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"Risk Management of Risk under the Basel Accord:
Forecasting Value-at-Risk of VIX Futures"

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Risk Management of Risk under the Basel Accord: Forecasting Value-at-Risk of VIX Futures*

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Abstract

The Basel II Accord requires that banks and other Authorized Deposit-taking Institutions (ADIs) communicate their daily risk forecasts to the appropriate monetary authorities at the beginning of each trading day, using one or more risk models to measure Value-at-Risk (VaR). The risk estimates of these models are used to determine capital requirements and associated capital costs of ADIs, depending in part on the number of previous violations, whereby realised losses exceed the estimated VaR. McAleer, Jimenez-Martin and Perez-Amaral (2009) proposed a new approach to model selection for predicting VaR, consisting of combining alternative risk models, and comparing conservative and aggressive strategies for choosing between VaR models. This paper addresses the question of risk management of risk, namely VaR of VIX futures prices. We examine how different risk management strategies performed during the 2008-09 global financial crisis (GFC). We find that an aggressive strategy of choosing the Supremum of the single model forecasts is preferred to the other alternatives, and is robust during the GFC. However, this strategy implies relatively high numbers of violations and accumulated losses, though these are admissible under the Basel II Accord.

Key words and phrases: Median strategy, Value-at-Risk (VaR), daily capital charges, violation penalties, optimizing strategy, aggressive risk management, conservative risk management, Basel II Accord, VIX futures, global financial crisis (GFC).

JEL Classifications: G32, G11, G17, C53, C22.

1. Introduction

Volatility derivatives have attracted much attention over the past years since they enable trading and hedging against changes in volatility. In 1993 the Chicago Board Options Exchange (CBOE) introduced a volatility index, VIX (Whaley, 1993), which was originally designed to measure the market expectation of 30-day volatility implied by at-the-money S&P100 option prices. In 2003, together with Goldman Sachs, CBOE updated VIX to reflect a new way to measure expected volatility, one that continues to be widely used by financial theorists.

The new VIX is based on the S&P500 Index, and estimates expected volatility by averaging the weighted prices of S&P500 puts and calls over a wide range of strike prices. Although many market participants considered the index to be a good predictor of short term volatility, daily or even intraday, it took several years for the market to introduce volatility products, starting with over the counter products such as variance swaps. The first exchange-traded product, VIX futures, was introduced in March 2004, and was followed by VIX options in February 2006. Both of these volatility derivatives are based on the VIX index as the underlying asset.

This paper focuses on the VIX futures market and addresses the question of risk management of risk when VIX futures are taken into account. McAleer, et al. (2009, 2010, 2011) analyse from a practical perspective, how the new market risk management strategies performed during the 2008-09 global financial crisis (GFC), and evaluate how the GFC affected the best risk management practices. Huskaj (2009) analyzes VIX futures from a different perspective, centered on the statistical properties of a different set of candidate forecasting models, and without focusing on the Basel II regulations, as is considered in this paper. McAleer and Wiphatthanananthakul (2010) examine the empirical behaviour of alternative simple expected volatility indexes, and compare them with VIX.

The GFC of 2008-09 has left an indelible mark on economic and financial structures worldwide, and left an entire generation of investors wondering how things could have become so severe. There have been many questions asked about whether appropriate regulations were in place, especially in the USA, to permit the appropriate monitoring and encouragement of (possibly excessive) risk taking.

The Basel II Accord was designed to monitor and encourage sensible risk taking using appropriate models of risk to calculate Value-at-Risk (VaR) and subsequent daily capital charges. When the Basel I Accord was concluded in 1988, no capital requirements were defined for market risk. However, regulators soon recognized the risks to a banking system if insufficient capital were held to absorb the large sudden losses from huge exposures in capital markets. During the mid-90's, proposals were tabled for an amendment to the 1988 Accord, requiring additional capital over and above the minimum required for credit risk. Finally, a market risk capital adequacy framework was adopted in 1995 for implementation in 1998.

The 1995 Basel I Accord amendment provides a menu of approaches for determining market risk capital requirements, ranging from simple to intermediate and advanced approaches. Under the advanced approach (that is, the internal model approach), banks are allowed to calculate the capital requirement for market risk using their internal models. The use of internal models was introduced in 1998 in the European Union. The 26 June 2004 Basel II framework, implemented in many countries in 2008 (though not yet in the USA), enhanced the requirements for market risk management by including, for example, oversight rules, disclosure, management of counterparty risk in trading portfolios.

VaR is defined as an estimate of the probability and size of the potential loss to be expected over a given period, and is now a standard tool in risk management. It has become especially important following the 1995 amendment to the Basel Accord, whereby banks and other Authorized Deposit-taking Institutions (ADIs) were permitted (and encouraged) to use internal models to forecast daily VaR (see Jorion (2000) for a detailed discussion). The last decade has witnessed a growing academic and professional literature comparing alternative modelling approaches to determine how to measure VaR, especially for large portfolios of financial assets.

The amendment to the initial Basel Accord was designed to encourage and reward institutions with superior risk management systems. A back-testing procedure, whereby actual returns are compared with the corresponding VaR forecasts, was introduced to assess the quality of the internal models used by ADIs. In cases where internal models lead to a greater number of violations than could reasonably be expected, given the confidence level, the ADI is required to hold a higher level of capital (see Table 1 for the penalties imposed under the Basel II

Accord). Penalties imposed on ADIs affect profitability directly through higher capital charges, and indirectly through the imposition of a more stringent external model to forecast VaR. This is one reason why financial managers may prefer risk management strategies that are passive and conservative rather than active and aggressive.

Excessive conservatism can have a negative impact on the profitability of ADIs as higher capital charges are subsequently required. Therefore, ADIs should perhaps consider a strategy that allows an endogenous decision as to how many times ADIs should violate in any financial year (for further details, see McAleer and da Veiga (2008a, 2008b), McAleer (2009), Caporin and McAleer (2009b) and McAleer et al. (2009)). This paper suggests alternative aggressive and conservative risk management strategies that can be compared with the use of one or more models of risk throughout the estimation and forecasting periods.

