causes the reduction in critical current, it is necessary to formulate the shape. Taking the width- and thickness- directions of the composite tape as the $x$- and $y$- axes, respectively, and the center of the composite tape as $x=y=0$ (Fig.1(c)), and denoting the $y$-coordinate of the boundary of the core (ABCDEFGHA in Fig.1(c)) as $y_{core}$, we formulated $y_{core}$ as a function of $x$ by 9th order polynomials, as follows [8,15]. The length unit is mm.

\[
\begin{align*}
\text{CDE in Fig.1: } y_{core} &= 0.0660214 - 0.0405173 + 1.03050x^2 - 4.05239x^3 \\
&\quad + 8.94014x^4 - 12.6431x^5 + 11.3895x^6 - 6.2952x^7 \\
&\quad + 1.87094x^8 - 0.235769x^9 \\
&\quad \text{ for } 0 \leq x \leq 1.95
\end{align*}
\]

ABC: symmetry with EDC with respect to $x=0$

AHGFE: symmetry with ABCDE with respect to $y=0$

Equation (1) is used later in Subsection 3.2 for description of the damage evolution and its relation to critical current.

For measurement of critical current $I_c$ under bending strain, the bending strain $\varepsilon_B$ ($=t/(2R)$, where $t$ is the sample thickness (0.23 mm) and $R$ is the radius of dies used to bend specimens, was given to the test specimens by 0 % with straight dies ($R=\infty$) and 0.338 % with dies of $R=34.0$ mm at room temperature. The $I_c$ (and $I_c/I_{c0}$) values at $\varepsilon_B=0$ and 0.338 % were measured for 36 test specimens at 77K in a self-field with a 1 $\mu$V/cm criterion. In addition, the $I_c$ values at $\varepsilon_B=0.290$ % were measured using dies with $R=19.8$ mm for 48 test specimens. The results at $\varepsilon_B=0.290$ % are used to examine the simulation result in Subsection 3.3.

3. Results and discussion

3.1 Description of measured $I_c$- and $I_c/I_{c0}$- values by regression analysis using three-parameter Weibull distribution function
Figure 2(a, b, c) show the probability density $f$ of the measured $I_c$ values at $\varepsilon_B=0$, 0.290 and 0.338%, respectively. The average critical current values, $I_{c,ave}$, were 69.1, 66.0 and 62.7 A, respectively. Figure 3 shows the change of $I_{c,ave}$ with $\varepsilon_B$. The open circles show the present result and the triangles show the values taken from our former work [15]. The solid curve shows the $I_{c,ave}$-$\varepsilon_B$ relation analyzed by the damage evolution model [7-9], whose details are shown later in Subsection 3.2. The average irreversible bending strain $\varepsilon_{B,irr,ave}$ was 0.243%. The experimental results were described well by the damage evolution model. The variation of distribution of $I_c$ values with $\varepsilon_B$ in the low strain region ($\varepsilon_B=0$-0.37% covering $\varepsilon_{B,irr,ave}=0.243$ % in a rectangle in Fig.3) is analyzed in detail in Subsection 3.3.

Distribution of $I_c$ values was formulated with the three-parameter Weibull distribution function [16], according to which the cumulative probability $F$ of $I_c$ is expressed by

$$F(I_c) = 1 - \exp\left\{-\left(\frac{I_c - I_{c,min}}{I_0}\right)^m\right\}$$

(2)

where $I_{c,min}$ is the minimum (lower limit) critical current, $I_0$ is the scale parameter and $m$ is the shape parameter. From the regression analysis of experimental results with Eq.(2), it was found that $I_{c,min}=63.2$, 55.2 and 53.4 A give the highest linearity between $\ln(1-F)^{-1}$ and $\ln(I_c-I_{c,min})$ for the experimental results at $\varepsilon_B=0$, 0.290 and 0.338%, respectively, as shown in Fig.2(a’, b’ ,c’). The estimated parameters of ($I_{c,min}$, $m$, $I_0$) for the distribution of measured $I_c$-values at $\varepsilon_B=0$, 0.290 and 0.338% were (63.2A, 5.85, 6.37A), (55.2A, 5.32, 11.7A) and (53.4A, 3.87, 10.3A), respectively. With these estimated parameter values, the cumulative probability $F$ expressed by Eq. (2) was converted to the density probability $f$ (frequency). The calculated $f$- $I_c$ relations are presented with solid curves in Fig.2(a,b,c), describing well the experimental results.

