TITLE:
Spontaneous mode-selection in the self-propelled motion of a solid/liquid composite driven by interfacial instability.

AUTHOR(S):
Takabatake, Fumi; Magome, Nobuyuki; Ichikawa, Masatoshi; Yoshikawa, Kenichi

CITATION:

ISSUE DATE:
2011-03-21

URL:
http://hdl.handle.net/2433/139584

RIGHT:
© 2011 American Institute of Physics
Spontaneous mode-selection in the self-propelled motion of a solid/liquid composite driven by interfacial instability

Fumi Takabatake,1 Nobuyuki Magome,2 Masatoshi Ichikawa,1 and Kenichi Yoshikawa1,a)
1Department of Physics, Graduate School of Science, Kyoto University, Kyoto 606-8502, Japan
2Institute for Integrated Cell-Material Sciences (iCeMS), Kyoto University, Kyoto 606-8501, Japan

(Received 30 September 2010; accepted 22 February 2011; published online 17 March 2011)

Spontaneous motion of a solid/liquid composite induced by a chemical Marangoni effect, where an oil droplet attached to a solid soap is placed on a water phase, was investigated. The composite exhibits various characteristic motions, such as revolution (orbital motion) and translational motion. The results showed that the mode of this spontaneous motion switches with a change in the size of the solid scrap. The essential features of this mode-switching were reproduced by ordinary differential equations by considering nonlinear friction with proper symmetry. © 2011 American Institute of Physics. [doi:10.1063/1.3567096]

I. INTRODUCTION

Thermal engines transfer chemical energy into mechanical energy through the generation of heat and obey the second law of thermodynamics: the upper limit of thermal efficiency is determined by the difference between the high and low temperatures. On the other hand, living organisms extract kinetic energy from chemical energy directly with a high energy-conversion efficiency under isothermal conditions. Despite previous extensive studies on motor proteins, our understanding of the underlying mechanisms remains at a primitive stage. Recent studies on the single-molecule observation of motor proteins have revealed that biological macromolecules work under highly fluctuating conditions. Thus, it may be of scientific value to clarify the generation of regular motion in a macroscopic system with fluctuating elements.

The spontaneous motion of liquid droplets, solid particles, and gels under nonequilibrium conditions has been investigated both experimentally and theoretically. Among the phenomena of spontaneous motions, the self-agitation of a fluid interface is well known and has been studied for over a century, especially in the field of chemical engineering in terms of the chemical Marangoni effect. The chemical Marangoni effect is induced by local variations in interfacial tension, which are caused by a chemical concentration gradient under isothermal conditions. Recently, it was found that irregular interfacial agitation caused by the Marangoni instability can be transformed to regular motion by choosing a suitable physical or chemical boundary condition.

In the present study, we found experimentally that a solid/liquid composite exhibits regular motion under spatially isotropic boundary conditions by breaking its anterior–posterior symmetry. The results showed that a composite driven by the chemical Marangoni effect assumes a certain mode of spontaneous motion depending on the relative size of the solid with respect to that of the liquid.

II. EXPERIMENTAL

A droplet of oleic acid and a solid sodium oleate (soap) were floated on water in a petri dish as shown in Fig. 1(a). Water was purified with a MilliQ filtering system (Millipore). The volume of the oil droplet was 100 μl. Oleic acid (analytical grade) was obtained from Wako Pure Chemical Industries, Japan. A solid column of sodium oleate with a desired axial length was chosen from a commercial sample (code 257-02) obtained from Nacalai Tesque [Fig. 1(b)]. The movement of the solid/liquid composite was captured by a digital video camera at 30 frames per second at room temperature, and then analyzed by an image-processing software (ImageJ). Example of the video frames is given in Fig. 1(c).