This paper defines risk management in terms of choosing sensibly from a variety of risk models, discusses the selection of optimal risk models, considers combining alternative risk models, discusses the choice between conservative and aggressive risk management strategies, evaluates the effects of the Basel II Accord on risk management of risk, examines how some risk management strategies performed during the 2008-09 GFC, and evaluates how the GFC affected risk management practices and daily capital charges.

The empirical results indicate that, when risk management is considered for VIX futures, the optimal strategy is to be aggressive rather than conservative. Specifically, this would involve a strategy of communicating to the national regulatory authority the Supremum of the point forecasts of the VaR models considered. This strategy tends to minimize the average daily capital charges, subject to staying within the limits of the number of violations that are permitted under the Basel II Accord.

The remainder of the paper is organized as follows. In Section 2 we present the main ideas of the Basel II Accord Amendment as it relates to forecasting VaR and daily capital charges. Section 3 reviews some of the most well-known univariate models of conditional volatility that are used to forecast VaR. In Section 4 the data used for estimation and forecasting are presented. Section 5 analyses the VaR forecasts before, during and after the 2008-09 GFC. Section 6 presents some concluding remarks.

2. Forecasting Value-at-Risk and Daily Capital Charges

In this section, we evaluate risk management of risk by applying the Basel II formulae to a period that includes the 2008-09 GFC. The Basel II Accord stipulates that daily capital charges (DCC) must be set at the higher of the previous day's VaR or the average VaR over the last 60 business days, multiplied by a factor (3+*k*) for a violation penalty, wherein a violation involves the actual negative returns exceeding the VaR forecast negative returns for a given day:

$$\mathrm{DCC}_t = \sup\left\{-(3+k)\overline{\mathrm{VaR}}_{60},\ -\mathrm{VaR}_{t-1}\right\} \tag{1}$$

where

DCC_t = daily capital charges, which is the higher of $-(3+k)\overline{\mathrm{VaR}}_{60}$ and $-\mathrm{VaR}_{t-1}$,

VaR_t = Value-at-Risk for day *t*,

$VaR_t = \hat{Y}_t - z_t \cdot \hat{\sigma}_t$,

$\overline{\mathrm{VaR}}_{60}$ = mean VaR over the previous 60 working days,

$\hat{Y}_t$ = estimated return at time t,

z_t = 1% critical value of the distribution of returns at time t,

$\hat{\sigma}_t$ = estimated risk (or square root of volatility) at time t,

$0 \le k \le 1$ is the Basel II violation penalty (see Table 1).

[Insert Table 1 here]

The formula given in equation (1) is contained in the 1995 amendment to Basel I, while Table 1 appears for the first time in the Basel II Accord in 2004. The multiplication factor (or penalty), *k*, depends on the central authority's assessment of the ADI's risk management

practices and the results of a simple backtest. It is determined by the number of times actual losses exceed a particular day's VaR forecast (Basel Committee on Banking Supervision (1996, 2006)).

The minimum multiplication factor of 3 is intended to compensate for various errors that can arise in model implementation, such as simplifying assumptions, analytical approximations, small sample biases and numerical errors that tend to reduce the true risk coverage of the model (see Stahl (1997)). Increases in the multiplication factor are designed to increase the confidence level that is implied by the observed number of violations at the 99% confidence level, as required by regulators (for a detailed discussion of VaR, as well as exogenous and endogenous violations, see McAleer (2009), Jiménez-Martin et al. (2009), and McAleer et al. (2010a)).

In calculating the number of violations, ADIs are required to compare the forecasts of VaR with realised profit and loss figures for the previous 250 trading days. In 1995, the 1988 Basel Accord (Basel Committee on Banking Supervision (1988)) was amended to allow ADIs to use internal models to determine their VaR thresholds (Basel Committee on Banking Supervision (1995)). However, ADIs that propose using internal models are required to demonstrate that their models are sound. Movement from the green zone to the red zone arises through an excessive number of violations. Although this will lead to a higher value of *k*, and hence a higher penalty, violations will also tend to be associated with lower daily capital charges. It should be noted that the number of violations in a given period is an important, though not the only, guide for regulators to approve a given VaR model.

VaR refers to the lower bound of a confidence interval for a (conditional) mean, that is, a "worst case scenario on a typical day". If interest lies in modelling the random variable, Y_t, it could be decomposed as follows:

$$Y_t = E(Y_t \mid F_{t-1}) + \varepsilon_t . \qquad (2)$$

This decomposition states that Y_t comprises a predictable component, $E(Y_t \mid F_{t-1})$, which is the conditional mean, and a random component, ε_t. The variability of Y_t, and hence its

distribution, is determined by the variability of ε_t. If it is assumed that ε_t follows a conditional distribution, such that:

$$\varepsilon_t \sim D(\mu_t, \sigma_t^2)$$

where μ_t and σ_t are the conditional mean and standard deviation of ε_t, respectively, these can be estimated using a variety of parametric, semi-parametric or non-parametric methods.

The VaR threshold for Y_t can be calculated as:

$$VaR_t = E(Y_t \mid F_{t-1}) - \alpha\sigma_t, \qquad (3)$$

where α is the critical value from the distribution of ε_t to obtain the appropriate confidence level. It is possible for σ_t to be replaced by alternative estimates of the conditional standard deviation in order to obtain an appropriate VaR (for useful reviews of theoretical results for conditional volatility models, see Li et al. (2002) and McAleer (2005), where several univariate and multivariate, conditional, stochastic and realized volatility models are discussed).