Probability density $f$ of $I_c/I_c0$ values measured at $\varepsilon_B=0.338$ % is shown in Fig.4(a). The cumulative probability $F(I_c/I_{c0})$ with the three-parameter Weibull distribution is expressed as
\[ F(I_c/I_{c0}) = 1 - \exp \left\{ - \left( \frac{I_c/I_{c0} - (I_c/I_{c0})_{\text{min}}}{(I_c/I_{c0})_0} \right)^m \right\} \]  

(3)

where \((I_c/I_{c0})_{\text{min}}\), \(m\) and \((I_c/I_{c0})_0\) are the lower limit of \(I_c/I_{c0}\) value, shape parameter and scale parameter, respectively. In a similar manner to the analysis of distribution of \(I_c\), the parameter values in Eq.(3) that fit to the experimental result were estimated by the regression analysis. We had \((I_c/I_{c0})_{\text{min}} = 0.715\), \(m = 6.06\) and \((I_c/I_{c0})_0 = 0.207\). The three-parameter Weibull distribution was suitable also for description of \(I_c/I_{c0}\) values, as shown by the high linearity between \(\ln \ln (1 - F)^{-1}\) and \(\ln (I_c/I_{c0})^{-0.715}\) (Fig. 4(b)).

The estimated distribution functions are used in Subsection 3.3 to examine whether the present simulation can reproduce well the experimental results or not.

3.2 Estimation of distribution functions of the \(\varepsilon_f - \varepsilon_r\) and \(\varepsilon_{B,\text{irr}}\) values

The relation of geometry of the cross-section to damage evolution is schematically shown in Fig.1(c). When \(\varepsilon_B\) reaches \(\varepsilon_{B,\text{irr}}\), damage to reduce critical current takes place in the outermost filaments existing at the maximum value of \(y_{\text{core}}\) (Eq.(1)), \(y_{\text{core,max}} = 0.101\) mm in the present sample). When \(\varepsilon_B\) is raised from \(\varepsilon_{B,\text{irr}}\) to \(\varepsilon_{B,i}\) and then to \(\varepsilon_{B,i+1}\), the damage front \(y_f\) moves from \(y_{\text{core,max}}\) to \(y_{f,i}\) and then to \(y_{f,i+1}\), resulting in reduction of the cross-sectional area of the Bi2223 filaments and hence of critical current. According the damage evolution model [7-9], \(\varepsilon_{B,\text{irr}}\) and \(I_c/I_{c0}\) are expressed by using the \(\varepsilon_f - \varepsilon_r\) value and Eq.(1), as follows.

\[ \varepsilon_{B,\text{irr}} = \left( \frac{t/2}{y_{\text{core,max}}} \right) (\varepsilon_f - \varepsilon_r) \]  

(4)

\[ \frac{I_c}{I_{c0}} = \begin{cases} 1 & \text{for } \varepsilon_B \leq \varepsilon_{B,\text{irr}} \\ 1 - 2 \int_0^{w_{\text{core}}/2} \left\{ y_{\text{core}} - \frac{(\varepsilon_f - \varepsilon_r)(t/2)}{\varepsilon_B} \right\} \frac{dx}{A_{\text{core}}} & \text{for } \varepsilon_B \geq \varepsilon_{B,\text{irr}} \end{cases} \]  

(5)
where \( A_{\text{core}} \) is the cross-sectional area of the core and \( W_{\text{core}} \) is the width of the core (distance between A and E in Fig.1(c)). As the \( \varepsilon_f-\varepsilon_i \) values are distributed among specimens, the \( \varepsilon_{B,\text{irr}} \) and \( I_c/I_{c0} \) values are also distributed among specimens (Eqs.(4) and (5)).