III. RESULTS AND DISCUSSIONS

Figure 2 exemplifies the characteristic modes of spontaneous motion of a solid/liquid composite on the water phase: (left) shadow graph images of the solid/liquid composite and (right) trajectory of the droplet-center over 5 s. The solid/liquid composite shows continuous motion by taking the solid column as the front side over a period of several minutes. Investigation on the moving composite indicates the occurrence of milky stream of surfactant micelles on the rear side opposite to the solid column, indicating the occurrence of eliminating or dispelling flow of the surfactant from the composite. When the axial length of the solid column is ca. 1 mm, the composite exhibits spinning motion, as shown in Fig. 2(a). When the axial length is ca. 3 mm, the composite exhibits translational motion accompanied by relatively slow spinning, as shown in Fig. 2(b). When the axial length is ca. 5 mm, the composite exhibits revolution (orbital motion) together with the spinning motion around its centroid [Fig. 2(c)], where the period of revolution is equal to that of the spinning motion.

The panels on the left in Figs. 3 1(a)–(c) show time traces of the angular velocity \( \omega \) and the speed \( |v| \) of motion over 30 s. The panels on the right show histograms of the angular velocity \( \omega \) over 60 s. The direction of spinning motion spontaneously switches in a stochastic manner, as shown in
When a scrap is placed on water, sodium oleate molecules are adsorbed at the interface and form a surfactant monolayer. This monolayer causes the interface to become unstable and drives the scrap to move. After 1–2 s, the interface is covered with a surfactant monolayer, which results in the termination of spontaneous motion.

Next, let us discuss the cause of the self-motion of a solid/liquid composite, in terms of surface instability. The oil/water interfacial tension near the solid is lowered because of the formation of a surfactant monolayer dissolved from the solid column. On the other hand, the interfacial tension far from the solid column remains high because of the acidic nature of the oil, oleic acid. Thus, it is expected that a spatial gradient of interfacial tension is generated around the composite, where the tension near the solid column is low and that far from the solid column is high. As is exemplified in Fig. 1(c), the surfactant molecules (oleate molecules) existing around the oil/water interface tend to escape toward...
the bulk aqueous solution by making an eliminating flow from the rear side. Due to such dissipative flow of the surfactant, we can consider that tangential stress is maintained around the interface, or the surface tension. In response to this stress, as depicted schematically in Fig. 4, shear flow concentration at the interface, or the surfactant concentration, at the oil/water boundary are released to the water phase from the backside of the droplet. As the result of competition between the surfactant-supply from the solid column and escape into the bulk solution, the spatial gradient of the surface tension is maintained. Through such dissipative mechanism, the composite continues self-propelled motion for a relatively long period (more than several minutes).

As described above, the gradient of the interfacial tension induces the Marangoni convection around the composite and exerts the propulsion force on the composite by the momentum transfer through the occurrence of shear flow parallel to the interface, as schematically shown in Fig. 4. The propulsion force acting on the composite may be simulated by solving the Stokes equation, taking account of the gradient of the oil/water interfacial tension, as a subject for future study. However, in order to grasp the essence on the bifurcation of self-propelled motion, here we would like to adopt a simple theoretical model that the translational motion of the composite is caused by the apparent driving force $F$, of which the magnitude $|F|$ is time-independent and the orientation exhibits time-dependence. Under such framework of the modeling, translational motion is expressed as

$$\frac{dr}{dt} = v; \quad m\frac{dv}{dt} = -\eta_r v + F \tag{1}$$

with

$$F = \begin{pmatrix} F_x \\ F_y \end{pmatrix} = \begin{pmatrix} F \cos \theta \\ F \sin \theta \end{pmatrix}, \quad F = |F|, \tag{2}$$

where $m$ is the mass of the composite, $r$ is the displacement of the center of mass, $v$ is the velocity of translational motion, and $\eta_r > 0$ is the friction coefficient. $F$ is the driving force due to the above-mentioned Marangoni stresses. In Eq. (2), $\theta$ is the angle of the axis that combines the centers of mass of the oil droplet and the solid scrap, as shown schematically in Fig. 5.

