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“Increasing Returns in Transportation and the Formation of Hubs”

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Abstract

The spatial structure of transport network is subject to increasing returns in transportation, distance and density economies. Transport costs between locations are thus in general endogenous, and are determined by the interaction between the spatial distribution of transport demand and these increasing returns, although such interdependence has long been ignored in regional models. By using a simple model, the present paper explains the formation of transport hubs endogenously, and shows how the balance of these two types of increasing returns influences the spatial distribution of transport hubs.

JEL Classifications: R12, R49

Keywords: Formation of a transport hub, Distance economies of transportation, Density economies of transportation

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1 Introduction

The transport cost has been undoubtedly one of the most essential elements in the urban and regional economics. But, in the formal models proposed so far in this literature, transport networks are either given exogenously or irrelevant. The present paper aims to disclose the role of two types of increasing returns in transportation, distance economies and density economies, in determining the location of a transport hub, and the spatial structure of the transport network. The former refers to a decrease in transport costs per distance by longer hauling, while the latter refers to a decrease in transport costs for a given link by a larger transport density on that link. Although these two increasing returns have been recognized as major causes for the formation of hubs and trunk links, to the best of my knowledge, they have never been simultaneously considered in explaining the spatial structure of transport network, not to mention the interdependency between the transport network structure and location behavior of economic agents.

As the formation of a transport hub necessarily requires concentration of transport demand, the industrial and population agglomerations influence their location. But, there is also an economic force working in the opposite direction. Namely, since the location of transport hubs, or more generally, the spatial structure of transport network is subject to the increasing returns specific to transportation, the size and location of industrial and population agglomerations are also influenced by the increasing returns in transportation. This latter channel has largely been ignored in the regional models for far.

For instance, it is hard to believe that the disproportionate growth of the largest city,

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1While the former type of models can be easily imagined, the latter case may require some explanation. There are two typical examples of this case. One is the two-region setup where the transport network consists of an only link between the two regions, and the other is the “system of cities” setup (introduced by Henderson [18]) in which interregional spatial structure is assumed away all together.

2Fuel efficiency is probably the most common source of the distance economy. But, time efficiency is also improved by longer hauling which justifies the services by express trains.

3Major sources of density economies are an increase in matching efficiency for transport services, and scale economies by using larger vehicles and airplanes for concentrating transport flows to a major hubs and trunk links.

4Some evidences have been reported for the presence of density economies in transportation. See Brueckner and Spiller [8], Brueckner, Dyer and Spiller [7], and Caves, Christensen and Thretheway [11] for the case of air transportation, and Braeutigam, Daughety and Turnquist [5, 6] for the case of rail transportation, and Mori and Nishikimi [28] for the case of ocean transportation.
Tokyo, in Japan during the last century was irrelevant to the advancement of the rail transport technology. In 1889, when it took 20 hours by rail to reach from Tokyo to the 550km apart next largest city, Osaka, the (population) size of Tokyo was only 10% larger than that of Osaka. When the travel time between the two cities reduced to 8 hours in 1935, it was 1.6 times larger. In 1965, right after the opening of the service of Shinkansen, the bullet train, halved the travel time, the size of Tokyo (17.9 million) had become twice as large as Osaka. When the travel time further reduced to 2.5 hours in 2005, Tokyo had grown to the size 2.7 times (33.3 million) as large as Osaka. It is to be noted that the distance between these two cities has been crucial to realize such drastic improvement in transport access for two reasons. First, to match the cost for the state-of-the-art mass transportation, each hub city needs to support a sufficiently large travel demand, which is possible only if they are reasonably far apart from one another so that they do not need to compete for their hinterlands (density economies). Second, a larger travel time reduction can be attained only by maintaining the maximum speed for a longer distance, which is only possible by stopping less for a longer distance (distance economies). It follows that the presence of these two increasing returns implies a certain unavoidable distance between major transport hubs.

The locations which happened to coincide with major express stops for Shinkansen obviously attracted a larger population. Indeed, between 1980 and 2005, among the 11 cities along the Shinkansen line connecting Tokyo and Osaka, the population growth rate of 4 cities, Tokyo, Osaka, Nagoya and Kyoto, at which the express trains stop was 31.2%, and was clearly higher than 20.3% for the case of the rest at which only local ones stop.

A similar argument applies at smaller spatial scales. In particular, the locations of subcenters within a large metro area typically coincide with hubs of the urban transport network. Since the urban transport modes (e.g., subways and busses) are also subject

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5The population sizes here are those of prefectures, since it is safe to assume that the metro areas of Tokyo and Osaka are roughly contained within their respective prefectures then.

6“Cities” and “metro areas” are used interchangeably throughout the paper. The definition of a city prior to 1980 is based on Eaton and Eckstein [13], while that in 1980 or later on the Urban Employment Area by Kanemoto and Tokuoka [24].

7Only those identified as metro areas are included.
to economies of density and distance, the spatial distribution of subcenters should also depend on these increasing returns in transportation.\(^8\)

The rest of the paper is organized as follows. It starts by reviewing the related literature in Section 2, and proceeds in Section 3 by presenting the general structure of transport technology subject to the distance and density economies. It is followed by the representation of the general transport network structure under a simple spatial distribution of transport demand in Section 4, and the definition of a transport network equilibrium in Section 5. In Section 6, by specifying transport technology explicitly, the mechanism in which the two increasing returns interact to pin down the location of a hub is presented. In Section 7, certain implications are discussed regarding the macro structure of a large hierarchical transport network. Finally, the paper is closed by the remarks on the basic results, limitations of the present analysis, and future research directions.