Some recent empirical studies (see, for example, Berkowitz and O'Brien (2001), Gizycki and Hereford (1998), and Pérignon et al. (2008)) have indicated that some financial institutions overestimate their market risks in disclosures to the appropriate regulatory authorities, which can imply a costly restriction to the banks trading activity. ADIs may prefer to report high VaR numbers to avoid the possibility of regulatory intrusion. This conservative risk reporting suggests that efficiency gains may be feasible. In particular, as ADIs have effective tools for the measurement of market risk, while satisfying the qualitative requirements, ADIs could conceivably reduce daily capital charges by implementing a context-dependent market risk disclosure policy. McAleer (2009) and McAleer et al. (2010a) discuss alternative approaches to optimize VaR and daily capital charges.

The next section describes several volatility models that are widely used to forecast the 1-day ahead conditional variances and VaR thresholds.

3. Models for Forecasting VaR

ADIs can use internal models to determine their VaR thresholds. There are alternative time series models for estimating conditional volatility. In what follows, we present several well-known conditional volatility models that can be used to evaluate strategic market risk disclosure, namely GARCH, GJR and EGARCH, with Gaussian, Student-*t* and Generalized Normal distribution errors, where the parameters are estimated.

These models are chosen as they are widely used in the literature. For an extensive discussion of the theoretical properties of several of these models, see Ling and McAleer (2002a, 2002b, 2003a) and Caporin and McAleer (2010b). As an alternative to estimating the parameters, we also consider the exponential weighted moving average (EWMA) method by Riskmetrics (1996) and Zumbauch, (2007) that calibrates the unknown parameters. We include a section on these models to present them in a unified framework and notation, and to make explicit the specific versions we are using. Apart from EWMA, the models are presented in increasing order of complexity.

3.1 GARCH

For a wide range of financial data series, time-varying conditional variances can be explained empirically through the autoregressive conditional heteroskedasticity (ARCH) model, which was proposed by Engle (1982). When the time-varying conditional variance has both autoregressive and moving average components, this leads to the generalized ARCH(*p*,*q*), or GARCH(*p*,*q*), model of Bollerslev (1986). It is very common in practice to impose the widely estimated GARCH(1,1) specification in advance.

Consider the stationary AR(1)-GARCH(1,1) model for daily returns, y_t:

$$y_t = \varphi_1 + \varphi_2 y_{t-1} + \varepsilon_t, \qquad |\varphi_2| < 1 \tag{4}$$

for $t = 1,...,n$, where the shocks to returns are given by:

$$\begin{aligned} \varepsilon_t &= \eta_t \sqrt{h_t}, \quad \eta_t \sim iid(0,1) \\ h_t &= \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1} \end{aligned} \tag{5}$$

and $\omega > 0, \alpha \geq 0, \beta \geq 0$ are sufficient conditions to ensure that the conditional variance $h_t > 0$. The stationary AR(1)-GARCH(1,1) model can be modified to incorporate a non-stationary ARMA(*p*,*q*) conditional mean and a stationary GARCH(*r*,*s*) conditional variance, as in Ling and McAleer (2003b).

3.2 GJR

In the symmetric GARCH model, the effects of positive shocks (or upward movements in daily returns) on the conditional variance, h_t, are assumed to be the same as the effect of negative shocks (or downward movements in daily returns) of equal magnitude. In order to accommodate asymmetric behaviour, Glosten, Jagannathan and Runkle (1992) proposed a model (hereafter GJR), for which GJR(1,1) is defined as follows:

$$h_t = \omega + (\alpha + \gamma I(\eta_{t-1}))\varepsilon_{t-1}^2 + \beta h_{t-1} \tag{6}$$

where $\omega > 0, \alpha \geq 0, \alpha + \gamma \geq 0, \beta \geq 0$ are sufficient conditions for $h_t > 0$, and $I(\eta_t)$ is an indicator variable defined by:

$$I(\eta_t) = \begin{cases} 1, & \varepsilon_t < 0 \\ 0, & \varepsilon_t \geq 0 \end{cases} \tag{7}$$

as η_t has the same sign as ε_t. The indicator variable differentiates between positive and negative shocks, so that asymmetric effects in the data are captured by the coefficient γ. For financial data, it is expected that $\gamma \geq 0$ because negative shocks have a greater impact on risk than do positive shocks of similar magnitude. The asymmetric effect, γ, measures the

contribution of shocks to both short run persistence, $\alpha + \gamma/2$, and to long run persistence, $\alpha + \beta + \gamma/2$.

Although GJR permits asymmetric effects of positive and negative shocks of equal magnitude on conditional volatility, the special case of leverage, whereby negative shocks increase volatility while positive shocks decrease volatility (see Black (1976) for an argument using the debt/equity ratio), cannot be accommodated, in practice (for further details on asymmetry versus leverage in the GJR model, see Caporin and McAleer (2010b)).

3.3 EGARCH

An alternative model to capture asymmetric behaviour in the conditional variance is the Exponential GARCH, or EGARCH(1,1), model of Nelson (1991), namely:

$$\log h_t = \omega + \alpha \left| \frac{\varepsilon_{t-1}}{h_{t-1}} \right| + \gamma \frac{\varepsilon_{t-1}}{h_{t-1}} + \beta \log h_{t-1} \quad |\beta| < 1 \tag{8}$$

where the parameters α, β and γ have different interpretations from those in the GARCH(1,1) and GJR(1,1) models.

EGARCH captures asymmetries differently from GJR. The parameters α and γ in EGARCH(1,1) represent the magnitude (or size) and sign effects of the standardized residuals, respectively, on the conditional variance, whereas α and $\alpha + \gamma$ represent the effects of positive and negative shocks, respectively, on the conditional variance in GJR(1,1). Unlike GJR, EGARCH can accommodate leverage, depending on the restrictions imposed on the size and sign parameters, though leverage is not guaranteed.

