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“Agency Contracts, Noncommitment Timing Strategies, and Real Options”

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Agency Contracts, Noncommitment Timing Strategies, and Real Options

Abstract

Given an owner’s noncommitment timing strategy and a manager’s hidden action, we consider how the optimal compensation contract for the manager is designed and how the corresponding timing decisions to launch the project and replace the manager are determined. Using a real options approach, we show that in comparison with the first-best case, the higher (lower)-quality project is launched later (at the same time as the first-best case), whereas the incumbent manager is replaced earlier. We also indicate that compared with the case of the owner’s commitment timing strategy and the manager’s hidden action, the higher (lower)-quality project is launched later (at the same time as the first-best case), whereas the incumbent manager is (is not necessarily) replaced later if the hidden-action problem is severe enough (is not severe enough). Unlike the folklore result of the standard moral hazard model, severance pay may serve to minimize the compensation for the manager’s loss of his option value caused by loss of corporate control by committing the owner to delaying replacement of the manager if the hidden-action problem is not too severe.

JEL Classification: D82, G30, G34, M51, M52.

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1. Introduction

In the corporate firm, timing of the commencement of the project and the replacement of the manager is often determined according to ex post economic situations. These ex post decisions are often chosen by an owner such as a venture capitalist, private equity fund, banks when a firm is being restructured, management buyout (MBO) firms, founder families or large dominant shareholders even in old established firms. In this case, if the manager needs to make a costly effort or firm-specific human capital investment before the project starts, and if the effort or investment is the manager’s private information (hidden action), the owner needs to design a managerial compensation contract that gives the manager an incentive to make an appropriate effort or investment. However, if the owner cannot precommit to the ex post timing decision to launch the project or to replace the manager, she may choose these ex post timing decisions opportunistically after the manager makes a costly effort or investment. Furthermore, because the compensation contract is determined before the timing strategies are executed, the contract will also affect the owner’s timing decisions in addition to the manager’s costly effort or investment. Hence, when the managerial compensation contract needs to be designed in the case of the owner’s noncommitment timing strategy and the manager’s moral hazard problem (abbreviated to the ‘noncommitment timing’ case), the effect of the compensation contract on the owner’s timing strategy needs to be considered, unlike the standard static contracting model that focuses on the interrelation between the compensation contract and the agent’s costly effort.

In this paper, under the noncommitment timing case, we explore how the optimal compensation contract is designed and how the corresponding timing decisions to launch the project and replace the manager are determined. The main questions this paper

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1 We have many practical examples in which the planned timing of project commencement is advanced or delayed, and where unpredicted replacement of the manager occurs.

2 For example, even in the recent financial crisis, many distressed financial institutions attempted to give predetermined compensation to their executives until the government and taxpayers criticized the amount of predetermined executive compensation as excessive.
addresses are:

(i) Compared with the first-best case, is the project launched earlier or later? In addition, is the manager replaced earlier or later?

(ii) Compared with the case of the owner’s commitment timing strategy and the manager’s moral hazard problem (abbreviated to the ‘commitment timing’ case), is the project launched earlier or later? In addition, is the manager replaced earlier or later?

(iii) Can the manager receive both on-the-job incentive pay and severance pay, unlike the folklore result of the standard moral hazard model?3

To achieve this objective, in this paper, we consider the owner’s noncommitment timing decisions to launch the project and replace the manager in the class of firms with a stochastic trend by extending the real options framework of Grenadier and Wang (2005) and Hori and Osano (2009). The commencement of the project needs a setup cost, which is regarded as sunk. As argued in the management literature, the replacement of the manager is equivalent to a significant strategy change by the firm.4 The strategy change involves spending large amounts on various adjustment costs, which can also be regarded as sunk. These sunk costs make the decisions regarding project commencement and managerial replacement irreversible. We also suppose that the costly specific human capital investment of the incumbent manager affects the likelihood of drawing a higher (or lower)-quality project, which determines the growth rate of the firm under the stochastic environment. For example, the manager may improve his management ability, engage in product innovation, or make improvements to existing production facilities. However,  

3The standard moral hazard model cannot explain why the manager receives both on-the-job incentive pay and severance pay. In the corporate governance literature, the same remark also holds. The exceptions are Almazan and Suarez (2003) and Inderst and Mueller (2006), which show that the combination of some degree of entrenchment and a sizeable severance package is desirable. Lambert and Larcker (1985), Knoeber (1986), Harris (1990) and Berkovich and Khanna (1991) attribute a similar role to golden parachutes in hostile tender offers.

4The management literature provides ample evidence that strategy changes are accompanied by the hiring of a new manager (see Hambrick, Geletkanycz and Fredrickson (1993), Baker and Duhaime (1997) and Gordon et al. (2000)), while CEO succession is one of the primary means by which firms adapt to major changes in their environments (see Tushman, Newman and Romanelli (1986) and Wiersema (1992)).
the manager’s investment decision is a hidden action. Thus, the owner needs to design a managerial compensation contract contingent on project quality in order to induce the manager to choose the efficient level of investment under agency problems. In fact, we suppose that the owner cannot precommit to the timing decision to launch the project or to replace the manager before the manager makes a costly investment. Thus, the owner also has an incentive to choose the timing of the launch of the project and replacement of the manager opportunistically.

The first main result of this paper relates to the efficiency of the timing decisions to launch the project and to replace the manager. Compared with the first-best case, the higher-quality project is launched later, whereas the lower-quality project is launched at the same time as the first-best case. Furthermore, the incumbent manager is replaced earlier.

The second main result is concerned with the effect of the noncommitment timing strategy. Compared with the case of the commitment timing case, the higher-quality project is launched later, whereas the lower-quality project is launched at the same time as the first-best case. In addition, the incumbent manager is (or is not necessarily) replaced later if the manager’s moral hazard problem is severe (or not severe enough).

The third main result is about severance pay. If the severity of the manager’s moral hazard problem is not too great, the owner may make a positive severance payment. Severance pay can become an instrument that commits the owner to replacing the manager later in order to minimize compensation for the loss of the manager’s option value at the loss of corporate control. Thus, unlike the standard moral hazard model, our approach clarifies the new role of severance pay that works through a change in the manager’s option value.

The reasoning behind these main results is explained as follows. First, suppose that the manager’s moral hazard problem is severe. Under the noncommitment timing strategy, the owner must consider the effects of the compensation contract on the trigger levels for commencement of the project and replacement of the manager because the owner cannot
be committed to the *ex ante* promised triggers. However, a positive nonsuccess reward (positive severance pay) for the manager is not only recognized as a sunk cost at the start of the project (at the replacement of the manager) from the owner’s *ex post* viewpoint, but also prevents the manager from choosing a higher level of investment. Thus, in the compensation contract components, the owner utilizes a success reward for the manager as an incentive to motivate the manager to choose the higher level of investment.

However, under the noncommitment timing case, the owner regards the success reward as an additional sunk cost at the start of the higher-quality project from her *ex post* viewpoint. Because the option value of the owner waiting to launch the project must then be even larger, she is forced to launch the higher-quality project later under the noncommitment timing case than under the first-best case or under the commitment timing case. By contrast, as the owner never utilizes the nonsuccess reward, it cannot be regarded as a sunk cost at the start of the project. Hence, the owner can launch the lower-quality project at the first-best time, which is the same as under the commitment timing case.

On the other hand, the replacement trigger, as well as the manager’s reward for success, can become an important incentive device. This is because the early replacement of the manager can directly increase the loss of his option value at his replacement, thereby serving to motivate him to select the higher level of investment. Thus, even though the severance pay is not used in the optimal contract, the owner replaces the incumbent manager earlier under the noncommitment timing case than under the first-best case. However, if the manager’s moral hazard problem is severe enough, the incumbent manager is replaced later under the noncommitment timing case than the commitment case. This is because the optimal replacement trigger under the *commitment* timing case is too low for the owner after the manager chooses the level of investment. Hence, replacing the incumbent manager later is more desirable under the *noncommitment* timing case.

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5Note that under the commitment timing case, the owner simultaneously chooses the compensation contract and timing decisions so that they are determined orthogonally. Hence, the owner never regards any of the compensation contract components as a sunk cost even though she utilizes it as an incentive device.
On the other hand, if the manager’s moral hazard problem is not severe enough, the optimal replacement trigger under the commitment timing case may be too high from the owner’s ex post viewpoint. This implies that, from the owner’s ex post viewpoint, replacing the incumbent manager earlier is more desirable. Then, it is possible that the incumbent manager is replaced earlier under the noncommitment timing case than under the commitment case.

Second, if the manager’s moral hazard problem is not too severe, the results of the commencement triggers of the higher- and lower-quality projects are the same as those attained if the manager’s moral hazard problem is severe enough, for the reasons argued above. However, to reduce the fixed base salary in this case, the owner now has an additional ex ante incentive to minimize compensation for the loss of the option value incurred by the manager at his replacement. Because the later replacement of the manager can reduce this compensation, the owner may make a positive severance payment in order to commit herself to delaying the replacement of the incumbent manager if she has an ex post incentive to replace the manager earlier.

This paper is related to the literature that extends the real options model to account for the issues of agency and information in corporations. Grenadier and Wang (2005) provided a model of investment timing with both moral hazard and information asymmetries, and analyzed the optimal contract problem.6 Hori and Osano (2009) extended the model of Grenadier and Wang (2005), and discussed the timing of the decision to replace the manager as an endogenous incentive mechanism. However, these two papers investigate the situation in which the owner can precommit to the triggers promised before the manager makes costly investments.7 As a result, these two papers suggest that if it is only the manager’s hidden action that causes the agency problem, both higher- and

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6 McDonald and Siegel (1986) and Dixit and Pindyck (1994) are the pioneers in the field of investment timing under the real options approach.

7 Hori and Osano (2009) also considered the possibility of renegotiation of the contract agreements and trigger strategies and characterized the renegotiation-proof contract, extending the framework of Fudenberg and Tirole (1990). Hori and Osano showed that all the triggers are fixed at the first-best levels under the renegotiation-proof contract. However, Hori and Osano still assume that the owner is committed to the renegotiated triggers after renegotiation.
lower-quality projects are launched at the first-best time. In addition, Hori and Osano indicate that if it is only the manager’s hidden action that causes the agency problem, the incumbent manager is replaced earlier under the optimal contract than under the first-best situation, although he receives no severance pay. By contrast, our present paper shows that (i) a higher-quality project is launched later under the optimal contract than under the first-best situation, although a lower-quality project is launched at the first-best time; (ii) the incumbent manager is still replaced prior to the first-best time, but is replaced even later under the noncommitment timing strategy when the manager’s moral hazard problem is severe; and (iii) the optimal severance pay may be positive.

In Grenadier and Wang (2005) and Hori and Osano (2009), the owner need not consider the effects of the compensation contract on the trigger levels because she can precommit to the ex ante promised triggers. Hence, it is efficient for the owner to start the higher-quality project at the first-best time because the success reward does not affect the trigger. On the other hand, in Hori and Osano (2009), the earlier replacement, as well as the manager’s reward for success, becomes an important incentive mechanism because it directly increases the loss of the option value to the manager on his replacement. Hence, the incumbent manager is replaced prior to the first-best time, although he receives no severance pay. By contrast, in our present model, the owner must consider the effects of the compensation contract on the trigger levels because she cannot precommit to the ex ante promised triggers. Thus, the owner is forced to start the higher-quality project after the first-best time in order to motivate the manager through the success reward because she regards the success reward as an additional sunk cost at the start of the project from her ex post viewpoint. Furthermore, if the manager’s moral hazard incentive is great enough, the incumbent manager is replaced even later under the noncommitment timing strategy because optimal replacement under the commitment timing strategy is too early for the owner after the manager chooses the level of investment. On the other hand, if the manager’s moral hazard incentive is not great, the owner may make a positive severance payment in order to minimize the compensation for the manager’s loss of his option value
when he is replaced.

