Torsion-Bar Antenna for Low-Frequency Gravitational-Wave Observations

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We propose a novel type of gravitational-wave antenna, formed by two bar-shaped test masses and laser-interferometric sensors to monitor their differential angular fluctuations. This antenna has a fundamental sensitivity to low-frequency signals below 1 Hz, even with a ground-based configuration. In addition, it is possible to expand the observation band to a lower limit determined by the observation time, by using modulation and up-conversion of gravitational-wave signals by rotation of the antenna. The potential sensitivity of this antenna is superior to those of current detectors in a 1 mHz–10 Hz frequency band and is sufficient for observations of gravitational waves radiated from in-spiral and merger events of intermediate-mass black holes.

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Introduction.—Direct observation of gravitational waves (GWs) is expected to provide new information that is independent of, or complementary to, astronomical observations by electromagnetic waves [1]. Since the wavelengths or frequencies of radiated GWs are determined by the scales of the sources, observation of GWs with various frequency bands is required for a clearer understanding of the hierarchical structure of the Universe.

Observations of high-frequency GWs above 10 Hz have been attempted mainly by using ground-based detectors: resonant-mass detectors and laser-interferometric detectors. A resonant-mass detector, typically a large aluminum bar, monitors its resonant oscillation excited by a tidal force of GWs at around 1 kHz [2]. On the other hand, a laser-interferometric GW detector monitors the changes in the optical path lengths in two orthogonal directions, with a wider observation band between 10 Hz and a few kilohertz [3]. Observations of low-frequency GWs have been tried by space-borne experiments and astrophysical observations: Doppler tracking and pulsar timing. In Doppler tracking, low-frequency GWs between 10^{-6} and 10^{-4} Hz have been searched by using the long baseline between Earth and a spacecraft cruising to the outer planets [4]. Pulsar timing uses stable radio pulsars as precise clock references. GWs propagating between Earth and the target pulsar would be detected as timing shifts in the pulses [5]. From long-term intermittent observations of pulsars for over 10 years, the upper limit on GW amplitude has been set at around 10^{-8} Hz.

For the direct detection of GWs and for new astrophysics and cosmology, several future detectors have been proposed. As ground-based detectors, advanced interferometric antenna projects such as Advanced Laser Interferometer Gravitational-Wave Observatory, Advanced VIRGO, and Large-Scale Cryogenic Gravitational-Wave Telescope [6] will be operational in this decade, and the Einstein Telescope [7] will follow them. In space, the Laser Interferometer Space Antenna (LISA) is expected to have good sensitivity at around 1 mHz [8]. The DECi-Hertz Interferometer Gravitational-wave Observatory and Big Bang Observer will be follow-up space missions of LISA, targeting around the 0.1 Hz band [9,10]. This band is also a target of Atomic Gravitational-Wave Interferometric Sensor, a novel detector that uses atomic interferometers [11]. In pulsar-timing observations, plans of pulsar-timing arrays have been proposed; their sensitivity will be improved by cross correlation of signals from multiple pulsars [12]. For extremely low frequencies, projects have been proposed for observing primordial GWs by using *B*-mode polarization of the cosmic microwave background [13].

In this Letter, we propose a novel type of GW antenna, named TOBA (torsion-bar antenna), for low-frequency observations; this antenna can be ground-based or used in a space-borne mission. In contrast to conventional detectors, it has a fundamental sensitivity to GWs below 1 Hz even with a ground-based design. As a space-borne mission, it will be a simpler mission with a single spacecraft. This antenna is formed by two bar-shaped test masses and laser-interferometric sensors to monitor their differential angular fluctuations. A characteristic feature of this antenna is that it can expand the observation band to lower frequencies by using modulation and up-conversion of GW signals by rotation of the antenna. A similar concept was proposed about 40 years ago as a heterodyne detector for circular-polarized GWs [14,15], in which modulation of the GW signal is used for the down-conversion and accumulation of GW effects. We reinvestigated the idea for the up-conversion of low-frequency GWs and estimated its potential sensitivity and scientific possibility in modern technologies.