As noted in McAleer et al. (2007), there are some important differences between EGARCH and the previous two models, as follows: (i) EGARCH is a model of the logarithm of the conditional variance, which implies that no restrictions on the parameters are required to ensure $h_t > 0$; (ii) moment conditions are required for the GARCH and GJR models as they are dependent on lagged unconditional shocks, whereas EGARCH does not require moment conditions to be established as it depends on lagged conditional shocks (or standardized

residuals); (iii) Shephard (1996) observed that $|\beta| < 1$ is likely to be a sufficient condition for consistency of QMLE for EGARCH(1,1); (iv) as the standardized residuals appear in equation (7), $|\beta| < 1$ would seem to be a sufficient condition for the existence of moments; and (v) in addition to being a sufficient condition for consistency, $|\beta| < 1$ is also likely to be sufficient for asymptotic normality of the QMLE of EGARCH(1,1).

The three conditional volatility models given above are estimated under the following distributional assumptions on the conditional shocks: (1) Gaussian, (2) Student-*t*, with estimated degrees of freedom, and (3) Generalized Normal. As the models that incorporate the t distributed errors are estimated by QMLE, the resulting estimators are consistent and asymptotically normal, so they can be used for estimation, inference and forecasting.

3.4 Exponentially Weighted Moving Average (EWMA)

As an alternative to estimating the parameters of the appropriate conditional volatility models, Riskmetrics (1996) developed a model which estimates the conditional variances and covariances based on the exponentially weighted moving average (EWMA) method, which is, in effect, a restricted version of the ARCH(∞) model. This symmetric approach forecasts the conditional variance at time t as a linear combination of the lagged conditional variance and the squared unconditional shock at time $t-1$. The EWMA model calibrates the conditional variance as:

$$h_t = \lambda h_{t-1} + (1-\lambda)\varepsilon_{t-1}^2 \qquad (9)$$

where λ is a decay parameter. Riskmetrics (1996) suggests that λ should be set at 0.94 for purposes of analysing daily data. As no parameters are estimated, there are no moment or log-moment conditions.

4. Data

The data used in estimation and forecasting are closing daily prices (settlement prices) for the 30-day maturity CBOE VIX volatility index futures (ticker name VX), and were obtained from the Thomson Reuters-Data Stream Database for the period 26 March 2006 to 10 January 2011. The settlement price is calculated by the CBOE as the average of the closing bid and ask quote so as to reduce the noise due to any microstructure effects. The contracts are cash settled on the Wednesday 30 days prior to the third Friday on the calendar month immediately following the month in which the contract expires. The underlying asset is the VIX index that was originally introduced by Whaley (1993) as an index of implied volatility on the S&P100. In 2003 the new VIX was introduced based on the S&P500 index.

VIX is a measure of the implied volatility of 30-day S&P500 options. Its calculation is independent of an option pricing model and is calculated from the prices of the front month and next-to-front month S&P500 at-the-money and out-the-money call and put options. The level of VIX represents a measure of the implied volatilities of the entire smile for a constant 30-day to maturity option chain. VIX is quoted in percentage points (for example, 30.0 VIX represents an implied volatility of 30.0%). In order to invest in VIX, an investor can take a position in VIX futures or VIX options.

Although VIX represents a measure of the expected volatility of the S&P500 over the next 30-days, the prices of VIX futures are based on the current expectation of what the expected 30-day volatility will be at a particular time in the future (on the expiration date). Although the VIX futures should converge to the spot at expiration, it is possible to have significant disparities between the spot VIX and VIX futures prior to expiration. Figure 1 shows the daily VIX futures index together with the 30 day maturity VIX futures closing prices. VIX has a correlation (0.96) with the 30-day maturity VIX futures. Regarding volatility, VIX futures prices tend to show significantly lower volatility than VIX, which can be explained by the fact that VIX futures must be priced in a manner that reflects the mean reverting nature of VIX. For the whole sample, the standard deviation is 11.12 for VIX and 9.55 for VIX futures prices.

In Figure 1 it can be seen that, from 2004 until 2007, the equity markets enjoyed a tranquil period, as did the VIX futures prices. After the first signs of a looming economic crisis, VIX futures rose sharply, in July 2007, after fluctuating around an average of 14. Following the Lehman Brothers collapse in September 2008, VIX futures appeared to jump to an all time

high of over 66.23 in November 20, 2008. VIX futures prices returned to around $20 in 2009, but with the Greek crisis in May 2010, the VIX futures prices again jumped to around $36.

If P_t denotes the closing prices of the VIX futures contract at time *t*, the returns at time *t* (R_t) are defined as:

$$R_t = 100 * \log\left(P_t / P_{t-1}\right). \tag{10}$$

Figures 2 shows the daily VIX futures returns, and the descriptive statistics for the daily returns are given in Table 2. The returns to the VIX futures are driven by changes in expectations of implied volatility. Figure 3 shows the histograms for the daily returns, together with the theoretical Gaussian and Student-t probability density functions and a kernel density estimator. The Student-t density fits the returns distributions better than does its Gaussian counterpart.

[Insert Figures 1-2 and Tables 2-3 here]

It is also interesting to examine the returns distributions for the three periods relating to before the GFC (January-August 2008), during the GFC (August 2008-March 2009), and after the GFC (March 2009- October 2010). As can be seen in Figure 4, there are changes in the shapes of the underlying probability density functions. We graph the empirical distributions, together with the Normal, Student-t and a kernel density estimator, for the three periods. Clearly, the shape of the densities changes from one period to another. All the sample distributions are positively skewed (0.88) and leptokurtic (8.21). The before GFC period exhibits the highest skewness (0.93), followed by the after GFC period (0.86), and with the during GFC period showing negative skewness (-0.15). The maximum and minimum (in parentheses) for the before, during and after GFC periods are, respectively, 16.25 (-8.55), 13.23 (-19.93) and 13.40 (-13.46), respectively. The returns for the three periods before, during and after the GFC show high kurtosis of 6.01, 4.25 and 4.90, respectively. The total period has the highest value kurtosis of 8.21.