If the distribution function of \( \varepsilon_f-\varepsilon_i \) is known in advance, the distributions of \( \varepsilon_{B,\text{irr}} \) and \( I_c/I_{c0} \) can be calculated by Eqs.(4) and (5), respectively, with \( y_{\text{core}} \) given by Eq.(1) and the measured geometrical values of \( y_{\text{core}, \text{max}}=0.101 \text{ mm}, \ t=0.230 \text{ mm}, \ A_{\text{core}}=0.660 \text{ mm}^2 \) and \( W_{\text{core}}=3.90\text{ mm} \). However, the distribution of \( \varepsilon_f-\varepsilon_i \) values is unknown. In the present work, the \( \varepsilon_f-\varepsilon_i \) values were back-calculated by substituting the measured \( I_c/I_{c0} \) values at \( \varepsilon_B=0.338 \% \) and the geometrical factors mentioned above into Eq.(5). The probability density \( f(\varepsilon_f-\varepsilon_i) \) of the obtained \( \varepsilon_f-\varepsilon_i \) values are presented in Fig.5(a).

The average of \( \varepsilon_f-\varepsilon_i \) values, \( (\varepsilon_f-\varepsilon_i)_{\text{ave}} \), was 0.214\%. The estimated \( \varepsilon_f-\varepsilon_i \) values were formulated also by the three-parameter Weibull distribution function, expressed by

\[
F(\varepsilon_f-\varepsilon_i) = 1 - \exp\left\{-\left(\frac{\varepsilon_f-\varepsilon_i-(\varepsilon_f-\varepsilon_i)_{\text{min}}}{(\varepsilon_f-\varepsilon_i)_0}\right)^m\right\} \quad (6)
\]

where \( F(\varepsilon_f-\varepsilon_i) \) is the cumulative probability, and \( (\varepsilon_f-\varepsilon_i)_{\text{min}}, \varepsilon_0 \) and \( m \) are the minimum value of \( \varepsilon_f-\varepsilon_i \), scale parameter and shape parameter, respectively. The parameters \( ((\varepsilon_f-\varepsilon_i)_{\text{min}}, \varepsilon_0 \) and \( m \) for the distribution of \( \varepsilon_f-\varepsilon_i \) values were estimated by the regression analysis to be 0.130\%, 0.0928 \% and 4.19, respectively. The estimated parameter values describe the experimental results as shown with a solid curve in Fig.5(a) and give a high linearity between \( \ln\ln(1-F)^{-1} \) and \( \ln\{\varepsilon_f-\varepsilon_i-(\varepsilon_f-\varepsilon_i)_{\text{min}}\} \) as shown in Fig.5(b).

The relation of \( \varepsilon_f-\varepsilon_i \) to \( \varepsilon_{B,\text{irr}} \) is expressed by Eq.(4). Substituting \( \varepsilon_f-\varepsilon_i=\varepsilon_{B,\text{irr}}y_{\text{core, max}}/(t/2) \) derived from Eq.(4) into Eq.(6), we have the cumulative distribution function of \( \varepsilon_{B,\text{irr}} \):