The problem we analyze is also related to the literature that extends the real options model to explain governance issues. Morellec (2004) and Lambrecht and Myers (2007) discussed corporate governance issues, such as optimal capital structure and takeovers. Dangl, Wu and Zechner (2008) also investigated how internal manager replacement and product market discipline interplay in a mutual fund company, although the managerial compensation contract is exogenously given. In addition, in their model, the replacement trigger is not used as an incentive instrument for motivating the manager. In fact, our paper is the first to examine how the optimal compensation contract and timing decisions on project launch and manager replacement are simultaneously determined within a real options framework if the owner cannot precommit to her timing decisions.  

With a static model of boards of directors, Almazan and Suarez (2003) suggested that severance pay plays a crucial role in solving a commitment problem by dissuading shareholders with strong boards from being too likely to replace a diligent manager or by dissuading an unsatisfactory manager from resisting being replaced by weak boards. Inderst and Mueller (2006) also indicated that severance pay reduces the manager’s incentive to entrench himself. In a model of the optimal termination of a long-term contract, Spear and Wang (2005) showed that the agent must be fired when he becomes too rich to be motivated to work diligently, and that if the agent is fired, he needs to be given a severance payment in order to be compensated for his promised utility. As the agent’s replacement happens when his utility has an income effect, the role of severance pay in Spear and Wang depends on the degree of risk aversion of the agent.

In contrast to these studies, in our paper, severance pay can induce the owner to commit herself to delaying replacement of the manager in order to minimize compensation for the loss of the manager’s option value at replacement of corporate control. Thus, in the present model, the role of severance pay is created by the owner’s commitment problem

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8 Hori and Osano (2008) examined the optimal compensation contract problem in which the manager chooses the timing of investment under agency conflicts and showed that restricted stock has an advantage over stock options.

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when determining the timing of a decision to replace the manager. Furthermore, in our model, the role of severance pay does not depend on the degree of risk aversion of the manager. Hence, our paper is complementary to these studies.

The rest of the paper is organized as follows. Section 2 presents the basic model. Section 3, as a benchmark, derives the first-best solution that corresponds to the standard real options case. Section 4 first examines the optimal timing decisions of the owner under agency conflicts, given the compensation contract, and then obtains the optimal compensation contract. The final section concludes the paper. Proofs of all propositions and lemmas are provided in Appendix A.

2. The Basic Environment

The basic setting of the model is similar to that of Hori and Osano (2009). We consider a firm that is entirely equity financed. There are three agents with risk-neutral preferences: an owner, an incumbent manager and new managers. The risk-free rate is $r$, at which investors may lend and borrow freely.

The owner offers a managerial contract to the incumbent manager at time zero and then chooses the time at which the manager commences a project and the time at which the manager is replaced by a new manager. The latter time may be interpreted as the time at which the owner is forced to make a project change. The firm incurs a setup cost, $K$, if a project is commenced, and a firing cost (exclusive of the severance payment), $C_F$, if the manager is replaced. These adjustment costs make the decisions regarding project commencement and managerial replacement irreversible.

The firm is run by the incumbent manager, who has no personal financial resources, a reservation utility of zero and limited liability. The manager incurs a cost in terms of the loss of corporate control $C_L$ if he is fired. The manager derives control benefits from retaining his office, and $C_L$ is a measure of the loss of these control benefits or the

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9 This is equivalent to assuming that there are several costs involved in the transition process and the corporate strategy change when the manager is replaced.
disutility cost that the manager incurs by losing the prestige of the managerial position or being forced to seek new employment.

The incumbent manager has a project that yields an instantaneous cash flow:

\[ R = (X + \theta_i)Y, \]  

where \( X \) is a deterministic component, \( \theta_i \) is a component that depends on an action related to the firm-specific human capital investment made by the manager and \( Y \) is a stochastic component. \( \theta_i \) takes on two possible values, \( \theta_1 \) or \( \theta_2 \), with \( \theta_1 > 0 > \theta_2 \). In addition, we assume that \( X + \theta_2 > 0 \).

The incumbent manager affects the probability of drawing \( \theta_1 \) or \( \theta_2 \) by investing in firm-specific human capital \( h (= H \) or \( L \)) at time zero. If the manager makes investment \( h = H \), he incurs an effort cost \( C_E \), but the probability of drawing \( \theta_1 \) (or \( \theta_2 \)) equals \( q_H \) (or \( 1 - q_H \)).\(^{10}\) If the manager chooses \( h = L \), he incurs no costs but decreases the probability of drawing \( \theta_1 \) from \( q_H \) to \( q_L \). Note that \( 1 > q_H > q_L > 0 \).

Let the value of \( Y \) evolve as a geometric Brownian motion:

\[ dY = \alpha Ydt + \sigma Ydz, \]  

where \( \alpha \in [(1/2)\sigma^2, r) \) is the instantaneous conditional expected percentage change in \( Y \) per unit of time, \( \sigma > 0 \) is the instantaneous conditional standard deviation per unit of time and \( dz \) is the increment of a standard Wiener process \( (dz \sim N(0, dt)) \). The restriction on the value of \( \alpha \) ensures that the firm is growing.\(^{11}\)

If the owner fires the incumbent manager, she hires a new manager from the pool of candidate successors. For simplicity, we assume that the firm’s instantaneous cash flow is the same for any new manager, and that \( \theta_i = 0 \) after the firm hires a new manager.\(^{12}\)

\(^{10}\)This may correspond to the case in which the manager improves management systems, engages in product innovation, or makes improvements to existing production facilities.

\(^{11}\)Note that \( d\left(E\log Y\right) = (\alpha - \frac{1}{2}\sigma^2)dt \) using Ito’s Lemma, where \( E \) is the expectation operator.

\(^{12}\)Similar assumptions were used in Hermalin and Weisbach (1998), Hermalin (2005) and Inderst and
Hence, the firm’s instantaneous cash flow under a new manager is

\[ R = XY. \]  

(3)

Because the firm’s instantaneous cash flow is the same for any new manager, the firm does not replace any new manager after replacing the incumbent manager. In addition, as \( \theta_1 > 0 \), the incumbent manager is never fired if he is known to draw \( \theta_1 \). Thus, we only consider the replacement of the incumbent manager with \( \theta_i = \theta_2 \). This implies that a new manager can pull the bad performance firm together after the replacement of the incumbent manager with \( \theta_i = \theta_2 \).

We assume that the incumbent manager’s choice of \( h \) is his private information (i.e., it is a hidden action), and that the contracting parties can observe the stochastic component of the firm’s instantaneous cash flow, \( Y \), as well as the firm’s whole instantaneous cash flow, \( R \), but cannot verify \( Y \) or \( R \) to the third parties. In addition, the contracting parties can observe the project quality (i.e., \( \theta_i \)) soon after the manager invests, and verify \( \theta_i \) to the third parties once the project is started. The unverifiability assumption about \( Y \) and \( R \) can be reasonable if the firm is a venture firm, a restructuring firm or a private firm. The verifiability assumption about \( \theta_i \) can be justified if the contracting parties cannot verify whether the manager succeeds in improving management systems, establishing product innovation or making improvements to existing production facilities until the project is started. All other variables, including the timing of the decision to start the project or replace the manager, are publicly observed, and the probability, \( (q_H, q_L) \), and the diffusion process of \( Y \) are common knowledge.

Because the incumbent manager’s choice of \( h \) is his private information, the owner needs to motivate him to make investment \( h = H \) at time zero by offering him a compensation contract that commits the owner to paying him at the commencement of the project

\cite{Mueller2006}. The assumption that the new manager cannot be hired by the firm at time 0 can be justified if the incumbent manager only has the ability to start the project or if the new manager only undertakes the activities after the project starts.
and at the time of his replacement. Specifically, the owner can make the compensation contract conditional upon the verifiable component of the project’s value at the time the project starts (performance pay) and the time the manager is replaced (severance pay). As $h$ has only two possible values, we focus on compensation contracts that yield the incumbent manager a fixed base pay of $W_0$ at time zero, a payment of $W_i$ ($i = 1, 2$) if the project is exercised and the project quality, $\theta_i$, is verified, and a severance pay of $S$ if the manager is replaced. Note that $W_0$ cannot be contingent on the project quality at time 0 because the project quality cannot be verified until the project starts. Also, note that only the manager with $\theta_i = \theta_2$ is fired because $\theta_1 > 0$ implies that the manager with $\theta_i = \theta_1$ is never fired. $W_1$ ($W_2$) may be interpreted as a reward for success (nonsuccess) paid to the incumbent manager. Because a new manager does not have a choice of investment, we can set his compensation, $W_N$, equal to 0 to simplify the analysis.

The equilibrium of the game is given as follows. (i) At time zero, the owner offers a compensation contract, $(W_0, W_1, W_2, S)$, to the incumbent manager in order to maximize her option value. The manager chooses whether to make investment $h = H$ or $h = L$ in order to maximize his option value. (ii) After the manager chooses $h = H$ or $h = L$, the owner pays $W_0$ and determines the timings of the project’s commencement and of the manager’s replacement. (iii) If the project is exercised, the owner pays $W_i$, conditional on $\theta_i$, to the incumbent manager. (iv) If the incumbent manager with $\theta_i = \theta_2$ is replaced, the owner pays him $S$, and hires a new manager from the pool of candidate successors.

The main difference between the present model and the former agency model with real options such as Grenadier and Wang (2005) and Hori and Osano (2009) is that the owner cannot precommit to the timing decision of the project’s commencement or of the manager’s replacement before the manager chooses the level of investment. Thus,

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13 Like Hermalin and Weisbach (1998), Hermalin (2005), Grenadier and Wang (2005), Spear and Wang (2005) and Inderst and Mueller (2006), we exclude the possibility of contract renegotiation. Hori and Osano (2009) discuss the possibility of contract renegotiation when the owner can be committed to the timing of the project’s commencement and the managerial replacement after renegotiation.

14 Although Hori and Osano (2009) do not consider $W_0$ in the compensation contract, the introduction of $W_0$ simplifies the analysis without affecting our main results.

15 The owner’s noncommitment to the timing strategy is caused by the assumption that $Y$ and $R$ are
the owner determines the timing of the project’s commencement and of the manager’s replacement after the manager chooses \( h = H \) or \( h = L \).

To simplify the analysis, we make the following assumptions.

**Assumption 1:** \( \frac{C_E}{q_H - q_L} > C_L \).

**Assumption 2:** \( \frac{C_E}{K} > -\frac{\theta_2}{X + \theta_2} \).

**Assumption 3:** The differences \( q_H - q_L \) and \( \theta_1 - \theta_2 \) are sufficiently large so that it is optimal to induce the manager to choose \( h = H \) both under the first-best solution and under the solution to the owner’s maximization problem in the hidden-action model.

Assumption 1 indicates that the cost–benefit ratio for the owner inducing the manager’s investment, \( \frac{C_E}{q_H - q_L} \equiv \frac{C_E}{\Delta q} \), is larger than the cost of the loss of corporate control incurred by the manager. Unless this assumption is satisfied, there is no possibility of moral hazard caused by the manager; thus, \( W_1 = W_2 = S = 0 \) becomes a solution. Assumption 2 implies that the ratio of the firing cost to the setup cost for the owner is larger than the loss ratio of the instantaneous cash flow under the manager with \( \theta_2 \). As verified in the subsequent sections, Assumption 2 ensures that the owner does not replace the incumbent manager until the project starts.