Regarding the returns volatility, several measures of volatility are available in the literature. In order to gain some intuition, we adopt the measure proposed in Franses and van Dijk (1999), wherein the true volatility of returns is defined as:

$$V_t = \left(R_t - E\left(R_t \mid F_{t-1}\right)\right)^2, \tag{11}$$

where F_{t-1} is the information set at time t-1. Figure 5 presents the square root of V_t in equation (11) as "volatilities". The series exhibit clustering that should captured by an appropriate time series model. Until January 2007, a month before the first reports of subprime losses, the volatility of the series seems to be stable. The volatility reached an all time peak on February 27, 2007, when it climbed to 0.26 (the median for the entire sample is 0.023), as the US equity market had its worst day in four years. Then it remained above historic levels, but the VIX futures volatility increases again after August 2008, due in large part to the worsening global credit environment, with a maximum again on November 3, 2008. Then the volatility remained low until the news about the sovereign debt crisis in the Euro zone created another spike in volatility in the first week of May, when the VIX futures reached 35 with a high volatility in returns.

5. VaR of VIX and Evaluation Framework

As discussed in McAleer et al. (2010c), the GFC has affected the optimal risk management strategies by changing the best model for minimizing daily capital charges in all the cases analyzed. The purpose of this section is to provide an analysis of risk management strategies when considering 30-day maturity VIX futures before, during and after the GFC.

ADIs need not restrict themselves to using only a single risk model. McAleer et al. (2010b) propose a risk management strategy that uses combinations of several models for forecasting VaR. It was found that an aggressive risk management strategy (namely, choosing the Supremum of VaR forecasts, or an upperbound) yielded the lowest mean capital charges and largest number of violations. On the other hand, a conservative risk management strategy (namely, by choosing the Infinum, or lowerbound) had far fewer violations, and correspondingly higher mean daily capital charges.

McAleer et al. (2010c) forecast VaR using ten single GARCH-type models with different error distributions. Additionally, they analyze twelve new strategies based on combinations of the previous standard single-model forecasts of VaR, namely: Infinum (0th percentile),

Supremum (100th percentile), Average, Median and nine additional strategies based on the 10th through to the 90th percentiles. This is intended to select a robust VaR forecast, irrespective of the time period, that provides reasonable daily capital charges and number of violation penalties under the Basel Accord. They found that the Median (50th percentile) is a GFC-robust strategy, in the sense that maintaining the same risk management strategy before, during and after the GFC leads to comparatively low daily capital charges and violation penalties under the Basel Accord.

In this section, we conduct a similar exercise to analyze the risk management performance of existing VaR forecasting models, as permitted under the Basel II framework, when applied to VIX futures prices.

5.1 Evaluating Risk Management Strategies

In Table 3 the performance criteria are calculated for each model and error distribution, and for each of the three sub-samples: before, during, and after the 2008-09 GFC. Based on the S&P500 peaks and troughs, before the GFC is prior to 11 August 2008, during the GFC is from 12 August 2008 through to 9 March 2009, and after the GFC is from 10 March 2009 onwards.

Table 3 shows the values of the criteria used for comparison of the strategies for forecasting the volatility and VaR of the VIX futures returns. The first column indicates the name of the model or combination of models that are used. The first row marks three groups of five columns each, corresponding to before, during and after the GFC. Each of the three groups of columns contains the average daily capital charges (AvDCC), the normalized number of violations, (NoV), failure rate (FailRa), accumulated losses, (AcLoss) and asymmetric linear tick loss function (AlTick) that are obtained by each strategy in each of the three periods related to the GFC.

Our basic criterion for choosing a strategy is minimizing the average daily capital charges subject to the constraint that the normalized number of violations (equivalently, the percentage of violations) is within the limits allowed under the Basel II Accord. Additionally, we consider the accumulated losses, which are not taken into account in the rules of Basel II,

but which might be considered in the future. In principle, low values of this criterion are desirable. We also consider the asymmetric loss tick function, which should be minimized.

The main conclusions can be summarized as follows:

1. **Before the GFC**, the best strategy for minimizing daily capital charges (DCC) is the Supremum. It has the lowest AvDCC (20.30) and the highest number of violations (4.78), but is still within the limits of the Basel II Accord. The Supremum also has the next to lowest asymmetric linear tick loss function (12.67). However, it also has the highest accumulated losses (1.85). The Supremum is clearly as the best strategy for forecasting the VaR of VIX futures before the GFC.

2. **During the GFC**, the Supremum has the lowest average daily capital charges (26.72) and highest (but admissible) number of violations (3.3). Moreover, it has one of the three lowest values of the AlTick (25.31). However, the Supremum shows the highest accumulated losses of all the models (25.31). In general, the Supremum is the optimal strategy for managing risk under the Basel II Accord during the GFC.

3. **After the GFC,** the Supremum has the lowest average daily capital charges (23.55), at the cost of the highest (but admissible) number of violations (1.6). Moreover, the Supremum has one of the three lowest asymmetric linear tick loss function values (39.74), at the cost of the highest accumulated losses (3.01).

The Supremum emerges as the optimal strategy for minimizing daily capital charges, at a cost in terms of the number of violations, and accumulated losses which are permissible under the Basel II Accord.

A comparison with a leading competitor, Riskmetrics, which is shown in the first row of Table 3, reveals that the Supremum consistently dominates Riskmetrics. The Supremum always has lower daily capital charges, with the same number of violations, across all the time periods that are considered, namely before, during and after the GFC.

In summary, the Supremum is the risk management strategy that performs the best across all the considered strategies and time periods. It is also a GFC-robust strategy, as defined in

McAleer et al. (2010c, 2011), meaning it is an optimal strategy that is valid before, during and after the GFC.