\[
F(\varepsilon_{B,\text{irr}}) = 1 - \left[1 - \exp\left\{-\left(\frac{y_{\text{core, max}}/(t/2)}{y_{\text{core, max}}/(t/2)}\right)\left(\frac{\varepsilon_{B,\text{irr}} - (\varepsilon_f-\varepsilon_i)_{\text{min}}}{y_{\text{core, max}}/(t/2)}\right)^m\right\}\right]^{m} \quad (7)
\]
Substituting \( y_{\text{core, max}} = 0.101 \) mm, \((\varepsilon - \varepsilon_r)_{\text{min}} = 0.130\%\), \( m = 4.19\), \( \varepsilon_0 = 0.0928 \) % and \( t = 0.230 \) mm into Eq. (7), we have the cumulative distribution \( F(\varepsilon_{B, \text{irr}}) \) of \( \varepsilon_{B, \text{irr}} \) for the present sample as shown in Fig. 6. The average of \( \varepsilon_{B, \text{irr}} \) values, \( \varepsilon_{B, \text{ave}} \), was 0.243%. When many specimens whose \( \varepsilon - \varepsilon_r \) values obey Eq. (6) are tested, the specimen with the lowest \( \varepsilon - \varepsilon_r \) value \( ((\varepsilon - \varepsilon_r)_{\text{min}} = 0.130\%) \) at \( y_{\text{core, max}} \) is damaged first among all specimens when \( \varepsilon_B \) reaches its \( \varepsilon_{B, \text{irr}-\text{value}} \) \( (=\varepsilon_{B, \text{irr, \text{min}}}/(t/2)/y_{\text{core, max}} = 0.148\%) \). No specimen is damaged up to \( \varepsilon_B = 0.148\% \). The number of damaged specimens increases with increasing \( \varepsilon_B \), in accordance with increase in \( F(\varepsilon_{B, \text{irr}}) \). At \( \varepsilon_B \), the specimens whose \( \varepsilon_{B, \text{irr}} \) values are lower than \( \varepsilon_B \) have been damaged. The fraction of the damaged specimens \( F_d(\varepsilon) \) is calculated by replacing \( \varepsilon_{B, \text{irr}} \) by \( \varepsilon_B \) in Eq. (7). Namely, the \( F_d(\varepsilon_B) - \varepsilon_B \) curve is the same as \( F(\varepsilon_{B, \text{irr}}) - \varepsilon_{B, \text{irr}} \) one. Accordingly, as indicated in Fig. 6, at \( \varepsilon_B = 0.20, 0.24, 0.29 \) and 0.32 % for instance, the fractions \( F_d \) of the damaged specimens increase to 0.051, 0.435, 0.97 and 0.9996, respectively. \( F_d(\varepsilon_B) - \varepsilon_B \) curve in Fig. 6 shows that, in the range of low bending strain (0-0.37% in this work, in which \( \varepsilon_{B, \text{irr, ave}} = 0.243\% \) is included), there exist three regions where no specimen is damaged (region (i) for \( \varepsilon_B = 0-0.15\% \)), damaged and non-damaged specimens co-exist (region (ii) for \( \varepsilon_B = 0.15-0.32\% \)) and almost all specimens are damaged (\( \varepsilon_B = 0.32-0.37\% \)).

3.3 Monte Carlo simulation to predict the distribution of \( I/I_{c0} \) and \( I_c \) values at arbitrary bending strain from the distribution of original critical current \( I_{c0} \) and distribution of \( \varepsilon - \varepsilon_r \) values estimated from the distribution of \( I_c/I_{c0} \) values measured at \( \varepsilon_B = 0.338 \) %

The distributions of the \( I/I_{c0} \) and \( I_c \) values were simulated by the following procedure in which the distributed \( \varepsilon - \varepsilon_r \) values given by Eq. (6) were combined with Eq. (5). As a first approximation, the distribution function of \( \varepsilon - \varepsilon_r \) at \( \varepsilon_B = 0.338 \) % was used for simulation of distribution of \( I_c \) values at arbitrary bending strain range (\( \varepsilon_B = 0-0.37\% \)).
(1) Determination of εᵢᵢ value of the i-specimen, (εᵢᵢ), where i is the specimen number among the total number 300 of simulated specimens: A random value R(i) was generated in the computer. Setting \( F(\varepsilon - \varepsilon_i) = R(i) \) and substituting the estimated values of \( (\varepsilon_i - \varepsilon_c)_{\text{min}} = 0.130 \% \), \( m = 4.19 \) and \( \varepsilon_0 = 0.0928 \% \) in Eq.(6), we had the \( \varepsilon_i - \varepsilon_c \) value of the i-specimen, (\( \varepsilon_i - \varepsilon_c \)).