3. **First-best Solution (The Standard Real Options Case)**

Before analyzing the equilibrium, we briefly review the first-best solution used as a benchmark. The first-best solution is derived by maximizing the option value to the owner at time zero, provided that the firm-specific human capital investment of the incumbent manager is publicly observable and contractible, that the manager is compensated for the loss of control benefits, \( C_L \), and that the owner can precommit to the timing of the project’s commencement and of the manager’s replacement before the manager chooses the level of investment. To derive the first-best solution, we use the backward induction method. First, we analyze the owner’s option value with respect to the replacement of the unverifiable so that the contract cannot stipulate the trigger points contingent on \( Y \) or \( R \).
incumbent manager with $\theta_2$, and then we discuss the owner’s option value with respect to the commencement of the project for each $\theta_i$ ($i = 1, 2$). Finally, we consider the owner’s option value at time zero.

Here, we summarize the results on the first-best replacement trigger level and the owner’s option value with respect to the replacement trigger.

**Proposition 1:** Let $Y_{rFB}^\ast$ denote the first-best trigger for the replacement of the incumbent manager with $\theta_2$, and let $V_r(Y)$ denote the corresponding owner’s option value. Then,

$$Y_{rFB}^\ast = \frac{\beta}{\beta - 1} \frac{r - \alpha}{-\theta_2}(C_F + C_L),$$

$$V_r(Y) = \left( \frac{Y}{Y_{rFB}^\ast} \right)^\beta \left( \frac{-\theta_2 Y_{rFB}^\ast}{r - \alpha} - C_F - C_L \right),$$

where $\beta = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{(\frac{\alpha}{\sigma^2} - \frac{1}{2})^2 + \frac{2\sigma^2}{\sigma^2}} (> 1)$.

This proposition suggests that, as $Y$ must be high enough to compensate for $C_F + C_L$, the owner does not replace the manager with $\theta_2$ until the first time at which $Y$ hits the trigger $Y_{rFB}^\ast$. The intuitive reason is that there is an opportunity cost associated with replacing the manager today that is created by irreversible replacement costs and the uncertain future values of $Y$; that is, the option value of waiting to replace the manager implies an action threshold where the expected value from replacing the manager, $\frac{-\theta_2 Y_{rFB}^\ast}{r - \alpha}$, exceeds the cost, $C_F + C_L$. This feature cannot be captured in the static model. Because the higher $Y_{rFB}^\ast$ corresponds to the replacement option being exercised at a later time, the higher $Y_{rFB}^\ast$ implies that the expected tenure of the manager becomes longer.

Next, we derive the results for the first-best commencement trigger levels and the corresponding owner’s option values with respect to the commencement triggers.

**Proposition 2:** (i) Let $Y_{c1}^{FB}$ denote the first-best trigger for the commencement of the project when $\theta_i = \theta_1$, and let $V_{c1}(Y)$ denote the corresponding owner’s option value.
Then,

\[ Y_{c1}^{FB} = \frac{\beta}{\beta - 1} \frac{r - \alpha}{X + \theta_1} K, \]  

(6)

\[ V_{c1}(Y) = \begin{cases} 
\left( \frac{Y}{Y_{c1}^{FB}} \right)^{\beta} \left[ \frac{(X + \theta_1)Y_{c1}^{FB}}{r - \alpha} - K \right] & \text{if } Y < Y_{c1}^{FB}, \\
\frac{(X + \theta_1)Y}{r - \alpha} - K & \text{if } Y_{c1}^{FB} \leq Y.
\end{cases} \]  

(7)

(ii) Let \( Y_{c2}^{FB} \) denote the first-best trigger for the commencement of the project when \( \theta_i = \theta_2 \), and let \( V_{c2}(Y) \) denote the owner’s option value. Then,

\[ Y_{c2}^{FB} = \frac{\beta}{\beta - 1} \frac{r - \alpha}{X + \theta_2} K, \]  

(8)

\[ V_{c2}(Y) = \begin{cases} 
\left( \frac{Y}{Y_{c2}^{FB}} \right)^{\beta} \left[ \frac{(X + \theta_2)Y_{c2}^{FB}}{r - \alpha} - K \right] + \left( \frac{Y}{Y_{c2}^{FB}} \right)^{\beta} \left( \frac{-\theta_2 Y_{c2}^{FB}}{r - \alpha} - C_F - C_L \right) - C_F - C_L & \text{if } Y < Y_{c2}^{FB}, \\
\frac{(X + \theta_2)Y}{r - \alpha} - C_F - C_L & \text{if } Y_{c2}^{FB} \leq Y < Y_{r}^{FB}, \\
XY - C_F - C_L & \text{if } Y_{r}^{FB} \leq Y.
\end{cases} \]  

(9)

This proposition indicates that if \( \theta_i = \theta_1 \), the owner does not start the project until the first time that \( Y \) reaches the trigger \( Y_{c1}^{FB} \), and that if \( \theta_i = \theta_2 \), the owner neither starts the project until \( Y \) first hits the trigger \( Y_{c2}^{FB} \) nor replaces the manager until \( Y \) first hits the trigger \( Y_{r}^{FB} \). The intuitive reason is that there is an opportunity cost associated with launching the project today that is created by the irreversible setup cost and the uncertain future values of \( Y \); that is, the option value of waiting to launch the project implies an action threshold where the expected value from launching the project, \( \frac{(X + \theta_i)Y_{c}^{FB}}{r - \alpha} \), exceeds the cost, \( K \). Because the higher \( Y_{c}^{*} \) corresponds to the commencement option being exercised at a later time, the higher \( Y_{c}^{*} \) implies that the project starts later.

Several remarks on Propositions 1 and 2 are in order. First, it follows from Assumption 2 that \( Y_{c2}^{FB} < Y_{r}^{FB} \). Hence, under the first-best solution, the owner does not replace the incumbent manager until the project starts. Second, the first-best timing of the project’s
commencement and the manager’s replacement do not depend on the initial value of $Y_0$ because of the time-consistent structure of our model. Hence, the first-best timings are determined independently of time, regardless of when the decisions are made.

4. The Optimal Compensation Contract and Trigger Strategies

In this section, we discuss the optimal compensation contract and trigger strategies under the moral hazard model given in Section 2, provided that the firm-specific human capital investment of the incumbent manager is privately observed only by the manager and that the owner cannot precommit to timing the project’s commencement or the manager’s replacement before the manager chooses the level of investment. In the subsequent analysis, we again work backwards to derive the optimal compensation contract and trigger strategies. First, taking the compensation contract as given, we explore the owner’s maximization problem with respect to the trigger points for launching the project and replacing the incumbent manager, and then examine the owner’s maximization problem with respect to the compensation contract at time zero under the moral hazard incentive of the incumbent manager.

4.1. The optimal trigger strategy and the owner’s option value for a given compensation contract.

Here, we summarize our results for the optimal replacement trigger level and the owner’s option value with respect to the replacement trigger.

**Proposition 3:** Let $Y_{r}^{*}$ denote the optimal trigger for the replacement of the incumbent manager, and let $\Pi_{r}^{O}(Y)$ denote the corresponding owner’s option value. Then,

\[
Y_{r}^{*} = \frac{\beta}{\beta - 1} \frac{r - \alpha}{-\theta_2} (S + C_F), 
\]

\[
\Pi_{r}^{O}(Y) = \left( \frac{Y_{r}^{*}}{Y_{r}} \right)^{\beta} \frac{S + C_F}{\beta - 1},
\]

where $\beta$ is the same as that defined in Proposition 1.
Several remarks are in order. First, this proposition implies that, as \( Y \) must be high enough to compensate for \( S + C_F \), the owner does not replace the manager with \( \theta_2 \) until the first time at which \( Y \) hits the trigger \( Y_r^* \). The intuitive reason is similar to that given below Proposition 1. Second, because the higher \( Y_r^* \) corresponds to the replacement option being exercised at a later time, the higher \( Y_r^* \) implies that the expected tenure of the manager becomes longer. Third, an increase in \( S \) increases the noncommitment trigger point of \( Y_r^* \). Because the owner cannot be committed to the ex ante promised replacement trigger, she regards the severance payment \( S \) as a sunk cost on the replacement of the manager in addition to the firing cost \( C_F \). Thus, the owner delays the replacement of the manager if \( S \) increases.

Next, we derive our results for the optimal commencement trigger levels and the owner’s option values with respect to the commencement triggers.

**Proposition 4:** (i) Let \( Y_{c1}^* \) denote the optimal trigger for the commencement of the project when \( \theta_1 = \theta_1 \), and let \( \Pi_{c1}^O(Y) \) denote the corresponding owner’s option value. Then,

\[
Y_{c1}^* = \frac{\beta}{\beta-1} X + \frac{r}{1} \frac{\theta_1}{(W_1 + K)},
\]

\[
\Pi_{c1}^O(Y) = \begin{cases} 
\frac{Y}{Y_{c1}^*} & \text{if } Y < Y_{c1}^*, \\
\frac{W_1 + K}{\beta-1} & \text{if } Y_{c1}^* \leq Y.
\end{cases}
\]

(ii) Let \( Y_{c2}^* \) denote the optimal trigger for the commencement of the project when \( \theta_1 = \theta_2 \), and let \( \Pi_{c2}^O(Y) \) denote the corresponding owner’s option value. Then,

\[
Y_{c2}^* = \frac{\beta}{\beta-1} X + \frac{r-\alpha}{1} \frac{\theta_2}{(W_2 + K)},
\]
Several comments about this proposition are in order. First, if \( \theta_i = \theta_1 \), the owner does not start the project until the first time that \( Y \) reaches the trigger \( Y_1^* \). Similarly, if \( \theta_i = \theta_2 \), the owner neither starts the project until \( Y \) first hits the trigger \( Y_2^* \) nor replaces the manager until \( Y \) first hits the trigger \( Y_r^* \). The intuitive reason is similar to that given below Proposition 2. Second, because the higher \( Y_i^* \) corresponds to the commencement option being exercised at a later time, the higher \( Y_i^* \) implies that the project starts later. Third, an increase in \( W_1 \) (or \( W_2 \)) increases the noncommitment trigger point of \( Y_1^* \) (or \( Y_2^* \)). Because the owner cannot be committed to the ex ante promised commencement triggers, she regards the success reward \( W_1 \) (or the non-success reward \( W_2 \)) as a sunk cost at the start of the project in addition to the setup cost \( K \). Thus, the owner delays the start of the project if \( W_1 \) (or \( W_2 \)) increases.

4.2. The owner’s maximization problem at time zero.—

Now, it follows that the owner’s option value is \( \Pi_{O1}^c(\hat{Y}) \) given by (13) if \( \theta_i = \theta_1 \), and \( \Pi_{O2}^c(\hat{Y}) \) given by (15) if \( \theta_i = \theta_2 \). Conditional on the incumbent manager making investment \( h = H \), the owner’s option value at time zero, \( \Pi^O(Y_0, q_H) \), is written as

\[
\Pi^O(Y_0, q_H) = q_H \left( \frac{Y_0}{Y_1^*} \right)^{\beta} \frac{W_1 + K}{\beta - 1} + (1 - q_H) \left[ \left( \frac{Y_0}{Y_1^*} \right)^{\beta} \frac{W_2 + K}{\beta - 1} + \left( \frac{Y_0}{Y_2^*} \right)^{\beta} \frac{S + C_F}{\beta - 1} \right] - W_0. \tag{16}
\]

Note that the owner makes the fixed base payment \( W_0 \) at time zero.