Principle of a torsion-bar antenna.—A TOBA is comprised of two bar-shaped test masses, arranged parallel to the x-y plane and orthogonal to each other (Fig. 1). Each bar is supported at its center, so as to rotate around the z axis. When GWs pass through this antenna, tidal forces by the GWs will appear as differential angular changes in these bars. These changes are extracted as a GW signal by using a sensitive sensor, such as a laser interferometer.

The rotation angle θ of a test-mass bar from the original position is obtained by the equation of motion

$$I\ddot{\theta} + \gamma\dot{\theta} + \kappa\theta = F_{\rm GW}(t), \tag{1}$$

where $F_{\rm GW}$ is the torque caused by the GW and γ and κ are the damping factor and spring constant of the restoring torque by the support, respectively. The moment of inertia of the test mass is expressed as $I = \int \rho(x^2 + y^2)dV$, where ρ and V are the density and volume of the test mass, respectively. Assuming that the antenna is much smaller than the wavelength of target GWs, the torque caused by a GW with an amplitude of h_{ij} is expressed as

$$F_{\rm GW}(t) = \tfrac{1}{4}q^{ij}\ddot{h}_{ij}(t), \tag{2}$$

by using dynamic quadrupole moment q^{ij} [16]. For bar rotation, $q_{11} = -q_{22} = -\int \rho(2xy)dV$ and $q_{12} = q_{21} = \int \rho(x^2 - y^2)dV$.

Here, we consider the response of a test-mass bar arranged along the x direction to GWs traveling along the z axis, $h_{11} = -h_{22} = h_+$ and $h_{12} = -h_{21} = h_\times$, where h_+ and $h_\times$ denote the amplitudes of two independent polarizations (plus and cross modes, respectively) of incident GWs. In an approximation that the test-mass bar freely rotate around the z axis ($\gamma \ll 1$ and $\kappa \ll 1$), Eq. (1) results in a simple equation, $\theta_1 = \alpha h_\times/2$, where α is a shape factor of the test mass; $\alpha = q_{12}/I \simeq 1$ in the case of a thin bar. Another test-mass bar, arranged along the y axis, rotates with an opposite sign as $\theta_2 = -\alpha h_\times/2$. The resultant output of the antenna is expressed as

$$\theta_{\rm diff}(t) \equiv \theta_1 - \theta_2 = \alpha h_\times(t). \tag{3}$$

GWs with a cross polarization are observed as differential angular fluctuations of the test-mass bars [17].

Now, we consider the situation that the antenna is rotating around the z axis with an angular velocity of $\omega_{\rm rot}$. In an approximation that rotation is sufficiently slow, the response of the antenna is expressed as

$$\ddot{\theta}_{\rm diff} = \alpha[\ddot{h}_\times \cos(2\omega_{\rm rot}t) + \ddot{h}_+ \sin(2\omega_{\rm rot}t)], \tag{4}$$

by calculating the torque, Eq. (2), in a coordinate rotating with the antenna. This indicates that the GW signal is modulated by the rotation; a GW signal with an angular frequency of ω_g is up- and down-converted to appear at $\omega_g \pm 2\omega_{\rm rot}$ frequencies. Equation (4) results in

$$\theta_{\rm diff} \simeq \alpha\left(\frac{\omega_g}{2\omega_{\rm rot}}\right)^2[h_\times \cos(2\omega_{\rm rot}t) + h_+ \sin(2\omega_{\rm rot}t)], \tag{5}$$

in the case of $\omega_g \ll \omega_{\rm rot}$. The low-frequency GW signal is up-converted to signals at an angular frequency of $2\omega_{\rm rot}$. Equation (5) also shows that two polarization components of incident GWs are extracted from two quadrature phases of the antenna output.