(2) Determination of \( I_i/I_{c0} \) value of the i-specimen, (\( I_i/I_{c0} \)): As shown in Subsection 3.2, at a given \( \alpha_B \), the specimens with \( \varepsilon_i - \varepsilon_c \) values lower than \( \alpha_B y_{\text{core, max}}/(t/2) \) have been damaged but not those with \( \varepsilon_i - \varepsilon_c \) values higher than it. In accordance with such a situation, when the \( (\varepsilon_i - \varepsilon_c)_i \) value obtained in (1) was higher than \( \alpha_B y_{\text{core, max}}/(t/2) \), the \( (I_i/I_{c0})_i \) was set to be 1 (unity). When the \( (\varepsilon_i - \varepsilon_c)_i \) was lower than \( \alpha_B y_{\text{core, max}}/(t/2) \), the \( (\varepsilon_i - \varepsilon_c)_i \) value, \( y_{\text{core}} \) (Eq.(1)), \( \alpha_B \) value and \( t = 0.23 \) mm were substituted into Eq.(5) and the \( I_i/I_{c0} \) value of the i-specimen was calculated.

(3) Determination of the \( I_c \) value of the i-specimen, (\( I_c \)): Figure 7 shows the plot of measured \( I_c \)-value at \( \alpha_B = 0.338 \% \) against the original critical current \( I_{c0} (= I_c \text{ at } \alpha_B = 0\%) \) for each specimen. The \( I_c \) values are almost independent of \( I_{c0} \) values. This suggests that the damage at \( \alpha_B = 0.338 \% \) took place almost evenly for high- and low- \( I_{c0} \) specimens and the correlation between the damage behavior and \( I_{c0} \) value is, if exists, small. The same feature has been observed for the sample from different supplier [7]. Based on these experimental results, the \( I_{c0,i} \) value was treated to be independent of \( I_{c,i} \) in the present simulation. The \( I_{c0,i} \) value of the i-specimen was obtained by generating another random value \( R'(i) \) different from \( R(i) \), setting \( F(I_{c0}(= I_c \text{ at } \alpha_B = 0\%)) = R'(i) \) and substituting the estimated values of \( I_{c,\text{min}} = 63.2 \text{A}, m = 5.85 \) and \( I_0 = 6.37 \text{A} \) for distribution of \( I_{c0} \) into Eq.(2). Then, the \( I_{c,i} \) value was obtained from the values of \( (I_i/I_{c0})_i \) and \( I_{c0,i} \).

(4) Determination of \( (I_i/I_{c0})_i \) and \( I_{c,i} \) values for \( i = 1 \) to 300 at \( eB = 0 \) to 0.37%: The procedure (1) to (3) were repeated for 300 times, and \( (I_i/I_{c0})_i \) and \( I_{c,i} \) values \( (i = 1 \text{ to } 300) \) were obtained for a given \( \alpha_B \). Such a simulation was carried out for \( \alpha_B = 0, 0.15, 0.17, 0.20, 0.23, 0.26, 0.29, 0.32, 0.338, 0.35 \) and 0.37%.

Figure 8 shows the representative simulation results for probability density \( f \) of the \( I_c \) and \( I_i/I_{c0} \) values, respectively. The regression curves of the experimental results (solid curves
in Fig.2(a), (b), and (c) for $I_c$ values at $\varepsilon_B=0$, 0.290 and 0.338 % and a solid curve in Fig.4(a) for $I/I_{c0}$ values at $\varepsilon_B=0.338 \%$ are redrawn with solid curves in Fig.8, for comparison. The experimental results were reproduced well. It is noted that the measured distribution of $I_c$ values at $\varepsilon_B=0.29$ % was described well, despite that the distribution of $\varepsilon_f$--$\varepsilon_r$ values estimated from the experimental data of $I/I_{c0}$ values at $\varepsilon_B=0.338 \%$ was used for simulation. This suggests that the distribution of $\varepsilon_f$--$\varepsilon_r$ values estimated at a limited bending strain can be employed without serious loss of accuracy for simulation to predict the distribution of $I_c$ (and $I/I_{c0}$) values at other bending strains, when the bending strain range in simulation is near to the bending strain at which $\varepsilon_f$--$\varepsilon_r$ values are estimated.