The manager’s option with respect to the commencement of the project for each \( \theta_i \) has a payoff function of \( W_i \), whereas his option with respect to his replacement has a payoff function of \( S - C_L \). Conditional on the manager investing \( h = H \), the value of his option
is given by

\[ \Pi^M (Y_0, q_H) = q_H \left( \frac{Y_0}{Y_{c1}} \right)^\beta W_1 + (1 - q_H) \left[ \left( \frac{Y_0}{Y_{c2}} \right)^\beta W_2 + \left( \frac{Y_0}{Y_r^*} \right)^\beta (S - C_L) \right] + W_0 - C_E. \]  

(17)

Note that the manager receives \( W_0 \), and incurs the effort cost \( C_E \) at time zero when he invests \( h = H \).

Because the owner’s and manager’s option values at time zero are expressed by (16) and (17) and the triggers \((Y_r^*, Y_{c1}^*, Y_{c2}^*)\) are provided by (10), (12) and (14), the owner’s maximization problem is now represented as follows: \(^{16}\)

\[ \max_{\{W_0, W_1, W_2, S\}} \Pi^O (Y_0, q_H) \text{ given by (16), } \]  

(18)

subject to (10), (12), (14), and

\[ q_H \left( \frac{Y_0}{Y_{c1}} \right)^\beta W_1 + (1 - q_H) \left[ \left( \frac{Y_0}{Y_{c2}} \right)^\beta W_2 + \left( \frac{Y_0}{Y_r^*} \right)^\beta (S - C_L) \right] - C_E \]

\[ \geq q_L \left( \frac{Y_0}{Y_{c1}} \right)^\beta W_1 + (1 - q_L) \left[ \left( \frac{Y_0}{Y_{c2}} \right)^\beta W_2 + \left( \frac{Y_0}{Y_r^*} \right)^\beta (S - C_L) \right], \]  

(IC)

\[ q_H \left( \frac{Y_0}{Y_r^*} \right)^\beta W_1 + (1 - q_H) \left[ \left( \frac{Y_0}{Y_{c2}} \right)^\beta W_2 + \left( \frac{Y_0}{Y_r^*} \right)^\beta (S - C_L) \right] + W_0 - C_E \geq 0, \]  

(IR)

\[ Y_r^* \geq Y_{c2}^*, \]  

(TR)

\[ W_0 \geq 0, \ W_1 \geq 0, \ W_2 \geq 0, \ S \geq 0. \]  

(LL)

Here, (IC) is the incentive compatibility constraint for the incumbent manager, which ensures that he prefers to invest \( h = H \) rather than \( h = L \). Notice that by choosing \( h = H \),

\(^{16}\)For simplicity, we neglect the nonnegativity constraints of \( Y_{c1} \), \( Y_{c2} \) and \( Y_r \). The difference between this problem and the standard contract problem is that the owner’s and manager’s payoffs are evaluated at the present value operator, and that the present value operator depends on the compensation contract.
the manager may receive the higher expected present value of compensation, but incurs the effort cost $C_E$. (IR) is the individual rationality constraint for the incumbent manager, which guarantees that the option value to him of accepting the contract is greater than or equal to the investment cost. Note that the individual rationality constraint for the new manager is always satisfied because he receives zero wages and incurs no investment or replacement costs. (TR) is the constraint for the triggers $Y_{c^2}$ and $Y_r$, which indicates that replacement of the incumbent manager does not occur before the project is started. Finally, (LL) denotes the limited liability constraints for the incumbent manager. Let $(W_0^*, W_1^*, W_2^*, S^*)$ denote the solution to problem (18).

4.3. The optimal contract and trigger strategies.—

To characterize the solution to problem (18), we simplify the problem by presenting the following lemmas.

**Lemma 1:** Under Assumption 1, $W_1^* > 0$.

**Lemma 2:** Under Assumptions 1 and 2, (IC) is always binding if (IR) is binding.

**Lemma 3:** Under Assumptions 1 and 2, (IC) is always binding if (TR) is binding.

Lemma 1 shows that $W_1^*$ must be positive in order to motivate the manager to choose $h = H$ under Assumption 1. Lemma 2 implies that (IR) is not binding if (IC) is not binding. The intuition for Lemma 2 is that if (IR) is binding while (IC) is not binding, the optimal solution involves $W_1^* = W_2^* = S^* = 0$, which contradicts Lemma 1 under Assumption 1. Finally, Lemma 3 suggests that (TR) is not binding if (IC) is not binding.

The intuition for Lemma 3 is that $W_1^*$ can be adjusted so that (IC) is always binding whenever (TR) is binding.

Now, using Lemmas 1–3, we obtain the following proposition.

**Proposition 5:** Suppose that Assumptions 1–3 hold.

(i) Suppose that

\[
\frac{q_L}{q_H - q_L} \frac{C_E}{C_L} > (Y_0)^{\beta} \left( \frac{\beta}{\beta + 1} \frac{r - \alpha}{-\theta_2} C_F \right)^{-\beta}.
\]

(19)
Then, the optimal compensation contract involves \((W_0^*, W_1^*, W_2^*, S^*)\) that satisfy

\[
W_1^* = - \left( \frac{Y_{c1}^*}{Y_r^*} \right)^\beta C_L + \left( \frac{Y_{c1}^*}{Y_0^*} \right)^\beta \frac{C_E}{q_H - q_L} > 0,
\]

\[
W_0^* = W_2^* = S^* = 0.
\]

The optimal triggers \((Y_r^*, Y_{c1}^*, Y_{c2}^*)\) are still given by \((10), (12)\) and \((14)\) for such \((W_1^*, W_2^*, S^*)\), respectively.

(ii) Suppose that \(q_L q_H - q_E C_L \leq (Y_0^*)^\beta \left( \frac{\beta}{\beta - 1} \frac{r - \alpha}{-\theta_2} C_F \right)^{-\beta} \).

(a) If \(S^* = 0\), then \((W_1^*, W_2^*, S^*, Y_r^*, Y_{c1}^*, Y_{c2}^*)\) are still characterized by part (i) of this proposition. Furthermore, \(W_0^*\) is expressed by the following relation for such \((W_1^*, Y_r^*, Y_{c1}^*)\):

\[
W_0^* = -q_H \left( \frac{Y_0^*}{Y_{c1}^*} \right)^\beta W_1^* + (1 - q_H) \left( \frac{Y_0^*}{Y_r^*} \right)^\beta C_L + C_E \geq 0.
\]

(b) If \(S^* > 0\), then \((W_1^*, W_2^*, S^*)\) satisfy the following relations:

\[
W_1^* = \left( \frac{Y_{c1}^*}{Y_r^*} \right)^\beta (S^* - C_L) + \left( \frac{Y_{c1}^*}{Y_0^*} \right)^\beta \frac{C_E}{q_H - q_L} > 0,
\]

\[
W_2^* = 0;
\]

and \((W_0^*, S^*)\) must also satisfy

\[
\frac{S^* - C_L}{S^* + C_F} = \frac{q_H W_1^*}{\beta - (1 - q_H) W_1^* - (1 - q_H)K} < 0,
\]

\[
W_0^* = -q_H \left( \frac{Y_0^*}{Y_{c1}^*} \right)^\beta W_1^* - (1 - q_H) \left( \frac{Y_0^*}{Y_r^*} \right)^\beta (S^* - C_L) + C_E > 0,
\]
or

\[ S^* = C_L - \left( \frac{Y^*_r}{Y_0} \right)^\beta \frac{q_L C_E}{q_H - q_L} < C_L; \]  
\[ W_0^* = 0. \]  

(26b) (27b)

The optimal triggers \((Y^*_r, Y^*_{c1}, Y^*_{c2})\) are still given by (10), (12) and (14) for such \((W_1^*, W_2^*, S^*)\), respectively.

**Corollary to Proposition 5:** Suppose that Assumptions 1–3 hold.

(i) Suppose that (19) holds. Then, \(Y^*_{c1} > Y_{FB}^*\), \(Y^*_{c2} = Y_{FB}^*\), and \(Y^*_r < Y_{FB}^*\).

(ii) Suppose that (22) holds. Then, \(Y^*_{c1} > Y_{FB}^*\), \(Y^*_{c2} = Y_{FB}^*\), and \(Y^*_r < Y_{FB}^*\). In addition, \(S^* < C_L\).

The intuition behind this proposition is as follows.\(^{17}\) Suppose that the cost–benefit ratio of inducing the manager’s investment, \(\frac{C_E}{q_H - q_L} = \frac{C_E}{q_H - q_L} \equiv \frac{C_E}{q_H - q_L}\), divided by the cost of the loss of corporate control compared with the likelihood of success when the manager does not invest, \(\frac{C_L}{q_L}\), is sufficiently large (that is, (19) holds). This corresponds to the case in which the manager has great incentives to avoid incurring the investment cost, but not much incentive to avoid being replaced; in other words, there is a severe moral hazard problem. In this case, using Lemmas 1–3 under Assumptions 1 and 2, the owner only needs to consider (IC) with (LL) of \(W_2\) and \(S\) to maximize her option value at time zero. Indeed, setting \(W_2 = S = 0\) maximizes the owner’s option value at time zero for any \(W_0 \geq 0\) and \(W_1 > 0\) because a decline in \(W_2\) (or \(S\)) decreases the sunk cost at the start of the project (or at the replacement of the manager) from the owner’s ex post viewpoint and thereby reduces the commencement trigger of the lower-quality project (or the replacement trigger).\(^{18}\) Similarly, setting \(W_2 = S = 0\) relaxes (IC) to the largest extent for any \(W_1 > 0\) because the manager then has the greatest incentive to invest \(h = H\) for

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\(^{17}\)The intuition behind the corollary of this proposition will be provided later, together with the intuition behind Proposition 7.

\(^{18}\)Note that \(\partial \Pi^O (Y_0, q_H) / \partial W_2 = - (1 - q_H) \left( \frac{Y_0}{Y^*_{c2}} \right)^\beta < 0\) using (14), and that \(\partial \Pi^O (Y_0, q_H) / \partial S = - (1 - q_H) \left( \frac{Y_0}{Y^*_r} \right)^\beta < 0\) using (10).
any $W_1 > 0$.\footnote{Let $\Phi(W_1, W_2, S) \equiv (q_H - q_L) \left( \frac{Y_0}{\gamma_0} \right)^{\beta} W_1 - (q_H - q_L) \left[ \left( \frac{Y_0}{\gamma_0} \right)^{\beta} W_2 + \left( \frac{Y_0}{\gamma_0} \right)^{\beta} (S - C_L) \right] - C_E$. Then, it follows from (10) that $\Phi(W_1, 0, 0) \geq \Phi(W_1, W_2, S)$ for any $W_1 > 0$ and any $(W_2, S) \geq (0, 0)$.} Note that the replacement trigger also becomes an important incentive device because it directly affects the loss of the option value to the manager at the loss of corporate control; thus, $S$ must be set equal to zero so that $\left( \frac{Y_0}{\gamma_0} \right)^{\beta} C_L$ is maximized. These arguments show that it is optimal for the owner to set $W_2^* = S^* = 0$. Because $(IC)$ must be binding under the case of Assumptions 1 and 2 where the ex ante moral hazard problem exists and the dismissal loss for the owner is relatively large, $W_1^*$ is determined by the binding $(IC)$ for $W_2^* = S^* = 0$, that is, (20). For such $(W_1^*, W_2^*, S^*)$, the owner determines the triggers optimally after the manager chooses the investment level. Thus, $(Y_r^*, Y_{c1}^*, Y_{c2}^*)$ are still given by (10), (12) and (14) for such $(W_1^*, W_2^*, S^*)$, respectively.