## 2 Related Literature

Regarding the impact of transport network structure on the location and pricing behaviors, there have been several important contributions. Hakimi [17] was the first to recognize that the set of vertices of a transport network includes the optimal location of firms in the classical *least cost approach* of Weber [41]. Louveaux, Thisse and Beguin [27] extended the Hakimi’s result by allowing distance economies, and showed further that a transshipment location may also be a candidate for the optimal location in the Weber problem above. Brueckner and Spiller [8] formalized the mechanism in which density economies lead to fare reduction in the trunk link of a given transport network. Krugman [26], Fujita and Mori [15], Mun [32], and Behrens, Lamorgese, Ottaviano and Tabuchi [3] investigated interaction between the hub-location advantage in an exogenous transport network and industrial/population agglomeration economies.\(^9\)

\(^8\)The existing literature explain the subcenter locations mainly as a result of the interaction between land market and transport costs without increasing returns in transportation (e.g., Ota and Fujita [34], Henderson and Mitra [19], Sasaki [35], Sasaki and Zhang [36, 37, 38]).

\(^9\)Konishi [25] also investigated the causality between population agglomeration and transport network structure, but in the absence of increasing returns to scale or externalities. In particular, he argued that
Endogenous formation of hub-and-spoke network structure in the presence of transport density economies was first formulated by Hendricks, Piccione and Tan [20]. But, no insight has been obtained regarding relative locations among hubs, as their model abstracted interregional space by assuming each pair of regions being equidistant. Mori and Nishikimi [28] proposed the first general equilibrium model with economies of transport density in which a hub formation and industrial location are endogenous. In the absence of distance economies in their model, however, it was not possible to investigate the spacing of hubs as in the present paper (for the reason to be clear in page 13). Finally, there are recent works by Behrens, Gaigne and Thisse [2] and Takahashi [40] which explicitly formulate the behavior of the transport sector. But, as their models are restricted to a two-region economy, the role of transport sector in shaping the transport network is yet to be studied.

3 The basic framework

Imagine a regional economy which extends over a one-dimensional location space, $X \equiv [0, F]$ as shown in Figure 1, where $F > 0$ is the fringe location of the region. At each location on $X$, some interregional transport demand is generated, where each shipment is of measure zero and independent from one another. For simplicity, assume that any shipment must go through the gateway hub located at the left end of the regional space, 0, before reaching the final destination (outside the region). Thus, all the transport demands in this regional economy are directed toward this gateway.

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population concentration at a hub in a given transport network is partly explained by a larger labor requirement for transport service at the hub, even in the absence of any increasing returns to scale.

10Scale economies in traffic density have long been recognized, e.g., O’Kelly [33] and Campbell [10]. But, it has not been explicitly modeled before their work.

11It is to be noted that both distance and density economies are essential in order to explain the spacing of hubs. In other words, the result of Mori and Nishikimi where only the density economies is considered cannot be straightforwardly extended into a more general location space with more than three regions.
From each location, $x \in X$, the gateway can be reached either directly by using only *individual transport mode*, or indirectly by using *mass transport mode* which connects transport hubs by *mass transport links*. The transport cost on the mass transport links is subject to the *mass transport technology*, otherwise it is subject to the *individual transport technology*. In the present model, the formations of transport hubs (except for the gateway hub) and mass transport links are endogenous. Once a transport link is formed, the transport rate (per shipment) on the link is given by $\tau(d, Q)$ which is a continuous function of the link length, $d$, and the transport density, $Q$, on the link.

Regarding the transport distance, this technology exhibits the following properties:

$$\tau(d, Q) > \tau(d', Q) > 0$$  \hspace{1cm} (1)

$$\frac{\tau(d, Q)}{d} < \frac{\tau(d', Q)}{d'}$$  \hspace{1cm} (2)

if $d > d' > 0$ for any $Q \in (0, \infty)$. Property (1) means that other things being equal, the transport rate is strictly higher for a longer transport link, while (2) expresses *economies of transport distance* (or *long hauling*), i.e., for a given transport density, the average transport cost (per distance) decreases in distance transported.\(^{12}\)

Regarding the transport density, while the aggregate transport costs is larger under a higher density, the transport rate is smaller, i.e.,

$$Q' \tau(d, Q') \leq Q \tau(d, Q)$$  \hspace{1cm} (3)

$$\tau(d, Q') > \tau(d, Q) > 0$$  \hspace{1cm} (4)

if $Q > Q' > 0$ for any $d \in (0, \infty)$. Property (4) expresses *economies of transport density* which are external to each shipper, i.e., they take $\tau$ as given and do not take into account their contribution to the density, $Q$.\(^{13}\)

\(^{12}\)The strict inequality in (2) does not need to hold for all $d > d' > 0$ to obtain the basic results below. But, it is assumed for simplicity of the argument.

\(^{13}\)The monotonicity in (3) can be strict. But, allowing the equality makes analysis much simpler without sacrificing the basic result. Refer to footnote 19 below in Section 6.
In reality, any single transport mode (e.g., bus, train, airplane, ship) exhibits the distance and density economies only in a limited range of the transport distance and density. Thus, the transport rate \( \tau \) here may be interpreted as the lower envelope of the transport rates which can be realized by all available mass transport modes. By imposing the properties given in (2) and (4), it is essentially assumed that there are large varieties of available transport modes, and in effect, increasing returns are at work for the entire range of distance and density levels with implicit modal choice.

Finally, the individual transportation is assumed to be linear in distance and independent of transport density, such that the transport cost for unit distance is a constant given by \( t > 0 \).