The variations of simulated average of $I_c$, $I_{c,\text{ave}}$, and average of $I/I_{c0}$, $(I/I_{c0})_{\text{ave}}$, with $\varepsilon_B$ are shown in Fig.9(a). The open circles show the measured $I_{c,\text{ave}}$ values for comparison. The solid curve shows the $I_{c,\text{ave}}$--$\varepsilon_B$ relation calculated by substituting $y_{\text{core}}$ (Eq.(1)), $I_{0,\text{ave}}=69.1$ A, $\varepsilon_{B,\text{irr,ave}}=0.243$ %, $(\varepsilon_f-\varepsilon_r)_{\text{ave}}=0.214$ %, $y_{\text{core,\text{max}}}=0.101$ mm, $t=0.230$ mm, $A_{\text{core}}=0.660$ mm$^2$, $W_{\text{core}}=3.90$mm into Eq.(5). This curve corresponds to the assumed case where all specimens have a common unique value of $\varepsilon_f$--$\varepsilon_r=0.214\%$ that corresponds to $\varepsilon_{B,\text{irr}}=0.243$ % (Eq.(4)).

In experiment, the coefficient of variation (COV=average/standard deviation) of critical current, COV($I_c$), was measured to be 0.0165, 0.0383 and 0.0427 at $\varepsilon_B=0$, 0.290 and 0.338 %, respectively. The COV of normalized critical current, COV($I/I_{c0}$), is zero at $\varepsilon_B=0$ %. It increased to 0.0400 at $\varepsilon_B=0.338$%. Both COV($I_c$) and COV($I/I_{c0}$) increased largely due to the difference in damage evolution among test specimens. The variation of COV($I_c$) with increasing $\varepsilon_B$ is also an important item for reliability and safe design. The measured and simulated COV($I_c$) and COV($I/I_{c0}$) values are plotted against $\varepsilon_B$ in Fig.9(b). The following statistical features in the low bending strain range are read from Figs.9(a) and (b).

(A) The $I_{c,\text{ave}}$--$\varepsilon_B$ and $(I/I_{c0})_{\text{ave}}$--$\varepsilon_B$ curves for $\varepsilon_B=0.15$ to around 0.30 % (broken curve in Fig.9(a)) simulated statistically with distributed $\varepsilon_f$--$\varepsilon_r$ values with an average of $(\varepsilon_f-\varepsilon_r)_{\text{ave}}=0.214\%$, are lower than the curves calculated non-statistically with a fixed value of
(\varepsilon - \varepsilon_r)_{ave} = 0.214\% \text{ (solid curve).} \text{ In the statistically analyzed } I_{c,ave} - \varepsilon_B \text{ and } (I_c/I_{c0})_{ave} - \varepsilon_B \text{ curves, the } I_{c,ave} \text{ and } (I_c/I_{c0})_{ave} \text{ values flatten off and approach 1 (unity) asymptotically with decreasing } \varepsilon_B. \text{ The minimum irreversible strain } \varepsilon_{B,irr,min} \text{ for distributed } \varepsilon - \varepsilon_r \text{ is calculated to be 0.148\% for } (\varepsilon - \varepsilon_r)_{min} = 0.130\% \text{ (Eq. (4))}, \text{ which is by 0.09\% lower than } \varepsilon_{B,irr,ave}. \text{ The largest difference in } I_{c,ave} \text{ and } (I_c/I_{c0})_{ave} \text{ between the statistical simulation and non-statistical calculation is found at around } \varepsilon_B = \varepsilon_{B,irr,ave} = 0.243\% \text{ at which around half of specimens are damaged (Fig.6). When } \varepsilon_B \text{ exceeds 0.3\% where most specimens are damaged, the difference in } I_{c,ave} \text{ and } (I_c/I_{c0})_{ave} \text{ between them becomes small.}