Next suppose that $C_E$ divided by $C_L$ is smaller (that is, (22) holds), so that the moral hazard problem is not severe. Then, the owner needs to consider (IR) in addition to the (IC) and (LL) of $W_2$ and $S$ when maximizing her option value at time zero. In this case, because the owner must compensate the manager for the loss of his option value at the loss of corporate control, $-\left( \frac{Y_0}{\gamma_0} \right)^{\beta} C_L$, through $W_0$ in order to induce the manager to participate in the contract relation, the owner has an incentive to minimize this compensation by raising $Y_r^*$ through an increase in $S$. However, such increases in $S$ and $Y_r^*$ raise $W_1$ and $Y_{c1}^*$ by making (IC) more stringent. If the former income effect of a decline in $W_0$ is dominated by the latter incentive effect, the optimal severance pay $S^*$ must be set equal to zero. Then, the results under (19) still hold, although $W_0^*$ must be positive because (IR) is binding. In contrast, if the former income effect of a decline in $W_0$ dominates the latter incentive effect, the optimal severance pay $S^*$ must then be positive. In this case, $S^*$ must be smaller than the cost of the loss of corporate control incurred by the manager, $C_L$. However, the effects of $W_1$ and $W_2$ are similar to those in the case of (19). Hence, $W_2^* = 0$. In addition, if $W_0^* > 0$, then $(W_1^*, S^*)$ are simultaneously determined by their first-order conditions and (IC) for $W_2^* = 0$. Thus, (24) and (26a) hold. On the other hand, if $W_0^* = 0$, then $(W_1^*, S^*)$ are simultaneously determined by
(IC) and (IR) for \( W^*_0 = W^*_2 = 0 \). Thus, (24) and (26b) hold. For such \((W^*_1, W^*_2, S^*)\), \((Y^*_r, Y^*_1, Y^*_2)\) are still given by (10), (12) and (14), respectively.

As an example, consider the parametric case in which \( r = 0.05, \alpha = 0.03, \sigma = 0.2, K = 100, C_F = 20, C_L = 30, C_E = 12, X = 4, \theta_1 = 0.5, \theta_2 = -0.5, q_H = 0.4 \) and \( q_L = 0.1 \). The risk-free rate \( r \), the drift term \( \alpha \) and the standard deviation \( \sigma \) follow Dixit and Pindyck (1994). We set the cost parameters \((K, C_F, C_L, C_E)\) so that \( K \) is considerably larger than any of \((C_F, C_L, C_E)\). The revenue parameters \((X, \theta_1, \theta_2)\) and the success probability \((q_H, q_L)\) are chosen to satisfy our assumptions for \((K, C_F, C_L, C_E)\). Because Proposition 5 shows that \( S^* \) can be positive only if \( \frac{\beta}{\beta - 1} \left( \frac{r - \alpha}{g_2} \right) \left( \frac{q_L - q_H}{q_H - q_L} \right) = 0.69312 \leq Y_0 \), we can neglect the case of \( Y_0 \leq 0.69312 \). Table 1 indicates the three possibilities for the optimal compensation contracts for different values of \( Y_0 \) (\( \geq 1.6 \)).\(^{20}\) Table 1A calculates \((W^*_0, W^*_1, W^*_2, S^*)\) and \( \Pi^O(Y_0, q_H) \) that correspond to the case of Proposition 5(ii)(a). Note that in this case, \( W^*_2 = S^* = 0 \). Table 1B represents \((W^*_0, W^*_1, W^*_2, S^*)\) and \( \Pi^O(Y_0, q_H) \) that correspond to the case of Proposition 5(ii)(b) when \((W^*_0, S^*)\) are given by (26a) (or (26a') in Appendix B) and (27a), whereas Table 1C indicates \((W^*_0, W^*_1, W^*_2, S^*)\) and \( \Pi^O(Y_0, q_H) \) that correspond to the case of Proposition 5(ii)(b) when \((W^*_0, S^*)\) are given by (26b) and (27b). Note that \( W^*_2 = 0 \) in the former case, whereas \( W^*_0 = W^*_2 = 0 \) in the latter case. In addition, \( S^* \) in Table 1B for any value of \( Y_0 \geq 2.2 \) is calculated by (26a') in Appendix B instead of (26a) because \( Y_0 \geq Y^*_{1,1} \) in these values of \( Y_0 \). Now, comparing \( \Pi^O(Y_0, q_H) \) for the three cases at each \( Y_0 \), we indicate that the optimal compensation contract involves \( S^* > 0 \) for any \( Y_0 \) listed in Table 1.

### 4.4. Comparison of the solution under the noncommitment timing case with the solution under the commitment timing case and with the first-best solution

To compare the solution under the noncommitment timing case with that under the commitment case, we summarize the optimal compensation contract \((W^*_0, W^*_1, W^*_2, S^*)\)

\(^{20}\)If \( Y^*_{1,1} < Y_0 \) or \( Y^*_{1,2} < Y_0 \), we can set \( Y^*_{1} = Y_0 \) or \( Y^*_{2} = Y_0 \). Even in this case, only (16) and (26a) need to be modified, whereas the remaining results are unchanged; see Appendix B.
and optimal trigger levels \((Y_r^{**}, Y_c^{**}, Y_c^{**})\) under the commitment timing case. To simplify the procedure, we impose the more stringent assumption instead of Assumption 2.

**Assumption 2’**: \(\frac{C_F}{R} - \frac{q_H}{1-q_H} \frac{C_L}{R} > -\frac{q_2}{X+q_2}\).

Then, using the results of Hori and Osano (2009), we obtain the following proposition.

**Proposition 6**: Suppose that Assumptions 1, 2’ and 3 hold.

(i) If \(\frac{q_H}{C_F} > (Y_0) \beta \left[ \frac{\beta - r - q_2}{1-q_H} (C_F - \frac{q_H}{1-q_H} C_L) \right]^{-\beta}\), then the optimal triggers are \(Y_{c1} = Y_{c1}^{FB}, Y_{c2} = Y_{c2}^{FB}\), and \(Y_r^{**} = Y_r^{FB}\). The optimal compensation is given by
\[
W_1^{**} = \left( \frac{Y_0^r}{Y_0} \right)^\beta \frac{C_F}{q_H - q_L} - \left( \frac{Y_0^r}{C_L} \right) \beta C_L > 0 \text{ and } W_0^{**} = W_2^{**} = S_2^{**} = 0.
\]

(ii) If \(\frac{q_H}{C_F} > (Y_0) \beta \left[ \frac{\beta - r - q_2}{1-q_H} (C_F - \frac{q_H}{1-q_H} C_L) \right]^{-\beta}\), then the optimal triggers are \(Y_{c1}^{**} = Y_{c1}^{FB}, Y_{c2} = Y_{c2}^{FB}\), and \(Y_r^{**} \leq Y_r^{FB}\). Furthermore, \(W_1^{**} > 0\) and \(W_2^{**} = S_2^{**} = 0\).

Now, using Propositions 5 and 6 with the Corollary to Proposition 5, the following proposition is established.

**Proposition 7**: Suppose that Assumptions 1, 2’ and 3 hold.

(i) Suppose that (19) holds.

(a) \(Y_{c1} > Y_{c1}^{**} = Y_{c1}^{FB}\) and \(Y_{c2} = Y_{c2}^{**} = Y_{c2}^{FB}\).

(b) If \(\frac{q_H}{C_F} > (Y_0) \beta \left[ \frac{\beta - r - q_2}{1-q_H} (C_F - \frac{q_H}{1-q_H} C_L) \right]^{-\beta}\), then \(Y_r^{**} < Y_r^{**} < Y_r^{FB}\). Otherwise, \(Y_r^{**}\) is smaller than \(Y_r^{FB}\), but can be larger or smaller than \(Y_r^{**}\).

(c) \(W_1 > 0, W_1^{**} > 0, \) and \((W_2^{**}, S^{**}) = (W_2, S) = (0,0)\).

(ii) Suppose that (22) holds. Then, the results of part (i) still hold, except that (a) \(Y_r^{**}\) is smaller than \(Y_r^{FB}\), but can be larger or smaller than \(Y_r^{FB}\), and (b) \(S^{**} > 0\) may hold.

If the owner cannot precommit to the commencement or replacement trigger promised before the manager chooses \(h = H\) or \(L\), the owner must design the optimal compensation contract \((W_1^*, W_2^*, S^*)\) by considering its effects on the ex post determination of the trigger levels of \((Y_r^*, Y_{c1}^*, Y_{c2}^*)\). Suppose that there is a severe moral hazard problem (19) holds) so that (IR) is never binding. Then, the owner pays the success reward \((W_1^* > 0)\), but does not pay the non-success reward or the severance pay \((W_2^* = S^* = 0)\). After the manager chooses the level of investment for \((W_1^*, W_2^*, S^*)\), the owner determines the commencement
and replacement triggers. In comparison with the first-best case, the owner starts the higher-quality project later, while commencing the lower-quality project at the same first-best time, and she replaces the incumbent manager earlier. By comparison with the commitment timing strategy, she launches the higher-quality project later, although she launches the lower-quality project at the same time as the first-best case. The owner also replaces the incumbent manager later under the noncommitment timing strategy if the moral hazard problem is severe enough ($\frac{q_{H}C_{F}}{q_{H}-q_{L}} > (Y_{0})^{\beta} \left[ \frac{\beta}{\beta-1} \frac{r-a}{Y_{0}} (C_{F} - \frac{q_{H}}{1-q_{H}}C_{L}) \right]^{-\beta}$).

Next, suppose that the moral hazard problem is not severe ((22) holds) so that (IR) is binding. The results of the commencement triggers in this case are similar to those in the case of (19). On the other hand, the owner not only pays the success reward but may also pay the severance pay, although she never pays the non-success reward. As the severance pay increases the owner’s sunk cost at the replacement of the manager from her ex post viewpoint, she wants to replace the incumbent manager even later. In fact, in this parametric case, the owner need not rely heavily on the replacement trigger in order to motivate the manager to choose the higher level of investment if the owner can precommit to her timing strategy. Hence, the owner may replace the incumbent manager earlier under the noncommitment timing case than under the commitment case. Irrespective of which result is obtained, under the noncommitment timing case, the optimal severance pay must be smaller than the loss of the option value to the manager at his replacement, as shown in (ii) of the Corollary to Proposition 5. Thus, the owner never replaces the incumbent manager later than the first-best replacement timing.

The theoretical implications and intuitions about this proposition are discussed as follows. First, Proposition 7 shows that the higher-quality project is launched later under the noncommitment timing case than under both the first-best case and the commitment timing case, regardless of the severity of the manager’s moral hazard problem. Although the owner incurs the success reward as a sunk cost at the start of the project in addition to the setup cost, she need not regard the success reward as an additional sunk cost from her ex post viewpoint under both the first-best and commitment timing strategies because
in these two cases the compensation contract is not considered or is determined simultaneously with the trigger strategies at time 0 before the manager chooses the level of investment. By contrast, the owner must regard the success reward as an additional sunk cost under the noncommitment timing case because she determines the commencement trigger of the higher-quality project after the manager chooses the level of investment. Hence, as the owner utilizes the success reward in order to motivate the manager to select the higher level of investment, the option value of waiting to launch the project is even larger from her ex post viewpoint under the noncommitment timing case. Thus, the owner is forced to delay the start of the higher-quality project under the noncommitment timing case. These results are in contrast to those attained not only in the case of “hidden action only” in Grenadier and Wang (2005), where the project quality is publicly observable, but also in Hori and Osano (2009). The two studies indicate that the higher-quality project is started at the same time as for the first-best case.

This difference depends on the fact that the present paper deals with the case in which the owner cannot precommit to the commencement trigger promised before the manager chooses investment, whereas the other two papers investigate the case in which the owner can do so. Hence, in the present paper, the owner is forced to start the higher-quality project after the first-best timing as long as she must use the success reward as an incentive device for the manager. By contrast, in the other two papers, it is always efficient for the owner to start the higher-quality project at the same time as for the first-best case because she can be committed to the commencement trigger of the higher-quality project even after the manager chooses the level of investment.