4 Transport network

Let \( K \equiv \{0,1,2,\ldots,K\} \) represent indices of hubs which exist in equilibrium (to be defined later), and \( H \equiv \{h_0,h_1,h_2,\ldots,h_K\} \) denote the set of their locations, where \( h_i \in \mathbf{X} \). Let \( h_0 \equiv 0 \) be the location of the gateway, and without loss of generality, assume that \( h_i < h_j \) if \( i < j \). Given \( K \) and \( H \), the set of locations, \( D_i \subset \mathbf{X} \), is called the direct (transport) hinterland of hub \( i \in K \), if hub \( i \) is the first hub among all the existing hubs, \( 0,\ldots,K \), through which the shipments from each location \( x \in D_i \) to the gateway pass. By definition, \( D_i \cap D_j = \emptyset \) if \( i \neq j \), and \( \bigcup_{i \in K} D_i = \mathbf{X} \). Under the given transport technology, it can be shown that each \( D_i \) is a continuous interval on \( \mathbf{X} \) if each shipment takes place along the cost minimizing route. Denote by \( b_i \) the boundary between the two adjacent direct hinterlands \( D_{i-1} \) and \( D_i \) of hubs \( i-1 \) and \( i \) for \( i = 1,\ldots,K \), let \( b_0 \equiv 0 \) and \( b_{K+1} \equiv F \) for notational convenience. Then, hubs and the boundaries of their direct hinterlands are aligned so that \( h_{i-1} < b_i \leq h_i \), \( D_i = [b_i,b_{i+1}) \) for \( i = 0,\ldots,K-1 \), and \( D_K = [b_{K-1},F] \).

If hub \( j \) is connected to hub \( i \) directly by a mass transport link, and all the transport flows passing \( j \) go through \( i \) before reaching the gateway, then call hub \( i \) the parent hub of hub \( j \). Denote by \( g(h_i) \) the location of the parent hub of hub \( i \), and define \( G \equiv \{g(h_1),g(h_2),\ldots,g(h_K)\} \). Also, denote by \( \ell_i \) the mass transport link between \( h_i \) and \( g(h_i) \),
and define $L \equiv \{\ell_1, \ell_2, \ldots, \ell_K\}$. If the set of direct hinterlands of hubs is denoted by $D \equiv \{D_0, D_1, \ldots, D_K\}$, then the physical structure of transport network $T$ can be identified by $\{H, G, D\}$, or equivalently by $\{H, L, D\}$.\(^{14}\)

Next, let $g^k(h_i)(k \geq 1)$ represent the $k$-times mapping of $h_i$ by $g$, when such mapping is well-defined, i.e., $g^k(x) \in H$. By definition, shipments from the direct hinterland, $D_j$, of hub $j$ to the gateway must pass hub $i$ if $g^k(h_j) = h_i$ for some integer $k \in [1, K)$. Hence, a transport network is hierarchical as there exists an order in which traffic flows through hubs toward the gateway, and the order can be defined relatively to the gateway. Namely, for hub $i \in K$, let $\lambda_i \in \{1, \ldots, K\}$ be an integer satisfying $g^{\lambda_i}(h_i) = h_0$. Then hub $i$ is said to be of order $\lambda_i$ relative to the gateway. Similarly, if for hub $j \in K$ there exists an integer $k \in K$ such that $g^k(h_j) = h_i$, then hub $j$ is said to be in the hinterland of hub $i$, and of order $k$ relative to hub $i$. A hub is said to be of higher order if it is more directly connected to the gateway, i.e., the value of the order is smaller.

Now, denote by $K_i(\subset K)$ the index set of hubs in the hinterland of hub $i$, i.e., $K_i \equiv \{j \in K | \exists k \in K \text{ s.t. } g^k(h_j) = h_i\}$. Then the total hinterland, $H_i$, of hub $i$ can be expressed by $\bigcup_{j \in K_i} D_j$,\(^{15}\) and accordingly, the transport density of hub $i$ is computed as $|H_i| \equiv \sum_{j \in K_i} |D_j|$, where $|D_j|$ represents the size of transport demand generated in $D_j$. It follows that the transport rate on the link $i$ is given by $\tau(|\ell_i|, |H_i|)$, where $|\ell_i| \equiv |h_i - g(h_i)|$.\(^{16}\)

An example of a three-order transport network with five hubs under the gateway is depicted in Figure 2 below. There are two first-order hubs, 1 and 3, where hub 1 is just a local hub near the gateway, whereas hub 3 is a major regional hub which collects transport demands generated in the hinterland, $H_2 \equiv D_2$ and $H_4 \equiv D_4 \cup D_5$, of its child hubs, 2 and 4, respectively, besides those generated in its own direct hinterland, $D_3$. Hub 4 is also a minor regional hub at which the transport flow coming from a local hub 5 merges.

Such a hierarchical transport network is ubiquitous in reality, but in order to explain this “agglomeration” of transport flows endogenously within a model, an explicit consideration

\(^{14}\)In a more general situation in which the destinations of transport flows are not unique, there could be multiple mass transport links from a given location. But, the present model focuses on a simplest setting such that it is not the case.

\(^{15}\)Note that the entire spatial economy, $X$, belongs to the transport hinterland of the gateway, i.e., $H_0 \equiv X$.

\(^{16}\)Here, $|\ell_i|$ represents the length of the link $i$, i.e., the distance between $h_i$ and $g(h_i)$. 

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of distance and density (or bulk-scale) economies is essential.