(B) \text{ COV}(I_c) \text{ and } \text{COV}(I_c/I_{c0}) \text{ can be obtained by the statistical simulation but not by the non-statistical calculation. The present statistical simulation revealed the following features for the variation of } \text{COV}(I_c) \text{ and } \text{COV}(I_c/I_{c0}) \text{ with } \varepsilon_B. \text{ (a) Both } \text{COV}(I_c) \text{ and } \text{COV}(I_c/I_{c0}) \text{ start to increase at } \varepsilon_B = \varepsilon_{B,irr,ave}. \text{ (b) The } \text{COV}(I_c) \text{ and } \text{COV}(I_c/I_{c0}) \text{ increase significantly with } \varepsilon_B \text{ in the neighborhood of } \varepsilon_{B,irr,ave} \text{ in accordance with the increase in fraction } F_d \text{ of damaged specimens. (c) The } \text{COV}(I_c) \text{ is affected by the distribution of } I_{c0} \text{ and that of extent of damage, while } \text{COV}(I_c/I_{c0}) \text{ is affected by the latter distribution. Accordingly, the } \text{COV}(I_c) \text{ is higher than } \text{COV}(I_c/I_{c0}) \text{ in the whole range of bending strain. (d) The difference between } \text{COV}(I_c) \text{ and } \text{COV}(I_c/I_{c0}) \text{ becomes small at high applied strain (>0.3\%), suggesting that the } \text{COV} \text{ for both } I_c \text{ and } I_c/I_{c0} \text{ values at high bending strains is governed mainly by the difference in damage evolution among specimens.}

4. Conclusions

(1) A simulation method was proposed for prediction of the distribution of } I_c \text{-values among many specimens at arbitrary bending strain within } 0 \sim 0.37\%, \text{ based on the damage evolution model that uses measured distribution of the normalized critical current } (I_c/I_{c0}) \text{-values at a bending strain (0.338\%)} \text{ and the distribution of the measured original critical current } I_{c0} \text{ values at zero bending strain. With this method, the experimental results for the measured } I_c \text{-values were reproduced well in the computer.}

(2) The simulation method proposed in (1) was applied for description of the } I_c \text{ and } I_c/I_{c0} \text{-
values near the average irreversible bending strain. The decreasing behavior of the average $I_c$- and $I_c/I_{c0}$ values and the increasing behavior of the coefficient of variation of $I_c$- and $I_c/I_{c0}$ values in the transition bending strain range where the fraction of damaged specimens increases were described satisfactorily.

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References


Figure Captions

Fig. 1 Transverse cross-section of the composite tape [8]. (a) shows the as-observed optical micrograph. (b) shows the modified micrograph, in which the thickness direction is enlarged by a factor of 3 from (a) and the core boundary is shown with a broken curve. (c) shows the relation of geometry of the cross-section to damage extension (when $\varepsilon_B$ reaches $\varepsilon_{B,irr}$, damage occurs first at $y_{core}=y_{core,max}$, and when $\varepsilon_B$ is raised from $\varepsilon_{B,irr}$ to $\varepsilon_{B,i}$ and then to $\varepsilon_{B,i+1}$, the damage front $y_f$ moves from $y_{core,max}$ to $y_{f,i}$ and then to $y_{f,i+1}$.)