As for the rotational motion, we adopt a term of nonlinear friction as follows:

$$\frac{d\theta}{dt} = \omega; \quad I\frac{d\omega}{dt} = -\eta_\omega (\omega) \omega + \xi_\omega (t), \tag{3}$$

where $\omega$ is the angular velocity and $I$ is the moment of inertia. $\eta_\omega (\omega)$ is the friction function that includes the contribution from the driving torque in addition to the usual positive friction. $\xi_\omega$ represents the random fluctuation due to the interfacial and fluidic instability. To interpret the essential features of mode-switching, we simply adopt a symmetry requirement:
As shown in the picture of Fig. 1(c), together with the experimental data of Fig. 3, the composite tends to cause rotational motion, accompanied by the spontaneous symmetry breaking. In order to take into account of such self-driving force on the rotation, we adopt negative rotational “friction,” $-\eta_2 \omega^2 < 0$. So as to prevent the diverge of the rotation due to the negative friction, we also add the quartic term, $\eta_3 \omega^4 > 0$. Thus, the friction function $\eta_\omega(\omega)$, including not only the frictional force but also the driving force due to Marangoni instability$^{10}$ together with geometrical asymmetry, can be described as

$$\eta_\omega(\omega) = \eta_1 - \eta_2 \omega^2 + \eta_3 \omega^4,$$  \hspace{1cm} (4)

with positive coefficients $\eta_1$, $\eta_2$, and $\eta_3$.

By nondimensionalizing procedure, the motion of the solid/liquid composite is transformed into the following equations:

$$\frac{d\bar{x}}{d\tilde{t}} = \ddot{\bar{x}}; \frac{d\bar{v}_x}{d\tilde{t}} = -\ddot{\bar{v}}_x + \bar{F} \cos \theta,$$  \hspace{1cm} (5)

$$\frac{d\bar{y}}{d\tilde{t}} = \ddot{\bar{y}}; \frac{d\bar{v}_y}{d\tilde{t}} = -\ddot{\bar{v}}_y + \bar{F} \sin \theta,$$  \hspace{1cm} (6)

$$\frac{d\tilde{\theta}}{d\tilde{t}} = \dot{\tilde{\omega}}; \frac{d\dot{\tilde{\omega}}}{d\tilde{t}} = -\left(\tilde{\eta}_1 - \tilde{\eta}_2 \tilde{\omega}^2 + \tilde{\eta}_3 \tilde{\omega}^4\right) \dot{\tilde{\omega}} + \tilde{\xi}_\omega,$$  \hspace{1cm} (7)

where $\tilde{t} = (m/\tilde{\eta}_\omega) t$ and $\bar{r} = \sqrt{T/m} \tilde{r}$. The variables in these equations, $\tilde{t}$, $\bar{x}$, $\bar{y}$, $\bar{v}_x$, $\bar{v}_y$, and $\dot{\tilde{\omega}}$, are dimensionless. The dimensionless friction function $\tilde{\eta}_\omega$ is

$$\tilde{\eta}_\omega(\tilde{\omega}) = \tilde{\eta}_1 - \tilde{\eta}_2 \tilde{\omega}^2 + \tilde{\eta}_3 \tilde{\omega}^4 = \tilde{\eta}_1 (1 - \alpha \tilde{\omega}^2 + \beta \tilde{\omega}^4),$$  \hspace{1cm} (8)

where $\alpha = \tilde{\eta}_2/\tilde{\eta}_1$ and $\beta = \tilde{\eta}_3/\tilde{\eta}_1$. In Eq. (8), the friction function becomes negative within a certain range of angular velocity $\omega$ if $\alpha^2/\beta > 4$, which implies that the active force is generated. Additionally, it is noted that the fixed points of Eq. (7) at $\tilde{\omega} = 0$ become more stable than a stable fixed point at $\tilde{\omega} = 0$ when $\alpha^2/\beta > 16/3$.

Figure 5 exemplifies the numerical results based on Eqs. (5)–(7). In numerical calculations, $\tilde{\xi}_\omega$ is simply taken as the white Gaussian noise whose dispersion is $s$. It is to be noted that the essential features of the experimental results as shown in Fig. 3 are reproduced by choosing appropriate parameters. In these simulations, to gain an insight on the mechanism of the mode bifurcation, we examine the effect of two parameters, $\alpha$ and $s$, while others are fixed to be constants.