Advantages of torsion-bar antenna.—A TOBA has significant features in both ground-based and space-borne designs. As a ground-based antenna, a TOBA is a novel approach to observe low-frequency GWs. In a usual ground-based interferometric antenna, a test-mass mirror is suspended as a pendulum to behave as a free mass in the horizontal plane. Conversely, it has almost no fundamental sensitivity to GWs below the resonant frequency of the pendulum (around 1 Hz). In a TOBA, a test-mass bar is supported as a torsion pendulum, with a low resonant frequency on the order of a few millihertz in the rotational degree of freedom. Thus, a TOBA has a fundamental sensitivity to low-frequency GWs.

The modulation and up-conversion scheme by antenna rotation is favorable for the observation of low-frequency GWs below a few millihertz. Here, we note that the observation run may be an intermittent one; the observation can be a series of data-taking operations with rotation and reverse rotation. The up-conversion of the GW signal is also advantageous from a practical perspective. Modulation prevents various types of low-frequency noises that are difficult to suppress, such as drifts of instruments caused by daily or seasonal changes in the environment and $1/f$ noises of electronics in sensors and controllers.

As a space antenna, a TOBA has good compatibility with spin-stabilized spacecraft. In a spacecraft, spinning itself is a simple and robust way to maintain its attitude with a gyro effect, without additional disturbances from attitude controllers. A TOBA, with its rotation axis aligned with that of the spacecraft spin, has a wide observation band from the low-frequency limit determined by the observation time. Another advantage in a space configuration is that the antenna is free from gravity-gradient and seismic noises caused by ground motions.

Sensitivity limits.—The fundamental sensitivity of a TOBA is limited by the thermal fluctuation of the bars, readout noise of the angular motion, and effects of the bar support, as detailed in Refs. [1,18]. We estimate the contributions of these noises in the case where a cylindrical bar is suspended as a torsion pendulum at its center, and its

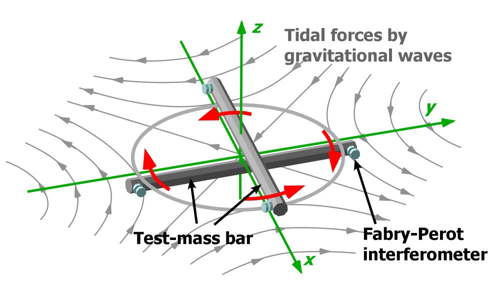

FIG. 1 (color online). Principle of a torsion-bar antenna. Two orthogonal bars feel differential torques by incident gravitational waves.

angular fluctuation is measured by using a laser interferometer at both edges.

The thermal fluctuation of the bar appears as differential displacements at both edges. This noise spectral density is described as $\delta\theta_{\rm mass} \simeq \frac{8}{L}\sqrt{\phi_{\rm mass} k_B T/(M\omega_{\rm bar}^2 \omega)}$ at a frequency below the lowest resonant angular frequency, $\omega_{\rm bar}$, of the bar. Here, $\phi_{\rm mass}$, M, L, T, ω, and k_B are the intrinsic mechanical loss angle, mass, length, temperature of the test-mass bar, angular frequency, and the Boltzmann constant, respectively.

The sensitivity of the antenna is also limited by the optical readout noise of the laser interferometer. It is a fundamental noise that originates from the quantum nature of the laser light; it appears as shot noise $\delta\theta_{\rm shot} = \frac{1}{2LN} \times \sqrt{\hbar c \lambda/(\pi P_{\rm in})}$ and radiation-pressure noise $\delta\theta_{\rm RP} = \frac{2LN}{I\omega^2} \times \sqrt{\pi \hbar P_{\rm in}/(c\lambda)}$ in a power spectral density. Here, $P_{\rm in}$, λ, $\hbar$, and c are the power and wavelength of the input laser beam, the reduced Planck constant, and the speed of light, respectively. The factor $N = 2\mathcal{F}/\pi$ is a bounce number in the Fabry-Perot cavities determined by a finesse $\mathcal{F}$. The standard quantum limit (SQL) is calculated as $\theta_{\rm SQL} = \sqrt{2\delta\theta_{\rm shot}\delta\theta_{\rm RP}} = \sqrt{2\hbar/I\omega^2}$.