Fig. 2 Histogram of the measured $I_c$-values at $\varepsilon_B=$(a) 0, (b) 0.290 and (c) 0.338 % and plot of $\ln\ln\{1-F(I_c)\}^{-1}$ against $\ln(I_c-I_{c,min})$ for the measured $I_c$-values at $\varepsilon_B=$(a’) 0, (b’) 0.290 and (c’) 0.338 %, where the $I_{c,min}$ values used here (63.2, 55.2 and 53.4 A at $\varepsilon_B=$(a”) 0, (b”) 0.290 and (c”) 0.338 %, respectively) were estimated as the values to give the highest linearity between $\ln\ln\{1-F(I_c)\}^{-1}$ and $\ln(I_c-I_{c,min})$. Solid curves in (a), (b) and (c) show the obtained regression curves.

Fig. 3 Variation of the measured $I_c$-ave with $\varepsilon_B$. Open circles and triangles show the experimental results of the present and former [15] works, respectively. Solid curve shows the $I_{c,ave}$-$\varepsilon_B$ relation analyzed by the damage evolution model shown later in Subsection 3.2. Distribution of $I_c$-values in low $\varepsilon_B$ region, indicated with a rectangle, is analyzed in detail in Subsection 3.3.

Fig. 4 Distribution of the normalized critical current ($I_c/I_{c,0}$) values measured at $\varepsilon_B=0.338\%$. (a) Probability density $f(I_c/I_{c,0})$. (b) Plot of $\ln\ln\{1-F(I_c/I_{c,0})\}^{-1}$ against $\ln(I_c/I_{c,0}-0.715)$ that best-fit to the experimental results. Solid curve in (a) shows the result of regression analysis.

Fig. 5 Distribution of the $\varepsilon_{f}-\varepsilon_{r}$ values at $\varepsilon_B=0.338\%$. (a) Probability density $f(\varepsilon_{f}-\varepsilon_{r})$. (a) Plot of $\ln\ln\{1-F(\varepsilon_{f}-\varepsilon_{r})\}^{-1}$ against $\ln(\varepsilon_{f}-\varepsilon_{r} - 0.130)$ that best-fits to the experimental results. Solid curve in (a) shows the result of regression analysis.
Fig.6 Variation of the cumulative probability $F(\varepsilon_{B,irr})$ with irreversible bending strain $\varepsilon_{B,irr}$, corresponding to the variation of the fraction $F_d$ (=ratio of the number of damaged specimens to that of all specimens) with bending strain $\varepsilon_B$. Regions (i), (ii) and (iii) refer to the bending strain ranges where no specimen is damaged ($\varepsilon_B<0.148\%$), non-damaged and damaged specimens decrease and increase, respectively ($0.148\%<\varepsilon_B<0.32\%$) and almost all specimens are damaged ($\varepsilon_B>0.32\%$).

Fig.7 Plot of the critical current $I_c$ at $\varepsilon_B=0.338\%$ against the original critical current $I_{c0}$ at $\varepsilon_B=0\%$.

Fig.8 Simulation result on the variation of (a to f) $I_c$- and (a’ to f’) $I_c/I_{c0}$- distributions with increasing bending strain, together with the regression curves of the experimental results (taken from Figs.2(a), (b) and (c) for $I_c$ values at $\varepsilon_B=0$, 0.290 and 0.338 % and from Fig.6(a) for $I_c/I_{c0}$ values at $\varepsilon_B=0.338\%$), for comparison.

Fig.9 Variations of (a) average critical current $I_{c,ave}$ and normalized critical current $(I_c/I_{c0})_{ave}$ and (b) coefficient of variation of $I_c$- and $I_c/I_{c0}$-values with bending strain $\varepsilon_B$. 
Fig. 1

Fig. 2
**Fig. 3**

![Graph showing the relationship between average critical current and bending strain.](image)

**Fig. 4**

- **a)** Probability density function $f(I_c/I_{c0})$ for $I_c = 0.338\%$.
- **b)** Plot of $\ln(1-F(I_c/I_{c0})^{-1}$ vs $\ln(I_c/I_{c0} - (I_c/I_{c0,\min})$, with parameters $m = 6.06$ and $(I_c/I_{c0})_0 = 0.207$.}

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Fig. 9