[Figure 2]

5 Transport network equilibrium

For a given transport network, $\mathcal{T}$, denote by $T(x)$ the equilibrium transport rate between the gateway and location $x \in X$. Since each shipment must take place along the cost minimizing route in equilibrium, it follows that for each $i \in \{1, \ldots, K\}$,

$$T(h_i) = T(g(h_i)) + \tau(\ell_i, |H_i|).$$

(5)

Under the cost minimization by individual shippers, it is also true that the transport rate at boundary, $b_i$, between the direct hinterlands of hubs $i-1$ and $i$ is the same for the route via either hub, and that each hub $i$ is contained in its direct hinterland, i.e., $h_i \in D_i$. It follows that

$$h_{i-1} < b_i \leq h_i \quad \forall i = 1, \ldots, K$$

(6)

and that

$$T(h_i) + t(h_i - b_i) = T(h_{i-1}) + t(b_i - h_{i-1}).$$

(7)

Since (7) can be equivalently written as

$$T(h_i) = T(h_{i-1}) + t(2b_i - h_{i-1} - h_i),$$

(8)

then by applying it recursively at each boundary, $b_j$, for $j = 1, \ldots, i-1$, the transport rate at each hub $i$ can be expressed as

$$T(h_i) = t \sum_{j=1}^{i} (2b_j - h_{j-1} - h_j).$$

(9)
The equilibrium condition (9) implies that

$$T(x) \leq tx \quad \forall x \in X.$$  \hfill (10)

A typical transport cost schedule is shown in Figure 3 which corresponds to the network depicted in Figure 2. Namely, it takes local minima at hub locations, and local maxima at the boundaries of their direct hinterlands. In particular, the location $h_3$ of the major regional hub offers the minimum transport rate within its total hinterland, $H_3 = \bigcup_{i=2}^{5} D_i$. This figure clearly indicates that the endogenous transport cost structure is very different from that under the individual linear transport technology, which in turn suggests a potentially large influence of increasing returns in transportation on the location behavior of economic agents.

[Figure 3]

In summary, the transport network equilibrium in the present context can be defined as follows.

**Definition 5.1 (Transport network equilibrium)** Transport network, $T$, is in equilibrium if and only if each shipment in $X$ takes place along the cost minimizing route in $T$, i.e., (5), (6) and (9) are satisfied, so that there is no incentive for unilateral deviation of individual shippers from $T$.

Obviously, this equilibrium concept is very restrictive in the sense that it does not allow for any collective deviation from the existing transport network structure. Thus, there is no possible mechanism which results in an active formation of a transport link/hub even if it is viable once realized. Nonetheless it is still worth investigating the properties of a “viable” location for a hub in the present context, and this task is pursued in the next section.
6 On the location of hubs

In order to illustrate the basic effects of distance and density economies on the location of hubs, the analysis in this section focuses on a simplest setting. Namely, besides the basic properties (1) through (4) for transport distance and density, the mass transport technology, \(\tau(\cdot)\), is assumed to satisfy the following conditions at the limits:

\[
\lim_{Q \downarrow 0} \tau(d, Q) > td \quad \forall d > 0 , \quad (11)
\]

\[
\lim_{Q \uparrow \infty} \tau(d, Q) \in [0, td) \quad \forall d > 0 , \quad (12)
\]

\[
\lim_{d \downarrow 0} \tau(d, Q) \in (0, \infty) \quad \forall Q > 0 . \quad (13)
\]

Properties (4) and (11) indicate that for any given transport distance, \(d > 0\), the individual transportation is more efficient than the mass transportation for a transport density close to zero, while the opposite is true for a sufficiently large transport density (due to density economies) under properties (4) and (12). Property (13) assumes the cost for stopping as one of the sources of distance economies.

Now, consider the situation in which the distribution of transport demand on \(X\) is uniform with density \(\nu > 0\), and no hub exists so that all shipments to the gateway generated in \(X\) rely on individual technology. In this context, it is possible to compute the viability of a hub if it were formed at each given location, \(h \in X\), and the (mass) transport link is formed from \(h\) to the gateway. This hub viability at a location, \(h \in X\), can be defined in terms of the transport rate on the potential transport link from \(h\) to the gateway under the present transport density, \(F - h\), without the hub, i.e.,

\[
\tau^*(h; F, \nu) \equiv \tau(h, \nu[F - h]) . \quad (14)
\]

Namely, if \(\tau^*(h; F, \nu) \leq th\), then there exists a unique equilibrium boundary of the direct hinterland, \(b \in (0, h)\), of this hub (so that the direct hinterland of hub \(h\) becomes \([b, F])\) as
depicted in Figure 4 for the case of a strict inequality.\footnote{If \( \tau^*(h; F, \nu) = th \) then \( b = h \).}

Using the concept of hub viability, the following proposition summarizes the condition under which a transport hub can be sustained in equilibrium at all.

**Proposition 6.1** For a given \( \nu \) [resp., \( F \)], there exists a threshold hinterland size, \( \tilde{F} \) [resp., transport demand density, \( \tilde{\nu} \)] such that there is a location at which a hub is viable if and only if \( F \geq \tilde{F} \) [resp., \( \nu \geq \tilde{\nu} \)]. Moreover, if \( F = \tilde{F} \) [resp., \( \nu = \tilde{\nu} \)], a hub is viable only at the (possibly multiple) critical location, \( \tilde{h} \) [resp., \( \tilde{\nu} \)], which satisfies \( \tau^*(\tilde{h}; F, \nu) = t\tilde{h} \) [resp., \( \tau^*(\tilde{h}; F, \nu) = t\tilde{h} \)] with its direct hinterland extending over an interval, \([\tilde{h}, \tilde{F}] \) [resp., \([\tilde{h}, F] \)]. If \( F > \tilde{F} \) [resp., \( \nu > \tilde{\nu} \)], a hub is viable at any location within a continuous vicinity of \( \tilde{h} \) [resp., \( \tilde{\nu} \)] such that \( \tau^*(h; F, \nu) \leq th \).