In a practical situation, the performance of an antenna is also limited by the effect of the test-mass support, which appears in nonzero damping and restoring torques described by γ and κ. With a finite restoring torque, the response of the antenna to GWs is degraded below the resonant angular frequency $\omega_{\rm sus} = \sqrt{\kappa/I}$. On the other hand, a damping torque causes thermal fluctuation; the spectrum is estimated to be $\delta\theta_{\rm sus} = \sqrt{4\gamma k_B T}/(I\omega^2)$, assuming viscous damping.

Sensitivity estimation.—Here, we estimate an example sensitivity of a TOBA to GWs, assuming parameters that will be realistic in the near future. In this design, the test mass is an aluminum bar with a length of 10 m, diameter of 0.6 m, mass of 7600 kg, and loss angle of 10^{-7}, kept at a temperature of 4 K. The moment of inertia is $I = 6.4 \times 10^4$ N m s^2. The bar is supported as a torsion pendulum with a resonant frequency of 1 mHz and a damping factor of $\gamma = 10^{-10}$ N m s. The bar rotational motion is measured by using a laser interferometer; short Fabry-Perot cavities with a finesse of 100 are formed at both ends of the bar. A Nd:YAG laser with a wavelength of 1064 nm and a power of 10 W is used as a laser source.

The thick black curve in Fig. 2 represents the antenna sensitivity estimated by a quadrature sum of the fundamental noises. The antenna has nearly the same sensitivity as in the case of rotated operation; the ω^{-2} noise dependence in low frequency follows the response degradation in the rotation mode, as shown in Eq. (5). Besides these noises, there will be practical noises of seismic disturbances and gravity-gradient noise in a ground-based configuration, as represented by the dotted lines. Figure 3 shows a comparison of the antenna sensitivity with other operating and proposed detectors.

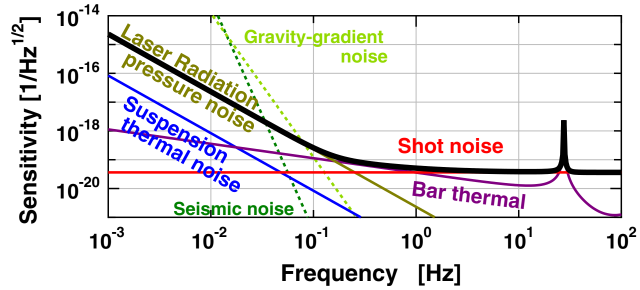

FIG. 2 (color online). Estimated sensitivity of a torsion-bar antenna (thick black curve). Limitations by fundamental noises are shown together.

With this sensitivity, the observation target of a TOBA are in-spiral and merger signals from compact binaries, continuous signals from pulsars, and stochastic background GWs from the early Universe. Among them, mergers of intermediate-mass black holes are possible sources; the observable range reaches 10 Gpc for a $10^5 M_\odot$ black-hole event with a one-year observation period until the merger, with the detection threshold being a signal-to-noise ratio (SNR) of 5 (Fig. 4, for sources at the optimal direction) [19]. In addition, a TOBA can provide a warning for a lower-mass binary merger prior to the observations by ground-based GW detectors and by electromagnetic counterparts [20]. For background GWs, the sensitivity with a one-year observation period by multiple antennas will be $\Omega_{\rm GW} \leq 10^{-7}$ around the 0.1 Hz frequency band for the dimensionless energy density ratio of GWs [4,5,21,22].