6. Conclusion

In the spectrum of financial assets, VIX futures prices are a relatively new financial product. As with any financial asset, VIX futures are subject to risk. In this paper we analyzed the performance of a variety of strategies for managing the risk, through forecasting VaR, of VIX futures under the Basel II Accord, before, during and after the global financial crisis (GFC) of 2008-09.

We forecast VaR using well known univariate model strategies, as well as new strategies based on combinations of risk models, that were proposed and analyzed in McAleer et al. (2009, 2010c, 2011).

The candidate strategies for forecasting VaR of the VIX futures, and for managing risk under the Basel II Accord, were several univariate models, such as Riskmetrics, GARCH, EGARCH and GJR, each subject to different error distributions. We also used several more sophisticated strategies that combined single models, such as the Supremum, Infinum, Average, Median and the 10th through 90th percentiles of the point values of the forecasts of the univariate models.

Our main criterion for choosing between strategies is minimizing the average daily capital charges subject to the constraint that the number of violations (equivalently, the percentage of violations) is within the limits allowed by the Basel II Accord. Additionally, we consider the accumulated losses and asymmetric loss tick function, each of which would desirably have low values.

The principal empirical conclusions of the paper can be summarized as follows:

1. **Before the GFC**, the Supremum has the lowest Average Daily Capital Charges (AvDCC) (20.30) and the highest (though admissible) number of violations (4.78) under the Basel II Accord.

2. **During the GFC**, the Supremum has the lowest AvDCC (26.72) and highest (but admissible) number of violations (3.3).

3. **After the GFC,** the Supremum has the lowest AvDCC (23.55), at the cost of the highest (but admissible) number of violations (1.6).

The Supremum dominates Riskmetrics consistently as it always has lower daily capital charges, with the same number of violations across all time periods: before, during and after the GFC.

The attraction for risk managers in using the Supremum strategy is that they do not need to keep changing the rules for generating daily VaR forecasts. The Supremum is an aggressive and profitable risk strategy for calculating VaR forecasts for VIX futures, both in tranquil and in turbulent times.

The idea of combining different VaR forecasting models is entirely within the spirit of the Basel Accord, although its use would require approval by the regulatory authorities, as for any forecasting model. This approach is not at all computationally demanding, even though several models need to be specified and estimated over time.