**Proof.** Note first that if a hub is viable at location \( h \in (0, F) \), then it must be true that \( \tau(h, \nu[F - h]) \leq th \). Define \( \tau^*(h; F, \nu) \) as in (14). Then, \( \tau^*(h) > th \) near \( h = 0 \) by (13) and the continuity of \( \tau^* \) (by the continuity of \( \tau \), i.e., there is no cost advantage for a hub formation in the vicinity of the gateway. Similarly, \( \tau^*(h) > th \) near \( h = F \) by (11) and the continuity of \( \tau^* \), i.e., there is no cost advantage for a hub formation in the vicinity of the fringe location, \( F \). But, as for \( h \in (0, F) \), if the transport density, \( F - h \), at \( h \) is sufficiently large (i.e., for a sufficiently large \( F \)), then \( \tau^*(h; F, \nu) < th \) by (4) and (12). Since \( \tau^*(h; F, \nu) \) is continuous and decreasing in \( F \), it follows that there exists a critical hinterland size, \( \tilde{F} \), and a hub location, \( \tilde{h} \in (0, \tilde{F}) \), such that \( \tau(\tilde{h}, \nu[F - \tilde{h}]) = t\tilde{h} \). For \( F > \tilde{F} \), again by the continuity of \( \tau^* \) and by (4) there is a continuous interval around \( \tilde{h} \) such that \( \tau^*(h; F, \nu) \leq th \). The result regarding \( \tilde{\nu} \) and \( \tilde{h} \) can be proved in a similar manner. Q.E.D.

Note that this result can be applied to check the viability of a new hub within the direct hinterland of any existing hub by re-interpreting the gateway location as that of this existing hub \( i \in K \) and the fringe location, \( F \), as the boundary location of the direct hinterland of this hub \( i \), i.e., \( b_{i-1} \) and \( b_i \). It is also possible to extend the viability concept to see the possible formation of a higher-order hub in a more general hierarchical network.
In that case, however, a more explicit equilibrium selection concept must be introduced as the formation of a higher-order hub may involve disappearance of some existing hubs.\textsuperscript{18} This more general case will be considered in a subsequent work discussed in Section 8.3.

To help understanding the hub viability under given levels of distance and density economies using a concrete example, a simple specification of the mass transport cost function can be given by

$$
\tau(d, Q) = \frac{\phi + \rho d}{Q} \quad (15)
$$

where $\phi/Q$ represents the fixed cost, and $\rho/Q$ the marginal cost on a given transport link with transport density, $Q$. A relatively larger value of $\phi$ [resp., $\rho$] implies larger distance [resp., density] economies.\textsuperscript{19} Under this specification, $\tau^*$ at the gateway, the fringe location, $F$, and an interior location, $h \in (0, F)$, take the values,

$$
\tau^*(0) = \frac{\phi}{vF} > 0, \quad (16)
$$
$$
\tau^*(F) \equiv \lim_{h \uparrow F} \tau^*(h) = \infty, \quad (17)
$$
$$
\tau^*(h) = \frac{\phi + \rho h}{v[F-h]}, \quad (18)
$$

respectively. Moreover, $\tau^*$ is increasing and convex:

$$
\frac{\partial \tau^*(h)}{\partial h} = \frac{1}{v} \left[ \frac{\rho}{F-h} + \frac{\phi + \rho h}{(F-h)^2} \right] > 0, \quad (19)
$$
$$
\frac{\partial^2 \tau^*(h)}{\partial h^2} = \frac{2(\phi + \rho F)}{v(F-h)^3} > 0. \quad (20)
$$

As indicated in Figure 5, the critical hinterland size, $\bar{F}$, the critical density of transport

\textsuperscript{18}If the new higher-order hub could offer a substantial cost reduction, it will attract transport demands in the hinterland of nearby lower-order hubs.

\textsuperscript{19}The specification in (15) is rather special in that condition (3) holds with equality. Alternatively, the mass transport technology can be given by, e.g., $\tau(d, Q) = \frac{\phi + \rho d}{Q}$ with $\alpha \in (0, 1)$ or $\tau(d, Q) = \alpha + \frac{\phi + \rho d}{Q}$, so that (15) holds with strict inequality. But, the analysis becomes far more complex without changing the basic results.
demand, \( \hat{v} \), as well as the critical hub locations, \( \tilde{h} \) and \( \hat{h} \), are determined uniquely as below:

\[
\begin{align*}
\tilde{F} &= 2\sqrt{f/v} + r/v \\
\tilde{h} &= \sqrt{f/v} \\
\hat{F} &= \frac{1}{F^2} \left( 2f + 2\sqrt{f} \sqrt{f + Fr + Fr} \right) \\
\hat{h} &= \sqrt{f/\hat{\nu}}
\end{align*}
\]

where \( f \equiv \phi/t \) and \( r \equiv \rho/t \) are the fixed and marginal cost for mass transportation relative to the rate of individual transportation. The interpretations of \( f \) and \( r \) are the same as those of \( \phi \) and \( \rho \). A larger value of \( f \) relative to \( r \) means relatively larger distance economies.

When the fixed cost, \( f \), for transportation is larger, the possible locations, \( \tilde{h} \) and \( \hat{h} \), of a hub is necessarily farther from the gateway to utilize distance economies. When \( \nu \) is larger, the density economies are relatively more pronounced, and hence, the mass transportation becomes efficient for a smaller transport distance. As a result, the hub location becomes closer to the gateway.