A comparison between Figs. 5(a) and 5(b) reveals modesswitching from rotational motion to translational motion, corresponding the experiments in (a) and (b) of Fig. 2. The larger value of $\alpha^2/\beta$ in (a) with respect to (b) in Fig. 5 is regarded as to represent a higher possibility of symmetry-breaking between clockwise and anticlockwise spinning motions under irregular fluctuation due to the interfacial instability. This matches with the experimental trend that small scrap tends to rotate faster. Since $m$ is fixed in the equations, the size change of the solid column is described as the relative value of mass of the solid column with that of the droplet, which affects $\sqrt{T/m}$, a coefficient of the length scale. Figure 5 is plotted in $\bar{r}$ space, i.e., each mass of the solid column proportional to the length $l$ in the experiment is used for the magnitude of the scale in the corresponding plot as modulus $\bar{r} \sim \sqrt{l}$.

FIG. 5. Numerical simulation based on Eqs. (5)–(7). The top panel shows a frame format of the model. Left: Time traces of the angular velocity $\tilde{\omega}$ and histograms of $\tilde{\omega}$ ($0 \leq \tilde{t} \leq 1000$). Right: Trajectory of the centroid of the composite in $\bar{r}$ space for a short period ($400 \leq \tilde{t} \leq 405$). The scales are the same on the four trajectories except for the inset. (a) $\alpha = 2.4$, $s = 0.3$. (b) $\alpha = 1.8$, $s = 0.3$. (c-1) $\alpha = 2.7$, $s = 0.5$. (c-2) $\alpha = 2.7$, $s = 0.3$. The other fixed parameters are $\tilde{\eta}_1 = 1$, $\beta = 1$, $\bar{F} = 0.1$. The initial values are $\bar{x}(0) = \bar{y}(0) = 0$, $\bar{v}_x(0) = \bar{v}_y(0) = 0$, $\dot{\tilde{\omega}}(0) = 0$, and $\tilde{\omega}(0) = 0$. 

Downloaded 09 May 2011 to 130.54.110.31. Redistribution subject to AIP license or copyright; see http://jcp.aip.org/about/rights_and_permissions
[in (a) \( l = 1 \), in (b) \( l = 3 \), and in (c) \( l = 5 \)]. Thus, motions in Figs. 5 (c-1, c-2) are regarded as rotation with larger orbital radii than that in (a). The larger value of \( s \) in (c-1) represents approximately a high possibility that the composite is subject to larger motional fluctuation due to the instability or external perturbations. This large fluctuation causes the flip–flop fluctuation of the direction of the orbital motion. When the sizes of the solid column and the droplet deviate largely from the similar magnitude, rotational motion of the composite shows bistability of clockwise and anticlockwise as in the experiments. Therefore, large enough unbalance between the sizes of the solid and the liquid in the composite, as in (a) and (c), corresponds to the state with large \( \alpha^2/\beta \), and the balanced condition as (b) is described as the effect of small \( \alpha^2/\beta \).

IV. CONCLUSION

We have experimentally demonstrated the appearance of mode-switching in the self-propelled motion of a solid/liquid composite. The driving force is the interfacial instability caused by nonuniformity in the chemical concentration or by spontaneous symmetry breaking the geometric configuration on the composite. Mode-switching, among spinning, translational motion, and orbital motion, is caused by the changes in size of the solid column. The essential features of the experimental observation have been reproduced by adopting a simple model with ordinary differential equations, by taking account of the scheme of spontaneous symmetry breaking. Since almost all self-moving organelles and cells exhibit a head–tail asymmetry, our simple experimental and numerical models may give deeper insight into mode-switching in living systems.

ACKNOWLEDGMENTS

We gratefully acknowledge the assistance of Professor K. I. Agladze. We also thank Dr. K. H. Nagai, Dr. Y. Sumino, Dr. N. Yoshinaga, and Dr. H. Kitahata for their helpful discussions and advice. This work was supported by a Sasagawa Scientific Research (Grant No. 22-220) from the Japan Science Society, by an iCeMS Cross-Disciplinary Research Promotion Project (OAP3-9), and by a Grant-in-Aid for Challenging Exploratory Research (Grant No. 22654046) from the Japan Society for the Promotion of Science.