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Figure 1

VIX and 30-day Maturity VIX Futures Closing Prices

26 March 2004 - 10 January 2011

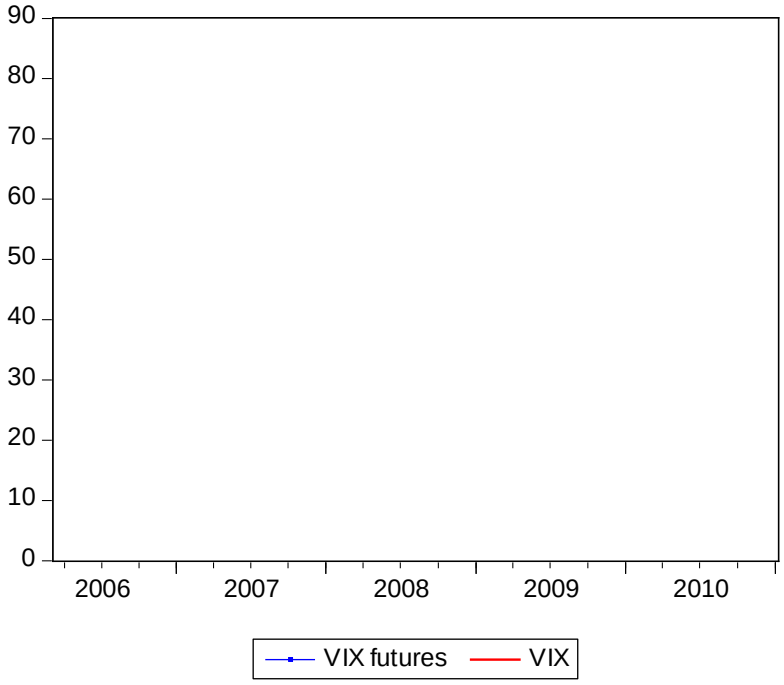


Figure 2

30-day Maturity VIX Futures Returns

26 March 2004 - 10 January 2011

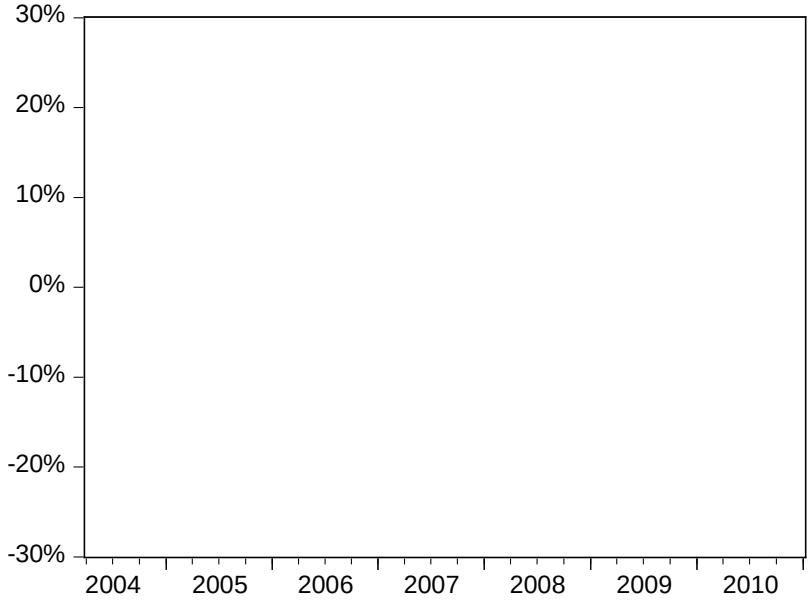


Figure 3

Histogram, Normal and Student-t Distributions and Kernel Density Estimator

For 30-day Maturity VIX Futures Returns

26 March 2006 - 10 January 2011

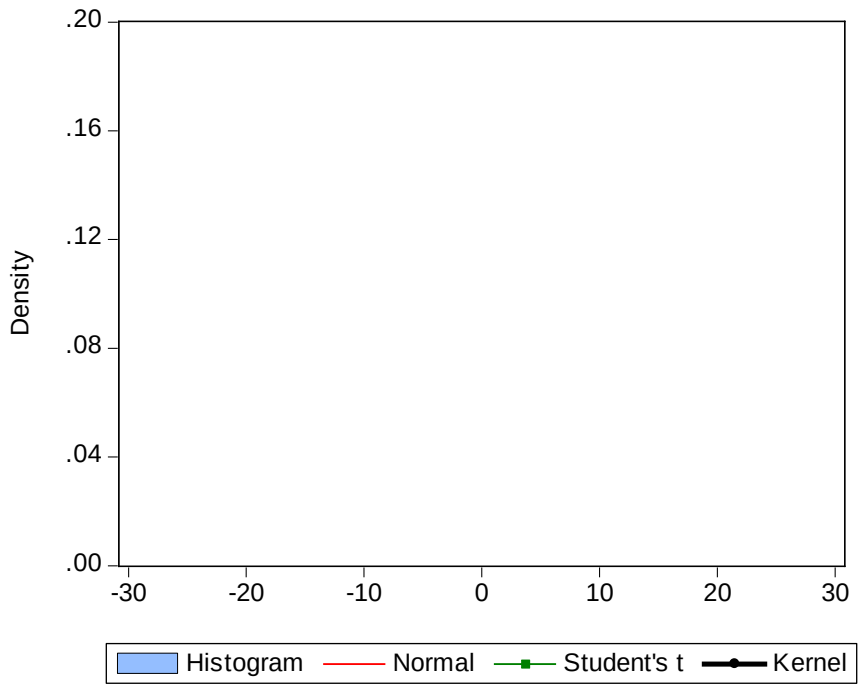


Figure 4

30-day Maturity VIX Futures Returns

Histogram, Normal, Student-t and Kernel Density Estimator

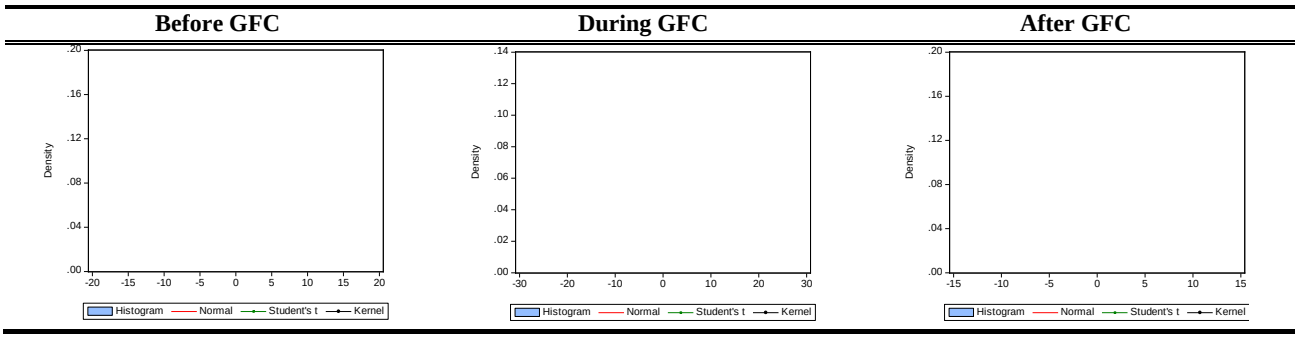


Figure 5

Volatility of 30-day Maturity VIX Futures Returns

26 March 2004 - 10 January 2011

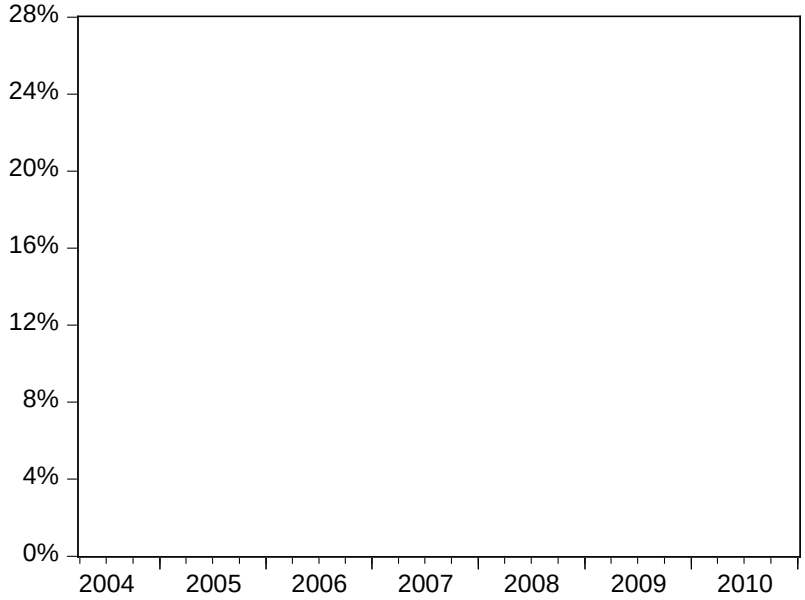
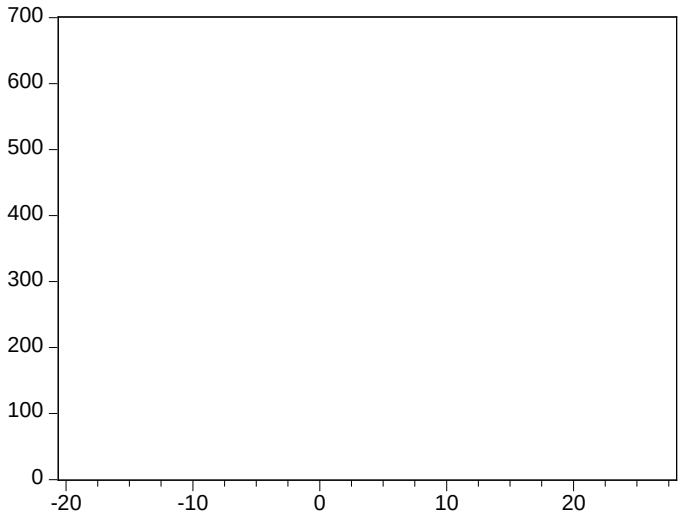