The viable location for a hub is thus determined by the balance between distance and density economies. If the former [resp., latter] is relatively stronger, the spacing of transport hubs tends to be larger [resp., smaller] as the transport efficiency increases relatively more by longer hauling [resp., carrying larger volume]. It is analogous to the agglomeration shadow around an existing industrial agglomeration suggested by the new economic geography models (e.g., Fujita and Krugman [14]) in which a new agglomeration is unlikely to form, since it is too close to the existing agglomeration in order for a firm to exercise monopoly power in the local market. In the present case, “agglomeration” is in terms of transport demand.

It is to be noted that both distance and density economies are necessary in order to investigate the spatial distribution of hubs. When transport demand is generated at each point in a continuous location space as in the present model, if there were no density
economies, then there would be no cost advantage of pooling transport demand, while if there were no distance economies, a continuum of hubs would form (i.e., hubs will form without leaving any space between them).

7 Implications to the structure of a transport network

In this section, preliminary implications for the size and spacing of hubs from the present model are discussed. Although a fuller analysis under more general equilibrium concepts is beyond the scope of the present paper, some interesting insights can still be obtained from the present simplistic setup.

In general, if density economies are more pronounced than distance economies, mass transportation tends to collect transport demand along the way as a local train stops every station in order to carry as many passengers as possible. Consequently, the hierarchy of hubs tends to deepen. In this "local train" network, the size distribution of hubs becomes relatively less skewed, as the size of a parent hub is larger than its child hub only by the size of shipments from its own direct hinterland.

If distance economies are more pronounced instead (say, when the cost of stopping is high, as for the case of airplanes), then the hierarchy is less likely to be formed among hubs, and each hub tends to be directly linked to the gateway. In that case, the traffic volume at the gateway will be disproportionately large as all the shipments gather there, while all the other hubs are small as they are of the lowest-order. The resulting size distribution of hubs will then be relatively more skewed.

Below, it is shown that for a sufficiently large spatial economy, the depth of a hub hierarchy would eventually hit its upper bound even if density economies are very strong (Section 7.1), while a higher-order hub would eventually develop, and the hub hierarchy would deepen, even if distance economies are very strong (Section 7.2). Hence, there is a tendency that the skewness of hub size distributions under different levels of distance and

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20The continuity of the location space is not essential in this argument as long as the location space consists of more than two locations.
density economies converge, though there may not be a common limit.

This possibility of the emergence of order in the size distribution of transport hubs may be in part responsible for the Rank-Size Rule for cities, a well-known log-linear relationship between the size and rank in terms of size of cities. In fact, it is not difficult to confirm in reality that most major cities are associated with major transport hubs. In the case of Japan, the Spearman’s rank correlation between the population size of a city and passenger traffic volume of the airport in that city in year 2000 is 0.645 (highly significant). 21

7.1 On the upper limit of the hub hierarchy depth

Consider a situation in which the spatial size of the economy expands, i.e., $F$ increases gradually. Then, a hub becomes viable first at $\tilde{h}_1 \equiv \tilde{h}$ when $F$ reaches $\tilde{F}_1 \equiv \tilde{F}$. Similarly, if $F$ reaches $\tilde{F}_2 \equiv h_1 + \tilde{F}$, the second hub becomes viable at $h_2 \equiv h_1 + \tilde{h}$. In this manner, it is possible to construct an equilibrium $i$-order “local train” hierarchical hub system with $i \ (\geq 1)$ hubs in which hubs are located at $h_i \equiv h_{i-1} + \tilde{h}$ for $F \geq \tilde{F}_i$, since there is no incentive for unilateral deviation of shippers from the existing transport pattern. This network structure is depicted in Figure 6.

[Figure 6]

Notice, however, that as the hierarchy of hubs deepens, the fixed cost of mass transportation accumulates, and at some point, there would emerge a possibility of cost reduction by directly connecting an existing hub $i \ (\geq 2)$ to the gateway by a trunk link so that all the shipments via hub $i$ would be transported to the gateway directly by a mass transport mode.

To see this, assume that hubs are formed at $h_i = h_{i-1} + \tilde{h}$ and $g(h_i) = h_{i-1}$, for all $i = 1, \ldots, K_F$, where $K_F$ is the ceiling of $F/\tilde{h}$. Now, define the relative cost for the direct trunk link from hub $n \ (\geq 2)$ to the gateway as

$$\Delta^* \tau(F) \equiv \frac{\phi + \rho n \tilde{h}}{H_n} \bigg/ \sum_{i=1}^{n} \frac{\phi + \rho \tilde{h}}{H_i}$$

21Airport size data are obtained from Civil Aviation Bureau of Japan [12].
where $H_i \equiv [b_i, F]$. Since $H_1 > H_i$ for all $i > 1$,

$$\Delta^* \tau(F) < \frac{\phi + \rho \tilde{h}}{H_n} \left/ \frac{\phi + \rho \tilde{h}}{nH_1} \right.$$

$$= \frac{\phi/n + \rho \tilde{h}}{H_n} \left/ \frac{\phi + \rho \tilde{h}}{H_1} \right.$$ 

$$= \frac{1/n + \rho \sqrt{tv/\phi H_1}}{1 + \rho \sqrt{tv/\phi}} H_n. \quad (26)$$

But, since

$$\frac{H_1}{H_n} < \frac{F}{F - n\tilde{h}} = \frac{F}{F - n\sqrt{tv} \sqrt{\phi}}, \quad (27)$$

and since $\lim_{F \to \infty} F/(F - n\tilde{h}) = 1$, it follows from (26) and (27) that

$$\lim_{F \to \infty} \Delta^* \tau(F) = \frac{1/n + \rho \sqrt{tv/\phi}}{1 + \rho \sqrt{tv/\phi}} < 1. \quad (28)$$