Second, Proposition 7 suggests that regardless of the severity of the manager’s moral hazard problem, the lower-quality project is launched under the noncommitment timing case at the same time as for the first-best case, which is exactly the same as that under the commitment timing case. Irrespective of whether the owner can precommit to the commitment trigger, she never utilizes the nonsuccess reward. This is because the nonsuccess reward forces the manager to choose the lower level of investment. As the nonsuccess
reward is set equal to zero, the optimal commencement trigger of the lower-quality project is set equal to the first-best one. This result is the same as that in the model of “hidden action only” in Grenadier and Wang (2005) and in Hori and Osano (2009).

Third, Proposition 7 indicates that the incumbent manager is replaced earlier under the noncommitment timing case than under the first-best case. Proposition 7 also shows that the incumbent manager is replaced later under the noncommitment timing case than under the commitment case if the manager’s moral hazard problem is severe enough, but not necessarily if the manager’s moral hazard problem is not severe enough. Hence, the result of the replacement trigger in the present model is different from that attained in Hori and Osano (2009), in which the owner can be committed to the replacement trigger.

Under the noncommitment trigger case, the owner must regard the severance pay as an additional sunk cost when determining the replacement trigger after the manager chooses the level of investment. However, if the manager’s moral hazard problem is severe, she never utilizes the severance pay that forces the manager to choose the lower level of investment. In fact, even if the severance pay is set equal to zero, the optimal replacement trigger is lower than the first-best trigger because the replacement trigger, as well as the manager’s reward for success, is used as an incentive device to motivate the manager to choose the higher level of investment. This is because the earlier replacement directly increases the loss of the option value to the manager at the loss of corporate control. Thus, whether the owner can or cannot precommit to the replacement trigger, the optimal replacement trigger is lower than the first-best trigger because the earlier replacement serves to motivate the manager to select the higher level of investment.

However, if the manager’s moral hazard problem is severe enough, the optimal replacement trigger under the commitment timing case is set too low from the owner’s viewpoint after the manager selects the level of investment. Thus, it is very costly from the owner’s ex post viewpoint to exercise the replacement trigger at such a low level because she incurs the dismissal cost as a sunk cost at the replacement of the manager. Hence, the optimal replacement trigger is higher under the noncommitment timing case than under
the commitment case if the manager’s moral hazard problem is severe enough. On the other hand, if the manager’s moral hazard problem is not severe enough, the optimal replacement trigger under the commitment timing strategy may be too high from the owner’s ex post viewpoint. This implies that, from the owner’s ex post viewpoint, replacing the incumbent manager earlier may be more desirable. Then, it is possible that the incumbent manager is replaced earlier under the noncommitment timing case than under the commitment case.

If the manager’s moral hazard problem is not severe, the owner must consider the constraint of (IR). Then, the owner may compensate the manager for the loss of his option value on his replacement through his fixed base salary, thereby inducing him to participate in the contract relation. However, the owner may also utilize the severance pay to the manager, because an increase in the severance pay can decrease the fixed base salary by reducing the manager’s option value compensation through an increase in the replacement trigger. As the severance pay is an additional sunk cost, the optimal replacement trigger may be even higher under the noncommitment timing case. However, under the commitment timing case, the owner need not heavily rely on the replacement trigger as an incentive device in this parametric situation, thereby raising the optimal replacement trigger. Therefore, the optimal replacement trigger may be higher under the commitment timing case than under the noncommitment case.

Finally, the severance pay can be positive under the noncommitment timing strategy if the manager’s moral hazard problem is not severe. This result is in contrast to that attained in Hori and Osano (2009), in which the severance pay must be set equal to zero. As the owner can be committed to the ex ante promised triggers in Hori and Osano, she need not consider the effect of the compensation contract on the replacement trigger. Hence, taking the replacement trigger level as given, the owner only needs to fix the severance pay at zero. The reason for this is that an increase in severance pay decreases the owner’s option value at time zero, whereas it tightens the incentive compatibility constraint for the manager. Under the limited liability constraint, severance pay must
be minimized and set equal to zero. By contrast, in the present model, the owner must consider the effect of the compensation contract on the replacement trigger because she cannot be committed to the ex ante promised triggers. In the absence of severance pay, the owner may have an ex post incentive to replace the manager earlier after he chooses the level of investment. However, if the owner has an incentive to minimize the compensation for the manager’s loss of control, she needs to delay the replacement trigger, thereby reducing the compensation for the manager’s loss of option value on his replacement. Hence, the owner may utilize the severance pay that forces her to replace the manager later. This finding suggests that severance pay plays the role of committing the owner to replacing the manager later and of reducing the manager’s option value compensation on his replacement in order to alleviate his disincentive to participate in the contract relation.

In a static model of boards of directors, Almazan and Suarez (2003) suggest that severance pay serves to moderate the temptations of the party with residual control rights on the replacement decision to behave opportunistically against the other. More specifically, in strong boards, severance pay protects the CEO from the shareholders’ tendency to replace him unnecessarily, whereas in weak boards, severance pay protects shareholders from the CEO’s tendency to resist his own fully justified replacement. Thus, severance pay may be a more efficient and cheaper instrument for providing effort incentives than the on-the-job incentive pay. As a result, in Almazan and Suarez, the role of the severance pay depends on both the owner’s and manager’s replacement incentives, and the manager’s effort incentives. Similarly, Inderst and Mueller (2006) indicate that severance pay reduces the manager’s incentive to entrench himself so that it prevents an irreversible investment that reduces the firm’s future value under a potential successor.21

In a model of optimal termination of a long-term contract, Spear and Wang (2005) show that the agent must be dismissed when he produces a bad output and becomes too poor to be punished effectively or when he becomes too rich to be motivated to work

21 Shleifer and Vishny (1989) also informally suggest that the severance pay prevents many entrenching investments.
diligently. In the first case, the principal needs to make no severance payment, whereas in the second case, the principal needs to make a severance payment in order to compensate the agent for his exogenous promised utility. In the second case, the principal replaces the incumbent agent with a new agent because motivating the incumbent agent is too costly. As the second case happens when the agent’s utility has an income effect, the role of severance pay in Spear and Wang depends on the degree of risk aversion of the agent.

Unlike these studies, in the present paper, the owner who cannot precommit to the ex ante promised replacement trigger may use a positive severance payment to induce herself to choose a later replacement of the manager if the manager’s moral hazard problem is not severe. Severance pay then serves to reduce the loss of the manager’s option value on replacement through an increase in the replacement trigger, thereby minimizing the compensation for the manager’s loss of corporate control. In addition, in our model, the role of severance pay does not depend on the degree of risk aversion of the manager.

5. Conclusion

Given the owner’s noncommitment timing strategy and the manager’s moral hazard problem, this article has examined how the optimal compensation contract is designed and how the corresponding timing decisions to launch the project and replace the manager are determined. Using the real options approach, we show that in comparison with the first-best case, the higher-quality project is launched later, although the lower-quality project is launched at the same time as the first-best case, and the incumbent manager is replaced earlier. Furthermore, we also indicate that, compared with the commitment timing case, the higher-quality project is launched later, whereas the incumbent manager is (or is not necessarily) replaced later if the manager’s moral hazard problem is severe enough (or not severe enough). Unlike the folklore result of the standard moral hazard model, severance pay can play the role of committing the owner to replacing the incumbent manager later in order to reduce the compensation to the manager for the loss of his option value for loss of corporate control if the severity of the manager’s moral hazard problem
is not great.
Appendix A

Proof of Proposition 1: It follows from (2) and (3) that $dR = \alpha R dt + \sigma R dz$. Thus, using Ito’s Lemma, $V_r(Y)$ satisfies the differential equation $\frac{1}{2} \sigma^2 Y^2 V_{YY}(Y) + \alpha Y V_r(Y) - r V_r(Y) = 0$, where $V_{YY} = dV_r/dY$, $V_{Yr} = d^2V_r/d^2Y$, and $V_r(0) = 0$.$^{22}$ Ruling out a speculative bubble and using $V_r(0) = 0$, we can show that the solution is determined by $V_r(Y) = A_r Y^\beta$, where $A_r$ is a positive constant parameter and $\beta = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{(\frac{\alpha}{\sigma^2} - \frac{1}{2})^2 + \frac{2\sigma}{\sigma^2}}$ ($> 1$).

Let $F_{c2}(Y)$ ($F_r(Y)$) denote the expected present value of the firm before (after) the incumbent manager with $\theta_2$ is replaced. The standard procedure (see Dixit and Pindyck (1994)) shows that $F_{c2}(Y) = \frac{(X + \theta_2)Y}{r - \alpha}$ and $F_r(Y) = \frac{\alpha Y}{r - \alpha}$. Note that $F_{c2}(Y) > 0$ and $F_r(Y) > 0$ from $r > \alpha$. The replacement of the manager is equivalent to investing in a project with the value of $F_r(Y)$, while incurring $C_F + C_L$, and abandoning a project with the value of $F_{c2}(Y)$.$^{23}$ This implies that the firm pays $C_F + C_L$ and invests in a project with the value of $\Delta_2 F(Y)$, where $\Delta_2 F(Y) = F_r(Y) - F_{c2}(Y)$. Thus, $\Delta_2 F(Y)$ is written as

$$\Delta_2 F(Y) = \frac{\theta_2 Y}{r - \alpha}.$$

The boundary conditions in this problem are $V_r(Y_{rFB}) = \Delta_2 F(Y_{rFB}) - C_F - C_L$ and $\frac{dV_r(Y_{rFB})}{dY} = \frac{d[\Delta_2 F(Y_{rFB})]}{dY}$. The first boundary condition is the value-matching condition, which states that the owner’s payoff is $\Delta_2 F(Y_{rFB}) - C_F - C_L$ at the date at which the option is exercised. The second boundary condition is the smooth-pasting condition, which ensures that the exercise trigger is chosen to maximize the owner’s option value. Combining these two conditions with $V_r(Y) = A_r Y^\beta$, we can derive (4) and (5), given in Proposition 1. ||

Proof of Proposition 2: (i) Again, $V_{c1}(Y)$ equals $A_{c1} Y^\beta$, where $A_{c1}$ is a positive constant parameter. The value-matching and smooth-pasting conditions in this case are $V_{c1}(Y_{c1FB}) = F_{c1}(Y_{c1FB}) - K$ and $\frac{dV_{c1}(Y_{c1FB})}{dY} = \frac{dF_{c1}(Y_{c1FB})}{dY}$, where $F_{c1}(Y) = \frac{(X + \theta_1)Y}{r - \alpha}$, as

$^{22}$Note that under the first-best solution, the incumbent manager is compensated for the loss of $C_L$.
argued in the proof of Proposition 1. Note that at the start of the project, the owner receives the value of the project \( F_{c_1}(Y_{c_1}^{FB}) \) after paying the exercise price \( K \). When \( \theta_i = \theta_1 \), the manager is not dismissed after the start of the project. Then, combining the two boundary conditions with \( V_{c_1}(Y) = A_{c_1}Y^\beta \), we can prove (6) and (7).