Table 1. Basel Accord Penalty Zones

Zone	Number of Violations	*k*
Green	0 to 4	0.00
Yellow	5	0.40
	6	0.50
	7	0.65
	8	0.75
	9	0.85
Red	10+	1.00

Note: The number of violations is given for 250 business days. The penalty structure under the Basel II Accord is specified for the number of violations and not their magnitude, either individually or cumulatively.

Table 2
30-day Maturity VIX Futures Returns: Histogram and Descriptive Statistics
26 March 2004 - 10 January 2011



Series: VIX Futures
Sample 26/03/2004 10/01/2011
Observations 1771

Mean	0.001859
Median	-0.210453
Maximum	25.81866
Minimum	-19.92871
Std. Dev.	3.412287
Skewness	0.882711
Kurtosis	8.210094
Jarque-Bera	2233.068
Probability	0.000000

Table 3. Comparing Alternative Models of Volatility of 30-day Maturity VIX Futures Returns

	Before GFC					During GFC					After GFC				
Model	**AvDCC**	**NoV**	**FailRa**	**AcLoss**	**AlTick**	**AvDCC**	**NoV**	**FailRa**	**AcLoss**	**AlTick**	**AvDCC**	**NoV**	**FailRa**	**AcLoss**	**AlTick**
RSKM	23.14	4.78	1.9%	1.72	13.44	31.96	3.3	1.33%	6.76	23.95	25.00	1.6	0.6%	0.81	39.60
GARCH	21.31	1.59	0.6%	1.42	12.78	27.58	3.3	1.33%	10.21	25.12	24.25	1.0	0.4%	1.83	39.68
GJR	21.62	1.59	0.6%	1.55	13.20	28.60	3.3	1.33%	10.73	26.24	24.73	1.0	0.4%	2.78	41.26
EGARCH	23.24	1.59	0.6%	1.40	13.37	28.85	3.3	1.33%	10.17	25.75	25.14	0.5	0.2%	2.12	41.29
GARCH_t	23.04	1.59	0.6%	0.76	13.19	33.14	3.3	1.33%	5.34	23.22	28.01	0.0	0.0%	0.00	43.77
GJR_t	23.41	1.59	0.6%	0.81	13.51	33.85	3.3	1.33%	5.97	24.30	28.40	0.5	0.2%	0.17	44.49
EGARCH_t	24.33	1.59	0.6%	0.14	13.20	35.45	1.7	0.67%	4.00	23.10	28.64	0.0	0.0%	0.00	44.67
GARCH_g	26.29	0.00	0.0%	0.00	14.16	36.85	1.7	0.67%	3.66	23.38	30.33	0.0	0.0%	0.00	47.40
GJR_g	27.02	0.00	0.0%	0.00	14.56	37.40	1.7	0.67%	4.06	24.19	30.86	0.0	0.0%	0.00	48.15
EGARCH_g	27.35	0.00	0.0%	0.00	14.68	38.48	1.7	0.67%	3.10	23.71	31.05	0.0	0.0%	0.00	48.45
Inf	28.30	0.00	0.0%	0.00	15.20	39.33	1.7	0.67%	3.10	24.14	31.58	0.0	0.0%	0.00	49.28
Sup	**20.30**	**4.78**	**1.9%**	**1.85**	**12.67**	**26.72**	**3.3**	**1.33%**	**10.78**	**25.31**	**23.55**	**1.6**	**0.6%**	**3.01**	**39.74**
Mean	23.83	1.59	0.6%	0.61	13.44	33.21	3.3	1.33%	6.08	23.98	27.64	0.5	0.2%	0.16	43.27
10th Per.	27.48	0.00	0.0%	0.00	14.80	38.46	1.7	0.67%	3.38	23.97	31.17	0.0	0.0%	0.00	48.65
20th Per.	26.20	0.00	0.0%	0.00	14.12	37.03	1.7	0.67%	3.83	23.71	30.40	0.0	0.0%	0.00	47.47
30th Per.	25.23	0.00	0.0%	0.00	13.56	35.93	1.7	0.67%	4.03	23.37	29.54	0.0	0.0%	0.00	46.13
40th Per.	24.18	1.59	0.6%	0.45	13.51	34.75	1.7	0.67%	4.62	23.35	28.67	0.0	0.0%	0.00	44.75
50th Per. (Median)	23.43	1.59	0.6%	0.79	13.48	33.50	3.3	1.33%	5.66	23.72	27.99	0.0	0.0%	0.00	43.70
60th Per.	22.96	1.59	0.6%	1.11	13.51	32.49	3.3	1.33%	6.37	23.87	26.86	0.5	0.2%	0.29	42.15
70th Per.	22.26	1.59	0.6%	1.41	13.40	30.69	3.3	1.33%	8.44	25.00	25.53	0.5	0.2%	1.06	40.77
80th Per.	21.51	1.59	0.6%	1.42	12.98	28.78	3.3	1.33%	10.16	25.75	24.70	1.0	0.4%	1.98	40.43
90th Per.	20.75	3.18	1.3%	1.50	12.60 -	27.48	3.3	1.33%	10.49	25.42	23.96	1.6	0.6%	2.71	40.05