Hence, if the hub hierarchy deepens as $F$ increases, for any given hub $n \geq 2$, the formation of a direct trunk link to the gateway would eventually become less costly than stopping at hub $n - 1$. As (26) indicates, if density economies are relatively more important than distance economies (i.e., $\phi$ is small relative to $\rho$), then such a trunk link is less effective for cost reduction. However, the analysis above suggests that for a sufficiently large spatial economy, for any given levels of distance and density economies (i.e., for any values of $\rho$ and $\phi$), the depth of the hub hierarchy has a limit, if an appropriate collective action (or in the presence of a large agent aiming for reducing total transport costs) is allowed as in reality. It corresponds to the fact that major trunk links (e.g., express and bullet trains, jumbo jet links) are only formed at a large interregional scale.

This result has an implication to the size distribution of transport hubs. When density economies dominates distance economies, the hub hierarchy tends to be deeper (e.g., trains tend to stop larger number of stations), so that the size distribution of hubs is relatively less skewed. But, the viability of major regional hubs which climb up the hub hierarchy for a larger spatial economy implies the tendency that the skewness of the size distribution increases (at the upper tail) as the spatial size of the economy increases.
7.2 On the lower limit of a hub hierarchy depth

Next, consider a situation in which each hub is directly linked to the gateway, so that the hub hierarchy is of only a single order. Such a configuration is likely to be an equilibrium when distance economies are very strong. But, for sufficiently remote hubs for which the distance cost dominates, there can be a substantial cost reduction by pooling transport demand among neighboring hubs (although such an action is infeasible by unilateral deviations of individual shippers/passengers).

To see this, consider a “hub-and-spoke” network so that local hubs are located at $h_1, h_2, \ldots$, each of which is directly connected to the gateway by a mass transport link, i.e., $g(h_i) = h_0$ and $H_i = D_i$ for all $i \geq 1$, as shown in Figure 7.

![Figure 7](image)

Now, define the relative transport cost from hub $n$ to the gateway when hub $n$ is connected to a neighbor hub $n - 1$ instead of the gateway as

$$\Delta T^{**}(h_n) \equiv \left[ \frac{\phi + \rho h_{n-1}}{D_{n-1} + D_n} + \frac{\phi + \rho (h_n - h_{n-1})}{D_n} \right] - \frac{\phi + \rho h_n}{D_n}. \quad (29)$$

In (29), the bracketed term represents the transport rate at hub $n$ when transport demands at hubs $n$ and $n - 1$ are pooled at hub $n - 1$, while the last term is the (current) transport rate on the direct link between hub $n$ and the gateway. Since (29) can be rewritten as

$$\frac{D_n}{\rho h_{n-1}} \Delta T^{**}(h_n) = \frac{D_n}{D_{n-1} + D_n} \left[ \frac{\phi}{\rho h_{n-1}} + 1 \right] - 1, \quad (30)$$

it follows that

$$\lim_{h_{n-1} \to \infty} \Delta T^{**}(h_n) < 0. \quad (31)$$

Hence, for a sufficiently remote hubs, $n$ and $n - 1$, a cost reduction is possible by pooling their transport demand.\(^{23}\)

\(^{22}\)The first [resp., second] term in the bracket is the transport rate between hub $n - 1$ and the gateway [resp., hubs $n$ and $n - 1$].

\(^{23}\)Obviously, for a hub to be viable at large $h_n$, the fringe location, $F$, should also be large.
If the distance economies are relatively stronger than density economies, there will be relatively small advantage of integrating transport demand among hubs, i.e., higher-order hubs is less likely to be formed. In particular, when these two hubs, \( n \) and \( n - 1 \), are located closely to the gateway, because of the fixed cost, \( \phi \), the integration of transport demand from both hubs is more costly than the direct shipment from each hub. But, when these two hubs are sufficiently far from the gateway, the fixed cost of the additional stopping will eventually become negligible, relative to the benefit of cost sharing on the long link to the gateway. It follows that as long as \( \rho \) (and \( \phi \)) is strictly positive, the formation of a higher-order hub in an area sufficiently far from the gateway would lead to a substantial cost reduction in the transportation from the peripheral region. Hence, under possibilities of some form of coalition leading to the formation of trunk links, it is likely that the hub hierarchy deepens as the economy expands spatially.

This result has an implication to the size distribution of transport hubs. When distance economies dominates density economies, the hub hierarchy tends to be shallower (e.g., airlines stop far less than trains before reaching the final destinations), so that the size distribution of hubs is relatively more skewed (e.g., only the gateway has disproportionately large traffic in the network depicted in Figure 7). But, the fact that the formation of higher-order hubs becomes possible for a large spatial economy implies that the skewness of the size distribution may decrease (at the upper tail) as the spatial size of the economy increases.

8 Concluding remarks

In this paper, a simple model of transport network formation was proposed to show that the increasing returns in transportation restricts the structure of transport network in its own right. In particular, it has been shown that the two types of increasing returns in transportation, i.e., distance and density economies, play a crucial role in determining the viable locations of transport hubs.

Under larger density economies, it is more efficient to stop more frequently to accumu-
late transport demand, hence the spacing between the parent and child hubs tends to be smaller. Under larger distance economies, it is more efficient to have a larger transport distance, hence a larger spacing between a parent and child hubs would result.