(ii) The value-matching and smooth-pasting conditions in this case are \( V_{c_2}(Y_{c_2}^{FB}) = F_{c_2}(Y_{c_2}^{FB}) - K \) and \( \frac{dV_{c_2}(Y_{c_2}^{FB})}{dY} = \frac{dF_{c_2}(Y_{c_2}^{FB})}{dY} \), where \( F_{c_2}(Y) = \frac{(X+\theta_2)Y}{r-\alpha} \), as argued in the proof of Proposition 1. When \( \theta_i = \theta_2 \), the manager needs to be replaced the first time that \( Y \) hits the trigger level \( Y_{r}^{FB} \). Given this, repeating a similar derivation procedure as that of (4)–(7), we can show (8) and (9). Note that \( Y_{c_2}^{FB} < Y_{r}^{FB} \) from Assumption 2. ||

**Proof of Proposition 3:** As argued in the proof of Proposition 1, \( \Pi_r^O(Y) \) satisfies the differential equation \( \frac{1}{2}\sigma^2Y^2\Pi_{Yr}^O(Y) + \alpha Y\Pi_{rY}^O(Y) - r\Pi_r^O(Y) = 0 \), where \( \Pi_{Yr}^O = \frac{d\Pi_r^O}{dY} \), \( \Pi_{rY}^O = d^2\Pi_r^O/d^2Y \), and \( \Pi_r^O(0) = 0. \) Ruling out a speculative bubble and using \( \Pi_r^O(0) = 0 \), we can show that the solution is determined by \( \Pi_r^O(Y) = B_rY^\beta \), where \( B_r \) is a positive constant parameter.

Let \( G_{c2}(Y) (G_r(Y)) \) denote the expected present value of the firm before (after) the incumbent manager with \( \theta_2 \) is replaced. The standard procedure (see Dixit and Pindyck (1994)) shows that \( G_{c2}(Y) = \frac{(X+\theta_2)Y}{r-\alpha} \) and \( G_r(Y) = \frac{XY}{r-\alpha} \). Note that \( G_{c2}(Y) > 0 \) and \( G_r(Y) > 0 \) from \( r > \alpha \). The replacement of the manager is equivalent to investing in a project with the value of \( G_r(Y) \), while incurring \( S + C_F \), and abandoning a project with the value of \( G_{c2}(Y) \). This implies that the firm pays \( S + C_F \) and invests in a project with the value \( \Delta_2G(Y) \), where \( \Delta_2G(Y) \equiv G_r(Y) - G_{c2}(Y) \). Thus, \( \Delta_2G(Y) \) is written as \( \Delta_2G(Y) = \frac{\theta_2Y}{r-\alpha} \).

The boundary conditions in this problem are \( \Pi_r^O(Y_r^*) = \Delta_2G(Y_r^*) - S - C_F \) and \( \frac{d\Pi_r^O(Y_r^*)}{dY} = \frac{d[\Delta_2G(Y_r^*)]}{dY} \). The first boundary condition is the value-matching condition, while the second boundary condition is the smooth-pasting condition. Combining these two conditions with \( \Pi_r^O(Y) = B_rY^\beta \), we can derive (10) and (11), given in Proposition 3. ||

\[^{24}\]\( \Pi_r^O(Y) \) satisfies the following Bellman equation: \( \Pi_r^O(Y) = E[\Pi_r^O(Y + dY)e^{-rdt}] \), where \( E \) is the expectation operator. Expanding the right-hand side of this equation with Ito’s Lemma and rearranging it as \( dt \to 0 \), we obtain the differential equation introduced here.
Proof of Proposition 4: (i) Again, \( \Pi^0_{c1}(Y) \) equals \( B_{c1}Y^\beta \), where \( B_{c1} \) is a positive constant parameter. The value-matching and smooth-pasting conditions in this case are
\[
\Pi^0_{c1}(Y^*_c) = G_{c1}(Y^*_c) - W_1 - K \quad \text{and} \quad \frac{d\Pi^0_{c1}(Y)}{dY} = \frac{dG_{c1}(Y)}{dY},
\]
where \( G_{c1}(Y) = \frac{(X+\theta_1)Y}{r-\alpha} \), as argued in the proof of Proposition 1. Note that at the start of the project, the owner receives the value of the project \( G_{c1}(Y^*_c) \) after paying the exercise price \( W_1 + K \). When \( \theta_i = \theta_1 \), the manager is not dismissed after the start of the project. Then, combining the two boundary conditions with \( \Pi^0_{c1}(Y) = B_{c1}Y^\beta \), we can prove (12) and (13).

(ii) The value-matching and smooth-pasting conditions in this case are
\[
\Pi^0_{c2}(Y^*_c) = G_{c2}(Y^*_c) - W_2 - K \quad \text{and} \quad \frac{d\Pi^0_{c2}(Y)}{dY} = \frac{dG_{c2}(Y)}{dY},
\]
where \( G_{c2}(Y) = \frac{(X+\theta_2)Y}{r-\alpha} \), as argued in the proof of Proposition 1. When \( \theta_i = \theta_2 \), the manager needs to be replaced the first time that \( Y \) hits the trigger level \( Y^*_r \). Given this, repeating a similar derivation procedure as that of (10)–(13), we can show (14) and (15). ||

Proof of Lemma 1: Suppose that \( W^*_1 = 0 \). As we can assume \( Y^*_r > Y_0 \) without loss of generality, it follows from Assumption 1 with \( q_H > q_L \) that (IC) is violated. ||

Proof of Lemma 2: Suppose that (IR) is binding if (IC) is not binding. Then,
\[
W_0 = -q_H \left( \frac{Y_0}{Y^*_c} \right)^\beta W_1 - (1 - q_H) \left[ \left( \frac{Y_0}{Y^*_c} \right)^\beta W_2 + \left( \frac{Y_0}{Y^*_r} \right)^\beta (S - C_L) \right] + C_E. \tag{A1}
\]
Substituting (A1) into (16), we obtain
\[
\Pi^0(X, Y_{c1}, q_H) = q_H \left( \frac{Y_0}{Y^*_c} \right)^\beta \frac{\beta W_1 + K}{\beta - 1}
+ (1 - q_H) \left[ \left( \frac{Y_0}{Y^*_c} \right)^\beta \frac{\beta W_2 + K}{\beta - 1} + \left( \frac{Y_0}{Y^*_r} \right)^\beta \left( \frac{\beta S + C_F}{\beta - 1} - C_L \right) \right] - C_E. \tag{A2}
\]
Partially differentiating $\widehat{\Pi}^{O}(Y_0, q_H)$ with respect to $W_1$ and $W_2$ using (12) and (14) yields

$$\frac{\partial \widehat{\Pi}^{O}(Y_0, q_H)}{\partial W_1} = q_H \left( \frac{Y_0}{Y_0^{*\dagger}} \right) \frac{\beta}{\beta - 1} \left( 1 - \frac{\beta W_1 + K}{W_1 + K} \right) < 0, \quad (A3)$$

$$\frac{\partial \widehat{\Pi}^{O}(Y_0, q_H)}{\partial W_2} = (1 - q_H) \left( \frac{Y_0}{Y_0^{*\dagger}} \right) \frac{\beta}{\beta - 1} \left( 1 - \frac{\beta W_2 + K}{W_2 + K} \right) < 0. \quad (A4)$$

Hence, $W_1 = W_2 = 0$ maximizes $\widehat{\Pi}^{O}(Y_0, q_H)$ under (LL) for any $S \geq 0$. Note that (IR) continues to be binding for $W_1 = W_2 = 0$ because $W_0$ is adjusted in accordance with (A1). As $W_1$ and $W_2$ decrease, $W_0 \geq 0$ also continues to hold as long as (IR) and $W_0 \geq 0$ initially hold. Furthermore, it follows from (10) and (14) with $W_2 = 0$ that for any $S \geq 0$:

$$Y^{*r} - Y^{*c} = \frac{\beta(r - \alpha)}{\beta - 1} \left( \frac{S + C_F}{-\theta_2} - \frac{K}{X + \theta_2} \right) > 0,$$

where the last inequality is obtained from Assumption 2. Hence, $W_1 = W_2 = 0$ satisfies (TR) for any $S \geq 0$ under Assumption 2. These findings imply that an optimal solution to (18) involves $W_1 = W_2 = 0$ as long as (IR) is binding while (IC) is not binding. However, $W_i^* = 0$ contradicts the result of Lemma 1. Therefore, (IR) is not binding if (IC) is not binding, which verifies the statement of this lemma. ||

Proof of Lemma 3: Suppose that (IC) is not binding. Under Assumptions 1 and 2, it follows from Lemmas 1 and 2 that $W_i^* > 0$ and that (IR) is not binding. In addition, it also follows from (16) that $\frac{\partial \Pi^{O}(Y_0, q_H)}{\partial W_2} = -(1 - q_H) \left( \frac{Y_0}{Y_0^{*\dagger}} \right) \frac{\beta}{\beta - 1} < 0$ using (14), and that $\frac{\partial \Pi^{O}(Y_0, q_H)}{\partial S} = -(1 - q_H) \left( \frac{Y_0}{Y_0^{*\dagger}} \right) \frac{\beta}{\beta - 1} < 0$ using (10). Thus, it is optimal to set $W_2 = S = 0$. Now, it follows from (10) and (14) that (TR) is not binding for $W_2 = S = 0$ under Assumption 2. Hence, the statement of this lemma is proved. ||

Proof of Proposition 5: (i) Suppose that (19) holds. We begin to solve problem (18) by dropping (IR) and (TR). After the solution under this assumption is obtained, we check whether the obtained solution satisfies (IR) and (TR). Because Lemmas 2 and 3 ensure that (IC) is always binding if either (IR) or (TR) is binding, we need not consider
the case in which (IR) or (TR) or both are binding while (IC) is not binding. Hence, the solution obtained is a solution to problem (18) if it satisfies (IR) and (TR).

First, let us notice that the solution must satisfy \( W_1^* > 0 \) from Lemma 1. Furthermore, we can show that \( W_2^* = S^* = 0 \). This result can be derived as follows. It follows from the proof of Lemma 3 that \( \frac{\partial H^c(Y_0, q_H)}{\partial W_2} < 0 \) and \( \frac{\partial H^c(Y_0, q_H)}{\partial S} < 0 \). In addition, let \( \Phi(W_1, W_2, S) \equiv (q_H - q_L) \left( \frac{Y_0}{Y_c} \right)^\beta W_1 - (q_H - q_L) \left( \frac{Y_0}{Y_r} \right)^\beta W_2 + \left( \frac{Y_0}{Y_r} \right)^\beta (S - C_L) - C_E \). Then, it follows from (10) that \( \Phi(W_1, 0, 0) \geq \Phi(W_1, W_2, S) \) for any \( W_1 > 0 \) and for any \( (W_2, S) \geq (0, 0) \). These findings imply that setting \( (W_2, S) \) equal to \( (0, 0) \) maximizes (18) subject to (IC), \( W_2 \geq 0 \) and \( S \geq 0 \) for any \( W_0 \geq 0 \) and \( W_1 > 0 \). Hence, we can set \( W_2^* = S^* = 0 \).

Now, we solve problem (18) by dropping (IR) and (TR) under \( W_2^* = S^* = 0 \). Then, (IC) must be binding with equality; otherwise, it follows from \( \frac{\partial \Pi_O(Y_0, q_H)}{\partial W_1} = -q_H \left( \frac{Y_0}{Y_c} \right)^\beta < 0 \) that \( W_1 = W_2 = S = 0 \) becomes the solution, which contradicts Lemma 1. Thus, \( W_1^* \) is determined by (20). Because \( Y_r^* > Y_0 \) without loss of generality, it follows from Assumption 1 that \( W_1^* > 0 \). As \( \frac{\partial H^c(Y_0, q_H)}{\partial W_0} < 0 \), it is also immediate that \( W_0^* = 0 \) if (IR) can be neglected. Given \( W_2^* = S^* = 0 \), we verify (21).

The remaining problem is to check whether the solution obtained \( (W_0^*, W_1^*, W_2^*, S^*) \) given by (20) and (21) satisfies (IR) and (TR). Rearranging (IR) with (20) and (21) yields

\[
q_H \left( \frac{Y_0}{Y_c} \right)^\beta W_1^* - (1 - q_H) \left( \frac{Y_0}{Y_r} \right)^\beta C_L - C_E = - \left( \frac{Y_0}{Y_r} \right)^\beta C_L + \frac{q_L C_E}{q_H - q_L} > 0, \tag{A5}
\]

where the last inequality is derived from (19) with (10) and (21). Hence, (IR) is satisfied. It also follows from (10), (14) and (21) that (TR) is satisfied under Assumption 2.