Discussions.—The potential sensitivity limit of a TOBA is described by the SQL of the optical readout system. The dependence of the SQL on the antenna parameters, $h_{\rm SQL} \propto 1/(M^{1/2}L)$, is the same as that for an interferometric antenna with an arm length of L and a mirror mass of M. Thus, the ultimate sensitivity will be reached by a laser-interferometric antenna, which can have a longer

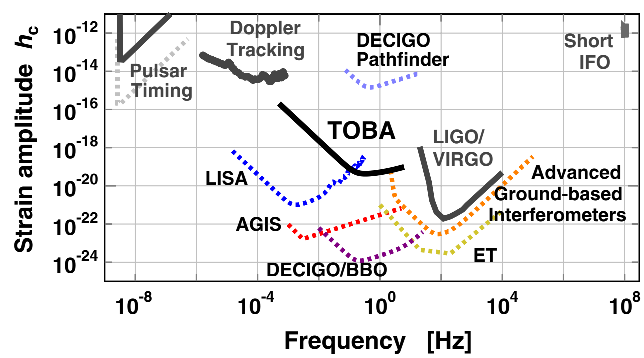

FIG. 3 (color online). Comparison of sensitivities in h_c, the characteristic amplitude of GWs defined as an amplitude with a bandwidth equal to the observation frequency [1]. The sensitivity of the TOBA is represented by a bold curve. Thick gray curves correspond to the current detectors [3–5,21], and the dotted curves correspond to designs of future projects [6–12].

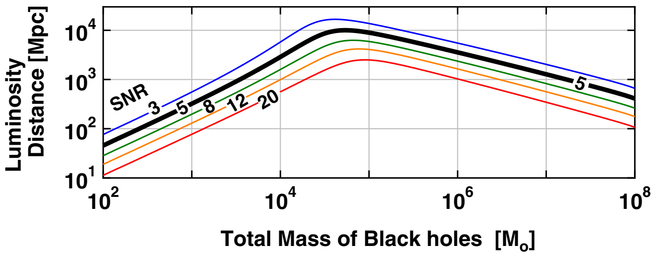

FIG. 4 (color online). Estimated SNR for a merger of equal-mass black holes with equal and parallel spin parameters of 0.5 [19], shown as a function of the total mass and the luminosity distance.

baseline length. The advantage of a TOBA is its configuration simplicity, the potential sensitivity in low frequencies even with a ground-based configuration, and the capability of an intermittent observation of low-frequency GWs with a modulation and up-conversion scheme.

A TOBA has an option to be a resonant antenna, in which two test-mass bars are connected by a shaft with a small spring constant [23]. Though the observation band is limited around the resonant frequency in this configuration, the requirement for the angular sensor is relaxed. Moreover, its sensitivity to low-frequency GWs can be enhanced by tuning the resonant frequency to twice the antenna rotation frequency. In such a case, a reduction of the thermal noise of the shaft is critical.

Besides the fundamental noises investigated in this work, there are many practical noises to be considered: additional noises in the angular sensors, Brownian fluctuation by residual gases, and noises due to electromagnetic fluctuations [24]. In a ground-based configuration, the simplicity of a TOBA is helpful for low-frequency isolations and common-mode reduction of seismic disturbances and for the reduction of gravity-gradient noises in an underground site. A space mission requires a reduction in the antenna size while maintaining its sensitivity by using advanced optical technologies. Optimization of the antenna parameters, implementation of advanced interferometric techniques, and investigations of these noise behaviors will be considered in future works.

Conclusion.—We propose a gravitational-wave antenna, a TOBA, comprised of bar-shaped test masses and sensors to monitor their differential angular motions. This antenna has a fundamental sensitivity to gravitational waves with frequencies lower than 1 Hz, which are inaccessible by current ground-based detectors. In order to investigate the concept and potential of a TOBA, we are developing a prototype ground-based detector [25]. In addition, we have developed a tiny module, called the SpaceWire Interface Demonstration Module, for space-related demonstrations; this module has been operated in a low-Earth orbit for one year [26].

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