This section closes the paper by discussing implications of the two increasing returns in transportation to the size and spacing of cities (Section 8.1), the limitations of the present analysis (Section 8.2), and future research directions (Section 8.3).

8.1 Implications to the size and spacing of cities

As transport hubs have a better transport access than non-hub locations, they are naturally more likely locations for industries and population. This result has an important implication to the economics of agglomeration in which the location (and the spatial distribution) of industrial and population agglomeration is a central subject. Other than a few specific exceptions (discussed in Section 2), either no increasing returns are assumed for transportation, or transport network structure is exogenous in this literature. But, the result of the present analysis indicates that increasing returns in transportation could be a major determinant of the size and location of economic agglomerations, as they determine endogenously the transport cost structure within the spatial economy.

The balance between distance and density economies not only affects the spacing of hubs, but also generates economic subregions in terms of an endogenous transport hinterland of a hub. Though the thorough investigation of the hierarchical transport network is beyond the scope of this paper, a preliminary analysis in Section 7 suggests that a wide spread transport demand makes possible the formation of a higher-order regional hub which requires both a large local hinterland and a large distance to the gateway. As a result, larger regional hubs would be formed farther apart from each other, which may in turn explain a larger spacing between major cities.

As transport costs account for increasing and non-negligible share in the international trade,\(^{24}\) the mechanism underlying the transport network formation should obtain a

\(^{24}\)E.g., Anderson and von Wincoop [1] estimated the ad valorem tax equivalent transport costs as 21 percent for industrialized countries. According to Hummels [22], the ratios between aggregate transport
serious attention in explaining the specialization pattern among countries as well.

8.2 Limitations of the present analysis

The present analysis is a modest step toward incorporating endogenous transport network structure to the general equilibrium urban and regional models. At this point, however, it only focuses on the most basic role of distance and density economies of transportation in determining the locations of hubs.

An obvious shortcoming of the present model is the lack of microeconomic foundation for these increasing returns. First, although density economies assumes underlying matching externalities between supply and demand of transport services (refer to footnote 3), the mechanism is completely in a black box. Second, while the ranges of both transport density and distance for which increasing returns are effective differ depending on the type of transport mode in reality, these increasing returns are assumed to be effective for the entire range of density and distance levels, i.e., the modal choice is implicit in the present model. Third, the “viability” of a hub at a given location only guarantees the cost reduction by the hub formation. But, it does not specify any concrete mechanism by which the hub is to be formed. This is indeed the difficulty that the modeling of transport network formation faces in general, as it must consider a collective decision making or the presence of large agents to coordinate the adjustment of a mass of transport flows, since the trunk link cannot start from a marginal size in a decentralized self-organization.25

Finally, in order to accomplish the ultimate objective mentioned above, it is also essential to endogenize the spatial distribution of transport demand, i.e., that of economic agents by explicitly modeling the production and consumption sectors.

25It is somewhat similar to the city formation as it requires a large agent in the “system of cities” model by Henderson [18]. An exceptional formulation has been proposed in the new economic geography by Fujita and Krugman [14] in which a city forms in a self-organizing manner from an arbitrarily small size in a continuous location space. But, the same technical trick cannot be applied to the transport network.
8.3 Future research directions

Despite many limitations listed in the previous section, there still are interesting extensions of the present model. An immediate agenda is to investigate the properties of equilibria in a large spatial economy. As the preliminary analysis in Section 7 indicated, there would likely exist equilibria with multi-order hierarchical transport network if the hinterland of the gateway grows large. While the non-linear effects of the two increasing returns make it difficult to identify all equilibria, it is still possible to find an equilibrium network structure for a spatially growing economy (i.e., in terms of an increasing $F$) by a myopic adjustment.

In the way that a viable hub location $\tilde{h}$ is identified when the spatial size of economy reaches a certain point, $\tilde{F}$, in Section 6, it is possible to ask if there is a potential trunk link from each hub (to another hub or the gateway) which if formed will reduce transport cost in the hinterland of the hub in question (or average transport cost of the entire economy). Such an adjustment is obviously ad-hoc compromise for computability. But, it is nonetheless useful as an initial attempt for explaining endogenous formation of a large transport network in the presence of distance and density economies.

It is also to be noted that in reality the locations of hubs of multiple transport modes tend to coincide, which in turn leads to the formation of a hub city. While the presence of large agents is particularly important in explaining the network structure of each specific transport mode, their spatial coordination may largely be subject to self-organization, where increasing returns in a given transport mode influences the network structure of another transport mode through the endogenous transport density. Since the coordination among different transport modes is far from perfect, the use of myopic adjustment is not necessarily unrealistic to explain the mechanism underlying the actual network structure.

A preliminary analysis in this direction indicates that in a large spatial economy, the size distributions of hubs exhibit a striking similarity across transport hinterlands of higher-order hubs (i.e., $H_i$ of hub $i$ in Section 4). A fuller investigation in this direction is to be conducted in a subsequent work.\textsuperscript{26}

\textsuperscript{26}The result exhibits a close resemblance to the similarity of size distributions of cities across subregions within a given economy reported for the cases of the US and Japan by Hsu, Mori and Smith [21].
References


Figure 1: Location space and transport demand

\[ g(h_1) = g(h_3) = h_0 \]

\[ g(h_2) = g(h_4) = h_3 \]

\[ g(h_5) = h_4 \]

Figure 2: Transport network
Figure 3: Transport cost schedule under the network in Figure 2

Figure 4: A viable location of a hub
Figure 5: Location of a hub

Order 1
Order 2
Order \( K-1 \)
Order \( K \)

Figure 6: A “local train” network

Figure 7: A “hub-and-spoke” network