(ii) Suppose that (22) holds. Again, given Lemmas 2 and 3, we begin to solve problem (18) by dropping (IR) and (TR). Applying the same procedure as that of part (i) in the proof of this lemma, we can obtain (20) and (21), and can show that (TR) is satisfied.

\[\text{Note that } \left( \frac{Y_0}{Y_r} \right)^\beta W_1 = -\frac{C_E}{q_H - q_L} - \left( \frac{Y_0}{Y_r} \right)^\beta C_L.\]
However, it follows from (10) and (20)–(22) that (IR) is satisfied only if $W_0 \geq \left(\frac{Y_0}{r_0}\right)^3 C_L - \frac{q_H C_F}{q_H - q_L} \geq 0$. As $\frac{\partial \Pi(Y_0, q_0)}{\partial W_0} < 0$, we must have $W_0^* = \left(\frac{Y_0}{r_0}\right)^3 C_L - \frac{q_H C_F}{q_H - q_L}$, which means that (IR) is binding. Hence, this is a contradiction.

Thus, in this parametric case, we may solve problem (18) by letting (IC) and (TR) bind while dropping (IR) or letting (IC) and (IR) bind while dropping (TR). However, in the former case, it is optimal to set $\lambda$, and hence we can neglect the former case, and focus on the latter case. Then, $W_0^*$ is derived from (A1) because (IR) is binding. Hence, the objective function is expressed by (A2) instead of (16). After the solution under this assumption is obtained, we check whether the solution obtained satisfies (TR).

First, let us again notice that $W_1^* > 0$ from Lemma 1. We next show $W_2^* = 0$, because $\frac{\partial \Pi(Y_0, q_0)}{\partial W_2} < 0$ from (A4) and because $\Phi(W_1, 0, S) \geq \Phi(W_1, W_2, S)$ for any $W_1 > 0$ and for any $(W_2, S) \geq (0, 0)$, and (TR) is satisfied for (10), (14) and $(W_2, S) = (0, 0)$ under Assumption 2. Because (TR) is not binding for $W_2 = S = 0$, this is a contradiction. Hence, we can neglect the former case, and focus on the latter case. Then, $W_0^*$ is derived from (A1) because (IR) is binding. Hence, the objective function is expressed by (A2) instead of (16). After the solution under this assumption is obtained, we check whether the solution obtained satisfies (TR).

Now, the first-order conditions to problem (18) with respect to $W_1$ and $S$ in this case are derived as follows:

\[
q_H \beta \frac{W_1^*}{W_1^* + K} = \lambda_1(q_H - q_L) \left(1 - \frac{\beta W_1^*}{W_1^* + K}\right) - \lambda_2 q_H \left(1 - \frac{\beta W_1^*}{W_1^* + K}\right), \tag{A6}
\]

\[
(1 - q_H) \beta \frac{C_L - S^*}{S^* + C_F} = \lambda_1(q_H - q_L) \left[1 + \frac{\beta (C_L - S^*)}{S^* + C_F}\right] + \lambda_2 (1 - q_H) \left[1 + \frac{\beta (C_L - S^*)}{S^* + C_F}\right] - \lambda_3, \tag{A7}
\]

where $\lambda_1$, $\lambda_2$ and $\left(\frac{Y_0}{r_0}\right)^3 \lambda_3$ are the nonnegative multipliers associated with (IC), $W_0^* \geq 0$ and $S^* \geq 0$.

If $W_0^* > 0$ and $S^* > 0$, that is, $\lambda_2 = \lambda_3 = 0$, it follows from (A6) and (A7) that the equality of (26a) holds. Because (IC) is binding, $(W_1^*, S^*)$ are then simultaneously
determined by (24) and (26a). Note that the inequality of (24) is verified because (A6) and (A7) imply that \[ \frac{\beta W^*_1}{W^*_1 + \kappa} = \frac{\lambda_1(q_H-q_L)}{q_H + \lambda_1(q_H-q_L)} > 0. \] Given (A1) for \( W^*_2 = 0, \) \( W^*_0 \) in this case is determined by (27a). To prove the inequality of (26a), suppose that \( S^* \geq C_L. \) Then, substituting (24) into (27a), we have \[ W^*_0 = -\left( \frac{\beta}{\beta} \right) (S^* - C_L) - \frac{q_L C_F}{q_H - q_L} < 0, \] which is a contradiction. This means that \( S^* < C_L, \) which verifies the inequality of (26a). Furthermore, it follows from (10), (14) and (25) that (TR) is satisfied under Assumption 2.

If \( W^*_0 = 0 \) and \( S^* > 0, \) then \((W^*_1, S^*)\) are given by (IC) and \( W^*_0 = 0 \) because (IC) is binding. Thus, given (A1), \((W^*_1, S^*)\) are then simultaneously determined by (24) and (26b). Note that the inequality of (24) is verified by substituting (26b) into (24). As \( W^*_0 = 0, \) (27b) is immediate. Furthermore, it follows from (10), (14) and (25) that (TR) is satisfied under Assumption 2.

If \( S^* = 0, \) then \( W^*_2 = S^* = 0. \) Because (IC) is binding, \((W^*_1, W^*_2, S^*)\) are determined by (20) and \( W^*_2 = S^* = 0. \) As (IR) is also binding, it follows from (A1) that \( W^*_0 \) is determined by (23). The inequality of (23) follows from (10), (20), (22) and \( S^* = 0. \) Then, it follows from (10), (14) and \( W^*_2 = 0 \) that (TR) is satisfied under Assumption 2.

Proof of Corollary to Proposition 5: (i) The statement of part (i) is evident from (4), (6), (8), (10), (12), (14), (20) and (21).

(ii) The statement of part (ii) is evident from (4), (6), (8), (10), (12), (14), (20), (21) and (24)–(26).

Proof of Proposition 6: Under the parametric case of Assumptions 1, 2’ and 3, Proposition 2 of Hori and Osano (2009) gives the optimal compensation contract and optimal triggers if the owner can precommit to the ex ante promised triggers. Although the present model includes the fixed base salary \( W_0 \) in the compensation contract, the role of \( W_0 \) only increases \( \Pi^O (Y_0, q_H) \) by relaxing (IR). In fact, in the solution given by Proposition 2 of Hori and Osano (2009), (IR) is not binding if \[ \frac{q_L C_F}{q_H - q_L} (C_F + C_L) > (Y_0)^{\beta} \left[ \frac{\beta^{\frac{r-\alpha}{\beta}}}{\beta-\alpha} \right]^{-\beta}. \] Because \( W_0 \) has no role in this region, the statement of part (i) of this proposition is immediate from Proposition 2 of Hori and Osano (2009).
Now, if \((Y_0)^{\beta} \left[ \frac{\beta - \alpha}{\beta - 1 - \theta_2} (C_F + C_L) \right]^{-\beta} \geq \frac{q_H - q_L}{q_H - q_L} C_F \), (IR) must be binding in the solution given by Proposition 2 of Hori and Osano (2009). Then, if both (IC) and (IR) are binding, \((W_1^{**}, Y_r^{**})\) must be adjusted to satisfy that (IC) and (IR) are binding. This causes a distortion that makes \(Y_r^{**}\) lower than \(Y_r^{FB}\). Hence, by setting \(W_0\) positive, the owner can relax (IR), thereby reducing the distortion of \(Y_r^{**}\) and increasing \(\Pi^O(Y_0, q_H)\). On the other hand, if (IR) is binding but (IC) is not binding, we can show \((Y_r^{**}, Y_{c1}^{**}, Y_{c2}^{**}) = (Y_r^{FB}, Y_{c1}^{FB}, Y_{c2}^{FB})\) by applying a procedure similar to that of Hori and Osano (2009). In both cases, we must have \(W_1^{**} > 0\) and can set \(W_2^{**} = S_2^{**} = 0\) without loss of generality. Thus, the statement of part (ii) of this proposition is obtained. ||

**Proof of Proposition 7**: (i) The statement of part (i) is evident from (4), (6), (8), (10), (12), (14), (20), (21) and Proposition 6. (ii) The statement of part (ii) is evident from (4), (6), (8), (10), (12), (14), (20), (21), (24)–(26) and Proposition 6. Note that \(\frac{q_H - q_L}{q_H - q_L} C_F < (Y_0)^{\beta} \left[ \frac{\beta - \alpha}{\beta - 1 - \theta_2} (C_F - \frac{q_H - q_L}{q_H - q_L} C_L) \right]^{-\beta}\) if (22) holds. ||

**Appendix B**

Note that we can set \(Y_{c1}^{*} = Y_0\) or \(Y_{c2}^{*} = Y_0\) if \(Y_{c1}^{*} < Y_0\) or \(Y_{c2}^{*} < Y_0\). Now, for the case of \(Y_{c1}^{*} < Y_0\), (16) is rewritten as

\[
\Pi^O(Y_0, q_H) = q_H \left[ \frac{(X + \theta_1)Y_0}{r - \alpha} - W_1 - K \right]
+ \begin{cases} 
(1 - q_H) \left[ (\frac{Y_0}{Y_{c2}^{*}})^{\beta} \frac{W_2 + K}{\beta - 1} + (\frac{Y_0}{Y_r^{*}})^{\beta} \frac{S + C_F}{\beta - 1} \right] - W_0 & \text{if } Y_{c2}^{*} > Y_0 \\
(1 - q_H) \left[ \frac{(X + \theta_2)Y_0}{r - \alpha} - W_2 - K + \left( \frac{Y_0}{Y_r^{*}} \right)^{\beta} \frac{S + C_F}{\beta - 1} \right] - W & \text{if } Y_{c2}^{*} \leq Y_0
\end{cases}
\]

(16')

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Then, (A6) is reduced to

\[ -q_H + \lambda_1(q_H - q_L) - \lambda_2 q_H = 0. \]  \hspace{1cm} (A6')

Suppose that \( W_0^* > 0 \) and \( S^* > 0 \). Then, it follows from (A6') and (A7) that \( \lambda_2 = \lambda_3 = 0 \), \( \lambda_1 = \frac{q_H}{q_H - q_L} \), and

\[ (1 - q_H) \beta \frac{C_L - S^*}{S^* + C_F} = \lambda_1(q_H - q_L) \left[ 1 + \beta \frac{C_L - S^*}{S^* + C_F} \right]. \]

Thus, (26a) is rewritten as

\[ \frac{C_L - S^*}{S^* + C_F} = \frac{q_H}{1 - 2q_H \beta}. \]  \hspace{1cm} (26a')

The remaining results for Proposition 5 are unchanged. ||
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1B. The case of Proposition 5(ii)(b) when $(W_0^*, S^*)$ are given by (26a) (or (26a')) and (27a)
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1C. The case of Proposition 5(ii)(b) when $(W^*_0, S^*)$ are given by (26b) and (27b)
Table 1A shows a compensation contract in the case of Proposition 5(ii)(a). Table 1B shows a compensation contract in the case of Proposition 5(ii)(b) when \((W_0^*, S^*)\) are given by (26a) (or (26a')) and (27a). Note that for \(Y_0 < 2.2\), \((W_0^*, S^*)\) are given by (26a) and (27a) because of \(Y^*_{c1} > Y_0\), whereas for \(Y_0 \geq 2.2\), \((W_0^*, S^*)\) are given by (26a') and (27a) because of \(Y^*_{c1} < Y_0\). Table 1C shows a compensation contract in the case of Proposition 5(ii)(b) when \((W_0^*, S^*)\) are given by (26b) and (27b).
References


